Functional Proofs of Goldbach Conjecture

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Abstract

Goldbach's Conjecture(GC) states that any even integer ≥ 4 can be represented by the sum of two prime numbers. This was conjectured by Christian Goldbach in 1742 and still remains unproved. In this thesis we proved GC by introducing, we called them, Goldbach Partition Model Table(GPMT) and Sieve Functions(SFs). GPMT is a 2-dimensional table of all possible pair of two numbers (x, $2n - x$), whose sum can be any even number $2n$. To functionally treat the sieve of Eratosthenes, we devised SFs that have sinusoidal symmetry and period properties. By using GPMT and the SFs, we could induce GC False Conditions(GCFC) that must be satisfied if GC is false. And we proved that GCFC can not be satisfied, so, GC is true.

1. Introduction

GC [1][2] states that any even number ≥ 4 is the sum of two prime numbers, like 22 = 3 + 19 = 5 + 17 = 11 + 11, 38 = 7 + 31 = 19 + 19. The conjecture has been shown to hold for all integers less than 4×10^{18} [3], but remains unproved despite various efforts [4][5][6].

Before we go further, let's define a basic terminology of GC.

Definition 1.1. Goldbach Partition(GP): A pair of two prime numbers (p, q) that satisfies $2n = p + q$, $n = 2, 3, 4, \ldots$.

In this thesis, we used Goldbach Partition Model Table(GPMT) and Sieve Functions(SFs). GPMT is a 2-dimensional arrangement of all possible GPs for a specific even number $2n$, and SFs are functional representation of the sieve of Eratosthenes. By using GPMT and SFs, we could visually understand the symmetry and period properties of GC, from which we derived GC False Conditions(GCFC) that must be satisfied if GC is false. And we proved that GCFC can not be satisfied, so, GC is true.

2. Goldbach Partition Model Table(GPMT)

Lemma 2.1. Possible GPs for $2n \geq 4$ have the form (x, $2n - x$), $x = 1, 2, 3, \ldots$.

Proof. GP is the sum of two primes $p + q = 2n$, so, $q = 2n - p$, i.e., (p, q) = (p, $2n - p$). So, possible GPs have the form(x, $2n - x$). ■

Definition 2.2. Goldbach Partition Model Table(GPMT): A table with all possible GPs of the form (x, $2n - x$), as in Table 1, and has the following properties.

Table 1 shows an example GPMT for n = 25, $2n$ = 50. Numbers 1, 2, …, 49 are arranged downward and upward. Apparently, $x + (2n - x) = 2n$, so, all possible GPs reside in the table. Numbers marked as red are prime numbers.

Table 1. Example GPMT for n = 25.

x	2n - x	2n = x + (2n – x)
1	49	
2	48	
3	47	
4	46	
5	45	
6	44	
7	43	
8	42	
9	41	
10	40	
11	39	
12	38	
13	37	50
14	36	
15	35	
16	34	
17	33	
18	32	
19	31	
20	30	
21	29	
22	28	
23	27	
24	26	
25	25	

Definition 2.3. Sieve set: Set of prime numbers in $2 \le p \le \sqrt{2n}$, required for the sieve of Eratosthenes, and is denoted by $S = \{p_1, p_2, \ldots, p_k\}$, $p_1 = 2$.

3. Definitions

3.1 Functions

The sieve of Eratosthenes is a traditional method for finding all prime numbers up to any given number. It does so by iteratively or periodically, removing the multiples of each seed prime in a sieve set, as shown in Figure 1.

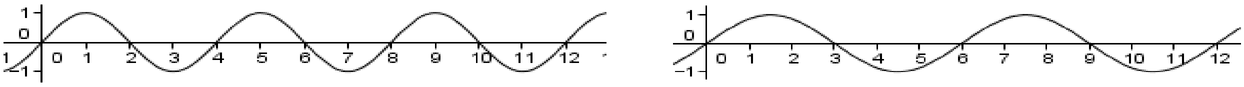
	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~	11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~	31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	39	~~40~~

Figure 1. Example Eratosthenes sieve.

In Figure 1, the multiples of 2, 3 and 5 is crossed by lines. The number of crosses means how many times a number has been the multiples of prime numbers 2, 3, 5, respectively.

To functionally represent the sieve of Eratosthenes, we introduce SFs and related functions.

Definition 3.1.1. Sieve Function(SF): A sine function, $f_i(x) = sin(\frac{\pi x}{p_i})$ where p_i is i'th prime number, as shown in Figure 2.



(a) $f_1(x) = \sin\left(\frac{\pi x}{2}\right), p_1 = 2.$ **(b)** $f_2(x) = \sin\left(\frac{\pi x}{3}\right), p_2 = 3.$

Figure 2. Example SFs.

Figure 2 (a) is SF for $p_1 = 2$ and (b) is for $p_2 = 3$. When $x = tp_i$, $t = 0, 1, 2, \ldots$, $f_i(x) = sin\left(\frac{\pi t p_i}{p_i}\right) = sin(\pi t) = 0$. Considering zeros of a SF as the sieved numbers, it is exactly same as the sieve of Eratosthenes, except when $t = 1$.

Definition 3.1.2. phased Sieve Function(pSF): A sine function $f_i(d_i - x)$, $d_i = 2n \ \ mod \ \ p_i$.

Definition 3.1.3. dual SF(dSF): A product of SF and pSF, $h_i(x) = f_i(x)f_i(2n - x) = f_i(x)f_i(d_i - x)$, $d_i = 2n \ \ mod \ \ p_i$.

Note that $d_i = 0$ when $p_i|2n$. Figure 3 depicts, $h_2(x) = f_2(x)f_2(50 - x) = f_2(x)f_2(2 - x) = sin(\frac{\pi x}{3})sin(\frac{\pi(2-x)}{3})$ and $h_3(x) = f_3(x)f_3(50 - x) = f_3(x)f_3(0 - x) = -sin(\frac{\pi x}{5})sin(\frac{\pi x}{5})$. In case of $h_3(x)$, the zeros are same as the zeros of $f_3(x)$.

(a) $h_2(x) = f_2(x)f_2(2 - x), d_2 = 2.$ (b) $h_3(x) = f_3(x)f_3(0 - x), d_3 = 0.$

Figure 3. dSF examples.

In Figure 3, we can see that dSF is also a periodic sinusoidal function with period p_i. A dSF is bisymmetric at $x = n$, as in Figure 4.

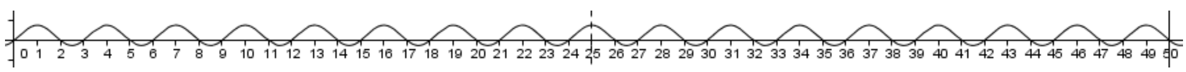


Figure 4. Example bisymmetry of a dSF, $h_2(x) = f_2(x)f_2(50 - x), n = 25$.

Definition 3.1.4. Composite Sieve Function(CSF): A product of SFs, $F_k(x) \ \ = \prod_{i=1}^{k} f_i(x)$, as in Figure 5.

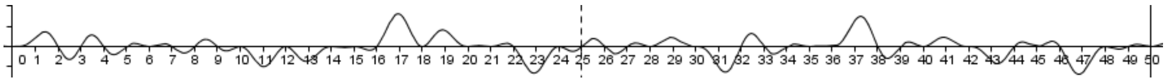


Figure 5. Example CSF, *n* = 25, *k* = 4, *S* = {2, 3, 5, 7}.

Definition 3.1.5. Composite phased Sieve Function(CpSF): A product of pSFs, $G_k(x) = \prod_{i=1}^{k} f_i(2n-x) = \prod_{i=1}^{k} f_i(d_i - x)$, where $d_i = 2n \mod p_i$.

Definition 3.1.6. Composite dual Sieve Function(CdSF): The product of dSFs, $H_k(x) = \prod_{i=1}^{k} h_i(x) = \prod_{i=1}^{k} f_i(x) f_i(2n-x) = F_k(x)F_k(2n-x)$.

We can see that CdSF is also bisymmetric at *x* = *n*, as in Figure 6.

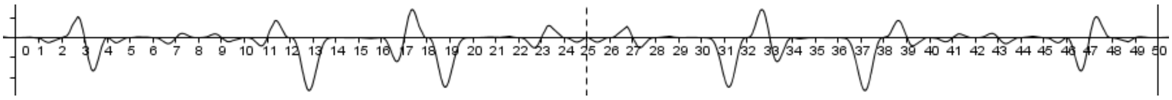
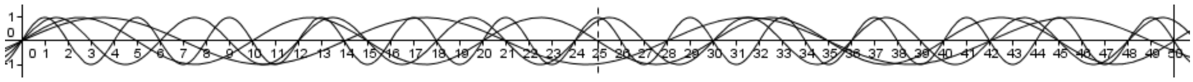


Figure 6. Example bisymmetry of a CdSF, $H_4(x) = \prod_{i=1}^{4} f_i(x) f_i(2n-x), n = 25$.

