Prove Collatz Conjecture via Operations for Integer's Expressions plus Few Integers

Abstract

First, let us set forth certain of basic concepts related to proving Collatz conjecture. After that, the author lists the mathematical induction that proves the conjecture and prepare several judging criteria. Then again classify unproved positive integers successively and prove an integer's expression therein for each time, until the last two integer's expressions are proven. The way be via operations and judging a result on a related operational route, such that the integer's expression fits the conjecture.

AMS subject classification: 11P32; 11A25; 11Y55

Keywords: Collatz conjecture; the rule of operations; the mathematical induction; judging criteria; classify positive integers; operational routes

1. Introduction

The Collatz conjecture is also called the *3x+1* mapping, *3n+1* problem, Hasse's algorithm, Kakutani's problem, Syracuse algorithm, Syracuse problem, Thwaites conjecture and Ulam's problem, etc.

But it remains a conjecture that has neither been proved nor disproved ever since named after Lothar Collatz in 1937; [1].

2. Certain of Basic Concepts

The Collatz conjecture states that take any positive integer n, if n is an even number, divide it by *2*; if n is an odd number, multiply it by *3* and add *1*, and repeat the process indefinitely, so no matter which positive integer you start with, you are always going to end up with *1*; [2].

We consider aforesaid operational stipulations as the rule of operations.

If you start with any positive integer⁄integer's expression to operate continually by the rule of operations, then continuous positive integers⁄ integer's expressions will be formed.

So, we reckon such continuous positive integers⁄integer's expressions plus arrows on a direction among them as an operational route.

In addition, let us use a capital letter with the subscript "*ie*" to express a positive integer's expression such as P_{ie}, C_{ie} etc.

If an operational route contains P_{ie}, then the operational route may be called "an operational route via P_{ie}".

In general, integer's expressions on an operational route have a common variable or many variables which can be converted into a variable.

3. The Mathematical Induction that Proves the Conjecture

The mathematical induction [3] that proves the conjecture is as follows:

(1) From 1→4→2→1; 2→1; 3→10→5→16→8→4→2→1; 4→2→1; 5→16→8→4→2→1; 6→3→10→5→16→8→4→2→1; 7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1; 8→4→2→1 and 9→28→14→7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1, it can be seen that every positive integer ≤ 9 fits the conjecture.

(2) Suppose that n fits the conjecture, where n is an integer ≥ 9.

(3) Prove that $n+1$ fits the conjecture likewise.

4. Several Judging Criteria

A certain result of operations which begin with each class of positive integers is judged by one of following criteria.

Theorem 1. If an operational route via P_{ie} has an integer's expression that is less than P_{ie}, and $n+1 \in P_{ie}$, then each and every integer's expression on the operational route, including $n+1$ within P_{ie} fits the conjecture.

For example, if let $P_{ie}=31+3^2\beta$ with $\beta \geq 0$, and $n+1 \in P_{ie}$, then from $27+2^3\beta \to 82+3\times2^3\beta \to 41+3\times2^2\beta \to 124+3^2\times2^2\beta \to 62+3^2\times2\beta \to \mathbf{31}+3^2\beta > \mathbf{27}+2^3\beta$, we get that each and every integer's expression on the operational route, including $n+1$ within P_{ie} fits the conjecture.

In addition, let $P_{ie}=5+2^2\mu$ with $\mu \geq 0$, and $n+1 \in P_{ie}$, then from $\mathbf{5}+2^2\mu \to 16+3\times2^2\mu \to 8+3\times2\mu \to \mathbf{4}+3\mu < \mathbf{5}+2^2\mu$, we get that each and every integer's expression on the operational route, including $n+1$ within P_{ie} fits the conjecture.

Proof. Suppose that an operational route via P_{ie} contains C_{ie}, and $C_{ie}<P_{ie}$, then when their common variable is equal to a certain fixed value such that $P_{ie}=n+1$, let $C_{ie}=m$, so there is $m<n+1$, and m fits the conjecture.

So, from $n+1$ can operate to m, or from m can operate to $n+1$, then start with m to continuously operate until 1, such that $n+1$ fits the conjecture.

