

# A fresh hope of proving Goldbach conjecture(1+1)

Li Ke

(Institute of Chemical Industry of Forest Products, CAF)

**Abstract:** Goldbach conjecture is known as the jewel in the crown of mathematics. At present, the research methods mainly include almost prime, exception set, three prime theorem and almost Goldbach problem. But none of them solved the problem. In 2019, Goldbach conjecture ushered in a new proof path – LiKe sequence. This method is feasible in theory. This paper briefly introduces the proof process of this method: by indirect transformation, Goldbach conjecture is transformed to prove that the LiKe sequence of prime sequence  $(3,5,7,\dots,P_n)$  must be greater than  $2 \times (P_{n+1}-1)$ . The problem is proved by proving the LiKe sequence  $> 3 \times P_n > 2 \times P_{n+1} > 2 \times (P_{n+1}-1)$  step by step. So that tells us that the Goldbach conjecture is true.

**Key words:** Goldbach conjecture; number theory; sequence; prove

## The definition of LiKe sequence:

Let  $2N$  represents even numbers and half of it is  $N$ , all odd primes less than  $N$  as the prime sequence  $(3,5,\dots,P_n)$ , only use these prime factors to represent composite numbers  $(Y|Y \in R, R=3^x 5^y \dots P_{n-1}^i P_n^j)$ . In these composite numbers, if there is a new sequence  $(Y_1, Y_2, \dots, Y_n)$ , make the inverse interval of the corresponding items is equal to the sequential interval of the items of the prime sequence  $(Y_1 - Y_2 = 2, \dots, Y_{n-1} - Y_n = P_n - P_{n-1})$ . We called  $(Y_1, Y_2, \dots, Y_n)$  is the corresponding LiKe sequence of the prime sequence  $(3,5,\dots,P_n)$ .

## Theorem:

**Given the odd primes sequence  $(3,5,7,\dots,P_n)$ , and the terms of its corresponding LiKe sequence can only be  $> 2(P_{n+1}-1)$ .**

Proof:

To prove this theorem, we must know the following two lemmas:

## Lemma 1:

**The minimum LiKe sequence of odd prime sequence  $(3,5,7,\dots,P_n)$  is greater than  $3 \times P_n$ .**

The LiKe sequence of (3) are (9), (27), ...,  $(3^n)$ . They all  $\geq 3 \times 3$ ;

The smallest LiKe sequence of (3, 5) is (25, 27).  $25 > 3 \times 5$ ;

The sequence of (3, 5, 7) has no LiKe sequence, the proof is easy:

Let  $(a,b,c)$  is a LiKe sequence of (3, 5, 7)

If  $u$  and  $v$  are made up of factor 3,5,7 and they are not relatively prime ( $v > u$ )

Suppose  $x$  is the greatest common divisor of  $u$  and  $v$

$x$  is divisible by at least one of 3,5, and 7

That is  $v-u$  can be divisible by  $x$ . But according to the questions  $v-u \in \{2,4\}$

So  $a,b,c$  must prime to each other, They can only be a permutation of  $(3^i, 5^j, 7^k)$

Because  $5^j \bmod 25 = 0, 7^k \bmod 25 \in \{24,18,1,7\}$

But LiKe sequence requires  $|7^k - 5^j| \in \{2,4\} \notin \{24,18,1,7\}$ , it's impossible.

So (3,5,7) has no LiKe sequence.

Similarly, it's not hard to prove there is no LiKe sequence of  $(3,5,7,\dots,P_n)$  less than  $3 \times P_n$ .

## Lemma 2:

**When  $N$  is large enough, there must be primes between  $N$  and  $1.5N$ .**

It's obvious true.

From the lemma 2 we can get:

When  $N$  is large enough, there must be:  $(3 \times P_n) / (2 \times P_{n+1}) > 1$

that is  $3 \times P_n > 2 \times P_{n+1}$

And because:  $2 \times P_{n+1} > 2 \times (P_{n+1}-1)$

So:  $3 \times P_n > 2 \times (P_{n+1}-1)$

From the lemma 1 we can get:

LiKe sequence  $> 3 \times P_n$

So: LiKe sequence  $> 2 \times (P_{n+1}-1)$

So the theorem is right.

Q.E.D

## Inference:

**Any sufficiently large even number  $2N$  minus an odd prime before  $N$  must have a prime number.**

Proof:

Because according to the theorem, the LiKe sequence of odd primes before  $N$  must be greater than  $2N$ .

So for odd prime sequence  $(3,5,\dots,P_n)$  before  $N$ , there must be prime in the reverse same-spaced sequence less than  $2N$ .

Q.E.D

**Note: The Inference is equivalent to Goldbach conjecture!**

## References:

[1]. 李科.哥德巴赫猜想研究新方法-数列法[J], 学习周报. 2019,(38):157.