Definition 3.1.7. Forward Sieve Function Set(FSFS): A set, $L_{fk} = \{f_1(x), f_2(x), \dots, f_k(x)\}$, as in Figure 7.

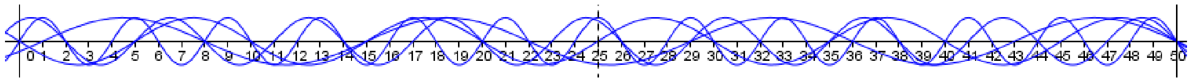


* $L_{fk} = \{f_1(x), f_2(x), f_3(x), f_4(x)\}$

Figure 7. Example FSFS, *n* = 25.

In Figure 7, $L_{fk} = \{sin\left(\frac{\pi x}{2}\right), sin\left(\frac{\pi x}{3}\right), sin\left(\frac{\pi x}{5}\right), sin(\frac{\pi x}{7})\}$, and the forward phase set is $D_{fk} = \{d_1, d_2, d_3, d_4\} = \{0, 0, 0, 0\}$.

Definition 3.1.8. Reverse Sieve Function Set(RSFS): A set, $L_{rk} = \{f_1(2n-x), f_2(2n-x), \dots, f_k(2n-x)\}$, as in Figure 8.

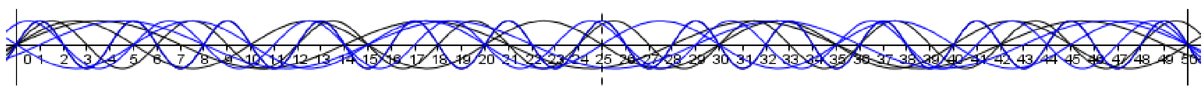


* $L_{rk} = \{f_1(2n-x), f_2(2n-x), f_3(2n-x), f_4(2n-x)\}$

Figure 8. Example RSFS, *n* = 25.

In Figure 8, $L_{rk} = \{sin\left(\frac{\pi(2n-x)}{2}\right), sin\left(\frac{\pi(2n-x)}{3}\right), sin\left(\frac{\pi(2n-x)}{5}\right), sin(\frac{\pi(2n-x)}{7})\}$, and the reverse phase set is $D_{rk} = \{d_1, d_2, d_3, d_4\} = \{0, 2, 0, 1\}$. Note that the reverse phase of $f_2(x)$ is always 0, because $d_1 = 2n \mod p_1 = 2n \mod 2 = 0$.

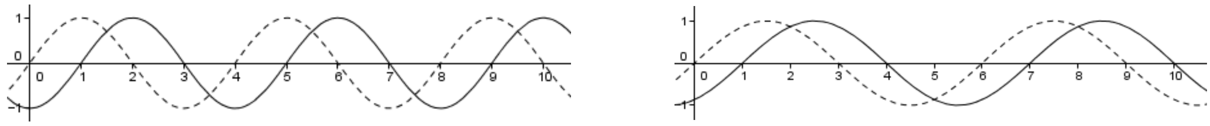
Definition 3.1.9. Total Sieve Function Set(TSFS): A set $L_{tk} = \{f_1(x), f_2(x), \dots, f_k(x), f_1(2n-x), f_2(2n-x), \dots, f_k(2n-x)\} = L_{fk} \cup L_{rk}$, as in Figure 9.



* $L_{tk} = L_{fk} \cup L_{rk}$

Figure 9. Example TSFS, *n* = 25.

Definition 3.1.10. Complementary SF: A SF whose zeros comprise all non-zeros of a SF.



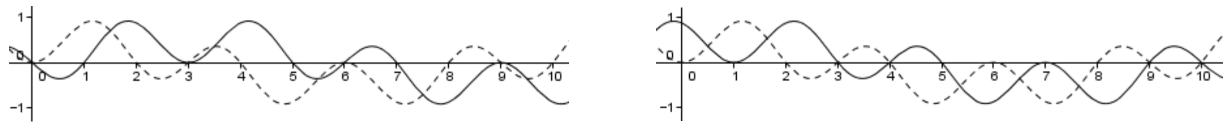
(a) Complementary SF of $f_1(x)$. **(b)** Complementary SF of $f_2(x)$ does not exist.

Figure 10. Graphs for complementary SF concept.

In Figure 10, $f_1(x-1)$ is a complementary SF of dashed graph $f_1(x) = sin\left(\frac{\pi x}{2}\right)$. But, $f_2(x-1)$ can not be a complementary SF of dashed graph $f_2(x)$, because zeros of $f_2(x-1)$ can not comprise all non-zeros of $f_2(x) = sin\left(\frac{\pi x}{3}\right)$.

Definition 3.1.11. Complementary CSF(CCSF): A CSF whose zeros comprise all non-zeros of a CSF.

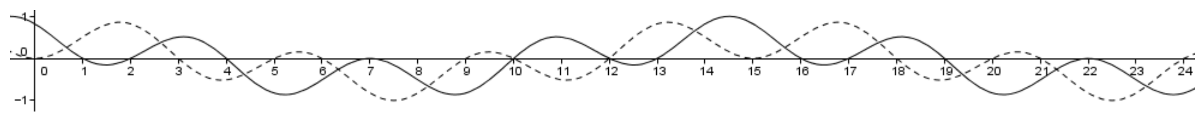
Figure 11 shows two CCSFs of $f_1(x)f_2(x)$, one is $f_1(x-1)f_2(x)$ and the other is $f_1(x-1)f_2(x-1)$. Dotted graphs are CSFs $f_1(x)f_2(x)$. We can see that the zeros of $f_1(x-1)f_2(x)$ and $f_1(x-1)f_2(x-1)$ comprise all non-zeros of $f_1(x)f_2(x)$, because of $f_1(x-1)$ is the complementary SF of $f_1(x)$.



(a) Dot CSF $f_1(x)f_2(x)$, CCSF $f_1(x-1)f_2(x)$. (b) Dot CSF $f_1(x)f_2(x)$, CCSF $f_1(x-1)f_2(x-1)$.

Figure 11. Example CCSFs.

Figure 12 shows a dotted CSF $f_2(x)f_3(x)$ and $f_2(x-1)f_3(x-2)$. We can see that zeros of $f_2(x-1)f_3(x-2)$ can not comprise all the odd non-zeros of $f_2(x)f_3(x)$, such as 11, 23. So, $f_2(x-1)f_3(x-2)$ can not be a CCSF of $f_2(x)f_3(x)$.



$*$ Dotted graph is CSF $f_2(x)f_3(x)$ and line graph is $f_2(x-1)f_3(x-2)$ which can't be a CCSF.

Figure 12. An example of CSF with no CCSF.

Definition 3.1.12. Zero function: Function $f_0(x) = sin(\pi x)$, whose zeros are all integers.

3.2 Properties of Functions

Definition 3.2.1. Orthogonality of numbers: Numbers are orthogonal to each other if they are mutually co-prime.

Definition 3.2.2. Orthogonality of sinusoidal functions: Sinusoidal functions are orthogonal to each other if their periods are mutually co-prime.

By definition 3.2.2, SFs, pSFs, dSFs, CSFs, CpSFs and CdSFs are orthogonal to each other if their periods are co-prime to each other.

Definition 3.2.3. Zero configuration: Zero distribution of FSFS L_{fk} = $\{f_1(x), f_2(x), \dots, f_k(x)\}$, in $0 \le x \le 2n$, where k is the k'th largest prime that is less than or equal to $\sqrt{2n}$.

Definition 3.2.4. Configuration range: The zero configuration range, $0 \le x \le 2n$.

Definition 3.2.5. Configuarion set: FSFS of a zero configuration. Configuration set is same as the SFs with sieve set $S = \{p_1, p_2, \dots, p_k\}$.

Definition 3.2.6. Configuration dimension: The number of SFs of a configuration set.

Definition 3.2.7. Phase set: A set of phases of pSFs, $D_k = \{d_1, d_2, \dots, d_k\}$, where $d_i = 2n \ mod \ p_i$.

4. Lemmas

[Lemmas on Periods and Phases]

Lemma 4.1. A SF, $f_i(x) = sin(\frac{\pi x}{p_i})$ is a periodic function with period p_i.

Proof. $f_i(x)$ is a sinusoidal function which zero repeats with interval p_i. So, $f_i(x)$ is a periodic function with period p_i. ■

Lemma 4.2. A phased Sieve Function(pSF), $f_i(d_i - x)$, $0 \le d_i < p_i$, is a periodic function with period p_i.

Proof. $f_i(x - d_i)$ is a sinusoidal function which zero repeats with interval p_i. So, $f_i(d_i - x)$ is a periodic function with period p_i. ■

Lemma 4.3. A dual SF(dSF), $h_i(x) = f_i(x)f_i(2n - x) = f_i(x)f_i(d_i - x)$, $d_i = 2n \ mod \ p_i$, is a periodic function with period p_i.