When their common variable is equal to each value, each value of each integer's expression can also be operated to 1, because each value of each integer's expression corresponds with a matched value of C_{ie}, so each and every integer's expression on the operational route, including $n+1$ within P_{ie} fits the conjecture.

Theorem 2. If an operational route via Q_{ie} and an operational route via P_{ie} intersect, and $n+1 \in P_{ie}$, and an integer's expression on the operational route via Q_{ie} is less than P_{ie}, then each and every integer's expression on these two operational routes, including $n+1$ within P_{ie} fits the conjecture.

For example, let $Q_{ie}=71+3^3\times 2^5\varphi$, and $P_{ie}=63+3\times 2^8\varphi$ where $\varphi\geq 0$, then from

$\mathbf{63}+3\times 2^8\varphi \to 190+3^2\times 2^8\varphi \to 95+3^2\times 2^7\varphi \to 286+3^3\times 2^7\varphi \to 143+3^3\times 2^6\varphi \to 430+3^4\times 2^6\varphi \to$

$215+3^4\times 2^5\varphi \to 646+3^5\times 2^5\varphi \to 323+3^5\times 2^4\varphi \to 970+3^6\times 2^4\varphi \to 485+3^6\times 2^3\varphi \to 1456+3^7\times 2^3\varphi$

$\to 728+3^7\times 2^2\varphi \to 364+3^7\times 2\varphi \to 182+3^7\varphi \to \ldots$

$\uparrow 121+3^6\times 2\varphi \leftarrow 242+3^6\times 2^2\varphi \leftarrow 484+3^6\times 2^3\varphi \leftarrow 161+3^5\times 2^3\varphi \leftarrow 322+3^5\times 2^4\varphi$

$\leftarrow 107+3^4\times 2^4\varphi \leftarrow 214+3^4\times 2^5\varphi \leftarrow \mathbf{71}+3^3\times 2^5\varphi \leftarrow 142+3^3\times 2^6\varphi \leftarrow \mathbf{47}+3^2\times 2^6\varphi < \mathbf{63}+3\times 2^8\varphi$,

we get that each and every integer's expression on these two operational routes, including $n+1$ within P_{ie} fits the conjecture.

Proof. Suppose that there is D_{ie} on an operational route via Q_{ie}, and $D_{ie}<P_{ie}$, and that the operational route via Q_{ie} and an operational route via P_{ie} intersect at A_{ie}, then when their common variable is given a certain fixed value such that $P_{ie}=n+1$, let $A_{ie}=\xi$ and $D_{ie}=\mu$, so there be $\mu<n+1$, and μ fits the conjecture.

Since ξ and μ belong to an operational route, and that μ fits the conjecture, then each and every integer on the operational route, including ξ fits the conjecture, according to the Theorem 1.

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220720-bqb84 Version 1 - Submitted to Proc. Amer. Math. Soc.

Since $n+1$ and ξ belong to an operational route, and that ξ fits the conjecture, then each and every integer on the operational route, including $n+1$ fits the conjecture, according to the Theorem 1.

When their common variable is equal to each value, each value of each integer's expression on these two operational routes can also be operated to *1,* because each value of each integer's expression corresponds with a matched value of D_{ie}, so each and every integer's expression on these two operational routes, including $n+1$ within P_{ie} fits the conjecture.

Lemma. If an operational route via Q_{ie} and an operational route via P_{ie} are in the indirect connection, and $n+1 \in P_{ie}$, and an integer's expression on the operational route via Q_{ie} is less than P_{ie}, then each and every integer's expression on these operational routes which intersect successively, including $n+1$ within P_{ie} fits the conjecture.

The indirect connection refers to the relation between two non-intersected operational routes on the premise that at least *3* operational routes intersect successively.

In addition to above, substitute *d, e, f, g,* etc. for *c* on the bunch of operational routes of $15+12c/19+12c$, that be actually for the sake of avoiding confusion, and be conducive convenience.

5. Classifications with Partial Proofs for Positive Integers

We divide positive integers into multilevel classes successively, then the integer $n+1$ is possibly included within any class of positive integers, thus each class of positive integers must be proved.

After positive integers are parted into a new class for each time, we need to find an integer's expression that is less than a kind within the new class, on operational routes via the kind, in order to follow the Theorem *1* and complete a partial proof.

