Notes on the Collatz conjecture II.

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August 2021

0- Abstract:

This paper is a resume of my investigation on the Collatz conjecture. We take a look to the hypothesis of infinite systems of iterations or combined iterations.

1- Introduction:

This is the second part paper, in the first part [1] I analyzed the different forms in the develop in the way of the functions. I stated two possibilities "Ideal conditions" and "Non-ideal conditions". Now we are going more away of these ideas, based in the classical form of the two possible modification functions and taking the definition form the statement of the problem of Wikipedia [2], we will call A to the $\frac{n}{2}$ if $n \equiv 0 (mod\, 2)$ function and B to the $3n+1$ if $n \equiv 1 (mod\, 2)$ function.

2- Best case possible.

First of all, we are going to see the best case, in which we have an iteration of A function. We can define this using the next formula:

$$(1.1) \quad 2n_m = 2n_{(m-1)} = 2n_{(m-2)} = \ldots = 2n_2 = 2n_1 = 2n_0$$

For $n \in \mathbb{N}$ and $m = \infty$
We can be more precise and define how this formula work with the following information:

$$(1.2) \quad n_a = 2n_{(a-1)}$$

For $a \in \mathbb{N}$
As we can see it does not matter when you start in this iteration you always end in 2 and then in number 1 in the fastest way possible.

3- Normal case.

A normal case in Collatz conjecture implies A and B function in different combinations, it will return you a 2 and then a final number 1 if and only if you have more A functions ($\frac{n}{2}$) than B functions ($3n+1$). This can be proved in a very simple way, if you start from a number 9: $9 \cdot 3 + 1 = 28 \Rightarrow 28 \div 2 = 14 \Rightarrow 14 \div 2 = 7$ And here we have the proof just seeing that 7 < 9. So in this case (as in any other case) if you have two consecutive A functions the end number is lower than the number you have before one B function. Generalizing $a \cdot 3 + 1 = b \Rightarrow b \div 2 = \frac{b}{2} \Rightarrow \frac{b}{2} \div 2 = \frac{b}{4} = c$ in any numbers c < a.

4- Worst case.

In this case you have the same A functions and B functions and there are combined one kind next of the other kind. This can be represented by the next formula:

$$(2.1) \quad 3n_1 + 1 = 2n_2 = 3n_2 + 1 = 2n_3 = 3n_3 + 1 = ... = 2n_m = 3n_m + 1 \simeq \infty$$

For $n \in \mathbb{N}$ and $m = \infty$

In this case, we need to restrict the formula to:

$$(2.2) \quad n_a = \frac{3n_{(a-1)} + 1}{2}$$

For $a \in \mathbb{N}$

The real question here is if this “dance” can be extended until infinity if we start in a finite number n. Whats more, There is a certain number n that necessarily prolongs this “dance” to infinity?.

5- References:

[1] Notes on the Collatz Conjecture. Millas Vera, Juan Elias. https://vixra.org/abs/2103.0198

[2] https://en.wikipedia.org/wiki/Collatz_conjecture