Proof. $h_i(x)$ is a product of two sinusoidal functions with same period p_i, so, $h_i(x)$ is a periodic function with period p_i. ■

Lemma 4.4. A CSF, $F_k(x) = \prod_{i=1}^{k} f_i(x)$, is a periodic function with period $Q_k = \prod_{i=1}^{k} p_i$.

Proof. $F_k(x)$ is the product of k periodic sine functions with period p_i, $1 \le i \le k$. So, $F_k(x)$ is a periodic function with period $Q_k = \prod_{i=1}^{k} p_i$. ■

Lemma 4.5. A Composite phased Sieve Function(CpSF), $G_k(x) = \prod_{i=1}^{k} f_i(d_i - x)$, where $d_i = 2n \ mod \ p_i$, is a periodic function with period $Q_k = \prod_{i=1}^{k} p_i$.

Proof. $G_k(x)$ is the product of k periodic sine functions with period p_i, $1 \le i \le k$. So, $G_k(x)$ is a periodic function with period $Q_k = \prod_{i=1}^{k} p_i$. ■

Lemma 4.6. A Composite dual Sieve Function(CdSF), $H_k(x) = \prod_{i=1}^{k} h_i(x)$, is a periodic function with period $Q_k = \prod_{i=1}^{k} p_i$.

Proof. $H_k(x)$ is the product of *k* periodic sine functions with period p_i, $1 \le i \le k$. So, $H_k(x)$ is a periodic function with period $Q_k = \prod_{i=1}^{k} p_i$. ■

Lemma 4.7. A CpSF $F_k(x)$ can have $Q_k = \prod_{i=1}^{k} p_i$ phases.

Proof. The period of $F_k(x)$ is Q_k, so, it can have $0 \sim (Q_k - 1)$ as its phases. ■

[Lemmas on Complementary Functions]

Lemma 4.8. Any SF $f_i(x)$ can not have a complementary SF except when $p_1 = 2$.

Proof. When $p_1 = 2$, $f_1(x)$ has a complementary SF $f_1(x-1)$. But, when $p_i \ge 3$, $f_i(x-d)$, $d = 1, 2, \ldots, p_i - 1$, can not be a complementary SF, because its zeros can not comprise all non-zeros of $f_i(x)$. ■

Lemma 4.9. Complementary SF or complementary CSF(CCSF), if any, must have the same period with SF or CSF.

Proof. SF and CSF are sinusoidal functions with finite period, so, they will repeat their non-zeros within the first period infinitely many times. To comprise all infinitely repeating non-zeros of SF or CSF, complementary SF or CCSF must have the same period with SF or CSF. ■

Lemma 4.10. The product of SF with complementary SF or the product of CSF with CCSF must comprise all integers as its zeros.

Proof. By definition, a complementary function comprises all non-zeros of SF or CSF as its zeros. So, there can not exist any non-zeros when two functions are multiplied. ■

Lemma 4.11. A dSF can not be a complementary dSF of other dSF.

Proof. A dSF has period p_i and other dSF has period $p_j \neq p_i$. So, by Lemma 4.9, a dSF can not be a complementary dSF of other dSF. ■

Lemma 4.12. A CdSF, $H_k(x)$, can not have another dSF as a complementary function.

Proof. The product of *k* dSFs is, $H_k(x) = \prod_{i=1}^{k} h_i = \prod_{i=1}^{k} f_i(x)\, f_i(2n - x)$. The period of $H_k(x)$ is $Q_k = \prod_{i=1}^{k} p_i$, which can not be same as the period of any other dSF. So, by Lemma 4.9, a CdSFs can not have another dSF as a complementary function. ■

[Lemmas on Orthogonality]

Lemma 4.13. Any orthogonal number can't be equal to any product of other orthogonal numbers.

Proof. If an orthogonal number is equal to any product of other orthogonal numbers, it means that that orthogonal number is not co-prime to other orthogonal numbers. So, it contradicts to Definition 3.2.1. ■

Lemma 4.14. Any orthogonal SF can't be equal to any product of other orthogonal SFs.

Proof. If an orthogonal SF is equal to any product of other orthogonal SFs, it means that the period of that orthogonal SF is not co-prime to other orthogonal SFs. So, it contradicts to Definition 3.2.2. ■

Lemma 4.15. Any product of orthogonal functions with prime periods can't be equal to the zero function, $f_0(x) = sin(\pi x)$.

Proof. Any product of orthogonal functions can't have the period 1, which is the period of zero function $f_0(x)$. So, Any product of orthogonal functions can't be equal to $f_0(x)$. ■

Lemma 4.16. Any product of orthogonal functions with prime periods can't sieve out all numbers.

Proof. Any product of orthogonal functions can't sieve out all numbers, because it can't be equal to the zero function $f_0(x)$. ■

[Lemmas on Zero Configuration]

Lemma 4.17. For a zero configuration there are only one configuration set with minimum configuration dimension.

Proof. A minimum configuration dimension is the minimum number of SFs to sieve all composite numbers in 2 ≤ x ≤ $\sqrt{2n}$. Adding a SF with seed $p_{k+1} > \sqrt{2n}$, does not affect the sieve result in 2 ≤ x ≤ $\sqrt{2n}$. So, for a zero configuration there are only one configuration set with minimum configuration dimension. ■

Appendix 1 shows the zero configuration of the window size 2n when n = 25. So, 2n = 50, k = 4, p_4 = 7, Q_4 = 2 * 3 * 5 * 7 = 210. Window ASP source program is also provided to generate sliding window tables.

5. GC False Conditions(GCFC)

Figure 13 depicts the positions of all prime numbers and their symmetry values in configuration range $0 \leq x \leq 2n$.

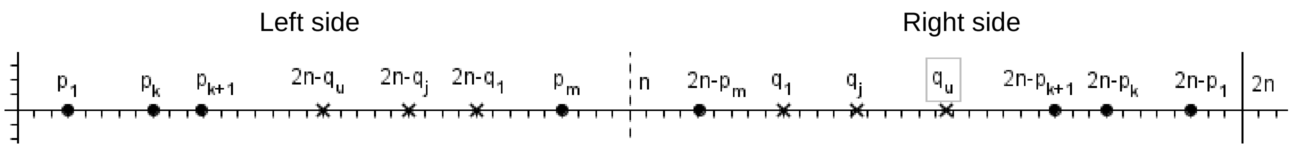


Figure 13. Symmetry property of GC.

In Figure 13, the sieve set is $S = \{p_1, p_2, \ldots, p_k\}$ and the maximum left side prime number is p_m. The symmetry position of left side prime number p_i is $2n - p_i$. Right side u prime numbers are denoted as q_j and their symmetry positions are denoted as $2n - q_j$. From the patterns of FSFS, RSFS and TSFS, we can derive the following GC false conditions.

5.1 GC False Conditions

Lemma 5.1.1. If GC is false, conditions on FSFS, RSFS and TSFS in Table 2 must be satisfied.

Table 2. GC false zero or non-zero conditions.

View	Condition ID	Left side conditions					Right side conditions				
		1	p_1~p_k	p_{k+1}~p_m	2n-q_j	compL	2n-1	2n-p_1~p_k	2n-p_{k+1}~p_m	q_j	compR
FSFS	C_f	×	○	×	○	○	dc	○	○	×	○
RSFS	C_r	dc	○	○	×	○	×	○	×	○	○
TSFS	C_t=C_f∪C_r	dc	○	○	○	○	dc	○	○	○	○
* ○: must be zero, ×: must not be zero, dc: don't care, compL/R: Left/Right side composite numbers											

Proof. The rationales of each conditions in Table 2 are as follows.

Table 3. Rationales of GC false zero or non-zero conditions.

View	Condition ID	Left side conditions			Right side conditions		
		Conditions		Rationale	Conditions		Rationale
FSFS	C_f	1	×	no $f_{p_i}(x)$ can pass 1	2n-1	dc	$f_{p_i}(x)$ can pass 2n-1
		p_1~p_k	○	$f_{p_i}(x)$, $1 \le i \le k$ pass p_1~p_k.	2n-p_1~p_k	○	if 2n-p_i is prime GC is true
		p_{k+1}~p_m	×	p_{k+1}~p_m are prime numbers	2n-p_{k+1}~p_m	○	if 2n-p_i is prime GC is true
		2n-q_j	○	if 2n-q_j is prime GC is true	q_j	×	q_j is a prime number
		compL	○	left side composite numbers	compR	○	right side composite numbers
RSFS	C_r	1	dc	symmetry point of 2n-1	2n-1	×	symmetry point of 1
		p_1~p_k	○	symmetry point of 2n-p_1~p_k	2n-p_1~p_k	○	symmetry point of p_1~p_k
		p_{k+1}~p_m	○	symmetry point of 2n-p_{k+1}~p_m	2n-p_{k+1}~p_m	×	symmetry point of p_{k+1}~p_m
		2n-q_j	×	symmetry point of q_j	q_j	○	symmetry point of 2n-q_j
		compL	○	symmetry point of compR	compR	○	symmetry point of compL
TSFS	C_t=C_f∪C_r	1	dc	zeros of FSFS and RSFS	2n-1	dc	zeros of FSFS and RSFS
		p_1~p_k	○		2n-p_1~p_k	○	
		p_{k+1}~p_m	○		2n-p_{k+1}~p_m	○	
		2n-q_j	○		q_j	○	
		compL	○		compR	○	

Table 2 is just a summary of the rationales of Table 3. So, if GC is false, conditions on FSFS, RSFS and TSFS in Table 2 must be satisfied. ■

The condition $C_t = C_f \cup C_r$ states that all numbers between 1 and $2n$, except 1, must be zeros of TSFS. If this can not be satisfied, GC is true.

