THE INTRODUCTION OF THE RULE OF 3x +1 PROBLEM

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Abstract

This article introduces a change rule of 3x+1 problem (Collatz conjecture), it's named LiKe's rule: For any positive integer, change by the Collatz conjecture, it will change to an odd number; the odd numbers will must change to a number of LiKe's second sequence $\{3^n-1 \mid n\in Z^+\}$; then this 3^n-1 will change to a smaller 3^n-1 and gradually decrease to 8(that is 3^2-1) then back to 1 in the end.

Keywords: 3x+1 problem; Collatz-Problem; Collatz conjecture; LiKe's rule; number theory

MSC2010: 11B83

0. Introduction

3x+1 problem(Collatz conjecture)[1]: If a positive integer x is odd then "multiply by 3 and add 1", while if it's even then "divide by 2", iterations of them, it will eventually reach the number 1. The Collatz function, as follows.

$$C(x) = \begin{cases} 3x + 1 & \text{if } x \equiv 1 \pmod 2 \\ x/2 & \text{if } x \equiv 0 \pmod 2 \end{cases}$$

As soon as this problem appeared, it becames popular all over the world, and teachers and students in both primary and secondary schools and colleges were fascinated by it. For nearly a century, mathematicians, physicists, computer scientists and others have studied this. It covers a wide range of mathematical fields, such as Number Theory, Ergodic Theory, Dynamical Systems, Mathematical Logic and the Theory of Computation, Stochastic Processes and Probability Theory, and Computer Science. Although achieved certain results, such as: J. C. Lagarias[2] or A.V. Kontorovich and J. C. Lagarias[3] and so on.

Terence Tao achieved significant result in 2019[4]. But in his paper his conclude is almost all, not all. And he admits the law is unsustainable.

The problem seems unsolvable, and no one can crack its secrets. Richard Guy said "Don't try to solve these problems!", Paul Erdos said "Hopeless. Absolutely hopeless."and "Mathematics is not yet ready for such problems." and so on.

But through my research, I discovered a special rule(LiKe's rule)[5]: For any positive integer, it must change to a number of 3^n-1 (LiKe's second sequence), and it will change to a smaller 3^n-1 then gradually decrease to 8 and back to 1. It might be helpful to the study this problem, so I decided to give a brief explanation. See below:

1. Even to Odd

For all even Numbers, as long as it changes according to the Collatz conjecture, see the table below.

Table 1: Change rule of odd Numbers

Even/E	E/2	E/2	E/2	E/2	$E/2^n$
2	1				
2^2	2	1			
6	3				
2^3	4	2	1		
10	5				
12	6	3			
14	7				
2^4	8	4	2	1	
18	9				
20	10	5			
22	11				
24	12	6	3		
26	13				
28	14	7			
30	15				
2^5	16	8	4	2	…
34	17				
E_x	…	…	…	…	…

Obviously, all of the Even Numbers will change to an Odd Number corresponding.

2. Odd to O_{n1}(1, 5, 17, 53, ⋯, O_{n1}, $2*3^n$-1)

2.1 The Collatz function

For all Odd numbers, multiply by 3 and add 1(3*O+1) must be an even, so (3*O+1)/2 is regarded as a one-step operation in this paper, so the Collatz function C(x) can be expressed as the following equation too.

$$T(x) = \begin{cases} (3x+1)/2 & \text{if } x \equiv 1 \pmod 2 \\ x/2 & \text{if } x \equiv 0 \pmod 2 \end{cases}$$

2.2 Definition

L_0(1, 3, 5, 7, ⋯, O, 2n-1) = {2n-1 | n∈Z^+} ⊂ Z^+, O_{01}=1;

L_1(5, 11, 17, ⋯, O_{1m}, 6n-1) = {6n-1 | n∈Z^+} ⊂ Z^+, O_{11}=5;

L_2(17, 35, 53, ⋯, O_{2m}, 18n-1) = {18n-1 | n∈Z^+} ⊂ Z^+, O_{21}=17;

L_3(53, 107, 161, ⋯, O_{3m}, 54n-1) = {54n-1 | n∈Z^+} ⊂ Z^+, O_{31}=53;

...

