Redundant Primes In Lemoine's Conjecture

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Abstract

Lemoine's conjecture (LC), still unsolved, states that all positive odd integer ≥ 7 can be expressed as the sum of a prime and an even semiprime. But do we need all primes to satisfy this conjecture? This work is devoted to selection of must-have primes and formulation of stronger version of LC with reduced set of primes.

1 Problem statement

Lemoine's conjecture (LC) [1] asserts that all positive odd integer $n \geq 7$ can be expressed as the sum of a prime p and an even semiprime $2q$: $n = 2m+1 = p+2q$ and let's call such pair (p,q) a Lemoine's Partition (LP) of n. Let's denote this relation as $LC(2m+1, p, 2q)$. Then LC can be written as (1):

$$\forall_{x>2, x\in\mathbb{N}} \exists_{p_1, p_2 \in \mathbb{P}} LC(2x+1, p_1, 2\times p_2) \quad (1)$$

But maybe we can formulate much stronger version of (1)? The set of prime numbers $\mathbb{P}$ is dense, number of $LP(n)$ is increasing with increasing n, thus question if stronger version LC is possible is legitimate (2):

$$\forall_{x>2, x\in\mathbb{N}} \exists_{p_1, p_2 \in \mathbb{R}} LC(2x+1, p_1, 2\times p_2) \quad (2)$$

where $|\mathbb{R}| < |\mathbb{P}|$. By design $\mathbb{R}$ contains primes only.

2 Algorithm

Elimination of primes must start from the fact that every prime is potentially required (Lemma 1).

Lemma 1. *Every prime is present is some Lemoine's partition of odd number.*

Proof. Let's assume that exists such prime which is not present in LC - in other words let's assume that exists such pair of primes (p,q) which is not producing odd number $n = 2p+q$. LC is for $n \geq 7$ so we eliminate $(2,2)$ as not applicable. Let's check all remaining cases. A) If both p and q are odd, then $n = 2p+q$ is odd, so all such primes are allowed and the smallest case $(3,3)$ gives $n = 9$, so again we are good. B) If $p = 2$ and q is odd prime then $n = 2p+q$ is odd too,with the smallest $n = 7$ for $(2,3)$. As a result in A) and B) we have covered all allowed pairs of primes (p,q) and in fact all primes ≥ 2 which is in contrary with initial assumption. □

Further elimination of primes from $\mathbb{P}$ to build $\mathbb{R}$ requires appropriate algorithm A which is able to resign from a given prime, even if it is present in some LPs. Such algorithm A could look as follows:

1. Let $\mathbb{R}$ is empty, a set of $\mathbb{R}_i$ is empty and $n = 5$. Let's assume that we break calculations at $n = n_{max} > 5$.
2. **New turn**: $n = n+2$.
3. Break calculations if $n > n_{max}$.
4. If $\mathbb{R}$ is sufficient to fulfill LC for all odd numbers q ($7 \leq q \leq n$), then go to **New turn**.
5. If we have a set of $\mathbb{R}_i$ and any $\mathbb{R}_j$ belonging to this set is sufficient to fulfill LC for all odd numbers q ($7 \leq q \leq n$), then $\mathbb{R} = \mathbb{R}_j$, we forget all $\mathbb{R}_i$ and go to **New turn**.
6. If not, find all $LP(n)$ and build as many candidates for $\mathbb{R}$ (let's call them $\mathbb{R}_i$) as required. As a base use either (as a first choice) $\mathbb{R}$ (if $\mathbb{R}_i$ does not exist) or all previous $\mathbb{R}_i$ (if they are present).
7. Go to **New turn**.

3 Results

Table 1 presents the very first rounds of algorithm eliminating primes required to satisfy LC - it is depicting current values of both $\mathbb{R}$ and $\mathbb{R}_i$, plus additional set $\mathbb{E}$ which contains primes that were present in LPs so far but can be eliminated without hurting LC.

Table 1: Results of the first rounds of A

n	$LP(n)$	$\mathbb{R}$	$\mathbb{R}_i$	$\mathbb{E}$
7	2×2+3	{2,3}	∅	∅
9	2×2+5 2×3+3	{2,3}	∅	{5}
11	2×2+7 2×3+5	∅	{2,3,7} {2,3,5}	∅
13	2×3+7 2×5+3	∅	{2,3,7} {2,3,5}	∅
15	2×2+11 2×5+5	{2,3,5}	∅	{7,11}

For instance, if we take into account all positive odd numbers $5 < n < 16$, then we need a set of primes $\{2,3,5\}$ to satisfy LC for all n checked so far and $\{7,11\}$ ($LC(11,2,7)$, $LC(13,3,7)$, $LC(15,2,11)$) are our candidates for elimination.