5.2 GCFC in View of Values of CSFs

Let's define two CSFs.

$$F_k(x) = \prod_{i=1}^{k} f_i(x) \tag{1}$$

$$F_m(x) = \prod_{i=1}^{m} f_i(x) \tag{2}$$

$F_k(x)$ is the product of SFs for the sieve set $S = \{p_1, p_2, \ldots, p_k\}$ and $F_m(x)$ is the product of SFs for all prime numbers less than *n*, $\{p_1, p_2, \ldots, p_k, \ldots, p_m\}$. Then, to make GC false the following functional value conditions must be satisfied.

Table 4. Functional value representation of GCFC.

$F_k(p_i) = 0, 1 \le i \le k$	(3-1)
$F_k(p_i) \ne 0, k+1 \le i \le m$	(3-2)
$F_m(p_i) = 0, 1 \le i \le m$	(3-3)
$F_k(2n - p_i) = 0, 1 \le i \le m$	(4-1)
$F_m(2n - p_i) = 0, 1 \le i \le m$	(4-2)
$F_k(q_j) \ne 0, 1 \le j \le u$	(5-1)
$F_m(q_j) \ne 0, 1 \le j \le u$	(5-2)
$F_k(2n - q_i) = 0, 1 \le j \le u$	(6-1)
$F_m(2n - q_i) = 0, 1 \le j \le u$	(6-2)

If all the above conditions can be satisfied GC is false, if not GC is true.

5.3 GCFC in GPMT View

The symmetricity of GCFC can be represented via GPMT without even numbers, as in Table 5 and 6. Table 5 is when *n* is even and Tabel 6 is when *n* is odd.

Table 5. Prime vs non-prime symmetry for GCFC, *n* = even.

X_{odd} $3 \le x \le n$	A X_{odd} condition	B Y_{odd} condition	Y_{odd} $n \le x \le 2n$	AB	$\lvert X_{odd} - Y_{odd} \rvert$	Remarks
-	n=even	n=even	-	○	-	• A, B: prime or non-prime condition • AB: logical product of A and B • $\lvert X_{odd} – Y_{odd} \rvert$: absolute gap between pair • GC false conditions - all primes must not symmetrically overlap.
n-1	●	○	n+1	○	2	
n-3	○	●	n+3	○	6	
n-5	●	○	n+5	○	10	
n-7	○	●	n+7	○	14	
n-9	●	○	n+9	○	18	
n-11	○	○	n+11	○	22	
n-13	○	●	n+13	○	26	
…	….	…	…	○	…	

7	●	○	2n-5	○	2n-12	- there must be no primes with distance 2, 6, 10, … bisymmetric at n
5	●	○	2n-3	○	2n-8	
3	●	○	2n-1	○	2n-4	
* legend: X_{odd}/Y_{odd}: odd numbers in $3 \leq x \leq n$/ $n \leq x \leq 2n$, ●: prime(true), ○: non-prime(false)						

Table 6. Prime vs non-prime symmetry for GCFC, *n* = odd.

X_{odd} $3 \leq x \leq n$	A X_{odd} condition	B Y_{odd} condition	Y_{odd} $n \leq x \leq 2n$	AB	$\lvert X_{odd} - Y_{odd}\rvert$	Remarks
n	○	○	n	○	0	• A, B: prime or non-prime condition • AB: logical product of A and B • $\lvert X_{odd} - Y_{odd}\rvert$: absolute gap between pair • GC false conditions - all primes must not symmetrically overlap. - there must be no primes with distance 0, 4, 8, … bisymmetric at n
n-2	●	○	n+2	○	4	
n-4	○	●	n+4	○	8	
n-6	●	○	n+6	○	12	
n-8	○	●	n+8	○	16	
n-10	●	○	n+10	○	20	
n-12	○	○	n+12	○	24	
n-14	○	●	n+14	○	28	
…	….	…	…	○	…	
7	●	○	2n-5	○	2n-12	
5	●	○	2n-3	○	2n-8	
3	●	○	2n-1	○	2n-4	
* legend: X_{odd}/Y_{odd}: odd numbers in $3 \leq x \leq n$/ $n \leq x \leq 2n$, ●: prime(true), ○: non-prime(false)						

6. ProofS OF GC

6.1 Proof by Orthogonality Property

Lemma 6.1.1. The product of all SFs in any TSFS(Definition 3.1.9) can't be equivalent to the zero function $f_0(x) = sin(\pi x)$ (Definition 3.1.12). So, the zeros of TSFS can't make all numbers in $1 \leq x \leq 2n$ as its zeros, i.e., GCFC can't be satisfied.

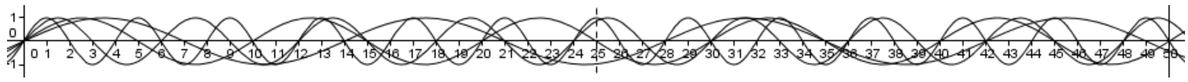
Proof. As in Lemma 5.1.1, to satisfy GCFC, all numbers in $1 \leq x \leq 2n$, must be the zeros of TSFS, which is equivalent to the zero function $f_0(x) = sin(\pi x)$. But, Lemma 4.15 states that any product of orthogonal functions with prime periods can't be equal to the zero function $f_0(x)$. So, the CdSF(Definition 3.1.6) $H_k(x) = \prod_{i=1}^{k} h_i(x) = \prod_{i=1}^{k} f_i(x)\, f_i(2n - x)$, which is the product of all SFs in TSFS, can't be same as $f_0(x)$, because $h_i(x)$ is an orthogonal function with a prime period. ∎

6.2 Proof by Functional Equivalence Contradiction

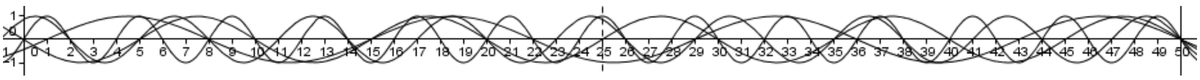
Figure 14 (a), (b) depicts SFs of FSFS(Definition 3.1.7) and RSFS(Definition 3.1.8) for $n = 25$. The configuration range(Definition 3.2.4) is $0 \leq x \leq 50$, so, the sieve set(Definition

2.3) is S = {2, 3, 5, 7} and $F_k(x) = F_4(x) = sin(\frac{\pi x}{2})sin(\frac{\pi x}{3})sin(\frac{\pi x}{5})sin(\frac{\pi x}{7}), F_k(2n-x) = F_4(2n-x) = sin(\frac{-\pi x}{2})sin(\frac{\pi(2-x)}{3})sin(\frac{-\pi x}{5})sin(\frac{\pi(1-x)}{7})$.

Figure 14 (c), (d) depicts SFs of FSFS and RSFS for 9 primes in $0 \le x \le 25$, which are {2, 3, 5, 7, 11, 13, 17, 19, 23}. So, $F_m(x) = F_9(x) = sin\left(\frac{\pi x}{2}\right) ... sin\left(\frac{\pi x}{23}\right), F_m(2n-x) = F_9(2n-x) = sin\left(\frac{-\pi x}{2}\right) ... sin(\frac{\pi(4-x)}{23})$.

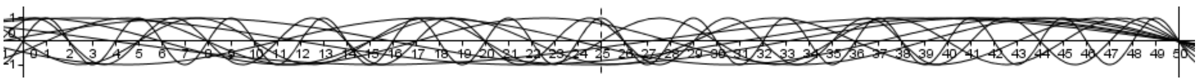


(a) $FSFS$: $f_1(x) \sim f_4(x)$.



(b) $RSFS$: $f_1(2n-x) \sim f_4(2n-x)$.

(c) $FSFS$: $f_1(x) \sim f_9(x)$.



(d) $RSFS$: $f_1(2n-x) \sim f_9(2n-x)$.

Figure 14. Example graphs of FSFS and RSFS.