$L_n(2*3^n-1, 2*2*3^n-1, 3*2*3^n-1, \ldots, O_{nm}) = \{2*3^n i-1 \mid n\in Z, i\in Z^+\} \subset Z^+$, $O_{n1}=2*3^n-1$;

$O_{n1}(1, 5, 17, 53, \cdots, O_{n1}, 2*3^n-1) = \{2*3^n-1 \mid n\in Z\} \subset Z^+$;

$LK_2(2, 8, 26, 80, \ldots) = \{3^n-1 \mid n\in Z^+\} \subset Z^+$;

$LC(1, 4, 13, 40, \ldots) = \{(3^n-1)/2 \mid n\in Z^+\} \subset Z^+$.

2.3 L_0 to L_1

Theorem 1: For all L_0, change by the Collatz conjecture, it will change to the sequence $L_1\{6n-1 \mid n\in Z^+\}$.

Proof:

It's incredibly easy to see that all even Numbers will change to odd Numbers by table 1. Through the following figure, it's also easy to see the change about L_0 to L_1.

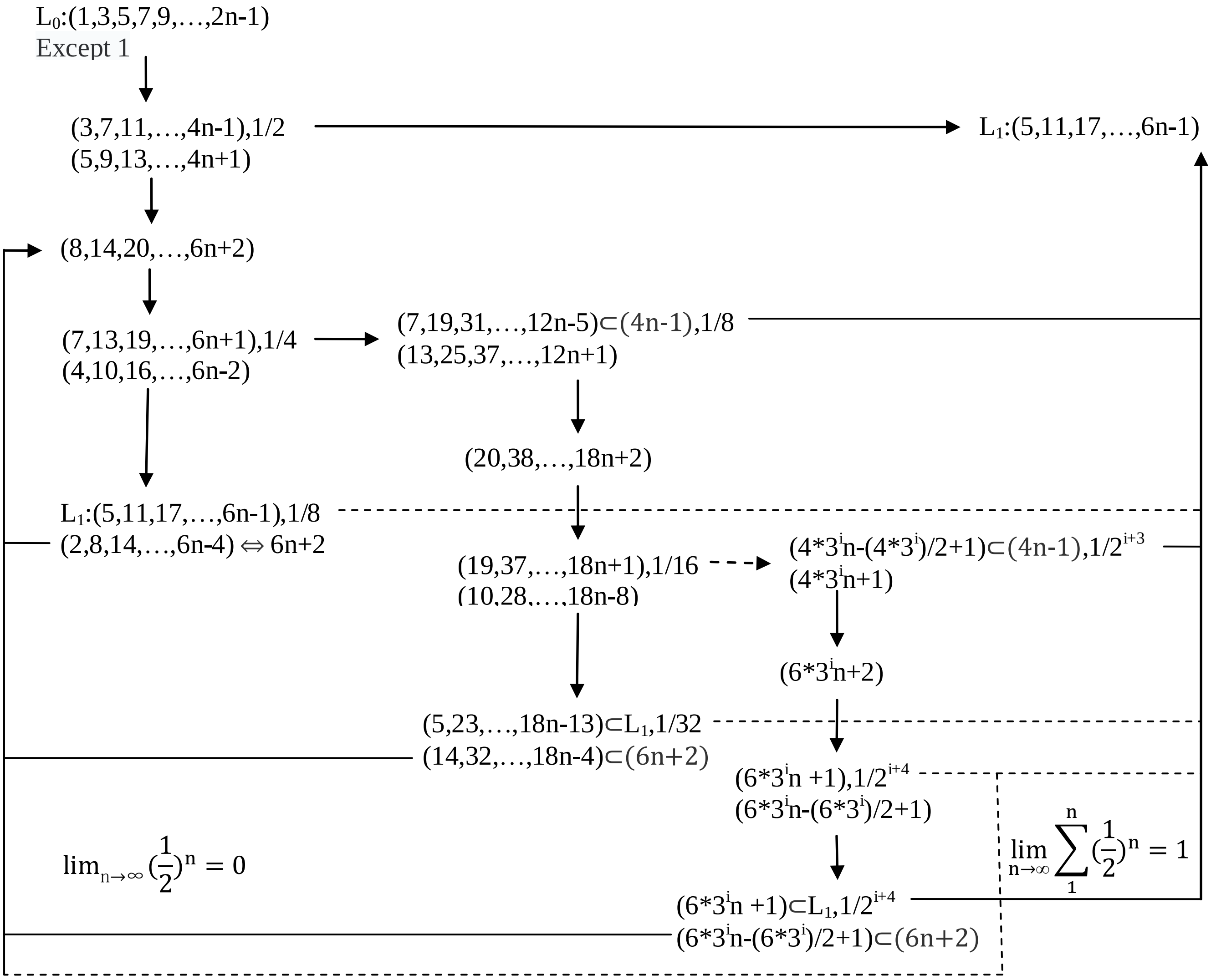

Figure 1: The path of $L_0\to L_1$

Obviously, L_0 will must change to L_1(except 1).

2.4 L_1 to L_n

Corollary 1.1: For all L_1, change by the Collatz conjecture, it will change to the sequence $L_n\{2*3^n i-1 \mid n \in Z, i \in Z^+\}$.

Proof:

Use the same method, the following figure can be obtained:

