FORMAL-LOGICAL ANALYSIS
OF THE STARTING POINT OF MATHEMATICAL LOGIC

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Abstract. The critical analysis of the starting point of mathematical (symbolic) logic is proposed. Methodological basis of the analysis is the unity of formal logic and rational dialectics. It is shown that mathematical logic represents incorrect mathematical theory. Arguments are as follows. (1) Pure mathematics operates on mathematical quantity. Pure mathematics and mathematical (symbolic) logic ignore the correct methodological basis (i.e., criterion of truth) of science. (2) Mathematics and mathematical (symbolic) logic abolishes (deletes) essence (central point) of formal logic: concept and proposition (as a logical form of verbal expression (statement) of thought). But thought without words does not exist. If concepts and propositions as meaning contents are removed from consideration, then formal logic is destroyed. (3) Mathematical (symbolic) logic replaces proposition by the property of proposition: “truth” or “false”. But the concepts “proposition” and “property of proposition” are not identical. In this case, the formal logic is destroyed. In addition, the concepts “proposition” and “property of proposition” do not represent mathematical quantities, and the concepts cannot be in mathematical formalism. (4) The starting point of mathematical (symbolic) logic represents several symbols that connect (join, unite) the words “truth” and “false” in truth tables. But words “truth” and “false” do not represent mathematical quantities, and words “truth” and “false” cannot be in mathematical formalism. The symbols are erroneously called propositional connectives. The symbols denote the words “negation”, “and”, “or”, “if, then”, “if and only if”, etc. (5) The symbols of common truth-functional operations are not mathematical symbols (i.e., symbols of mathematical operations). The definition of symbols is based on set theory. But set theory is an erroneous theory. Therefore, the use of these symbols in mathematical (quantitative) expressions is a gross mistake. (6) The essences of formal logic and mathematics are different. The qualitative aspect (i.e., meaning content) is the essence of formal logic; the quantitative (i.e., numerical) aspect is the essence of mathematics. Since formal logic has no quantitative aspect, and mathematics has no qualitative aspect, the union (join, conjunction, combination, synthesis) of formal logic and mathematics is a gross methodological error.

Thus, mathematical logic is a thoughtless, absurd theory in science.

Keywords: formal logic, mathematical logic, pure mathematics, philosophy of mathematics, methodology of mathematics, history of mathematics, higher education; history of science;, dialectics, epistemology.

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Introduction

The truth or falsity of scientific theories depends on the methodological basis used. The correct methodological basis (as the criterion of truth) of science is the unity of formal logic and rational dialectics. If a theory is based on the correct methodological basis, then the theory satisfies the criterion of truth. In this case, the theory is true. If a theory does not meet the criterion of truth, then the theory is false. The truth (validity, verity) of the theory is the property of the theory. The falsity of the theory means the absence of this property.

Mathematics in initial point and in the final analysis is quantitative science (quantitative method of cognition): the science of numbers and calculations. Pure mathematics operates on mathematical quantity and abstract numbers. Mathematical quantity has no qualitative determinacy. Therefore, abstract numbers have no names and represent the values of the mathematical quantity.

Mathematical quantity and abstract numbers have no measure (measure is the philosophical category that designates inseparable unity (connection) of the qualitative and quantitative determinacy of an object) [1-48]. Consequently, mathematical quantity and abstract numbers cannot be used to describe reality. As is shown in works [1-48], pure mathematics (including set theory) is not based on the correct methodological basis. This signifies that pure mathematics (including set theory) is not a correct science.

As is known, mathematical (symbolic) logic as a branch of modern pure mathematics represents the following: (a) the unity of pure mathematics and formal logic; (b) "a subfield of mathematics exploring the applications of formal logics to mathematics" (Wikipedia); (c) mathematical representation (formulation) of formal logic; (d) mathematical methods of research of ways of reasoning (conclusions); (e) a mathematical theory of deductive methods of reasoning. Mathematical logic (symbolic logic) as a theory was created and developed by famous mathematicians (George Boole, Augustus De Morgan, Giuseppe Peano, Ernst Zermelo, David Hilbert, Kurt Gödel, Abraham Robinson, Paul Cohen, Gottlob Frege, Charles Peirce, Bertrand Russell and others) in the 19-20 centuries [49-59]. But this historical fact is not a scientific proof of the truth of mathematical logic as a theory. To date, there are no scientific works in the world literature, devoted to the analysis of this theory within the correct methodological basis.