Here, we copied the following four conditions from Table 4 for convenience sake.

$$F_k(2n - p_i) = 0, 1 \le i \le m \quad (4\text{-}1)$$

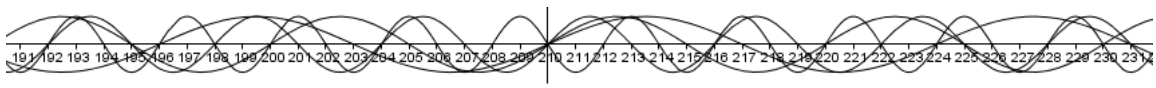
$$F_m(2n - p_i) = 0, 1 \le i \le m \quad (4\text{-}2)$$

$$F_k(2n - q_i) = 0, 1 \le j \le u \quad (6\text{-}1)$$

$$F_m(2n - q_i) = 0, 1 \le j \le u \quad (6\text{-}2)$$

In Figure 12 (a), all $(2n - p_i), 1 \le i \le m$ lies between $n \sim 2n$. To satisfy (4-1), (4-2), (6-1) and (6-2) for *all* prime numbers between $0 \sim 2n$, $F_k(x)$ must functionally equivalent to $F_m(x)$ in $n \le x \le 2n$.

But, the graphs of Figure 14 (b) can not occur at 2n = 50. The node point, where all k graphs of $f_i(x)$ pass, only occurs at $x = Q_k = \prod_{i=1}^{k} p_i$, as in Figure 15.



$* f_1(x) \sim f_4(x), Q_4 = \prod_{i=1}^{4} p_i = 2 * 3 * 5 * 7 = 210.$

Figure 15. Periodic occurrences of node point of SFs.

Also, the graphs of Figure 14 (d) can not occur at 2*n* = 50. The node point, where all *m* graphs of $f_i(x)$ pass, only occurs at $x = Q_m = \prod_{i=1}^{m} p_i$.

Lemma 6.2.1. The functional value representation (4-1) and (4-2) causes functional symmetry contradiction. So, GCFC can't be satisfied.

Proof. Conditions (4-1) and (4-2) cause functional symmetry contradictions, because they are obviously the functional symmetries of (1) and (2). But, those functional symmetries can't occur at 2*n*, they occur only at $x = Q_m = \prod_{i=1}^{m} p_i$ and $x = Q_m = \prod_{i=1}^{m} p_i$, respectively. So, (4-1) and (4-2) can't be satisfied for ***all*** p_i, they can only be satisfied for some p_i. ■

In algebraic view, (4-1) and (6-1) mean that there are p_j and q_j that satisfy the following equations for ***all*** prime numbers in $0 \leq x \leq 2n$.

$$p_j \mid 2n - p_i, 1 \leq j \leq k,\ 1 \leq i \leq m \tag{7}$$

$$p_j \mid 2n - q_i, 1 \leq j \leq k,\ 1 \leq i \leq u \tag{8}$$

To make GCFC false, both the algebraic and functional conditions must be satisfied. It is hard to algebraically prove whether there exists such *n* that satisfies (7) and (8). But, it is obvious that (4-1) and (4-2) causes functional contradiction.

6.3 Proof by Complementary Function Concept

Lemma 6.3.1. Any CdSF(Definition 3.1.6) $H_{k-1}(x)$ can not have dSF(Definition 3.1.3) $h_k(x)$, as its complementary function. So, GCFC can't be satisfied.

Proof. Lemma 4.9 states that a complementary SF or complementary CSF(CCSF) must have the same period with SF or CSF. So, any CdSF $H_{k-1}(x)$ can not have a dSF $h_k(x)$, as its complementary function. That is to say, $H_k(x) = h_k(x)H_{k-1}(x)$ can not comprise all integers in configuration range $0 \leq x \leq 2n$ as its zeros. So, GCFC can not be satisfied. ■

To help understand we added an example case at Appendix B.

7. Conclusion

In this thesis, we devised GPMT and SFs to functionally understand the various symmetry and period properties of GC. They have the properties of sinusoidal functions, from which we could derive the GC false conditions(GCFC) of Lemma 5.1.1. We proved that GCFC can not be satisfied in three points of view, orthogonality property view, functional symmetry contradiction view and complementary function concept view. So, GC is true.

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Appendix A. Sliding Window Tables and ASP Program Source.

A.1. Zero Configuration of Sliding Window 2n, *n* = 25.

From	To	Phase	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49
0	50	0 0 0		3	5	7		11	13		17	19		23			29	31			37		41	43		47	
160	210	1 0 6		163		167			173			179	181					191	193		197	199					
0	50	0 0 0		3	5	7		11	13		17	19		23			29	31			37		41	43		47	
2	52	2 2 2	3	5	7		11	13		17	19		23			29	31			37		41	43		47		
4	54	1 4 4	5	7		11	13		17	19		23			29	31			37		41	43		47			53
6	56	0 1 6	7		11	13		17	19		23			29	31			37		41	43		47			53	
8	58	2 3 1		11	13		17	19		23			29	31			37		41	43		47			53		
10	60	1 0 3	11	13		17	19		23			29	31			37		41	43		47			53			59
12	62	0 2 5	13		17	19		23			29	31			37		41	43		47			53			59	61
14	64	2 4 0		17	19		23			29	31			37		41	43		47			53			59	61	
16	66	1 1 2	17	19		23			29	31			37		41	43		47			53			59	61		
18	68	0 3 4	19		23			29	31			37		41	43		47			53			59	61			67
20	70	2 0 6		23			29	31			37		41	43		47			53			59	61			67	
22	72	1 2 1	23			29	31			37		41	43		47			53			59	61			67		71
24	74	0 4 3			29	31			37		41	43		47			53			59	61			67		71	73
26	76	2 1 5		29	31			37		41	43		47			53			59	61			67		71	73	
28	78	1 3 0	29	31			37		41	43		47			53			59	61			67		71	73		
30	80	0 0 2	31			37		41	43		47			53			59	61			67		71	73			79
32	82	2 2 4			37		41	43		47			53			59	61			67		71	73			79	
34	84	1 4 6		37		41	43		47			53			59	61			67		71	73			79		83
36	86	0 1 1	37		41	43		47			53			59	61			67		71	73			79		83	
38	88	2 3 3		41	43		47			53			59	61			67		71	73			79		83		
40	90	1 0 5	41	43		47			53			59	61			67		71	73			79		83			89
42	92	0 2 0	43		47			53			59	61			67		71	73			79		83			89	
44	94	2 4 2		47			53			59	61			67		71	73			79		83			89		
46	96	1 1 4	47			53			59	61			67		71	73			79		83			89			
48	98	0 3 6			53			59	61			67		71	73			79		83			89				97
50	100	2 0 1		53			59	61			67		71	73			79		83			89				97	
52	102	1 2 3	53			59	61			67		71	73			79		83			89				97		101
54	104	0 4 5			59	61			67		71	73			79		83			89				97		101	103
56	106	2 1 0		59	61			67		71	73			79		83			89				97		101	103	
58	108	1 3 2	59	61			67		71	73			79		83			89				97		101	103		107
60	110	0 0 4	61			67		71	73			79		83			89				97		101	103		107	109
62	112	2 2 6			67		71	73			79		83			89				97		101	103		107	109	
64	114	1 4 1		67		71	73			79		83			89				97		101	103		107	109		113
66	116	0 1 3	67		71	73			79		83			89				97		101	103		107	109		113	
68	118	2 3 5		71	73			79		83			89				97		101	103		107	109		113		
70	120	1 0 0	71	73			79		83			89				97		101	103		107	109		113			
72	122	0 2 2	73			79		83			89				97		101	103		107	109		113				
74	124	2 4 4			79		83			89				97		101	103		107	109		113					
76	126	1 1 6		79		83			89				97		101	103		107	109		113						
78	128	0 3 1	79		83			89				97		101	103		107	109		113							127
80	130	2 0 3		83			89				97		101	103		107	109		113							127	
82	132	1 2 5	83			89				97		101	103		107	109		113							127		131
84	134	0 4 0			89				97		101	103		107	109		113							127		131	
86	136	2 1 2		89				97		101	103		107	109		113							127		131		
88	138	1 3 4	89				97		101	103		107	109		113							127		131			137

