

Beyond A Quartic Polynomial Modeling of the DNA Double-Helix Genetic Code

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Abstract

By combination of finite number theory and quantum information the complete quantum information in the *DNA* genetic code has been made likely by *Planat et al.* (2020). In the present contribution a varied quartic polynomial contrasting the polynomial used by *Planat et al.* is proposed that considered apart from the golden mean also the fifth power of this dominant number of nature to adapt the code information. The suggested polynomial is denoted as $g(x) = x^4 - x^3 - (4 - \varphi^2)x^2 + (4 - \varphi^2)x + 1$, where $\varphi = \frac{\sqrt{5}-1}{2}$ is the golden mean. Its roots are changed to more golden mean based ones in comparison to the *Planat* polynomial. The new coefficients $4 - \varphi^2$ instead of 4 would implement the fifth power of the golden mean indirectly applying $4 - \varphi^2 = 3 + \varphi = \sqrt{13 + \varphi^5} = \sqrt{2 + \varphi^{-5}} = 3.6180\dots$ As an outlook it should be emphasized that the connection between genetic code and resonance code of the *DNA* may lead us to a full understanding of how nature stores and processes compacted information and what indeed is consciousness linking everything with each other suggestedly mediated by all-pervasive dark constituents of matter respectively energy.

Keywords: *DNA* Genetic Code, *DNA* Resonance Code, Quartic Polymial, Golden Mean, Silver Mean, Fifth Power of the Golden Mean, *Fiboacci* number 13, α -Helix, Icosahedron Equation, Number Theory, Quantum Computation, Consciousness, Dark Energy.

1. Introduction

In a more literary parlance, the genetic code of *DNA* is the very language of life, made up of four nucleotides (A = adenine, C = cytosine, G = guanine, T = thymine) combined to 64 three-letter words, the codons. The code can be read by cellular complexes, the ribosomes, and the information translated to produce proteins. In a brilliant contribution *Planat et al.* set out unrevealing the complete quantum information in the *DNA* genetic code by combining finite number theory and quantum information [1]. The cited authors have used a quartic polynomial containing the golden mean φ in its structure (see [Appendix](#)). Intuitively, the present author suspects that in addition to the golden mean, also the fifth power of this number should play an important role in mapping the genetic code owed to the fact that this

number is observed extremely frequently in nature, covering examples from microscopic to cosmic scale [2] [3]. So, one may argue that this feature should also be found in some way in the genetic code. A little varied quartic polynomial can deliver such an implementation of φ^5 by replacing the starting polynomial coefficients adequately. This was described in Chapter 2. In times of a viral pandemic it is essential to know the genetic code of *DNA* respectively surface proteins in order to effectively block the replication capacity of any pathogenic virus via this route. A further practical aspect of research in this direction is to pave the way for quantum computation under ambient conditions. Recently, a very creative review for understanding of what may be important when dealing with such suggestions has been given by *Savelyev et al.* connecting the *DNA* resonance code to the neural code [4]. When translating such connections in the parlance of phase transition, again one comes back to the golden ratio respectively its fifth power [2], both considered as fundamental numbers of nature.

The presented contribution is a short excerpt from a more comprehensive review coming soon about new aspects of helical structures in general [5]. Therein a didactic explanation for the characteristic diffraction pattern of a helix (double-helix) will be presented in detail.

2. Varying the Quartic Polynomial $f(x)$

The starting *Planat* quartic polynomial (*PQP*) given in [1] is

$$f(x) = x^4 - x^3 - 4x^2 + 4x + 1 \quad (1)$$

Its curve is illustrated in [Figure 1](#) and its four roots (zeros) are presented in [Table 1](#) according to [1]. What happens, if the polynomial would be slightly varied by replacing the coefficients 4 by $4 - \varphi^2 = 3 + \varphi$, where φ is the golden mean: $\varphi = \frac{\sqrt{5}-1}{2}$?

