Beal's Conjecture is Tenable

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Abstract

In this article, first classify A, B and C according to their odevity, then, two types of $A^X+B^Y \neq C^Z$ are excluded. Next, several kinds of $A^X+B^Y=C^Z$ under the given requirements are exemplified. After that, $A^X+B^Y \neq C^Z$ is divided into 4 inequalities under the known requirements. And then, by applying the odd-even relations of integers concerned in the symmetry, two inequalities are proved by the mathematical induction. Then again, other two inequalities are too proved by the method of splitting integers and then merging them. Finally, reach the conclusion that Beal's conjecture is tenable via the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y \neq C^Z$ under the given requirements.

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1. Introduction

Beal's conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C

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must have a common prime factor.

The conjecture was discovered by Andrew Beal in 1993. Later, it was announced in December 1997 issue of the Notices of the American Mathematical Society, [1]. However, it remains still a conjecture that has neither been proved nor disproved.

Let us regard limits of values of A, B, C, X, Y and Z in the indefinite equation $A^X+B^Y=C^Z$ as given requirements for indefinite equations and inequalities concerned after this.

2. Selection of Combinations of Values of A, B and C

First, classify A, B and C according to their respective odevity, then, following two types of $A^X+B^Y\neq C^Z$ are excluded:

1) A, B and C are all odd numbers.

2) A, B and C are two even numbers and an odd number.

After that, merely continue to have following two types which contain $A^X+B^Y=C^Z$ under the given requirements:

1) A, B and C are all positive even numbers.

2) A, B and C are two positive odd numbers and one positive even number.

3. Exemplifying $A^X+B^Y=C^Z$ under the Given Requirements

For the indefinite equation $A^X+B^Y=C^Z$ that satisfies aforesaid either qualification, in fact, it has many solutions with A, B and C to positive integers, as shown in following examples.

If A, B and C are all positive even numbers, let A=B=C=2, X=Y≥3, and

$Z=X+1$, so $A^X+B^Y=C^Z$ are changed to $2^X+2^X=2^{X+1}$. Then, $A^X+B^Y=C^Z$ under the circumstances have one set of solution with A, B and C as 2, 2 and 2, and that A, B and C have one common prime factor 2.

In addition, let $A=B=162$, $C=54$, $X=Y=3$ and $Z=4$, so $A^X+B^Y=C^Z$ are changed to $162^3+162^3=54^4$. Then, $A^X+B^Y=C^Z$ under the circumstances have one set of solution with A, B and C as 162, 162 and 54, and that A, B and C have common prime factors 2 and 3.

If A, B and C are two odd numbers and one even number, let $A=C=3$, $B=6$, $X=Y=3$ and $Z=5$, so $A^X+B^Y=C^Z$ are changed to $3^3+6^3=3^5$. Then, $A^X+B^Y=C^Z$ under the circumstances have one set of solution with A, B and C as 3, 6 and 3, and that A, B and C have one common prime factor 3.

In addition, let $A=B=7$, $C=98$, $X=6$, $Y=7$ and $Z=3$, so $A^X+B^Y=C^Z$ are changed to $7^6+7^7=98^3$. Then, $A^X+B^Y=C^Z$ under the circumstances have one set of solution with A, B and C as 7, 7 and 98, and that A, B and C have one common prime factor 7.

It follows that the indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either qualification can be established, but only if there is at least one common prime factor in A, B and C.

4. Divide $A^X+B^Y\neq C^Z$ into Four Inequalities under the Known Requirements

As mentioned above, if we can prove $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor, then the conjecture must be true.

Since A, B and C have a prime factor of 2 in common where A, B and C are all even numbers, so the case where A, B and C have not a common prime factor can only occur if A, B and C are two odd numbers and one even number.

If A, B and C do not have a common prime factor, then any two of them do not have a common prime factor either, because if two have a common prime factor and another do not, then it will result in $A^X+B^Y\neq C^Z$ according to the unique factorization theorem of natural number.