In the following paragraphs, the classes and kinds of positive integers are expressed by integer's expressions.

Proof. As positive integers≤ 9 have been proven to fit the conjecture in section *2*, so first divide integers>9 into even numbers and odd numbers.

For even numbers $2k$ with $k>4$, from $2k\to k<2k$, we get that if $n+1\in 2k$, then $2k$ and $n+1$ fit the conjecture, according to the Theorem *1*.

For odd numbers >9, divide them into $11+4k$ and $13+4k$, where $k\geq 0$.

For $13+4k$, from $13+4k\to 40+12k\to 20+6k\to 10+3k<13+4k$, we get that if $n+1\in 13+4k$, then $13+4k$ and $n+1$ fit the conjecture, according to the Theorem *1*.

Further divide $11+4k$ into $11+12c$, $15+12c$ and $19+12c$, where $c\geq 0$.

For $11+12c$, from $7+8c\to 22+24c\to 11+12c>7+8c$, we get that if $n+1\in 11+12c$, then $11+12c$ and $n+1$ fit the conjecture, according to the Theorem *1*.

Prove $15+12c$ and $19+12c$, where $c\geq 0$, this is focuses of this article, thus we need to make specially a main proof, *ut infra*.

6. Prove that 15+12c and 19+12c Fit the Conjecture

For $15+12c/19+12c$ where $c\geq 0$, continue to operate it until which find and deduce all integer's expressions which are monogamously less than all kinds of *15+12c/19+12c* on operational routes of *15+12c/19+12c*, so as to accord with judging criteria one-on-one.

Firstly, start with *15+12c* to operate continuously by the rule of operations, as listed below.

15+12c→46+36c→23+18c→70+54c→35+27c ♣

d=2e+1: 29+27e (**1**)   e=2f: 142+486f→71+243f ♥
♣35+27c↓→c=2d+1: 31+27d↑→d=2e: 94+162e→47+81e↑→e=2f+1:64+81f (**2**)
c=2d: 106+162d→53+81d↓→d=2e+1:67+81e↓→e=2f+1:74+81f (**3**)
d=2e:160+486e♦   e=2f: 202+486f→101+243f ♠

g=2h+1: 200+243h (**4**)   **…**
♥ 71+243f↓→f=2g+1:157+243g↑→g=2h: 472+1458h→236+729h↑→ **…**
f=2g: 214+1458g→107+729g↓→g=2h+1: 418+729h↓→**…**
g=2h: 322+4374h→**… …**

g=2h: 86+243h (**5**)
♠101+243f↓→f=2g+1:172+243g↑→g=2h+1:1246+1458h→**…**
f=2g: 304+1458g→152+729g ↓→**…**
…

…
♦160+486e→80+243e↓→e=2f+1: 970+1458f→485+729f↑→ **…**   **…**
e=2f:40+243f↓→f=2g+1:850+1458g→ 425+729g↑→ **…**
f=2g: 20+243g↓→g=2h: 10+243h (**6**)   **…**
g=2h+1:790+1458h→395+729h↑→ **…**

Annotation:
(1) Each of letters c, d, e, f, g, h, etc on listed above operational routes expresses each of natural numbers plus 0.
(2) There are ♣↔♣, ♥↔♥, ♠↔♠, and ♦↔♦ on above operational routes.
(3) Aforesaid two points are suitable to latter operational routes of 19+12c similarly.

First, define a term. That is, if an operational result is less than a kind of *15+12c/19+12c*, and it occurs first on the operational route via the kind of *15+12c/19+12c*, then we call the operational result "№1 satisfactory operational result" about the kind of *15+12c/19+12c*.

Accordingly, the author first emphatically concludes following *3* kinds of *15+12c* derived from №1 satisfactory operational results on the bunch of operational routes of *15+12c* to fit the conjecture.

1). From c=2d+1 and d=2e+1 to get c=2d+1=2(2e+1)+1=4e+3, then there are $15+12c=51+48e=51+3\times2^4e\to154+3^2\times2^4e\to77+3^2\times2^3e\to232+3^3\times2^3e\to116+3^3\times2^2e\to58+3^3\times2e\to29+27e$ where the mark (**1**).