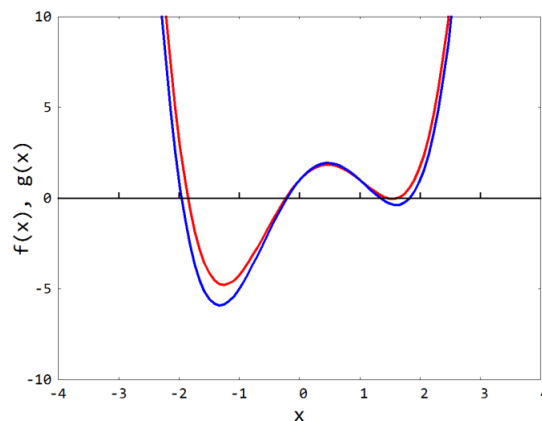


Figure 1. Comparison of the polynomials $f(x)$ (blue) and $g(x)$ (red).

The coefficients now used possibly represent limits and can be recast by

$$4 - \varphi^2 = \sqrt{13 + \varphi^5} = 3.6180339887 \dots \quad (2)$$

In this way one would implement the fifth power of the golden mean indirectly. The new polynomial is denoted as $g(x)$ and is given by

$$g(x) = x^4 - x^3 - (4-\varphi^2)x^2 + (4 - \varphi^2)x + 1 \quad (3)$$

Its roots are changed to more golden mean based ones in comparison to the starting polynomial (see [Table 1](#)). Both quartic polynomials can be compared in [Figure 1](#). The curve for $g(x)$ is somewhat flattened and compressed in comparison to the starting curve.

Remarkably, for $g(x)$ two zeros r_2 and r_4 meet at $x = \varphi^{-1}$. This flatten out of the curve around zero may be important, when considering the polynomial as a template for a potential energy model of the *DNA* double helix and critical behavior. The maximum of $g(x)$ at $x = 0.4661 \approx 2\varphi^3$ has, similar to the $f(x)$ polynomial, a value of

$$g_{max} = 1.846288 \dots \approx \varphi^{-1} + \varphi^3. \quad (4)$$

Table 1. Roots of the quartic polynomials $f(x)$ and $g(x)$

Roots	$f(x)$		$g(x)$	
r_1	$-2\cos\left(\frac{\pi}{15}\right)$	-1.956295...	$\approx -(\varphi^{-1} + \varphi^3)$	-1.8516618...
r_3	$-2\cos\left(\frac{7\pi}{15}\right)$	-0.209056...	$\approx -\varphi^3$ $= -0.2360679 \dots$	-0.2283086...
r_2	$-2\cos\left(\frac{11\pi}{15}\right)$	1.338261...	$-2 \cos\left(\frac{12\pi}{15}\right) = \varphi^{-1}$	1.61803398...
r_4	$-2\cos\left(\frac{13\pi}{15}\right)$	1.827091...		

The effect of the varied coefficients is a pronounced presence of the golden mean hierarchy compared to the starting polynomial. Consequences of the variation of the quartic polynomial with respect to group theoretical changes (entries of the character table) may not be trivial and should be worked out and evaluated by experts. The basic group-theoretical approach [1] was the creation a group G of order 240 by linking the finite group Z_5 of order 5 with the binary octahedral $2O$ group of order 48.

3. Helical Structures

Helical structures dominate or life in the form of the α -helix of proteins [6] [7] [8] or double-helices composed of double-stranded molecule chains of nucleic acids such as *DNA* [9]. Helical curling of cellulosic micro-fibrils leads to spring-like elastic behavior [10]. Interesting is that helical left-handedness and its opposite can be realized, when a single cellulosic fiber is supported at both ends [10]. Designers have many learnt from the multilayer tube-like structure of wood cell walls leading to superior properties of this natural material [11]. Let's stick with technical aspects such as making torque from wind the wake behind multi-bladed

rotors can be described by interlaced helical vortices [12]. But also liquid water forms helices of pentagonal or hexagonal symmetry [13]. When subtracting from the regular tetrahedron angle of 109.47° the angle $\arcsin(\varphi^5) = 5.17^\circ$, where φ is the golden mean, then the bonding angle of H_2O is quite well adapted giving 104.30° . This connection to the golden mean hierarchy is intriguing. It is reasonable to assume that water may be involved in two ways in the origin of life, as a liquid medium and as helical scaffold mediating the first assembling of a double-helix of nucleic acids. In contrast, the participation of structurally disordered clay minerals in this process is very unlikely. The fifth power of the golden mean φ^5 in turn is frequently connected to phase transformation from microscopic to cosmic scale [2] [14]. This number may be characterized as a universal one [15]. On the border between hexagonal and pentagonal respectively *Fibonacci* number governed symmetry our existence is located. Numerical it may be emphasized by the relation $\varphi^5 = 5\varphi - 3$.