There is no doubt that the following two inequalities, taken together, are sufficient to replace $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C are two odd numbers and one even number without a common prime factor.

1). $A^X+B^Y\neq(2W)^Z$, i.e. $A^X+B^Y\neq 2^Z W^Z$;

2). $A^X+(2W)^Y\neq C^Z$, i.e. $A^X+2^Y W^Y\neq C^Z$.

In above these inequalities, A, B and C are positive odd numbers; $X\geq 3$, $Y\geq 3$, $Z\geq 3$ and $W\geq 1$; and that three terms within each inequality have not a prime factor in common.

Continue to divide $A^X+B^Y\neq 2^Z W^Z$ into the following two inequalities:

(1) $A^X+B^Y\neq 2^Z$;

(2) $A^X+B^Y \neq 2^Z O^Z$.

Continue to divide $A^X+2^Y W^Y \neq C^Z$ into the following two inequalities:

(3) $A^X+2^Y \neq C^Z$;

(4) $A^X+2^Y O^Y \neq C^Z$.

In above four inequalities, A, B, C and O are positive odd numbers; $X\geq 3$, $Y\geq 3$ and $Z\geq 3$; and that three terms in each inequality have not a common prime factor.

And regard above these qualifications as the known requirements for inequalities or indefinite equations concerned after this.

As thus, the proof for $A^X+B^Y \neq C^Z$ under the given requirements can be changed to prove above four inequalities under the known requirements.

5. Main Bases that Prove two Leading Inequalities

Before the proof begins, it is necessary to state some basic conceptions in order to regard them as main bases that prove two leading inequalities.

First of all, at positive half line of the number axis, if any even point is taken as a center of symmetry, then odd points on the left side of the center of symmetry and odd points concerned on the right side are one-to-one bilateral symmetric , [2].

Like that, in the sequence of natural numbers, if any even number is taken as a center of symmetry, then odd numbers smaller than the even number are one-to-one symmetric to the partially odd numbers greater than the even number.

Take any one of $2^{H-1}W^V$ as a center of symmetry, then two distances from the center of symmetry to each other's-symmetric two odd points or two odd numbers are two equilong line segments at positive half line of the number axis or two same differences in the sequence of natural numbers, where H, W and V are integers, and W≥1, H≥3 and V≥1.

Thus, several conclusions can be drawn from the interrelation *inter* three integers concerned in the sequence of natural numbers, *ut infra*.

Conclusion 1˙ The sum of two bilateral symmetric odd numbers is equal to the double of the even number that serves as the center of symmetry, in the sequence of natural numbers.

Conclusion 2˙ The sum of two non-symmetric odd numbers is not equal to the double of the even number that serves as the center of symmetry, in the sequence of natural numbers.

Conclusion 3˙ If the sum of two odd numbers is equal to the double of an even number, then these two odd numbers are in the symmetry whereby the even numbe to serve as the center of symmetry, in the sequence of natural numbers.

Conclusion 4˙ If the sum of two odd numbers is not equal to the double of an even number, then these two odd numbers are not in the symmetry whereby the even number to serve as the center of symmetry, in the sequence of natural numbers.

In addition, any odd number can be represented as one of O^V, where O is an odd number and $V\geq 1$. Also, when V=1 or 2, you can write O^V as $O^{1\sim 2}$.

In following paragraphs, let us prove each of aforementioned four inequalities one by one.

6. Proving $A^X+B^Y\neq 2^Z$ under the Known Requirements

We sue 2^{Z-1} as each center of symmetry about odd numbers to prove $A^X+B^Y\neq 2^Z$ under the known requirements by the mathematical induction.

(1) When Z-1=2, 3, 4, 5 and 6, bilateral symmetric odd numbers on two sides of each symmetric center 2^{Z-1} are listed successively below.

1^6, 3, (2^2), 5, 7, (2^3), 9, 11, 13, 15, (2^4), 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127

As listed above, it can be seen that there are no two of O^V with $V\geq 3$ in two places of each pair of bilateral symmetric odd numbers whereby 2^{Z-1} to serve as each center of symmetry, where Z-1=2, 3, 4, 5 and 6.