Due to $29+27e<51+48e$, we get that if there is $n+1\in51+48e$, then 51+48e and $n+1$ fit the conjecture, according to the Theorem *1*.

2). From c=2d+1, d=2e and e=2f+1 to get c=2d+1=4e+1=4(2f+1)+1=8f+5, then there are $15+12c=75+96f=75+3\times2^5f\to226+3^2\times2^5f\to113+3^2\times2^4f\to340+3^3\times2^4f\to170+3^3\times2^3f\to85+3^3\times2^2f\to256+3^4\times2^2f\to128+3^4\times2^1f\to64+81f$ where the mark (**2**).

Due to $64+81f<75+96f$, we get that if there is $n+1\in75+96f$, then *75+96f* and $n+1$ fit the conjecture, according to the Theorem *1*.

3). From c=2d, d=2e+1 and e=2f+1 to get c=2d=4e+2=4(2f+1)+2=8f+6, then there are $15+12c=87+96f=87+3\times2^5f\to262+3^2\times2^5f\to131+3^2\times2^4f\to394+3^3\times2^4f\to197+3^3\times2^3f\to592+3^4\times2^3f\to296+3^4\times2^2f\to148+3^4\times2^1f\to74+81f$ where the mark (**3**).

Due to $74+81f<87+96f$, we get that if there is $n+1\in87+96f$, then *87+96f* and $n+1$ fit the conjecture, according to the Theorem *1*.

Like that, each reader can also emphatically conclude other 3 kinds of *15+12c* derived from №1 satisfactory operational results on the bunch of operational routes of *15+12c* to fit the conjecture, according to the Theorem *1*. They are:

4). Pursuant to c=2d+1, d=2e, e=2f, f=2g+1 and g=2h+1, you can get *15+12c=315+384h* derived from *200+243h* where the mark (**4**);

5). Pursuant to c=2d, d=2e+1, e=2f, f=2g+1 and g=2h, you can get

15+12c=135+384h derived from *86+243h* where the mark (**5**);

6). Pursuant to c=2d, d=2e, e=2f, f=2g and g=2h, you can get *15+12c=15+384h* derived from *10+243h* where the mark (**6**).

Secondly, start with *19+12c* to operate continuously by the rule of operations, as listed below.

19+12c→58+36c→ 29+18c→ 88+54c→ 44+27c ♣

d=2e: 11+27e (**α**)   e=2f:37+81f (**β**)
♣ 44+27c↓→c=2d: 22+27d↑→d=2e+1:148+162e→74+81e↑→e=2f+1:466+486f ♥
c=2d+1: 214+162d→107+81d↓→d=2e:322+486e ♠
d=2e+1:94+81e↓→e=2f:47+81f (γ)
e=2f+1:526+486f ♦

g=2h: 119+243h (**δ**)   …
f=2g+1:238+243g↑→g=2h+1:1444+1458h→722+729h↑→…
♥466+486f→233+243f↑→f=2g: 700+1458g→350+729g↓→g=2h+1:3238+4374h↓
g=2h: 175+729h↓→…   …
…

g=2h+1:172+243h (**ε**)
f=2g: 101+243g↑→g=2h: 304+1458h→…
e=2f+1:202+243f↑→f=2g+1:1336+1458g→**…**
♠322+486e→161+243e↑→e=2f:484+1458f→**…**

♦526+486f→263+243f↓→f=2g: 790+1458g→**…**
f=2g+1: 253+243g↓→g=2h+1: 248+243h (ζ)
g=2h: 760+1458h→**…**

As listed above, the author first emphatically concludes following 3 kinds of *19+12c* derived from №1 satisfactory operational results on the bunch of operational routes of *19+12c* to fit the conjecture.

1). From c = 2d and d = 2e to get c = 2d = 4e, then there are 19+12c = 19+48e = $19+3\times2^4e \rightarrow 58+3^2\times2^4e \rightarrow 29+3^2\times2^3e \rightarrow 88+3^3\times2^3e \rightarrow 44+3^3\times2^2e \rightarrow 22+3^3\times2e \rightarrow 11+27e$ where the mark (**α**).