The purpose of this work is to analyze the starting point of mathematical logic within the correct methodological basis: the unity of formal logic and rational dialectics. Reliable sources of methodological basis are courses in formal logic and rational dialectics.

1. The essence of formal logic

1) By definition, formal logic is the science of the laws of correct thinking. The starting point and fundamental element of formal logic is a concept. A concept is a form of thought that expresses the essential features of objects and phenomena. A concept is expressed in a word or in several words (grammatical sentences). Concepts (thoughts) cannot be expressed without words and grammatical sentences.

2) The basis of formal logic is a system of concepts. The connection of concepts forms the structure of the system. The connection of concepts is expressed by the following words: "is", "is not", "if… is…, then…", "if… is not…, then…", "consequently".

3) Proposition as a logical form of verbal expression (utterance) of thought is the essence of formal logic. The definition of proposition is the following: proposition is a statement (i.e., the act of thinking and verbal expression of thought) about the existence or non-existence of an object or phenomenon; proposition is a statement about the properties of an object or phenomenon of reality; proposition is expressed in the statement of the existence or absence of certain features of objects and phenomena. A proposition connects concepts that logically express objects. There are no true propositions that connect concepts without objects. Also, there are no true propositions that connect objects without concepts of objects (in this case, the connection between objects is not a logical connection!). Therefore, a proposition has the following two properties: (a) the property of assertion or negation; (b) the property of truth or false. This property is expressed in the following words: "truth" or "false".

4) The connection (combination) of propositions, which represents deriving (extracting) a new proposition from one or more propositions, is called inference. The new proposition is

called a conclusion (in Latin: conclusio). Those propositions from which a new proposition is derived (extracted, follows) are called premises (in Latin: praemissae). The relation between premises and conclusion is the relation between cause and effect. Inference is based on the law of sufficient reason.

5) Inferences are divided into the following two groups: direct inferences and mediated inferences. If a conclusion (proposition) is made from only one premise (proposition), then the inference is called direct inference. If a conclusion (proposition) is made from several premises (propositions), then the inference is called mediated inference.

6) Such expressions (word combinations) as “predicate logic”, “quantificational logic”, “propositional logic”, “inferencial logic”, “logic of justification”, “logic of evidence”, “class logic”, “epistemic logic ”,“logic of truth”, “feature logic”,“action logic”, “machine logic”, “logic of reasoning”, “decision logic”, “logic of strict implication”, “feature logic”, “logic of whole”, “logic of part”, etc. do not exist (are not allowed) in formal logic. In the point of view of formal logic, these expressions are absurd.

Thus, formal logic has no quantitative aspect.

2. The essence of mathematics

Starting point of mathematics is the art of computing (calculating). The numbers represent the initial and terminal point of mathematics. “The practical application of the results of theoretical mathematical research is impossible without expressing the results in numerical form” (Russian Wikipedia). Mathematical research is based on the basic laws of formal logic: the law of identity; the law of absence of contradiction; the law of the excluded third; the law of sufficient reason.

1) Applied mathematics is used in theoretical physics. For example, the physical relationship

$$v_M = \frac{S_M}{t}$$

(where S_M is a distance traveled by the material point M; t is time (i.e., the universal informational quantity) of the motion of the material point M)
is the mathematical definition of the speed v_M of motion of the material point M.

2) In the point of view of formal logic, physical quantities v_M and S_M express concrete concepts: the concept v_M is the concept of the speed of motion of the material point M; the concept S_M is the concept of the distance traveled by the material point M. In the point of view of rational dialectics, physical quantities v_M and S_M have measures as the unities of qualitative and quantitative determinacy. In other words, quantities v_M and S_M have dimensions $"\frac{meter}{second}"$ and $"meter"$, respectively.

3) Physical quantities v and S also have measures and dimensions $"\frac{meter}{second}"$ and $"meter"$, respectively. In the point of view of formal logic, the quantities v and S express abstract concepts, because properties of the quantities v and S do not belong to a concrete object. If the quantities v and S did not have dimensions (i.e. properties), then the quantities v and S would be called mathematical quantities.

4) The central point of pure mathematics is the expression $y = f(x)$, where x and y are mathematical quantities; f is a symbol of the functional connection of variables x и y. The quantities x and y have no dimensions. The quantities x and y take only numerical values. Numerical values of mathematical quantities x and y represent unnamed neutral numbers (i.e.,

numbers without names and signs "+" and "−") [28, 31-38, 40]. Abstract numbers have no names and represent the values of a mathematical quantity. For example,

$$x = \frac{p\ (meter)}{q\ (meter)} = \frac{p\ (kilogram)}{q\ (kilogram)} = \frac{p\ (second)}{q\ (second)}.$$

Unnamed numbers cannot be used in practice to describe reality.