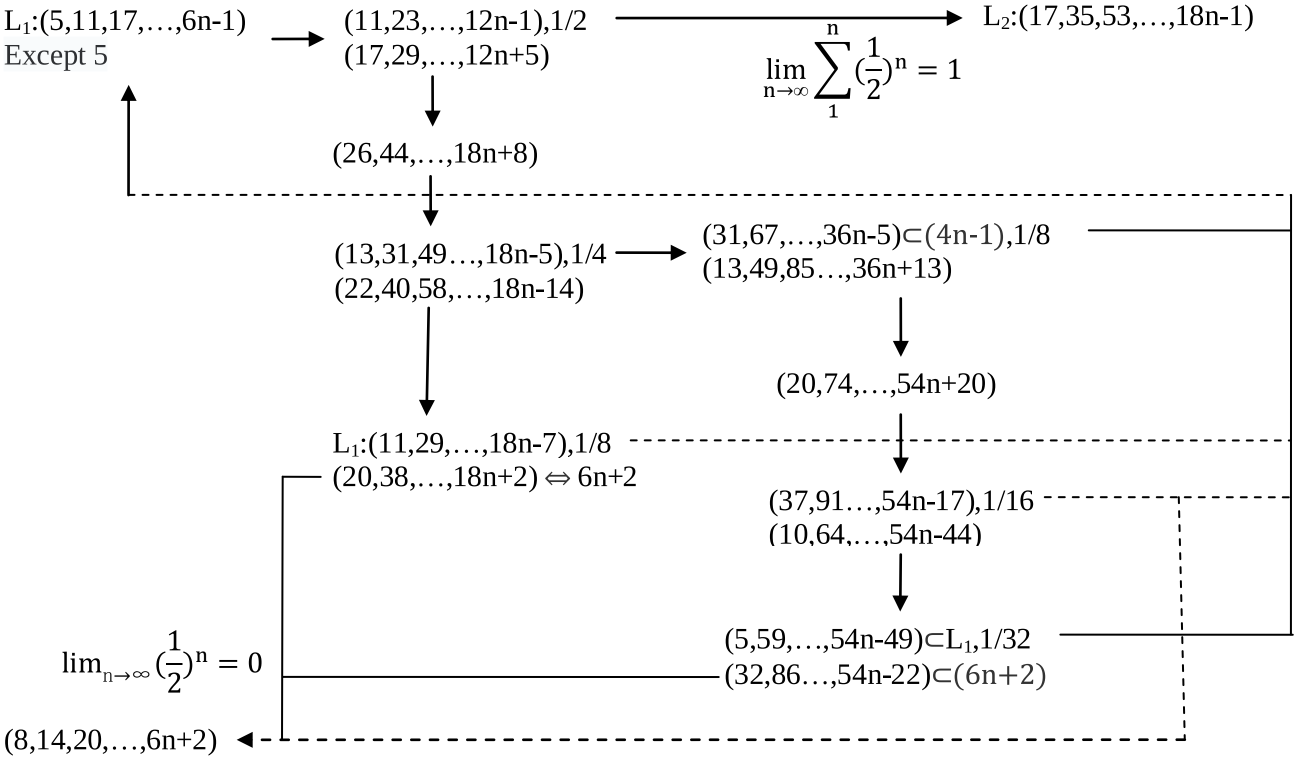

Figure 2: The path of $L_1 \rightarrow L_2$

Obviously, L_1 will translate to L_2(except 5).

Similarly:

L_2 will translate to L_3 (except 17);

L_3 will translate to L_4 (except 53);

…

Obviously, all the except numbers are the first number of $L_0(2n-1)$, $L_1(6n-1)$, ⋯, $L_n(2*3^n-1)$. They are $O_{n1}(1, 5, 17, 53, \cdots, O_{n1}, 2*3^n-1)$.

2.5 L_n to O_{n1}

Theorem 2: For all $O_{n1}(2*3^n-1)$, change by the Collatz conjecture, it will must change to a new $O_{n1}(2*3^n-1)$.

Proof:

All the except numbers (1, 5, 17, 53, ⋯) are the items of $O_{n1}(2*3^n-1)$ too. So all odd numbers must translate to O_{n1}.

If you want to learn more, you can refer to the video in reference [6].

3. Smaller and smaller

3.1 O_{n1} to O_{n1}

To sum up, it is easy to know: O_{n1} must translate to O_{n1}.

3.2 Smaller and smaller

Corollary 2.1: If O_{n1} change to O_{m1}, the m must be less than n.