Turning to inorganic structures, in 1977 I solved the hexagonal palatinate crystal structure of $\text{Cs}_2[\text{Pt}(\text{CN})_4]\cdot\text{H}_2\text{O}$, in which planar $\text{Pt}(\text{CN})_4$ groups are helical winding down the c axis by means of a 6_1 screw axis [16]. It represents a nice example of helicity in inorganic matter. At that time, I have discussed the Pt orbital overlap of this interesting inorganic compound with *Nobel laureate Linus Pauling* who solved the structure of the α -helix of proteins 26 years before [6] [7] [8].

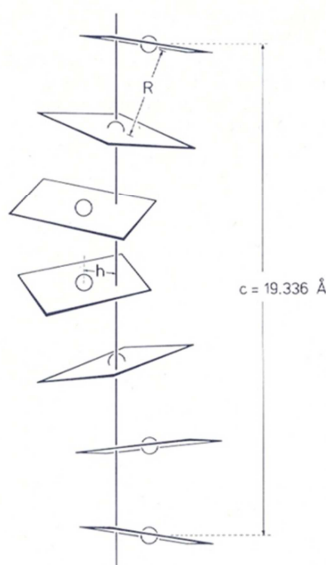


Figure 2. Helix of $\text{Pt}(\text{CN})_4$ squares screwed down the c axis of $\text{Cs}_2[\text{Pt}(\text{CN})_4]\cdot\text{H}_2\text{O}$ [16].

Since that time I was fascinated by the helical structure of matter, but found no time to think about it anymore considering crystal-chemical, biochemical as well as mathematical aspects.

4. A Number-Theoretical Golden Helix Gimmick

Now we turn to the α -helix peculiarities of proteins where 18 subunits perform 5 turns to reach an identical position when projected down the fiber axis [6] [7]. This means one needs

3.6 subunits per turn. This number is nearby another golden mean derived number (see Equation 2)

$$3 + \varphi = 3.6180339887 \dots = \sqrt{13 + \varphi^5} = \sqrt{2 + \varphi^{-5}} \quad (5)$$

In case of the α -helix, when tentatively using 3.61803... subunits per turn instead of 3.6, one would end with $18 + \varphi^5$ subunits after 5 turns

$$5(2 + \varphi^{-1}) = (\varphi^5 + \varphi^{-5})\varphi^{-1} = 18 + \varphi^5 = 18.0901699 \dots \quad (6)$$

and again the fifth power of the golden mean would be involved.

What marginal change would such approach of helix breathing produce in the *Fourier* transform possibly overseen as yet by the interpretation of the diffraction patterns showing quite broad smeared out diffraction spots? An elaborated study of structural variations of *B-DNA* by interpreting features of its diffraction pattern was recently given in [17].

The value of $\sqrt{3 + \varphi} = \sqrt{1 + \varphi^{-2}}$ is also related to an icosahedron as its circumradius when this polyhedron with an edge length of $a = 2$ is centered at the origin [18] [3]. It may be considered as a very important distance frequently found in the following reflections.

Turning to five-fold symmetrical nucleic acid structures discussed in [1], the regular pentagon with side length a has an area of A_5 calculated by [18]

$$A_5 = \frac{5}{4\varphi^2\sqrt{3+\varphi}} a^2 = 1.720477403 a^2 \quad (7)$$

Another example should be considered with respect to the seven-fold symmetry of complexes like the *Lsm1-7* one recently analyzed by *Planat et al.* [20]. The geometry of a heptagon delivers a connection to the golden mean. The heptagon area A_7 with side length a yields also about $\sqrt{3 + \varphi}$ in its expression

$$A_7 = \frac{7}{4\tan\left(\frac{180}{7}\right)} a^2 = 3.633912 a^2 \approx (3 + \varphi) a^2 = 3.618033987 a^2 \quad (8)$$

$$\text{and} \quad \tan\left(\frac{180}{7}\right) = 0.481574618 \approx \ln(\varphi^{-1}) = 0.481211826 \quad (9)$$