5) In the point of view of rational dialectics, mathematical quantities x and y have no the measure as the unity of qualitative and quantitative determinacy. In the point of view of formal logic, quantities and numbers without qualitative determinacy (dimension) cannot be expressed in any concepts. Really, a concept expresses the essential features of objects and phenomena. But a mathematical quantity does not express the essential features of objects, because concepts of objects and phenomena do not exist in mathematics [28, 31-38, 40]. Therefore, a mathematical quantity cannot be defined (expressed) as a concept. This means that mathematical quantities are only letter symbols for unnamed neutral numbers. Consequently, mathematical quantities cannot be considered within the framework of formal logic [1-49].

6) Mathematical expressions are constructed in the following typical way:

$$x = x,\ \ ax = ax,\ \ ax + b = ax + b,\ y = ax + b, \dots\ \ y = f(x)$$

where a and b are the numbers; the variables x (argument) and y (function) take numerical values. The symbol f signifies set of mathematical operations on the argument x. The symbols for mathematical operations are as follows: "+", "−", "·", ":". The symbol "=" signifies the word “is”. The symbol "≠" signifies the word “is not”. The symbols for comparison of numbers are as follows: "=", "≠", ">", "<", "≥", "≤". The double symbol "≥" signifies the words “equal-to-or-greater-than”; the double symbol "≤" signifies the words “equal-to-or-less-than”. (The meanings of the words “less than” and “equal to” (also, “greater than” and “equal to”) are not intersected: there is no partial coincidence (intersection) between the meanings of these terms. Therefore, the word “or” has only one meaning: separation meaning).

The punctuation marks ",", ";" are symbols of enumeration in sequence of numbers. The punctuation mark "." is the symbol of cessation of enumeration. The brackets $(\ )$, $[\ ]$, $\{\ \}$ are symbols of indication (designation) of order of operations. The symbols ",", ";", ".", $(\ )$, $[\ ]$, $\{\ \}$ are separation symbols. The symbols "∪" and "∩" are not symbols of mathematical operations, because the concept “set” is not a mathematical concept (“set” is a formal-logical concept).

The symbols "+", "−", "·", ":", ",", ";", ".", $(\ )$, $[\ ]$, $\{\ \}$ represent the only and complete set (exhaustive set, full set) of symbols for mathematical operations on unnamed numbers. This means that these symbols represent a single and complete set (exhaustive set, full set) of symbols for mathematical operations on mathematical quantities in the final analysis.

Thus, the essence of pure mathematics is as follows: pure mathematics abstracts away from the essential properties (qualitative determinacy) of quantities. Pure mathematics operates only with unnamed numbers within the framework of the basic laws of formal logic (i.e., the law of identity, the law of the absence of contradiction, the law of the excluded middle, the law of sufficient reason). The unnamed numbers and the listed symbols are the essence of pure mathematics.

Formal logic operates with concepts. Concepts and propositions are the essence of formal logic. There is no connection (relationship, dependence) between unnamed numbers and concepts (propositions). Mathematical expressions are not propositions. Consequently, there is no connection (relations) between the essence of pure mathematics and the essence of formal logic. The connection (relation) between the essence of pure mathematics and the essence of formal logic would exist if these essences were identical.

Thus, pure mathematics has no qualitative aspect [1-49].

3. The essence of mathematical logic

By definition, mathematical logic is: (a) a mathematical representation (formulation) of formal logic; (b) the unity (combination) of pure mathematics and formal logic. To join (combine) pure mathematics and formal logic, one must remove concepts from formal logic. If one removes concepts from formal logic, then the following basic laws of thinking remain: (1) the law of identity, (2) the law of absence of contradiction, (3) the law of the excluded third, (4) the law of sufficient reason. But if one replaces concepts by the words "truth" and "false", then the laws of formal logic lose meaning.

1) As is known, the starting point of mathematical logic is the following statements [49-59].

"One of the popular definitions of logic is that it is the analysis of methods of reasoning. In studying these methods, logic is interested in the form rather than the content of the argument. For example, consider the two arguments:

1. All men are mortal Socrates is a man. Hence, Socrates is mortal.
2. All cats like fish. Silly is a cat. Hence, Silly likes fish.