From	To	Phase			1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49
90	140	0	0	6				97		101	103		107	109		113							127		131			137	139
92	142	2	2	1			97		101	103		107	109		113							127		131			137	139	
94	144	1	4	3		97		101	103		107	109		113							127		131			137	139		
96	146	0	1	5	97		101	103		107	109		113							127		131			137	139			
98	148	2	3	0		101	103		107	109		113							127		131			137	139				
100	150	1	0	2	101	103		107	109		113							127		131			137	139					149
102	152	0	2	4	103		107	109		113							127		131			137	139					149	151
104	154	2	4	6		107	109		113							127		131			137	139					149	151	
106	156	1	1	1	107	109		113							127		131			137	139					149	151		
108	158	0	3	3	109		113							127		131			137	139					149	151			157
110	160	2	0	5		113							127		131			137	139					149	151			157	
112	162	1	2	0	113							127		131			137	139					149	151			157		
114	164	0	4	2							127		131			137	139					149	151			157			163
116	166	2	1	4						127		131			137	139					149	151			157			163	
118	168	1	3	6					127		131			137	139					149	151			157			163		167
120	170	0	0	1				127		131			137	139					149	151			157			163		167	
122	172	2	2	3			127		131			137	139					149	151			157			163		167		
124	174	1	4	5		127		131			137	139					149	151			157			163		167			173
126	176	0	1	0	127		131			137	139					149	151			157			163		167			173	
128	178	2	3	2		131			137	139					149	151			157			163		167			173		
130	180	1	0	4	131			137	139					149	151			157			163		167			173			179
132	182	0	2	6			137	139					149	151			157			163		167			173			179	181
134	184	2	4	1		137	139					149	151			157			163		167			173			179	181	
136	186	1	1	3	137	139					149	151			157			163		167			173			179	181		
138	188	0	3	5	139					149	151			157			163		167			173			179	181			
140	190	2	0	0					149	151			157			163		167			173			179	181				
142	192	1	2	2				149	151			157			163		167			173			179	181					191
144	194	0	4	4			149	151			157			163		167			173			179	181					191	193
146	196	2	1	6		149	151			157			163		167			173			179	181					191	193	
148	198	1	3	1	149	151			157			163		167			173			179	181					191	193		197
150	200	0	0	3	151			157			163		167			173			179	181					191	193		197	199
152	202	2	2	5			157			163		167			173			179	181					191	193		197	199	
154	204	1	4	0		157			163		167			173			179	181					191	193		197	199		
156	206	0	1	2	157			163		167			173			179	181					191	193		197	199			
158	208	2	3	4			163		167			173			179	181					191	193		197	199				
160	210	1	0	6		163		167			173			179	181					191	193		197	199					

A.2. Geometric Zero Configuration of A.1.

From	To	Phase			1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49
0	50	0	0	0		●	●	●		●	●		●	●		●			●	●			●		●	●		●	
160	210	1	0	6		●		●			●			●	●					●	●		●	●					
0	50	0	0	0		●	●	●		●	●		●	●		●			●	●			●		●	●		●	
2	52	2	2	2	●	●	●		●	●		●	●		●			●	●			●		●	●		●		
4	54	1	4	4	●	●		●	●		●	●		●			●	●			●		●	●		●			●
6	56	0	1	6	●		●	●		●	●		●			●	●			●		●	●		●			●	
8	58	2	3	1		●	●		●	●		●			●	●			●		●	●		●			●		
10	60	1	0	3	●	●		●	●		●			●	●			●		●	●		●			●			●
12	62	0	2	5	●		●	●		●			●	●			●		●	●		●			●			●	●
14	64	2	4	0		●	●		●			●	●			●		●	●		●			●			●	●	

From	To	Phase			1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49
16	66	1	1	2	●	●		●			●	●			●		●	●		●			●			●	●		
18	68	0	3	4	●		●			●	●			●		●	●		●			●			●	●			●
20	70	2	0	6		●			●	●			●		●	●		●			●			●	●			●	
22	72	1	2	1	●			●	●			●		●	●		●			●			●	●			●		●
24	74	0	4	3			●	●			●		●	●		●			●			●	●			●		●	●
26	76	2	1	5		●	●			●		●	●		●			●			●	●			●		●	●	
28	78	1	3	0	●	●			●		●	●		●			●			●	●			●		●	●		
30	80	0	0	2	●			●		●	●		●			●			●	●			●		●	●			●
32	82	2	2	4			●		●	●		●			●			●	●			●		●	●			●	
34	84	1	4	6		●		●	●		●			●			●	●			●		●	●			●		●
36	86	0	1	1	●		●	●		●			●			●	●			●		●	●			●		●	
38	88	2	3	3		●	●		●			●			●	●			●		●	●			●		●		
40	90	1	0	5	●	●		●			●			●	●			●		●	●			●		●			●
42	92	0	2	0	●		●			●			●	●			●		●	●			●		●			●	
44	94	2	4	2		●			●			●	●			●		●	●			●		●			●		
46	96	1	1	4	●			●			●	●			●		●	●			●		●			●			
48	98	0	3	6			●			●	●			●		●	●			●		●			●				●
50	100	2	0	1		●			●	●			●		●	●			●		●			●				●	
52	102	1	2	3	●			●	●			●		●	●			●		●			●				●		●
54	104	0	4	5			●	●			●		●	●			●		●			●				●		●	●
56	106	2	1	0		●	●			●		●	●			●		●			●				●		●	●	
58	108	1	3	2	●	●			●		●	●			●		●			●				●		●	●		●
60	110	0	0	4	●			●		●	●			●		●			●				●		●	●		●	●
62	112	2	2	6			●		●	●			●		●			●				●		●	●		●	●	
64	114	1	4	1		●		●	●			●		●			●				●		●	●		●	●		●
66	116	0	1	3	●		●	●			●		●			●				●		●	●		●	●		●	
68	118	2	3	5		●	●			●		●			●				●		●	●		●	●		●		
70	120	1	0	0	●	●			●		●			●				●		●	●		●	●		●			
72	122	0	2	2	●			●		●			●				●		●	●		●	●		●				
74	124	2	4	4			●		●			●				●		●	●		●	●		●					
76	126	1	1	6		●		●			●				●		●	●		●	●		●						
78	128	0	3	1	●		●			●				●		●	●		●	●		●							●
80	130	2	0	3		●			●				●		●	●		●	●		●							●	
82	132	1	2	5	●			●				●		●	●		●	●		●							●		●
84	134	0	4	0			●				●		●	●		●	●		●							●		●	
86	136	2	1	2		●				●		●	●		●	●		●							●		●		
88	138	1	3	4	●				●		●	●		●	●		●							●		●			●
90	140	0	0	6				●		●	●		●	●		●							●		●			●	●
92	142	2	2	1			●		●	●		●	●		●							●		●			●	●	
94	144	1	4	3		●		●	●		●	●		●							●		●			●	●		
96	146	0	1	5	●		●	●		●	●		●							●		●			●	●			
98	148	2	3	0		●	●		●	●		●							●		●			●	●				
100	150	1	0	2	●	●		●	●		●							●		●			●	●					●
102	152	0	2	4	●		●	●		●							●		●			●	●					●	●
104	154	2	4	6		●	●		●							●		●			●	●					●	●	
106	156	1	1	1	●	●		●							●		●			●	●					●	●		
108	158	0	3	3	●		●							●		●			●	●					●	●			●
110	160	2	0	5		●							●		●			●	●					●	●			●	
112	162	1	2	0	●							●		●			●	●					●	●			●		
114	164	0	4	2							●		●			●	●					●	●			●			●
116	166	2	1	4						●		●			●	●					●	●			●			●	

From	To	Phase			1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49
118	168	1	3	6					●		●			●	●					●	●			●			●		●
120	170	0	0	1				●		●			●	●					●	●			●			●		●	
122	172	2	2	3			●		●			●	●					●	●			●			●		●		
124	174	1	4	5		●		●			●	●					●	●			●			●		●			●
126	176	0	1	0	●		●			●	●					●	●			●			●		●			●	
128	178	2	3	2		●			●	●					●	●			●			●		●			●		
130	180	1	0	4	●			●	●					●	●			●			●		●			●			●
132	182	0	2	6			●	●					●	●			●			●		●			●			●	●
134	184	2	4	1		●	●					●	●			●			●		●			●			●	●	
136	186	1	1	3	●	●					●	●			●			●		●			●			●	●		
138	188	0	3	5	●					●	●			●			●		●			●			●	●			
140	190	2	0	0					●	●			●			●		●			●			●	●				
142	192	1	2	2				●	●			●			●		●			●			●	●					●
144	194	0	4	4			●	●			●			●		●			●			●	●					●	●
146	196	2	1	6		●	●			●			●		●			●			●	●					●	●	
148	198	1	3	1	●	●			●			●		●			●			●	●					●	●		●
150	200	0	0	3	●			●			●		●			●			●	●					●	●		●	●
152	202	2	2	5			●			●		●			●			●	●					●	●		●	●	
154	204	1	4	0		●			●		●			●			●	●					●	●		●	●		
156	206	0	1	2	●			●		●			●			●	●					●	●		●	●			
158	208	2	3	4			●		●			●			●	●					●	●		●	●				
160	210	1	0	6		●		●			●			●	●					●	●		●	●					

Explanations on tables A.1 and A.2 are as follows.

- **Horizontal arrangement.**
  - **From:** Start value of silding window.
  - **To:** End value of sliding window.
  - **Phase:** Phase of each pSFs, $d_i = 2n \bmod p_i$, $2 \leq i \leq k$.
  - **Head row numbers:** Odd numbers between 1 ~ 2n.
- **Vertical arrangement.**
  - Sliding window from 0 ~ 2n to (Q_4 – 2n) ~ Q_4.
- **Other cells.**
  - **number:** Prime numbers between 0 ~ Q_4.
  - **●:** Prime numbers are replaced by ● to show geometric relationships between sliding windows.
- **First two red letter rows.**
  - **First row:** The first sliding window.
  - **Secondt row:** The last sliding window.
  - The first row and the second row can not satisfy GCFC.

We can see the following characteristics from tables A.1.1. and A.1.2.