O_{n1} will not only must translate to O_{n1}, it must change to a smaller one. For proof, see the table 2.

Table 2: Change rule of O_{n1}

O_{n1}	T(x)	C(x)	T(x)	C(x)
5	8	4	2	1
17	26	13	20	10
53	80	40		
161	242	121	182	91
485	728	364		
1457	2186	1093	1640	820
4373	6560	3280		
$2*3^n$-1	3^{n+1}-1	$(3^{n+1}-1)/2$	$(9^n-1)/4$	$(9^n-1)/8$

Accordingly, all of $2*3^n-1$, $3^{n+1}-1$, $(3^{n+1}-1)/2$, $(9^n-1)/4$ and $(9^n-1)/8$ are equivalent, they're named "LiKe's step". $3^{n+1}-1$(or 3^n-1) is the easiest of them all, it's named "LiKe's second sequence"(LK_2). Obviously, Just need to prove one of them can't get bigger. Here, I choose $(3^{n+1}-1)/2$.

$(3^{n+1}-1)/2$: 1, 4, 13, 40, 121, 364, 1093, 3280, 9841, 29524, 88573, ⋯.

Scrutinize, it is not difficult to find:

1=3*0+1；

4=3*1+1；

13=3*4+1；

⋯

The first term is x, the second term has to be 3x+1.

So $(3^{n+1}-1)/2$ is named "LiKe-Collatz number"(LC).