So there are $A^X+B^Y\neq 2^3$, $A^X+B^Y\neq 2^4$, $A^X+B^Y\neq 2^5$, $A^X+B^Y\neq 2^6$ and $A^X+B^Y\neq 2^7$ under the known requirements according to have got Conclusion 2.

(2) When Z-1=K with $K\geq 6$, we suppose that there are $A^X+B^Y\neq 2^{K+1}$ under the known requirements.

(3) When Z-1=K+1, we need to prove that there are $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

Proof· Suppose that A^X and B^Y are two bilateral symmetric odd numbers whereby 2^K to serve as the center of symmetry, then there are $A^X+B^Y=2^{K+1}$ according to have got Conclusion 1.

While, there are $A^X+B^Y\neq 2^{K+1}$ under the known requirements in line with second step of the mathematical induction. Namely there are no two of O^V with V≥3 in two places of each pair of bilateral symmetric odd numbers whereby 2^K to serve as the center of symmetry.

So let us tentatively regard A^X as one of O^V with V≥3, and regard B^Y as one of $O^{1\sim 2}$, i.e. let X≥3 and Y=1 or 2.

Taken one with another, if there are $A^X+B^Y=2^{K+1}$, then A^X and B^Y must be two bilateral symmetric odd numbers whereby 2^K to serve as the center of symmetry, and at least one of Y and X in this situation is equal to 1 or 2.

If you change the above requirements even a little, you will inevitably lead to $A^X+B^Y\neq 2^{K+1}$. Vice versa, there are surely $A^X+B^Y=2^{K+1}$ under the known requirements except for Y, and Y=1 or 2.

Then, there are $A^X+(A^X+2B^Y)=2^{K+2}$ under the known requirements except for Y, and Y=1 or 2, so A^X and A^X+2B^Y are surely two bilateral symmetric odd numbers whereby 2^{K+1} to serve as the center of symmetry according to have got Conclusion 3.

But then, since there are $A^X+B^Y\neq 2^{K+1}$ under the known requirements, thus there are $A^X+(A^X+2B^Y)\neq 2^{K+2}$ under the known requirements, then A^X and A^X+2B^Y are not two symmetric odd numbers whereby 2^{K+1} to serve as the center of symmetry, according to have got Conclusion 4.

In any case, the sum of A^X+2B^Y is an odd number, so let A^X+2B^Y be equal to O^E, where O is still an odd number, and E is the exponent of O.

After this substitution, on the one hand, there are $A^X+(A^X+2B^Y)= A^X+O^E=2^{K+2}$ under the known requirements except for Y, and Y=1 or 2, and that A^X and O^E are two bilateral symmetric odd numbers whereby 2^{K+1} to serve as the center of symmetry.

On the other hand, there are $A^X+(A^X+2B^Y)=A^X+O^E\neq 2^{K+2}$ under the known requirements, yet A^X and O^E are not two symmetric odd numbers whereby 2^{K+1} to serve as the center of symmetry. In this case, whatever positive integer E is equal to, including each of E≥3, it can satisfy $A^X+O^E\neq 2^{K+2}$.

Even though there are two of O^E derived from A^X+2B^Y, because values of Y in two A^X+2B^Y are in different limits, i.e. Y≥3 in $A^X+(A^X+2B^Y)= A^X+O^E\neq 2^{K+2}$ and Y=1 or 2 in $A^X+(A^X+2B^Y)=A^X+O^E=2^{K+2}$, so A^X+2B^Y within $A^X+(A^X+2B^Y)=A^X+O^E\neq 2^{K+2}$ are greater than A^X+2B^Y within $A^X+(A^X+2B^Y) =A^X+O^E=2^{K+2}$. That is to say, O^E within $A^X+O^E\neq 2^{K+2}$ is greater than O^E within $A^X+O^E=2^{K+2}$.