Due to 11+27e<19+48e, we get that if there is $n+1\in 19+48e$, then *19+48e* and *n+1* fit the conjecture, according to the Theorem *1*.

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2). From c=2d, d=2e+1 and e=2f to get c=2d=2(2e+1)=4e+2=8f+2, then there are $19+12c=43+96f=43+3\times2^5f\to130+3^2\times2^5f\to65+3^2\times2^4f\to196+3^3\times2^4f\to98+3^3\times2^3f$ $\to49+3^3\times2^2f\to148+3^4\times2^2f\to74+3^4\times2^1f\to37+81f$ where the mark (**β**).

Due to 37+81f<43+96f, we get that if there is $n+1\in43+96f$, then *43+96f* and *n+1* fit the conjecture, according to the Theorem *1*.

3). From c=2d+1, d=2e+1 and e=2f to get c=2d+1=4e+3=8f+3, then there are $19+12c=55+96f=55+3\times2^5f\to166+3^2\times2^5f\to83+3^2\times2^4f\to250+3^3\times2^4f\to125+3^3\times2^3f\to$ $376+3^4\times2^3f\to188+3^4\times2^2f\to94+3^4\times2^1f\to47+81f$ where the mark (**γ**).

Due to 47+81f < 55+96f, we get that if there is $n+1\in55+96f$, then *55+96f* and *n+1* fit the conjecture, according to the Theorem *1*.

Like that, each reader can also emphatically conclude other 3 kinds of *19+12c* derived from №1 satisfactory operational results on the bunch of operational routes of *19+12c* to fit the conjecture, according to the Theorem *1*. They are:

4). Pursuant to c=2d, d=2e+1, e=2f+1, f=2g+1 and g=2h, you can get *19+12c=187+384h* derived from *119+243h* where the mark (**δ**);

5). Pursuant to c=2d+1, d=2e, e=2f+1, f=2g and g=2h+1, you can get *19+12c=271+384h* derived from *172+243h* where the mark (**ε**);

6). Pursuant to c=2d+1, d=2e+1, e=2f+1, f=2g+1 and g=2h+1, you can get *19+12c=391+384h* derived from *248+243h* where the mark (**ζ**).

So far, if we take into account what the author and readers have done, then there are clearly 6 kinds of *15+12c/19+12c* to fit the conjecture.

In fact, all kinds of *15+12c/19+12c* on an operational route via each of the 6 kinds of *15+12c/19+12c* fit the conjecture, according to the Theorem *1*.

It follows that if *n+1* belongs within any kind of *15+12c/19+12c* derived from №1 satisfactory operational result, then that kind of *15+12c/19+12c* and *n+1* fit the conjecture.

Further, you will find that not only one kind of *15+12c/19+12c* derives from №1 satisfactory operational result, but also at least two kinds of *15+12c/19+12c* derive also from such a satisfactory operational result, such as $15+12(4+2^{55}\times3^{2}y)$ and $15+12(8+2^{32}\times3^{17}y)$ derived from $61+2^{3}\times3^{37}y$.

In some cases, an operational route of *15+12c* and an operational route of *19+12c* can coincide partially or intersect from each other, such as start with $15+12(1+2^{57}y)$ to operate, via five steps to get $19+12(1+2^{54}\times3^{2}y)$.

Due to $c\geq1$, there are infinitely more odd numbers of *15+12c/19+12c*, whether they belong to infinite or finite more kinds, boil down to they can only be on the bunch's operational routes of *15+12c/ 19+12c*. And that variables of kinds of *15+12c/19+12c* within any limits on the bunch's operational routes are either one and the same, or can be converted into a common variable, otherwise, any kind of *15+12c/19+12c* is in the hide always.

As thus, not only allows you to compare the size between an operational result and a kind of *15+12c/19+12c,* but also let you know that operational routes via any kind of *15+12c/19+12c* belong to a small part that forms the bunch of operational routes of *15+12c/19+12c*, whether this kind of

15+12c/19+12c is proven or unproved. However, such a small part is essential too.

So, for every two of the bunch's operational routes, either both directly intersect or indirectly connect, since they can be extended enough.