① As windows slide by 2, the prime numbers also slide diagonally.

② Empty cells are the zeros of CpSFs.

③ The zero configuration is unique for each windows.

A.3. Window ASP Program Source for Sliding Window Tables.

note) How to invoke: URL/SourceProgramName.asp?n=25

```
<!-- metadata type="typelib"
              file="c:\Program Files\Common Files\System\ado\msado15.dll" -->
<html>
<head>
   <title>gc</title>
   <meta http-equiv="Content-Type" content="text/html; charset=euc-kr">
   <STYLE TYPE="text/css">
<!--
   td {font-size: 8pt; color: #000000;}
-->
</STYLE>
</head>

<%
'//* ASP source for sliding window table.
Function isPrime(num)
If num =< 2 then
 isPrime = false
ElseIf num = 2 Then
 isPrime = true
Else
 isPrime = true
 For a = 2 To num \ 2
  If num Mod a = 0 Then
   isPrime = false
   exit for
  End If
 Next
End If
End Function

'//* main
Server.ScriptTimeOut = 30
dim p(50000), phase(50000, 20), zero(50000)
n = CInt(Request.QueryString("n")) 'n >= 25. URL/SourceProgram.asp?n=25
if isNull(n) or n<25 then n=25
n2 = n * 2
rn2 = int(sqr(n2))
cnt = 0
Q = 2
FOR i=1 to rn2
```

```
  if isPrime(i) then
   cnt = cnt + 1
   p(cnt) = i
   Q = Q * i
  end if
 NEXT
 response.write "* n=" & n & ", 2n=" & n2 & ", max p=" & p(cnt) & ", Q=" & Q & " : "
 FOR i=1 to cnt
  response.write p(i) & " "
 NEXT

 response.write "<br><br>"

 '//* prime numbers, phases
 FOR i=0 to Q
  FOR j=1 to cnt
   phase(i, j) = i mod p(j)
  NEXT
  IF isPrime(i) THEN
   zero(i) = I 'display numbers indtead of ●
 '   zero(i) = "●"
  ELSE
   zero(i) = ""
  END IF
 NEXT

 '//* sliding window table
 response.write "<table><tr bgcolor=eeeeee align=center><td>From</tf><td>To</td><td colspan=" & cnt &
">phase</td>"
 FOR i=1 to n2 step 2
  response.write "<td width=13>" & i
 NEXT
 FOR i=0 to Q-n2 step Q-n2
  xfrom = i
  xto = xfrom + n2
  response.write "<tr align=center><td>" & xfrom & "<td>" & xto
  FOR j=1 to cnt
   response.write "<td>" & phase(i, j)
  NEXT
 col = -1
 FOR k=xfrom+1 to xto step 2
   col = col + 1
   if col mod 5 = 0 or i mod 5 = 0 then
    bg = "pink"
   else
    bg = "ffffff"
   end if
   if IsNumeric(zero(k)) or zero(k)="●" then
    color = "black"
   else
    color = "dddddd"
   end if
   response.write "<td bgcolor=" & bg & "><font color=" & color & ">" & zero(k)
```

```
 NEXT
NEXT

FOR i=0 to Q step 2
 xfrom = i
 xto = xfrom + n2
 response.write "<tr align=center><td>" & xfrom & "<td>" & xto

 FOR j=1 to cnt
  response.write "<td>" & phase(i, j)
 NEXT
col = -1
FOR k=xfrom+1 to xto step 2
  col = col + 1
  if col mod 5 = 0 or i mod 5 = 0 then
   bg = "yellow"
  else
   bg = "ffffff"
  end if
  if IsNumeric(zero(k)) or zero(k)="●" then
   color = "black"
  else
   color = "dddddd"
  end if
  response.write "<td bgcolor=" & bg & "><font color=" & color & ">" & zero(k)
NEXT
NEXT
response.write "</td></tr></table>"
%>
```

Appendix B. Example RSFS Sieve of Prime Numbers in in $n \leq x \leq 2n$.

B.1. Table of the Sieve of Prime Numbers via RSFS, *n* = 25.

seq	phases			3	5	7	11	13	17	19	23	29	31	37	41	43	47	pnum
	3	5	7															
1	0	0	0	x3	x5	x7	11	13	17	19	23	29	31	37	41	43	47	11
2	0	0	1	x3	x5	7	11	13	17	19	23	x7	31	37	41	x7	47	10
3	0	0	2	x3	x5	7	11	13	17	19	x7	29	31	x7	41	43	47	10
4	0	0	3	x3	x5	7	11	13	x7	19	23	29	x7	37	41	43	47	10
5	0	0	4	x3	x5	7	x7	13	17	19	23	29	31	37	41	43	47	11
6	0	0	5	x3	x5	7	11	13	17	x7	23	29	31	37	41	43	x7	10
7	0	0	6	x3	x5	7	11	x7	17	19	23	29	31	37	x7	43	47	10
8	0	1	0	x3	5	x7	x5	13	17	19	23	29	x5	37	x5	43	47	9
9	0	1	1	x3	5	7	x5	13	17	19	23	x7	x5	37	x5	x7	47	8
10	0	1	2	x3	5	7	x5	13	17	19	x7	29	x5	x7	x5	43	47	8
11	0	1	3	x3	5	7	x5	13	x7	19	23	29	x5	37	x5	43	47	9
12	0	1	4	x3	5	7	x5	13	17	19	23	29	x5	37	x5	43	47	10
13	0	1	5	x3	x7	7	x5	13	17	x7	23	29	x5	37	x5	43	x7	7
14	0	1	6	x3	5	7	x5	x7	17	19	23	29	x5	37	x5	43	47	9
15	0	2	0	x3	5	x5	11	13	x5	19	23	29	31	x5	41	43	x5	9
16	0	2	1	x3	5	x5	11	13	x5	19	23	x7	31	x5	41	x7	x5	7
17	0	2	2	x3	5	x5	11	13	x5	19	x7	29	31	x5	41	43	x5	8
18	0	2	3	x3	5	x5	11	13	x5	19	23	29	x7	x5	41	43	x5	8
19	0	2	4	x3	5	x5	x7	13	x5	19	23	29	31	x5	41	43	x5	8
20	0	2	5	x3	x7	x5	11	13	x5	x7	23	29	31	x5	41	43	x5	7
21	0	2	6	x3	5	x5	11	x7	x5	19	23	29	31	x5	x7	43	x5	7
22	0	3	0	x3	5	x7	11	x5	17	19	x5	29	31	37	41	x5	47	9
23	0	3	1	x3	5	7	11	x5	17	19	x5	x7	31	37	41	x5	47	9
24	0	3	2	x3	5	7	11	x5	17	19	x5	29	31	x7	41	x5	47	9
25	0	3	3	x3	5	7	11	x5	x7	19	x5	29	x7	37	41	x5	47	8
26	0	3	4	x3	5	7	x7	x5	17	19	x5	29	31	37	41	x5	47	9
27	0	3	5	x3	x7	7	11	x5	17	x7	x5	29	31	37	41	x5	x7	7
28	0	3	6	x3	5	7	11	x5	17	19	x5	29	31	37	x7	x5	47	9
29	0	4	0	x3	5	x7	11	13	17	x5	23	x5	31	37	41	43	47	10
30	0	4	1	x3	5	7	11	13	17	x5	23	x5	31	37	41	x7	47	10
31	0	4	2	x3	5	7	11	13	17	x5	x7	x5	31	x7	41	43	47	9
32	0	4	3	x3	5	7	11	13	x7	x5	23	x5	x7	37	41	43	47	9
33	0	4	4	x3	5	7	x7	13	17	x5	23	x5	31	37	41	43	47	10
34	0	4	5	x3	x7	7	11	13	17	x5	23	x5	31	37	41	43	x7	9
35	0	4	6	x3	5	7	11	x7	17	x5	23	x5	31	37	x7	43	47	9
36	1	0	0	3	x5	x3	11	x3	17	x3	23	29	x3	x3	41	x3	47	7
37	1	0	1	3	x5	x3	11	x3	17	x3	23	x7	x3	x3	41	x3	47	6
38	1	0	2	3	x5	x3	11	x3	17	x3	x7	29	x3	x3	41	x3	47	6
39	1	0	3	x7	x5	x3	11	x3	x7	x3	23	29	x3	x3	41	x3	47	5
40	1	0	4	3	x5	x3	x7	x3	17	x3	23	29	x3	x3	41	x3	47	6
41	1	0	5	3	x5	x3	11	x3	17	x3	23	29	x3	x3	41	x3	x7	6
42	1	0	6	3	x5	x3	11	x3	17	x3	23	29	x3	x3	x7	x3	47	6
43	1	1	0	3	5	x3	x5	x3	17	x3	23	29	x3	x3	x5	x3	47	6
44	1	1	1	3	5	x3	x5	x3	17	x3	23	x7	x3	x3	x5	x3	47	5
45	1	1	2	3	5	x3	x5	x3	17	x3	x7	29	x3	x3	x5	x3	47	5
46	1	1	3	x7	5	x3	x5	x3	x7	x3	23	29	x3	x3	x5	x3	47	4
47	1	1	4	3	5	x3	x5	x3	17	x3	23	29	x3	x3	x5	x3	47	6
48	1	1	5	3	x7	x3	x5	x3	17	x3	23	29	x3	x3	x5	x3	x7	4
49	1	1	6	3	5	x3	x5	x3	17	x3	23	29	x3	x3	x5	x3	47	6
50	1	2	0	3	5	x3	11	x3	x5	x3	23	29	x3	x3	41	x3	x5	6