Both have the same form: All **A** are **B**. **S** is an **A**. Hence, **S** is a **B**. The truth or falsity of the particular premises and conclusions is of no concern to logicians. They want to know only whether the premises imply the conclusion. The systematic formalization and cataloguing of valid methods of reasoning are a main task of logicians. If the work uses mathematical techniques or if it is primarily devoted to the study of mathematical reasoning, then it may be called mathematical logic. We can narrow the domain of mathematical logic if we define its principal aim to be a precise and adequate understanding of the notion of mathematical proof.

Impeccable definitions have little value at the beginning of the study of a subject. The best way to find out what mathematical logic is about is to start doing it, and students are advised to begin reading the book even though (or especially if) they have qualms about the meaning and purpose of the subject.

Although logic is basic to all other studies, its fundamental and apparently self-evident character discouraged any deep logical investigations until the late 19th century. Then, under the impetus of the discovery of non-Euclidean geometry and the desire to provide a rigorous foundation for calculus and higher analysis, interest in logic revived. This new interest, however, was still rather unenthusiastic until, around the turn of the century, the mathematical world was shocked by the discovery of the paradoxes – that is, arguments that lead to contradictions.

Sentences may be combined in various ways to form more complicated sentences. We shall consider only truth-functional combinations, in which the truth or falsity of the new sentence is determined by the truth or falsity of its component sentences.

Negation is one of the simplest operations on sentences. Although a sentence in a natural language may be negated in many ways, we shall adopt a uniform procedure: placing a sign for negation, the symbol $\neg$, in front of the entire sentence. Thus, if **A** is a sentence, then $\neg$**A** denotes the negation of **A**. The truth-functional character of negation is made apparent in the following truth table:

A	$\neg$**A**
T	**F**
F	**T**

When **A** is true, $\neg$**A** is false; when **A** is false, $\neg$**A** is true. We use **T** and **F** to denote the truth values true and false.

Another common truth-functional operation is the conjunction: ‘and’. The conjunction of sentences **A** and **B** will be designated by $\mathbf{A} \wedge \mathbf{B}$ and has the following truth table:

A	**B**	$\mathbf{A} \wedge \mathbf{B}$
T	**T**	**T**
F	**T**	**F**
T	**F**	**F**
F	**F**	**F**

$\mathbf{A} \wedge \mathbf{B}$ is true when and only when both **A** and **B** are true. **A** and **B** are called the conjuncts of $\mathbf{A} \wedge \mathbf{B}$. Note that there are four rows in the table, corresponding to the number of possible assignments of truth values to **A** and **B**.

In natural languages, there are two distinct uses of ‘or’: the inclusive and the exclusive. According to the inclusive usage, ‘**A** or **B**’ means ‘**A** or **B** or both’, whereas according to the exclusive usage, the meaning is ‘**A** or **B**, but not both’. We shall introduce a special sign, $\vee$, for the inclusive connective. Its truth table is as follows:

A	**B**	$\mathbf{A} \vee \mathbf{B}$
T	**T**	**T**
F	**T**	**T**
T	**F**	**T**
F	**F**	**F**

Thus, $\mathbf{A} \vee \mathbf{B}$ is false when and only when both **A** and **B** are false. ‘$\mathbf{A} \vee \mathbf{B}$’ is called a disjunction, with the disjuncts **A** and **B**.

Another important truth-functional operation is the conditional: ‘if **A**, then **B**’. Ordinary usage is unclear here. Surely, ‘if **A**, then **B**’ is false when the antecedent **A** is true and the consequent **B** is false. However, in other cases, there is no well-defined truth value. For example, the following sentences would be considered neither true nor false:

1. If $1+1=2$, then Paris is the capital of France.
2. If $1+1 \neq 2$, then Paris is the capital of France.
3. If $1+1 \neq 2$, then Rome is the capital of France.

Their meaning is unclear, since we are accustomed to the assertion of some sort of relationship (usually causal) between the antecedent and the con-sequent. We shall make the convention that ‘if **A**, then **B**’ is false when and only when **A** is true and **B** is false. Thus, sentences 1-3 are assumed to be true. Let us denote ‘if **A**, then **B**’ by ‘$\mathbf{A} \Rightarrow \mathbf{B}$’. An expression ‘$\mathbf{A} \Rightarrow \mathbf{B}$’ is called a conditional. Then $\Rightarrow$ has the following truth table:

A	**B**	$\mathbf{A} \Rightarrow \mathbf{B}$
T	**T**	**T**
F	**T**	**T**
T	**F**	**F**
F	**F**	**T**

This sharpening of the meaning of ‘if **A**, then **B**’ involves no conflict with ordinary usage, but rather only an extension of that usage.