Naturally, all kinds of *15+12c/19+12c* are on the bunch of operational routes of *15+12c/19+12c* unquestionably.

In the following paragraphs, the author tries to prove that every kind of *15+12c/19+12c* fits the conjecture from opposite directions from each other.

Firstly, start with proven 6 kinds of *15+12c/19+12c* to endlessly expand limits of proven kinds of *15+12c/19+12c.*

According to the Theorem *1*, all kinds of *15+12c/19+12c* on an operational route via each of proven 6 kinds of *15+12c/19+12c* fit the conjecture. Then, all kinds of *15+12c/19+12c* on the 6 operational routes are turned into proven kinds of *15+12c/19+12c.*

According to Theorems *1* and *2* plus the Lemma, kinds of *15+12c/ 19+12c* on an operational route via each and every proven kind of *15+12c/19+12c* and on operational routes whose each intersects directly the operational route or indirectly connects to the operational route, all fit the conjecture. Then, all such kinds of *15+12c/19+12c* are turned into proven kinds of *15+12c/19+12c.*

The rest can be deduced by analogy, then limits of proven kinds of *15+12c/19+12c* are getting bigger and bigger, until all kinds of *15+12c/*

19+12c on the bunch of operational routes of *15+12c/19+12c* are proved to fit the conjecture.

Secondly, the proof in the opposite direction of the above proof is that start with any unproved kind of *15+12c/19+12c* to do continuous operations, according to the rule of operations, in order to encounter or find a №1 satisfactory operational result about the unproved kind of *15+12c/19+12c*.

First of all, how do you present an unproved kind of *15+12c/19+12c* ?

As all kinds of *15+12c/19+12c* can be expressed into *15+12px/19+12px,* where p is an integer, and $p>1$; x is a variable of integer, and $x\neq c$.

So, after p is assigned to an integer >1, if *15+12px/19+12px* is not a proven kind of *15+12c/19+12c,* then it is exactly an unproved kind of *15+12c/19+12c,* merely there are several proven kinds of *15+12c/19+12c* up to now.

As listed above, there are *6* operational routes whose each has a proven kind of *15+12c/19+12c.*

Moreover, all operational routes whose each has a unproved kind of *15+12c/19+12c* and *6* operational routes whose each has a proven kind of *15+12c/19+12c* form the bunch of operational routes of *15+12c/19+12c*, while the correlation between an unproved kind of *15+12c/19+12c* and a proven kind of *15+12c/19+12c* is realized by at least an operational route via both, and there is at least one of the following three situations, such that the unproved kind of *15+12c/19+12c* is proved to fit the conjecture.

(1) An operational route via an unproved kind of *15+12c/19+12c* has an integer's expression that is less than the unproved kind, then the unproved kind of *15+12c/19+12c* fits the conjecture, according to the Theorem *1*.

(2) An operational route via an unproved kind of *15+12c/19+12c* and an operational route via a proven kind of *15+12c/19+12c* intersect, then the unproved kind of *15+12c/19+12c* fits the conjecture, according to the Theorem 2.

(3) An operational route via an unproved kind of *15+12c/19+12c* and an operational route via a proven kind of *15+12c/19+12c* are in the indirect connection, then the unproved kind of *15+12c/19+12c* fits the conjecture, according to the Lemma.

Apply the above established practices on and on, then all unproved kind of *15+12c/19+12c* on the bunch of operational routes of *15+12c/19+12c* are proved to fit the conjecture.

Now that we have done bidirectional proofs, such that all kind of *15+12c/19+12c* on the bunch of operational routes of *15+12c/19+12c* have been proved to fit the conjecture, so if *n+1* belongs within a kind of *15+12c/19+12c* therein, then *n+1* within the kind fits the conjecture.

7. Make a Summary and Reach the Conclusion

To sum up, *n+1* has been proved to fit the conjecture whether *n+1* belongs within what kind of odd numbers, or it is exactly an even number.

We can also prove integers *n+2*, *n+3* etc. up to every integer > *n+1* to fit the conjecture in the light of the old way of doing the thing.

The proof was thus brought to a close. As a consequence, the Collatz conjecture is tenable.

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20 Jul 2022 06:32:58 PDT Alg+NT+Comb+Logi
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