The relative area difference can be approximated by

$$\frac{A_7 - A_5}{A_5} = 1.11169295 \approx \sqrt{1 + \varphi^3} = 1.1117859. \quad (10)$$

Another example from number theory where the golden mean in form of the square root term $\sqrt{3 + \varphi}$ is involved but not yet expected is the series of heptagonal numbers generated when heptagons with ascending size are combined [19]. Such figurative heptagonal numbers are generated by the formula [19]

$$P_n = \frac{5n^2 - 3n}{2} = 1, 7, 18, 34, 55, 81 \dots \quad (11)$$

indicating a parity sequence of odd, odd, even, even ... and successive *Lucas* numbers 7 and 18 respectively *Fibonacci* numbers 34 and 55.

The sum of reciprocals of this number series shows a result in full glory, if we recast for the first time the already known expression [19] using the golden mean as $\varphi = \frac{\sqrt{5}-1}{2}$

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{1}{P_n} &= \frac{2}{3} \left(\frac{\pi}{2} \frac{\varphi}{\sqrt{1+\varphi^{-2}}} + \ln(5) + \varphi^{-1} \ln(\sqrt{1+\varphi^2}) - \varphi \cdot \ln(\sqrt{1+\varphi^{-2}}) \right) \quad (12) \\ &= 1.3227792531 \dots \approx \frac{2}{3} \ln(5) + \frac{1}{4}\end{aligned}$$

Alternatively it applies because $\sqrt{1+\varphi^2} = \varphi\sqrt{1+\varphi^{-2}} = \varphi\sqrt{3+\varphi}$

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{1}{P_n} &= \frac{2}{3} \left(\frac{\pi}{2} \frac{\varphi}{\sqrt{3+\varphi}} + \ln(5) + \frac{\ln(\varphi)}{\varphi} + \ln(\sqrt{3+\varphi}) \right) \quad (13) \\ &= \frac{2}{3} \left(\frac{\pi}{2} \frac{\varphi}{\sqrt{3+\varphi}} + \ln(5 \cdot \varphi^{1/\varphi}) + \ln(\sqrt{3+\varphi}) \right)\end{aligned}$$

where
$$\frac{\ln(\sqrt{3+\varphi})}{\ln(5 \cdot \varphi^{1/\varphi})} = 2.289438326 \approx 5 \cdot \varphi^{1/\varphi} = 2.29520192 \quad (14)$$

and further
$$\frac{1}{2} \frac{(\sqrt{3+\varphi})}{5 \cdot \varphi^{1/\varphi}} = 0.414367253 \approx (\sqrt{2} - 1) = 0.414213562 \quad (15)$$

$\sqrt{2} - 1$ represents the silver mean here approximated by an expression for the golden mean.

It indicates that even in the case of seven-fold symmetry a sophisticated numerical relationship to φ , the omnipresent golden code number of life, is given and should be considered in future work on the genetic code and resonance behavior of life.

5. Simple Didactic Approach for the Fourier Transform of a Helix

The *Fourier* transform of a helical structure resulting in the integral representation of *Bessel* functions of different order was first introduced in 1952 by *Cochran* [7] and can be studied in detail in the monograph ‘Direct Analysis of Diffraction by Matter’ by *Hosemann* and *Bagchi* [21]. Another recent contribution by *Latychevskaia* and *Fink* [22] is recommended as well as that given by *Strutz* in German [23].