Hence, LiKe-Collatz sequence can be expressed as:

$(3x+1, 9x+4, 27x+13, 3^4x+40, 3^5x+121, \cdots, 3^nx+(3^n-1)/2)$ （$x=(3^i-1)/2$）

The formula of "LiKe-Collatz number" is: $3^nx+(3^n-1)/2$

Also∵According to the rule of the corazz conjecture, All the change paths of 3x+1are as follows:

Table 3: All the changes of LiKe-Collatz number

<table>
<tr><td rowspan="16">3x+1
even：up
odd：below</td><td rowspan="8">$\frac{3x+1}{2}$</td><td rowspan="4">$\frac{3x+1}{4}$</td><td rowspan="2">$\frac{3x+1}{8}$</td><td>⋯</td></tr>
<tr><td>⋯</td></tr>
<tr><td rowspan="2">$\frac{9x+7}{8}$</td><td>⋯</td></tr>
<tr><td>⋯</td></tr>
<tr><td rowspan="4">$\frac{9x+5}{4}$</td><td rowspan="2">$\frac{9x+5}{8}$</td><td>⋯</td></tr>
<tr><td>⋯</td></tr>
<tr><td rowspan="2">$\frac{27x+19}{8}$</td><td>⋯</td></tr>
<tr><td>⋯</td></tr>
<tr><td rowspan="8">$\frac{9x+4}{2}$</td><td rowspan="4">$\frac{9x+4}{4}$</td><td rowspan="2">$\frac{9x+4}{8}$</td><td>⋯</td></tr>
<tr><td>⋯</td></tr>
<tr><td rowspan="2">$\frac{27x+16}{8}$</td><td>⋯</td></tr>
<tr><td>⋯</td></tr>
<tr><td rowspan="4">$\frac{27x+14}{4}$</td><td rowspan="2">$\frac{27x+14}{8}$</td><td>⋯</td></tr>
<tr><td>⋯</td></tr>
<tr><td rowspan="2">$\frac{81x+46}{8}$</td><td>⋯</td></tr>
<tr><td>⋯</td></tr>
</table>

Accordingly,

$$\frac{3^j x + a}{2^i} \neq 3^n x + (3^n - 1)/2$$

∴ 3x+1is never translate to bigger ones 9x+4, 27x+121, 3^4x+40, 3^5x+13, ⋯.

∴ According to 3.1: 3x+1will translate to a smaller LiKe-Collatz number. So 3^n-1 will translate to a smaller 3^m-1(m＜n) until 8, 2, 1.

q.e.d!

4. Roadmap

To sum up, the path of change can be seen in the diagram below, which is simple and clear.