When A^X within $A^X+O^E\neq 2^{K+2}$ and A^X within $A^X+O^E=2^{K+2}$ are identical one and O within $A^X+O^E\neq 2^{K+2}$ is equal to O within $A^X+O^E=2^{K+2}$, E within

$A^X+O^E\neq 2^{K+2}$ is greater than E within $A^X+O^E=2^{K+2}$ surely.

Thus it can be seen, values of E within $A^X+O^E\neq 2^{K+2}$ both contain $E\geq 3$ and are all greater than values of E within $A^X+O^E=2^{K+2}$.

As stated, now that there are $A^X+B^Y=2^{K+1}$ under the known requirements except for Y, and Y=1 or 2, then likewise deduce $A^X+O^E=2^{K+2}$ under the known requirements except for E, and E=1 or 2 by the same method.

Or rather, E within $A^X+O^E=2^{K+2}$ can only be equal to 1 or 2 due to have supposed $X\geq 3$ at the head. Yet, for E within $A^X+O^E\neq 2^{K+2}$ under the known requirements, if A^X and O^E are not in the symmetry whereby 2^{K+1} to serve as the center of symmetry, then O^E can be any odd number; if A^X and O^E are in the symmetry whereby 2^{K+1} to serve as the center of symmetry, then allow only $E\geq 3$ in which case $X\geq 3$, since when E=1 or 2, it can lead to $A^X+O^E=2^{K+2}$.

For the inequality $A^X+O^E\neq 2^{K+2}$, substitute B for O, since either B or O can express every positive odd number; in addition, substitute Y for E where $E\geq 3$, and $Y\geq 3$, then we get $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

In this proof, if B^Y is one of O^V with $V\geq 3$, then A^X is surely one of $O^{1\sim 2}$, or A^X and B^Y are two of $O^{1\sim 2}$, and yet conclusions concluded finally from both are one and the same with $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

So much for, the author has proven that when Z-1=K+1 with $K\geq 6$, there are $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

By the preceding way, we can continue to prove that when Z-1=K+2, K+3…up to each of integers greater than K+1, there are $A^X+B^Y\neq 2^{K+3}$, $A^X+B^Y\neq 2^{K+4}$ … up to general $A^X+B^Y\neq 2^Z$ under the known requirements.

7. Proving $A^X+B^Y\neq 2^ZO^Z$ under the Known Requirements

Use $2^{Z-1}O^Z$ as the center of symmetry about odd numbers concerned to prove successively $A^X+B^Y\neq 2^ZO^Z$ under the known requirements by the mathematical induction, and emphasize that O≥3.

(1) When O=1, $2^{Z-1}O^Z$ i.e. 2^{Z-1}, as has been proved, there are $A^X+B^Y\neq 2^Z$ under the known requirements, in №6 section.

(2) When O=J and J is an odd number≥1, $2^{Z-1}O^Z$ i.e. $2^{Z-1}J^Z$, we suppose that there are $A^X+B^Y\neq 2^ZJ^Z$ under the known requirements.

(3) When O=K and K=J+2, $2^{Z-1}O^Z$ i.e. $2^{Z-1}K^Z$, we need to prove that there are $A^X+B^Y\neq 2^ZK^Z$ under the known requirements.

Proof· Under the premise of X≥3, suppose that A^X and B^Y are two bilateral symmetric odd numbers whereby $2^{Z-1}J^Z$ to serve as the center of symmetry, then there are $A^X+B^Y=2^ZJ^Z$ according to have got Conclusion 1.

And yet, there are $A^X+B^Y\neq 2^ZJ^Z$ under the known requirements in line with second step of the mathematical induction.

Obviously there are $A^X+B^Y=2^ZJ^Z$ under the known requirements except for Y, and Y=1 or 2.

