Representing a cube as the sum of three cubes

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ABSTRACT: For the first time, the article proposes a formula allowing to represent a cube of a natural number as a sum of cubes of three natural numbers.

KEY WORDS: sum of three cubes, representation, diaphontic equation.

The representation of an integer as the sum of three cubes is an open problem in mathematics. Variants of the Diophantine equation $x^3 + y^3 + z^3 = n$ have been studied by many scientists, for example [1, 2, 3, 4, 5].

Roger Heath-Brown in 1992 suggested that any number n not giving a remainder of 4 or 5 when divided by 9 has infinitely many representations as sums of three cubes [4]. The search for integers representable as the sum of three cubes and unknown options for representing integers as the sum of three cubes is carried out using computer networks, for example, in 2019, Andrew Booker found a representation in the form of the sum of three cubes for the number 33 [5].

Until the present work, there was not a single formula that would allow one to obtain infinite integers representable as the sum of cubes of three integers. We have established that the cube of any natural number of the form 9t can be represented as a sum of cubes of three integers by the formula

$$(9t)^3 = (8t)^3 + (6t)^3 + t^3, \text{ where } t \in \mathbb{N}.$$

Example. For t = 11 we get

$$(9 \cdot 11)^3 = (8 \cdot 11)^3 + (6 \cdot 11)^3 + 11^3; \quad 99^3 = 88^3 + 66^3 + 11^3.$$

References

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