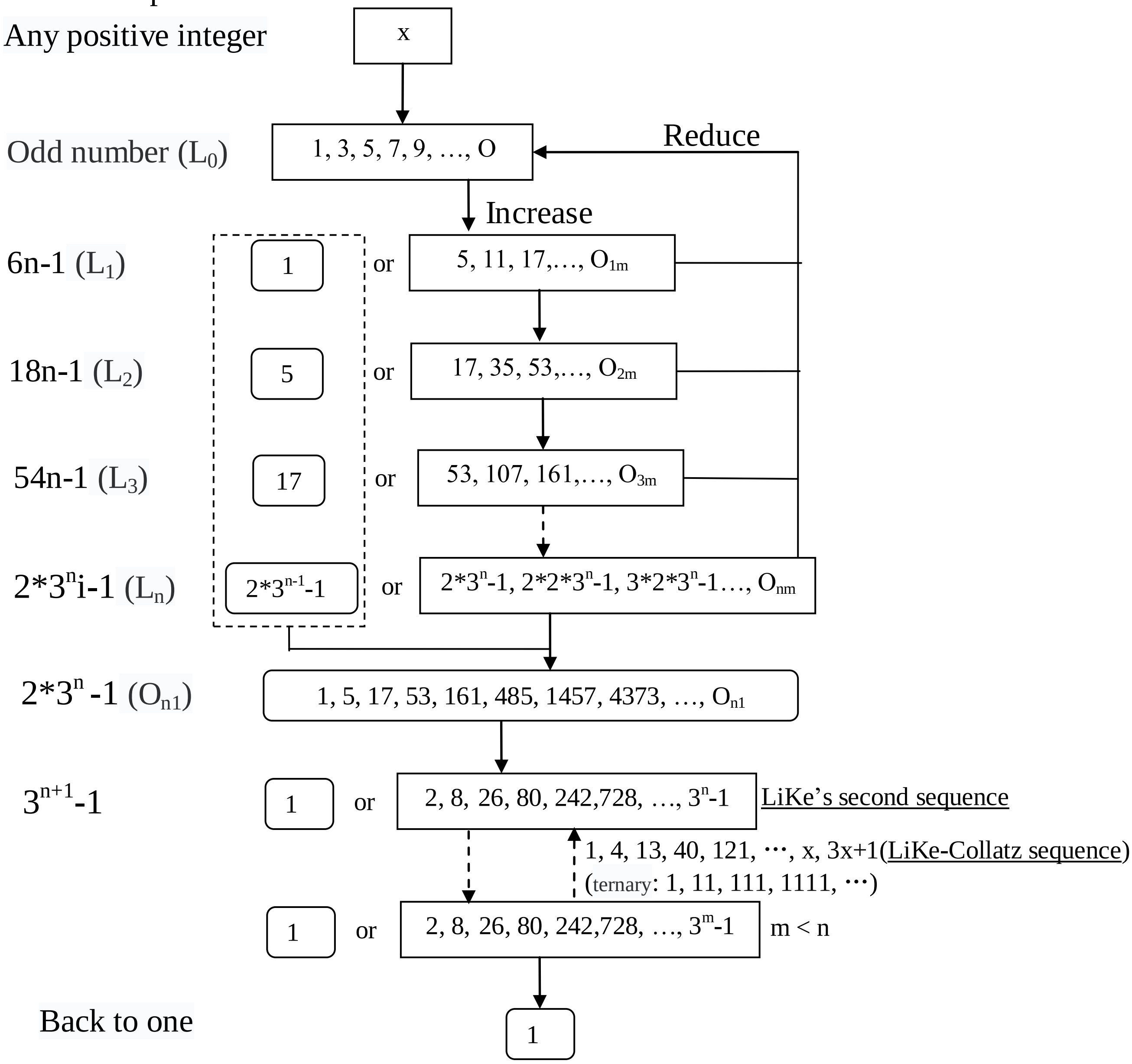

Figure 4: The change path of LiKe's Rule

If the value of C(x) is taken as the ordinate and the number of change steps is taken as the abscissa, the function C(x) can be expressed as the following figure.

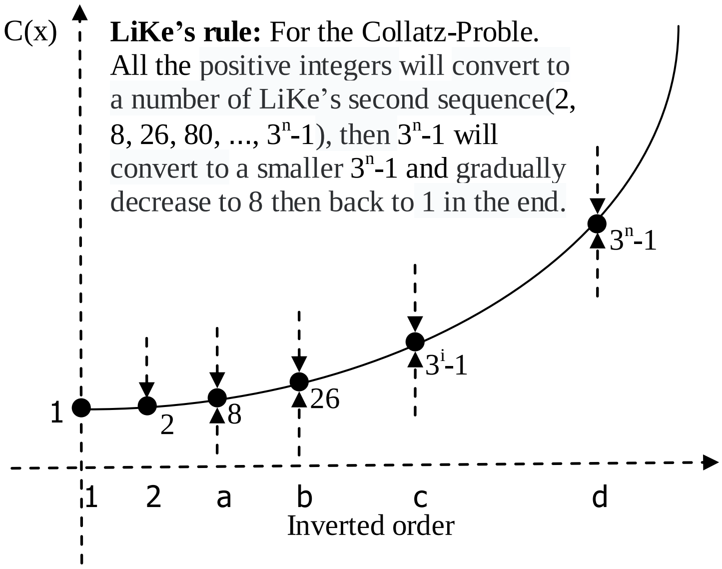

Figure 5: The change rule of Collatz conjecture: LiKe's Rule

Acknowledgments. The author was supported by Natural Science Dream Grants WDM2019002.

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