Our simplified but didactic approach is as follows. If one looks through the projection of a helical structure perpendicular to the filament direction, one will see the mirror image on the back at the helix diameter distance to the front image, somewhat offset in filament direction depending of the helix winding parameters. One can perform the *Fourier* summation to obtain diffraction intensities exemplified by the projected hexagonal nets of a special *Fibonacci* micro-tubule with a 13 steps full turn. The starting small cell is helically wound at the twist angle α . It needs 13-times the small cell lattice parameter a_{sub} to reach again identity with a

large cell lattice point. In this way the diameter of the smallest geometrically possible microtubule is calculated to be $d_1 \approx 13 a_{sub} \cdot \cos(\alpha)/\pi = 3.5 \cdot a/\pi$. Interesting is $d_6 = 21 \cdot \frac{a}{\pi}$ showing a further *Fibonacci* number. By a full turn one gains a height in filament direction of

$$h_1 = \frac{\sqrt{39}}{2} a_{sub} = \frac{\sqrt{3}}{2} \sqrt{13} \cdot a_{sub} = 3.1225 \cdot a_{sub} \approx \pi \cdot a_{sub}. \quad (16)$$

More detail of the chosen example can be found in [24] [25]. However, when again choosing in equation (13) a value of $\sqrt{13 + \varphi^5}$ instead of $\sqrt{13}$ by a small change of the twist, then $\frac{h_1}{a_{sub}} = 3.1333$ would better approximate π regardless whether this can be significant in any way.

The example simplifies helically twisted tubulin microtubules that also show strongest reinforcement of the ordered pattern when the proto-filament number n equals 13 [26] [27]. Interestingly, according to *Penrose* and *Hamedoff* tubulin microtubules are considered as the locus where information is stored and assessed, what constitutes our consciousness and cognition [28] [29]. This supports the proposal for a twisted *Fibonacci* net as displayed in [Figure 3](#) respectively [Figure 4](#) as a quantum computer bio-template. Information may be preserved in tubulin molecules by structural adaption and sequence fluctuations disrupting the helical coherence.

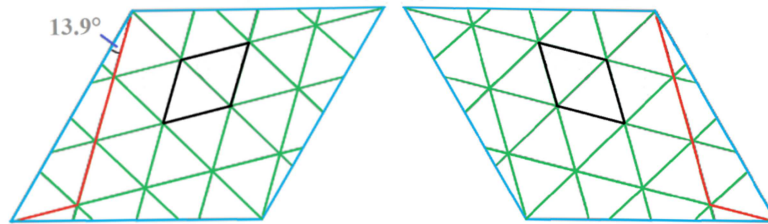


Figure 3. *Fibonacci* arrangement of a hexagonal net and its mirror image [2] [24] [25]
The light-blue outlined unit-cell contains 13 sub-cells, offset by an angle of $\alpha = 13.9^\circ$.

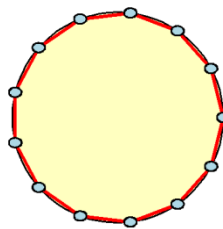


Figure 4. Helically twisted *Fibonacci* microtubule projected down the filament direction with 13 light-blue atoms or atom groups on sub-lattice positions.

When removing from the *Fibonacci* net some atomic stripes one can simulate a double helix. Detailed intensity calculation of such a construct will be given in a forthcoming contribution [5] showing the characteristic X shaped cross of diffraction intensity distribution of a double helix similar to the electron diffraction pattern of B-DNA fibers [30] respectively to the

historical X-ray pattern (photo 51) of B-DNA taken by the felt *Nobel* laureate of hearts *Rosalind Franklin* [31]. When one works with the projected curved net instead of a flat net, the reflections smear out as result of the introduced coordinate statistics. [Figure 3](#) may also help to understand in which way inverted repeat of nucleotides works capable of self-complementary base pairing.

6. Beyond the Icosahedron Equation

If the genetic information is ultimately coiled and compacted into an icosahedral shape as found in viruses, then another function may come into play, represented by the icosahedron equation mapping for instance the positions of the face centers of an icosahedron with unit in-radius projected onto a complex plane where z are the coordinates [32][33] [34] [35]

$$H(z, 1) = z^{20} - 228z^{15} + 494z^{10} + 228z^5 + 1 \quad (17)$$

The roots accordingly correspond to the locations of the face midpoints on the *Riemann* sphere. Interestingly but not surprisingly, the coefficient ratio yields $494/228 = 13/6$ and contains the *Fibonacci* numerator 13. The coefficients 228 as well as 494 can be replaced by a golden mean based approximation besides number 13

$$228 \approx \frac{4}{3} \left(13 + \frac{\varphi^4}{\sqrt{3+\varphi}} \right)^2 = 228.00022 \quad (18)$$

$$228 \approx \frac{4}{3} \left(13 + \frac{1}{13} \right)^2 = 228.00789 \quad (19)$$

$$494 \approx \frac{2}{9} \cdot 13 \left(13 + \frac{1}{13} \right)^2 = 494.01709 \quad (20)$$