Figures 1, 2 and 3 are depicting findings after analyzing odd numbers $7 \leq n \leq 2.5 \times 10^4 + 1$. First of all, function of required primes increases but slower than $\Pi(n)$.

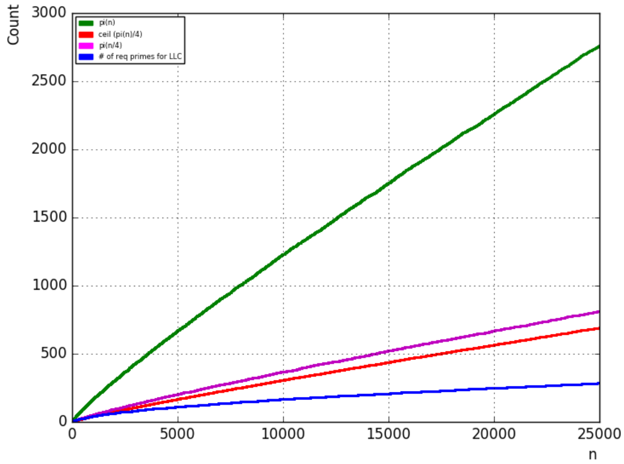

Figure 1: Number of required primes for LC vs. other functions ($7 \leq n \leq 2.5 \times 10^4 + 1$)

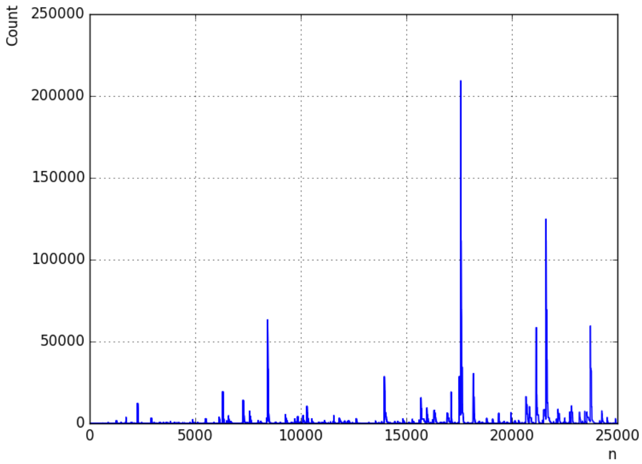

Figure 2: Number of $\mathbb{R}_i$ ($7 \leq n \leq 2.5 \times 10^4 + 1$)

It is also interesting that algorithm A has sporadic congestions in terms of number of $\mathbb{R}_i$ - generally its number at a given time is low but there are quite frequent situation when it is high, even exceeding 200000 (this means that we have 200000+ subsets containing candidates for $\mathbb{R}$) - it happens when there are still some $\mathbb{R}_i$ and new n requires new prime to be used which is multiplying number of $\mathbb{R}_i$ in the next round of A. Surprisingly, shortly after number of $\mathbb{R}_i$ is decreasing to much smaller values. Congestions could be better handled (from memory utilization standpoint) if in A instead of seperate lists (paths) we have a tree (where branches/leaves represent variable part added on top of a common base).

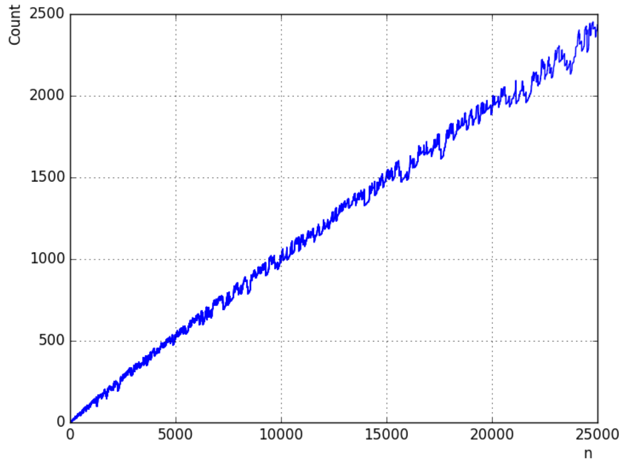

Figure 3: Number of eliminated primes ($7 \leq n \leq 2.5 \times 10^4 + 1$)