seq	phases			3	5	7	11	13	17	19	23	29	31	37	41	43	47	pnum
	3	5	7															
51	1	2	1	3	5	x3	11	x3	x5	x3	23	x7	x3	x3	41	x3	x5	5
52	1	2	2	3	5	x3	11	x3	x5	x3	x7	29	x3	x3	41	x3	x5	5
53	1	2	3	x7	5	x3	11	x3	x5	x3	23	29	x3	x3	41	x3	x5	5
54	1	2	4	3	5	x3	x7	x3	x5	x3	23	29	x3	x3	41	x3	x5	5
55	1	2	5	3	x7	x3	11	x3	x5	x3	23	29	x3	x3	41	x3	x5	5
56	1	2	6	3	5	x3	11	x3	x5	x3	23	29	x3	x3	x7	x3	x5	5
57	1	3	0	x5	5	x3	11	x3	17	x3	x5	29	x3	x3	41	x3	47	6
58	1	3	1	x5	5	x3	11	x3	17	x3	x5	x7	x3	x3	41	x3	47	5
59	1	3	2	x5	5	x3	11	x3	17	x3	x5	29	x3	x3	41	x3	47	6
60	1	3	3	x5	5	x3	11	x3	x7	x3	x5	29	x3	x3	41	x3	47	5
61	1	3	4	x5	5	x3	x7	x3	17	x3	x5	29	x3	x3	41	x3	47	5
62	1	3	5	x5	x7	x3	11	x3	17	x3	x5	29	x3	x3	41	x3	x7	4
63	1	3	6	x5	5	x3	11	x3	17	x3	x5	29	x3	x3	x7	x3	47	5
64	1	4	0	3	5	x3	11	x3	17	x3	23	x5	x3	x3	41	x3	47	7
65	1	4	1	3	5	x3	11	x3	17	x3	23	x5	x3	x3	41	x3	47	7
66	1	4	2	3	5	x3	11	x3	17	x3	x7	x5	x3	x3	41	x3	47	6
67	1	4	3	x7	5	x3	11	x3	x7	x3	23	x5	x3	x3	41	x3	47	5
68	1	4	4	3	5	x3	x7	x3	17	x3	23	x5	x3	x3	41	x3	47	6
69	1	4	5	3	x7	x3	11	x3	17	x3	23	x5	x3	x3	41	x3	x7	5
70	1	4	6	3	5	x3	11	x3	17	x3	23	x5	x3	x3	x7	x3	47	6
71	2	0	0	3	x3	x7	x3	13	x3	19	x3	x3	31	37	x3	43	x3	6
72	2	0	1	3	x3	7	x3	13	x3	19	x3	x3	31	37	x3	x7	x3	6
73	2	0	2	3	x3	7	x3	13	x3	19	x3	x3	31	x7	x3	43	x3	6
74	2	0	3	x7	x3	7	x3	13	x3	19	x3	x3	x7	37	x3	43	x3	5
75	2	0	4	3	x3	7	x3	13	x3	19	x3	x3	31	37	x3	43	x3	7
76	2	0	5	3	x3	7	x3	13	x3	x7	x3	x3	31	37	x3	43	x3	6
77	2	0	6	3	x3	7	x3	x7	x3	19	x3	x3	31	37	x3	43	x3	6
78	2	1	0	3	x3	x7	x3	13	x3	19	x3	x3	x5	37	x3	43	x3	5
79	2	1	1	3	x3	7	x3	13	x3	19	x3	x3	x5	37	x3	x7	x3	5
80	2	1	2	3	x3	7	x3	13	x3	19	x3	x3	x5	x7	x3	43	x3	5
81	2	1	3	x7	x3	7	x3	13	x3	19	x3	x3	x5	37	x3	43	x3	5
82	2	1	4	3	x3	7	x3	13	x3	19	x3	x3	x5	37	x3	43	x3	6
83	2	1	5	3	x3	7	x3	13	x3	x7	x3	x3	x5	37	x3	43	x3	5
84	2	1	6	3	x3	7	x3	x7	x3	19	x3	x3	x5	37	x3	43	x3	5
85	2	2	0	3	x3	x5	x3	13	x3	19	x3	x3	31	x5	x3	43	x3	5
86	2	2	1	3	x3	x5	x3	13	x3	19	x3	x3	31	x5	x3	x7	x3	4
87	2	2	2	3	x3	x5	x3	13	x3	19	x3	x3	31	x5	x3	43	x3	5
88	2	2	3	x7	x3	x5	x3	13	x3	19	x3	x3	x7	x5	x3	43	x3	3
89	2	2	4	3	x3	x5	x3	13	x3	19	x3	x3	31	x5	x3	43	x3	5
90	2	2	5	3	x3	x5	x3	13	x3	x7	x3	x3	31	x5	x3	43	x3	4
91	2	2	6	3	x3	x5	x3	x7	x3	19	x3	x3	31	x5	x3	43	x3	4
92	2	3	0	x5	x3	x7	x3	x5	x3	19	x3	x3	31	37	x3	x5	x3	3
93	2	3	1	x5	x3	7	x3	x5	x3	19	x3	x3	31	37	x3	x5	x3	4
94	2	3	2	x5	x3	7	x3	x5	x3	19	x3	x3	31	x7	x3	x5	x3	3
95	2	3	3	x5	x3	7	x3	x5	x3	19	x3	x3	x7	37	x3	x5	x3	3
96	2	3	4	x5	x3	7	x3	x5	x3	19	x3	x3	31	37	x3	x5	x3	4
97	2	3	5	x5	x3	7	x3	x5	x3	x7	x3	x3	31	37	x3	x5	x3	3
98	2	3	6	x5	x3	7	x3	x5	x3	19	x3	x3	31	37	x3	x5	x3	4
99	2	4	0	3	x3	x7	x3	13	x3	x5	x3	x3	31	37	x3	43	x3	5
100	2	4	1	3	x3	7	x3	13	x3	x5	x3	x3	31	37	x3	x7	x3	5
101	2	4	2	3	x3	7	x3	13	x3	x5	x3	x3	31	x7	x3	43	x3	5
102	2	4	3	x7	x3	7	x3	13	x3	x5	x3	x3	x7	37	x3	43	x3	4
103	2	4	4	3	x3	7	x3	13	x3	x5	x3	x3	31	37	x3	43	x3	6

seq	phases			3	5	7	11	13	17	19	23	29	31	37	41	43	47	pnum
	3	5	7															
104	2	4	5	3	x3	7	x3	13	x3	x5	x3	x3	31	37	x3	43	x3	6
105	2	4	6	3	x3	7	x3	x7	x3	x5	x3	x3	31	37	x3	43	x3	5

Explanations on tables B.1 are as follows.

- **Header and the first row.**
  - **seq:** Sequence.
  - **phases:** Phases of each seed 3, 5, 7. Phase of 2 is always 0, so, is omitted.
  - **3, 5, …, 47:** Prime numbers in the zero configuration range $0 \leq x \leq 50$.
  - **pnum:** Number of remaining prime numbers.
- **Other cells.**
  - **x3, x5, x7:** Prime numbers are sieved out by the phased SF of 3, 5, 7, respectively. For example, at seq 105, x3 for prime number 5 means that 5 is sieved out by $sin\left(\frac{\pi(2-x)}{3}\right)$, where the phase value of seed 3 is 2.
- **Actual case for n = 25.**
  - **phase for seed 3:** $d_2 = 2n \ mod \ 3 = 50 \ mod \ 3 = 2$.
  - **phase for seed 5:** $d_2 = 2n \ mod \ 5 = 50 \ mod \ 5 = 0$.
  - **phase for seed 7:** $d_2 = 2n \ mod \ 7 = 50 \ mod \ 7 = 1$.
  - The actual case for phase set $D_3 = \{2, 0, 1\}$ is marked as blue rectangle**.**
- **The significance of B.1.**
  - To make GC false, all prime numbers in the zero configuration range $0 \leq x \leq 2n$ must be the zeros of RSFS. So, RSFS must be the complementary function set of the FSFS.
  - Table B.1 shows all possible phase set $D_3 = \{d_2, d_3, d_4\}$, where $d_2 = 0 \sim 2, d_3 = 0 \sim 4, d_4 = 0 \sim 6$, so, there are 3 * 5 * 7 = 105 phase combinations. It shows that any combination of phases can not sieve out all prime numbers in the zero configuration range $0 \leq x \leq 50$.