Thus, there are $A^X+[B^Y+2^Z(K^Z-J^Z)]=(A^X+B^Y)+2^ZK^Z-2^ZJ^Z=2^ZK^Z$ under the known requirements except for Y, and Y=1 or 2, and that A^X and

$B^Y+2^Z(K^Z-J^Z)$ are two bilateral symmetric odd numbers whereby $2^{Z-1}K^Z$ to serve as the center of symmetry according to have got Conclusion 3.

In addition, there are $A^X+B^Y\neq 2^ZJ^Z$ under the known requirements, hereby, we conclude $A^X+[B^Y+2^Z(K^Z-J^Z)]=(A^X+B^Y)+2^ZK^Z-2^ZJ^Z\neq 2^ZK^Z$ under the known requirements, so A^X and $B^Y+2^Z(K^Z-J^Z)$ are not two symmetric odd numbers whereby $2^{Z-1}K^Z$ to serve as the center of symmetry according to have got Conclusion 4.

In that case, let the odd number $B^Y+2^Z (K^Z-J^Z)$ be equal to D^E, where D is a positive odd number, and E is the exponent of D.

After this substitution, on the one hand, there are $A^X+[B^Y+2^Z(K^Z-J^Z)]= A^X+D^E=2^ZK^Z$ under the known requirements except for Y, and Y=1 or 2, and that A^X and D^E are two bilateral symmetric odd numbers whereby $2^{Z-1}K^Z$ to serve as the center of symmetry.

On the other hand, there are $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E\neq 2^ZK^Z$ under the known requirements, so A^X and D^E are not two symmetric odd numbers whereby $2^{Z-1}K^Z$ to serve as the center of symmetry. In this case, whatever positive integer E is equal to, including each of E≥3, it can satisfy $A^X+D^E\neq 2^ZK^Z$.

Even though there are two of D^E derived from $B^Y+2^Z(K^Z-J^Z)$, because values of Y in two $B^Y+2^Z(K^Z-J^Z)$ are in different limits, i.e. Y≥3 in $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E\neq 2^HK^Z$ and Y=1 or 2 in $A^X+[B^Y+2^Z(K^Z-J^Z)]= A^X+D^E=2^ZK^Z$, so $B^Y+2^Z(K^Z-J^Z)$ within $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E\neq 2^ZK^Z$

are greater than $B^Y+2^Z(K^Z-J^Z)$ within $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E=2^ZK^Z$.

Namely D^E within $A^X+D^E\neq 2^ZK^Z$ is greater than D^E within $A^X+D^E=2^ZK^Z$.

When A^X within $A^X+D^E\neq 2^ZK^Z$ and A^X within $A^X+D^E=2^ZK^Z$ are identical one and D within $A^X+D^E\neq 2^ZK^Z$ is equal to D within $A^X+D^E=2^ZK^Z$, E within $A^X+D^E\neq 2^ZK^Z$ is greater than E within $A^X+D^E=2^ZK^Z$ surely.

Thus it can be seen, values of E within $A^X+D^E\neq 2^ZK^Z$ both contain $E\geq 3$ and are greater than values of E within $A^X+D^E=2^ZK^Z$.

As stated, now that there are $A^X+B^Y=2^ZJ^Z$ under the known requirements except for Y, and Y=1 or 2, then likewise deduce $A^X+D^E=2^ZK^Z$ under the known requirements except for E, and E=1 or 2, by the same method.

Or rather, E within $A^X+D^E=2^ZK^Z$ can only be equal to 1 or 2 due to we have supposed $X\geq 3$ at the head. Yet, for E within $A^X+D^E\neq 2^ZK^Z$ under the known requirements, if A^X and D^E are not in the symmetry whereby $2^{Z-1}K^Z$ to serve as the center of symmetry, then D^E can be any odd number; if A^X and D^E are in the symmetry whereby $2^{Z-1}K^Z$ to serve as the center of symmetry, then allow only $E\geq 3$ in which case $X\geq 3$, since when E=1 or 2, it can lead to $A^X+D^E=2^ZK^Z$.