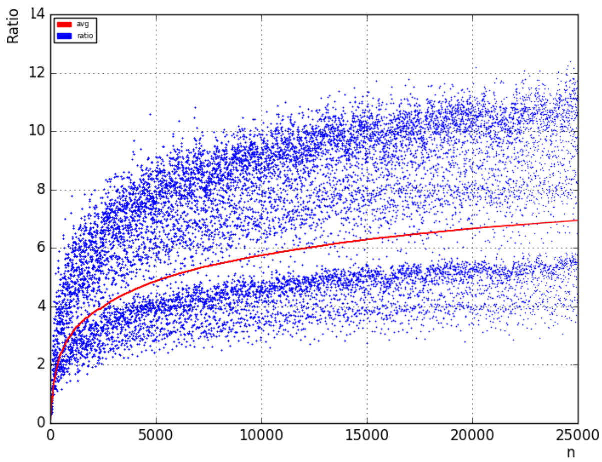

Figure 4: Ratio of number of eliminated primes to number of LPs, including average value ($7 \leq n \leq 2.5 \times 10^4 + 1$)

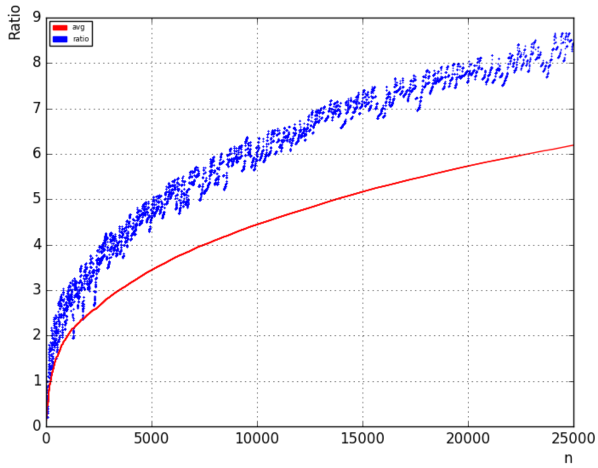

Figure 5: Ratio of number of eliminated primes to number of required primes, including average value ($7 \leq n \leq 2.5 \times 10^4 + 1$)

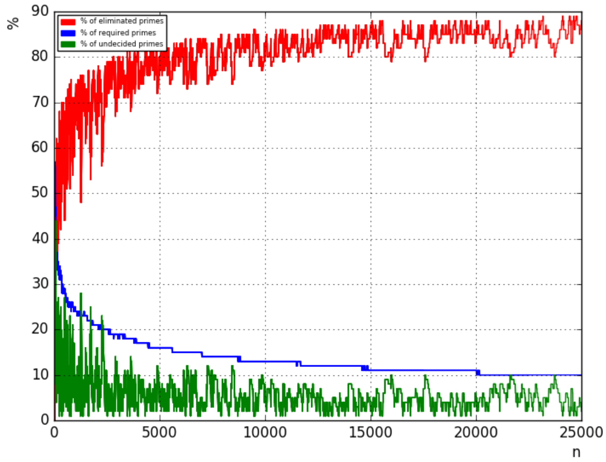

Figure 6: Percentages of required, eliminated and undecided primes to all primes possible at a given point ($7 \leq n \leq 2.5 \times 10^4 + 1$)

4 Summary and next steps

Executed experiments, run for all odd $n \leq 2.5 \times 10^4 + 1$, confirmed that we do not need entire set of primes to satisfy LC. Appendix A lists eliminated primes after checking all odd numbers $7 \leq n \leq 2.5 \times 10^4 + 1$ - the smallest eliminated prime is 11. Of course, exercised cases do not proof that eventually such set exists for all even n but observed trends (Figure 3, Figure 4, Figure 5, Figure 6) give strong foundation that such set exists indeed and conjecture (2) is true.

References

[1] Emile Lemoine, *L'intermédiare des mathématiciens* 1 (1894), 179; ibid 3 (1896), 151.