Based on the preliminary work by *Gordon* [36], *Klein* showed the connection between the regular icosahedron, one of the five *Platonic* bodies, and the solution of the quintic polynomial. Instead of following *Klein's* quintic icosahedral solution, the substitution of the complex variable $z^5 \rightarrow x$ formally leads to a quartic polynomial

$$H(x, 1) = x^4 - 228x^3 + 494x^2 + 228x + 1 \quad (21)$$

$$\approx x^4 - \frac{4}{3} \left(13 + \frac{1}{13} \right)^2 x \left(x^2 - \frac{13}{6} x - 1 \right) + 1 \quad (22)$$

The four root of this polynomial have been calculated giving

$$x_3 = -\frac{1}{x_2}, \quad x_4 = -\frac{1}{x_1} \quad (23)$$

$$x_3 = 2.58365 \approx 228 - \frac{4}{3} 13^2 = 2.66666 \quad (24)$$

$$x_4 = 225.80782 \approx \frac{4}{3} 13^2 = 225.33333 \quad (25)$$

In addition it yield

$$\sum_{i=1}^4 x_i = 228 \quad (26)$$

The reader is frequently confronted with the *Fibonacci* number 13, which obviously plays an important role besides φ and φ^5 when assessing bio-coding and related storage and processing of information.

Turning to another polynomial equation corresponding to the vertices of an icosahedron [35]

$$w(x) = x(x^{10} + 11x^5 - 1) \quad (27)$$

Now again substituting z for x^5 in the term in brackets and solving the resulting quadratic equation (only bracket term)

$$z^2 + 11z - 1 = 0, \quad (28)$$

where
$$11 = \varphi^{-5} - \varphi^5 \quad (29)$$

one can confirm the two roots as $z_1 = \varphi^5$ respectively $z_2 = -\varphi^{-5}$ (see also [2]).

7. Self-Duality as a Suggested Property for Image Storage, Memory and Consciousness

Self-duality is a very important concept of mathematics as well as physics and should be applied to biophysics respectively biochemistry in order to understand the very stable information storage within the human brain. Self-dual objects include polyhedra, graphs, configurations and codes. The skeleton of a self-dual polyhedron is a self-dual graph. For instance, self-dual solitons as self-reinforcing and shape-maintaining wave packets are expected to be very stable. Among the self-dual polyhedra are the regular tetrahedron respectively pyramids in general. Also the *Kummer* quartic surface with 16 nodes and 16 singular tangent conics, believed by *Planat et al.* [20] to explain in part the *DNA* genetic code, is self-dual [37].

The information storage respectively processing certainly doesn't work via simple bits or qubits but via complex pictures generated by superposition of stable (soliton-like) wave packets, burned into a multiplex microtubule array or a comparable matrix. This happens against the background of the omnipresent dark energy with all the information collected up to then. Each speedy moving body or particle is surrounded by dark matter respectively energy [14], and when seeking for an assumed resonance field that links *DNA* or tubulin resonances with the work of the brain according to [4], the first candidate to fulfill all conditions is simply dark energy.

8. What is the Code for an Assumed Electron Helix?

Importantly, also elementary particles such as the electron are thought to exhibit a helical structure [38] [39] indicating that nature used this principle of curling again and again [40]. Remarkably, the arithmetic mean between the proposed maximal superluminal electron velocity and the minimal one yields $\bar{v} = \frac{c}{\varphi}$. The application of *IRT* transformations [14] to the problem would certainly present the electron helix approach correctly and in full glory. We will work on this. The frequency of electron's *zitterbewegung* is found to equal that of the

composing photons [39]. Besides the results given in [40] [35] and contrasting the *QED* coding that uses an unbelievable cascade of *Feynman* diagrams [41] [42] [43] one should try to encode and understand a numerical golden mean relation for the electron as a quite stable helically wound compacted entity where Δg_e is the gyromagnetic correction factor of the electron and 34 is a *Fibonacci* number

$$\Delta g_e \approx \frac{\varphi - \exp(-\varphi)}{34 + \varphi^5} = 0.00231829 \quad (30)$$

The experimental value of Δg_e was determined slightly larger [44], but a new relativistic correction was proposed shifting this value towards the considered smaller one [40].