For the inequality $A^X+D^E\neq 2^ZK^Z$, substitute B for D, since either B or D can express each of positive odd numbers; in addition, substitute Y for E where $E\geq 3$, and $Y\geq 3$, then we get $A^X+B^Y\neq 2^ZK^Z$ under the known requirements.

In this proof, if B^Y is one of O^V with $V\geq 3$, then A^X is one of $O^{1\sim 2}$ surely, or

A^X and B^Y are two of $O^{1\sim2}$, and yet conclusions concluded finally from both are one and the same with $A^X+B^Y\neq2^ZK^Z$ under the known requirements.

On balance, the author has proven $A^X+B^Y\neq2^ZK^Z$ with K=J+2 under the known requirements.

By the preceding way, we can continue to prove that when O=J+4, J+6… up to each of odd numbers greater than J+2, there are $A^X+B^Y\neq2^Z(J+4)^Z$, $A^X+B^Y\neq2^Z(J+6)^Z$ up to general $A^X+B^Y\neq2^ZO^Z$ under the known requirements.

8. Proving $A^X+2^Y\neq C^Z$ under the Known Requirements

Proof· If you want to transform A^X+2^Y into one of O^V with V≥3, an efficient way to do that is to divide either item within A^X+2^Y into the sum of some same powers, after that, apportion averagely another item in accordance with the number of the same powers to each of the same powers. If a number after the apportionment has not a fractional part, then the sum of the number plus one power may form one of O^V with V≥3. If a number after the apportionment has a fractional part, then the sum of the number plus one power can not form one of O^V with V≥3.

Here reconfirm that A^X and 2^Y have not a common prime divisor.

Then, from $A^X= A^{X-K} + A^{X-K} + A^{X-K} +\ldots = A^K (A^{X-K})$ where K is an integer and X>K≥1, we get that the number of A^{X-K} is A^K, and A^K is an odd number.

At present, we divide 2^Y into A^K parts, i.e. $2^Y/A^K$. It is obvious that $2^Y/A^K$ is not an integer, so the sum of $A^{X-K} +2^Y/A^K$ is not an integer either.

On the other, from $2^Y=2^{Y-K}+2^{Y-K}+2^{Y-K}+\ldots=2^K(2^{Y-K})$ where K is an integer and $Y>K\geq 1$, we get that the number of 2^{Y-K} is 2^K, and 2^K is an even number. Like that, we divide A^X into 2^K parts, i.e. $A^X/2^K$. Yet $A^X/2^K$ is not an integer, so the sum of $2^{Y-K}+A^X/2^K$ is not an integer either.

Either the sum of $A^{X-K}+2^Y/A^K$ or the sum of $2^{Y-K}+A^X/2^K$ is not an integer, then $(A^{X-K}+2^Y/A^K)^N$ or $(2^{Y-K}+A^X/2^K)^N$ is not an integer either, where N is an integer$\geq$1. Nevertheless, C^Z is an integer.

That is to say, there are $A^X+2^Y=(A^{X-K}+2^Y/A^K)^R\neq C^Z$ or $A^X+2^Y=(2^{Y-K}+A^X/2^K)^R\neq C^Z$ in which case R is equal to a corresponding value that matches K, such that $A^X+2^Y=(A^{X-K}+2^Y/A^K)^R$ or $A^X+2^Y=(2^{Y-K}+A^X/2^K)^R$ exist. Obviously, R is not an integer here, since $2^Y/A^K$ or $A^X/2^K$ is not an integer.

Therefore, there are $A^X+2^Y\neq C^Z$ under the known requirements.

9. Proving $A^X+2^YO^Y\neq C^Z$ under the Known Requirements

***Proof*·** As previously done, divide either item within $A^X+2^YO^Y$ into the sum of some same powers, after that, apportion averagely another item in accordance with the number of the same powers to each of the same powers. If a number after the apportionment has not a fractional part, then the sum of the number plus one power may form one of O^V with $V\geq 3$. If a number after the apportionment has a fractional part, then the sum of the number plus one power can not form one of O^V with $V\geq 3$.