A List of redundant primes

Based on empirical verification done for all odd numbers $7 \leq n \leq 2.5 \times 10^4 + 1$:

[11, 17, 19, 41, 43, 61, 83, 89, 101, 103, 109, 113, 127, 139, 149, 151, 157, 167, 191, 197, 227, 233, 257, 263, 269, 271, 277, 283, 307, 313, 347, 373, 383, 397, 409, 419, 421, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 509, 541, 557, 563, 571, 577, 587, 599, 601, 617, 619, 631, 641, 653, 659, 673, 677, 691, 701, 719, 727, 733, 739, 743, 751, 769, 773, 787, 811, 827, 839, 857, 859, 863, 877, 881, 887, 907, 911, 919, 941, 953, 967, 971, 977, 983, 991, 997, 1013, 1021, 1031, 1033, 1039, 1051, 1063, 1069, 1087, 1093, 1097, 1103, 1109, 1123, 1129, 1151, 1181, 1187, 1193, 1213, 1217, 1223, 1229, 1231, 1249, 1259, 1277, 1279, 1283, 1289, 1291, 1303, 1307, 1319, 1321, 1327, 1367, 1373, 1409, 1427, 1429, 1439, 1447, 1451, 1471, 1481, 1483, 1487, 1489, 1493, 1511, 1531, 1553, 1559, 1567, 1571, 1579, 1597, 1601, 1607, 1609, 1613, 1619, 1627, 1637, 1657, 1663, 1667, 1693, 1697, 1699, 1709, 1721, 1723, 1741, 1747, 1777, 1783, 1787, 1823, 1847, 1861, 1867, 1871, 1873, 1877, 1889, 1901, 1907, 1913, 1931, 1933, 1951, 1973, 1979, 1987, 1993, 2003, 2011, 2017, 2027, 2029, 2039, 2053, 2069, 2081, 2083, 2087, 2089, 2099, 2111, 2113, 2129, 2137, 2141, 2143, 2161, 2179, 2203, 2207, 2213, 2221, 2237, 2239, 2243, 2251, 2267, 2269, 2281, 2293, 2297, 2333, 2339, 2341, 2347, 2351, 2357, 2371, 2377, 2383, 2389, 2393, 2411, 2417, 2423, 2437, 2447, 2459, 2467, 2473, 2477, 2503, 2521, 2531, 2539, 2543, 2549, 2551, 2557, 2579, 2591, 2593, 2609, 2617, 2633, 2647, 2657, 2659, 2663, 2677, 2683, 2687, 2689, 2693, 2699, 2707, 2713, 2731, 2741, 2749, 2753, 2767, 2789, 2797, 2819, 2833, 2837, 2843, 2851, 2857, 2861, 2879, 2887, 2897, 2903, 2909, 2917, 2927, 2939, 2953, 2957, 2963, 2969, 2999, 3001, 3011, 3037, 3041, 3049, 3061, 3067, 3083, 3089, 3109, 3137, 3163, 3169, 3181, 3187, 3191, 3203, 3217, 3221, 3229, 3257, 3259, 3301, 3307, 3313, 3319, 3323, 3329, 3331, 3343, 3347, 3359, 3361, 3373, 3391, 3407, 3413, 3433, 3449, 3461, 3463, 3467, 3469, 3491, 3499, 3511, 3517, 3527, 3533, 3539, 3541, 3559, 3571, 3581, 3593, 3607, 3613, 3617, 3623, 3631, 3637, 3643, 3671, 3673, 3677, 3691, 3701, 3709, 3719, 3727, 3767, 3769, 3779, 3793, 3797, 3803, 3821, 3823, 3833, 3847, 3851, 3853, 3863, 3911, 3917, 3923, 3929, 3931, 3943, 3947, 3967, 3989, 4001, 4003, 4019, 4021, 4027, 4049, 4051, 4057, 4073, 4091, 4099, 4111, 4127, 4129, 4133, 4157, 4159, 4201, 4211, 4217, 4229, 4231, 4241, 4253, 4259, 4261, 4271, 4273, 4283, 4297, 4327, 4337, 4349, 4357, 4363, 4391, 4397, 4421, 4423, 4441, 4447, 4451, 4457, 4481, 4483, 4493, 4507, 4513, 4517, 4519, 4523, 4547, 4549, 4561, 4567, 4583, 4591, 4597, 4621, 4637, 4639, 4643, 4649, 4657, 4673, 4679, 4703, 4721, 4723, 4751, 4759, 4783, 4789, 4793, 4799, 4801, 4813, 4817, 4861, 4871, 4877, 4889, 4903, 4909, 4919, 4933, 4937, 4943, 4951, 4967, 4969, 4973, 4987, 4993, 4999, 5003, 5009, 5011, 5021, 5023, 5039, 5051, 5077, 5081, 5087, 5107, 5113, 5119, 5147, 5153, 5167, 5171, 5179, 5189, 5209, 5227, 5231, 5233, 5261, 5273, 5279, 5281, 5297, 5333, 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