Function of two variables and sequences eventually periodic.

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Summary

I present an algorithm that defines a function generator of sequences eventually periodic, with eligible cycle values and starting with any integer.

Keywords

Function of two variables, sequences eventually periodic, Collatz conjecture.

Description

The sequences generated with this function will be eventually periodic, whose cycle we can choose by assigning a value to m.

Let (k,m) $\in \mathbb{Z}$, this algorithm is defined as the function f (k,m), such that:

$$f(k,m) = \begin{cases} (k-m)/2, & \text{if (k,m) have the same parity.} \\ (3k+1+m)/2, & \text{if (k,m) have opposite parity.} \end{cases}$$

Dom f(k,m) = (k+m) > 0.

For $\forall$ (k,m) $\in \mathbb{Z}$, in a finite number of iterations, k(n) = 1-m.

Properties

1 - All sequences will be eventually periodic, of period 2, (p(1)=2-m, p(2)=1-m).

2 - Sequences with the same value of (k+m) will have same number of elements and same distance between them, which will be equal to the distance between the values of m.

$$k(n)-k_1(n)=m-m_1 \iff k+m=k_1+m_1$$

Examples :	k(37)+m(28) = 65	37, 70, 21, 46, 9, 28, 0, -14, -21, -17, -11, -2, -15, -8, -18, -23, -20, -24, -26, -27.
	k(243)+m(-178) = 65	243, 276, 227, 252, 215, 234, 206, 192, 185, 189, 195, 204, 191, 198, 188, 183, 186, 182, 180, 179.
	k(65)+m(0) = 65	65, 98, 49, 74, 37, 56, 28, 14, 7, 11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1.

There are 20 elements in each sequence.

3 - In all sequences, difference between first element k and last one k(n), is k(n)=k+m-1.

$$k-k(n)=k+m-1$$

Matrices M(n)

With all possible values of k and m, we form a matrix with two rows and infinite columns. In the first row, the integers ordered written, with the positive numbers at the right of zero, representing the possible values of k.
In the second row, the integers ordered written, with the positive numbers at the left of zero, representing the values of m.

A part or section of the matrix with the values from -5 to 7 for k and from 6 to -6 for m:

k	. . . -5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	. . .
m	. . . 6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	. . .

Matrix M(1), in which k+m = 1 in each column.

A part or section of the matrix with the values from 10 to 22 for k and from 6 to -6 for m:

k	. . .	10	11	12	13	14	15	16	17	18	19	20	21	22	. . .
m	. . .	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	. . .

Matrix M(16), because in each column k+m = 16.

The elements of this matrix are the same, but in the matrix M(16) the first row has moved, until k(16) coincides with m(0), to visualize that in all columns k+m=16.

Sets C(n)

All the sequences generated with the values of k and m of each column of the matrix M(n), they have the same number of elements and there is the same distance between them. We call the set of these sequences C(n), where n=k+m.

Example:

With the values of the columns of the matrix M(16), the function will generate infinite sequences that will form set C(16).

$$
C(16) \left\{ \begin{array}{lrrrr}
\ldots & & & & \\
(10, & 2, & -2, & -4, & -5); \\
(11, & 3, & -1, & -3, & -4); \\
(12, & 4, & 0, & -2, & -3); \\
(13, & 5, & 1, & -1 & -2); \\
(14, & 6, & 2, & 0, & -1); \\
(15, & 7, & 3, & 1, & 0); \\
(16, & 8, & 4, & 2, & 1); \\
(17, & 9, & 5, & 3, & 2); \\
(18, & 10, & 6, & 4, & 3); \\
(19, & 11, & 7, & 5, & 4); \\
(20, & 12, & 8, & 6, & 5); \\
(21, & 13, & 9, & 7, & 6); \\
(22, & 14, & 10, & 8, & 7); \\
\ldots & & & &
\end{array} \right\}
$$

There are infinite possible results for (k+m), which will form infinite sets C(n), with the same properties.

Examples

If we want to form a sequence that reach 45, we will assign to m the value of -44 and will apply the following function, iteratively, until reach k(n)=1-m:

$$
f(k,m) = \begin{cases} (k+44)/2, & \text{if } (k,m) \text{ have the same parity.} \\ (3k-43)/2, & \text{if } (k,m) \text{ have opposite parity.} \end{cases}
$$

Because Dom = (k+m) > 0 $\rightarrow$ k ≥ 45.

Sequence started with k = 74, m = -44:

74, 59, 67, 79, 97, 124, 84, 64, 54, 49, 52, 48, 46, 45, 46, 45, . . .

Sequence started with k = 12795, m = -44:

12795, 19171, 28735, 43081, 64600, 32322, 16183, 24253, 36358, 18201, 27280, 13662, 6853, 10258, 5151, 7705, 11536, 5790, 2917, 4354, 2199, 3277, 4894, 2469, 3682, 1863, 2773, 4138, 2091, 3115, 4651, 6955, 10411, 15595, 23371, 35035, 52531, 78775, 118141, 177190, 88617, 132904, 66474, 33259, 49867, 74779, 112147, 168199, 252277, 378394, 189219, 283807, 425689, 638512, 319278, 159661, 239470, 119757, 179614, 89829, 134722, 67383, 101053, 151558, 75801, 113680, 56862, 28453, 42658, 21351, 32005, 47986, 24015, 36001, 53980, 27012, 13528, 6786, 3415, 5101, 7630, 3837, 5734, 2889, 4312, 2178, 1111, 1645, 2446, 1245, 1846, 945, 1396, 720, 382, 213, 298, 171, 235, 331, 475, 691, 1015, 1501, 2230, 1137, 1684, 864, 454, 249, 352, 198, 121, 160, 102, 73, 88, 66, 55, 61, 70, 57, 64, 54, 49, 52, 48, 46, 45, 46, 45, . . .

For every integer k ≥ 45, the iteration under this transformation will end in 46, 45.

If we want the sequence that reach - 100, we will assign to m the value of 101 and the iteration under this transformation, for every integer k ≥ -100, will end in -99, - 100.

$$f(k,m) = \begin{cases} (k-101)/2, & \text{if } (k,m) \text{ have the same parity.} \\ (3k+102)/2, & \text{if } (k,m) \text{ have opposite parity.} \end{cases}$$

Because Dom=(k+m) > 0 $\rightarrow$ k ≥ -100.

Sequence started with k = 21, m = 101:

21, -40, -9, -55, -78, -66, -48, -21, -61, -81, -91, -96, -93, -97, -99, -100, -99, -100, . . .

Sequence started with k = 0, m = 101:

0, 51, -25, -63, -82, -72, -57, -79, -90, -84, -75, -88, -81, -91, -96, -93, -97, -99, -100, -99, -100, . . .

Conclusion

Any integer k ∈ ℤ of the domain, subjected to the transformation of the function of iterated way, it will always reach k(n) = 1-m.

With this function we can determine the integer that each sequence will reach, then of a finite number of iterations, depending on the value that we assign to m ∈ ℤ, of the domain.

Collatz conjecture will hold for every value of k ∈ ℤ, because in all sets C(n) exist a sequence, generated with the value of m=0, that will reach k(n)=1-m, that is, 1.

Online calculator of the function, sequence generator: www.riodena.es

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