First we reconfirm that 2^YO^Y and A^X are two co-prime integers according to the previous rule, so there is no integral multiple's relation between $2^Y O^Y$ and A^X.

Then, from $A^X = A^{X-K} + A^{X-K} + A^{X-K} + \ldots = A^K (A^{X-K})$ where K is an integer and $X > K \geq 1$, we get that the number of A^{X-K} is A^K, and A^K is an odd number.

At present, we divide $2^Y O^Y$ into A^K parts, i.e. $2^Y O^Y / A^K$. It is obvious that $2^Y O^Y / A^K$ is not an integer, so the sum of $A^{X-K} + 2^Y O^Y / A^K$ is not an integer.

On the other, from $2^Y O^Y = 2^{Y-K} O^{Y-K} + 2^{Y-K} O^{Y-K} + 2^{Y-K} O^{Y-K} + \ldots = 2^K O^K (2^{Y-K} O^{Y-K})$ where K is an integer and $Y > K \geq 1$, we get that the number of $2^{Y-K} O^{Y-K}$ is $2^K O^K$, and $2^K O^K$ is an even number.

Like that, we divide A^X into $2^K O^K$ parts, i.e. $A^X / 2^K O^K$. Yet $A^X / 2^K O^K$ is not an integer, so the sum of $2^{Y-K} O^{Y-K} + A^X / 2^K O^K$ is not an integer either.

Either the sum of $A^{X-K} + 2^Y O^Y / A^K$ or the sum of $2^{Y-K} O^{Y-K} + A^X / 2^K O^K$ is not an integer, then $(A^{X-K} + 2^Y O^Y / A^K)^N$ or $(2^{Y-K} O^{Y-K} + A^X / 2^K O^K)^N$ is not an integer either, where N is an integer ≥ 1. Nevertheless, C^Z is an integer.

That is to say, there are $A^X + 2^Y O^Y = (A^{X-K} + 2^Y O^Y / A^K)^R \neq C^Z$ or $A^X + 2^Y O^Y = (2^{Y-K} O^{Y-K} + A^X / 2^K O^K)^R \neq C^Z$ in which case R is equal to a corresponding value that matches K, such that $A^X + 2^Y O^Y = (A^{X-K} + 2^Y O^Y / A^K)^R$ or $A^X + 2^Y O^Y = (2^{Y-K} O^{Y-K} + A^X / 2^K O^K)^R$ exist. Obviously, R is not an integer here, since $2^Y O^Y / A^K$ or $A^X / 2^K O^K$ is not an integer.

Therefore, there are $A^X + 2^Y O^Y \neq C^Z$ under the known requirements.

10. Make A Summary and Reach the Conclusion

To sum up, the author has proven every kind of $A^X + B^Y \neq C^Z$ under the given requirements plus which A, B and C have not a common prime factor in №6, №7, №8 and №9 sections.

Then again, the author has given examples to have proven $A^X+B^Y=C^Z$ under the given requirements plus which A, B and C have at least a common prime factor in №3 section.

In these circumstances, so long as make a comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements, we can immediately reach such a conclusion that an indispensable prerequisite of the existence of $A^X+B^Y=C^Z$ under the given requirements is the very which A, B and C must have a common prime factor.

The proof was thus brought to a close. As a consequence, Beal's conjecture is tenable.

11. Proving Fermat's Last Theorem from Proven Beal's Conjecture

Fermat's last theorem is a special case of Beal's conjecture, [3]. If Beal's conjecture turns out to be true, then let X=Y=Z, so $A^X+B^Y=C^Z$ are going to be changed to $A^X+B^X=C^X$.

Furthermore, you divide three terms of $A^X+B^X=C^X$ by greatest common divisor of the three terms, then you will get a set of solution of positive integers without common prime factor.

Obviously, the conclusion is in contradiction with proven Beal's conjecture.

As thus, we have proved Fermat's last theorem by reduction to absurdity as easy as pie.

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