If we proceed with this helix quantum-vortex approach we may at least understand electron's pairing in superconductors, too [45]. Historically, already in 1978 *L. S. Levitt* had raised the question whether the photon is a double helix [46]. A photon with high energy decompose by interaction with matter to electron and positron, both helically twisted.

However, if one could ultimately prove the double helix structure of the electron, it would seem natural to think further of its possible fine structure making one electron not equal to another one, despite their same mass and bulk properties. It would revolutionize our ideas when it comes to storing and processing extra-orbitant amounts of information.

9. Helical Helix Construct

Single respectively double-strand helices may not only be found in nature as rotational blurred linearly aligned fibers. The fiber axis of a helix or double-helix may itself be proposed in turn helically wound to strengthen its spring-like elastic behavior suggested for application in building constructions. Such construct would not represent the densest packing, but an extremely flexible one. I am not sure whether nature used this construct, for instance as *DNA* partial packing in viruses or in the structure of the electron, but designers could certainly gain profit in its application. Also energy harvesting may be possible when such construct was set in vibration by wind or water.

10. Conclusion

The given suggestions are intended to provide an expanded view on the already complicated matter of coding important helical structures of life and matter exemplified by the *DNA* or the beautiful electron. The duality of matter and surrounding waves or matter and dark matter leaves fingerprints in such structures in form of numerical reciprocity given by the golden mean $\varphi^{-1} = \varphi + 1 = \emptyset$, besides $\varphi^{-5} + \varphi^5 = 11$ (phase transitions from microscopic to cosmic scale, dimension of superstring theory) or $\frac{2}{3} \left(13 + \frac{1}{13}\right)^2 = 228$ (coefficient of the icosahedron equation). Number 13 is also found as proto-filament number in helically twisted tubulin microtubules as the locus where human information storage and assessment is suspected. Philosophical one could say that life beats 13. The reciprocity symmetry principles may find application in topological quantum computers based on *Fibonacci* anyons simulating processes which determine the speed and storage capacity of human mind.

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Appendix

Golden Mean Beauty and Its Intrinsic Reciprocity Property [40]

The golden mean or golden ratio φ is an omnipresent number in nature, found in the architecture of living creatures as well as human buildings, music, finance, medicine, philosophy, and of course in physics and mathematics [47][48]. It is the most irrational number known and a number-theoretical chameleon with a self-similarity property. On the other hand, its infinite continued fraction representation is the simplest of all and is represented by [49]

$$\varphi = \frac{\sqrt{5}-1}{2} = \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}} \quad (31)$$

It impressively underlines the fractal character of this number. Most obviously, the golden mean mediates stability of a system, because only 'particles' as the center of gravity of vibrations with most irrational winding survive. Important relations involving φ are summarized below. However, to prevent confusion, in textbooks of mathematics the reciprocal value for φ is frequently used.

$$\varphi = \frac{\sqrt{5}-1}{2} = 0.618033988 \dots \quad \varphi^{-1} = 1 + \varphi = \frac{\sqrt{5}+1}{2} = 1.618033988 \dots \quad (32)$$

$$\varphi^2 = 1 - \varphi = 0.381966011 \dots \quad \varphi^{-2} = 2 + \varphi = 2.618033988 \dots \quad (33)$$

$$(\varphi^2 + 1)^{-1} = (\varphi^{-2} + 1)/5 \quad (34)$$

or equivalently $(\varphi^{-2} + 1)^{-1} = (\varphi^2 + 1)/5 \quad (35)$

$$\varphi^5 = \frac{1-\varphi}{1+\varphi}\varphi^2 = 0.090169943 \dots \quad (36)$$

$$\frac{\varphi^5}{2} + \frac{5}{2}\varphi^2 = 1 \quad (37)$$

$$\varphi^3 + 2\varphi^2 = 1 \quad (38)$$