A justification of the truth table for $\Rightarrow$ is the fact that we wish ‘if **A** and **B**, then **B**’ to be true in all cases. Thus, the case in which **A** and **B** are true justifies the first line of our truth table for $\Rightarrow$, since (**A** and **B**) and **B** are both true. If **A** is false and **B** true, then (**A** and **B**) is false while **B** is true. This corresponds to the second line of the truth table. Finally, if **A** is false and **B** is false, (**A** and **B**) is false and **B** is false. This gives the fourth line of the table.

Let us denote ‘**A** if and only if **B**’ by ‘**A** $\Leftrightarrow$ **B**’. Such an expression is called a biconditional. Clearly, ‘**A** $\Leftrightarrow$ **B**’ is true when and only when **A** and **B** have the same truth value. Its truth table, therefore is:

A	**B**	**A** $\Leftrightarrow$ **B**
T	**T**	**T**
F	**T**	**F**
T	**F**	**F**
F	**F**	**T**

The symbols $\neg, \wedge, \vee, \Rightarrow$ and $\Leftrightarrow$ will be called propositional connectives. Any sentence built up by application of these connectives has a truth value that depends on the truth values of the constituent sentences. In order to make this dependence apparent, let us apply the name statement form to an expression built up from the statement letters **A**, **B**, **C**, and so on by appropriate applications of the propositional connectives” [54].

4. Critical analysis of the starting point of mathematical logic

1) As is known, the scope (volume) of general concepts is expressed in the form of a class. The logical class is a collection of objects that have common essential features (characteristics). As a consequence, these objects are covered by the general concept. One class is superior to another class if it includes another class together with other classes. The class that is superior to another is called a genus. The class that is inferior to the genus is called a species.

2) In the point of view of formal logic, the division of propositions is based on the existence of an essential feature (property) in one group of propositions and the absence of this feature (property) in another group of propositions. The proposition has the following essential feature: the property of truth or falsity. The property of falsity is the absence of the property of truth. There is the logical relation of disagreement (contradiction) between true propositions and false propositions: the feature of true negates the feature of falsity. The property of true or falsity of a proposition is determined within the framework of the correct methodological basis: the unity of formal logic and rational dialectics. The unity of formal logic and rational dialectics is the criterion of truth. The property of true or false is expressed by the following words: “truth” or “false”. The properties “truth” and “false” are contradictory. Therefore, the set of propositions can be divided into two non-overlapping classes: the “class of true propositions” and the “class of false proposition”:

$$A_T = \{a_1,\ a_2, \ldots, a_n, \ldots\}, \quad a_n \notin B_F\,;$$
$$B_F = \{b_1, b_2, \ldots, b_m, \ldots\}, \quad b_m \notin A_T$$

where A_T is the “class of true propositions”, B_F is the “class of false proposition”, the element a_n is the true proposition, the element b_m is a false proposition.

The classes A_T and B_F are subordinate classes (subclasses) of the generic class C. Therefore, the volumes (scopes) of propositions are connected by the following relationship:

$$V_C = V_{A_T} + V_{B_F}\,.$$

This relationship corresponds to the following diagram:

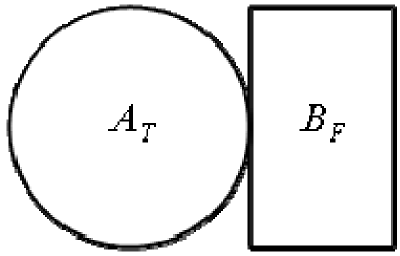

Figure 1. Graphical interpretation of the relationship $V_C = V_{A_T} + V_{B_F}$.

Hence it follows that the standard relationships

$$C = A_T \cup B_F ,$$
$$C = A_T \cap B_F$$

and corresponding diagrams

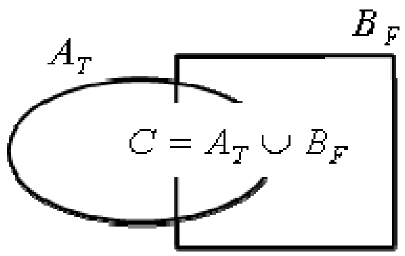

Figure 2. Graphical interpretation of the relationship $C = A_T \cup B_F$.

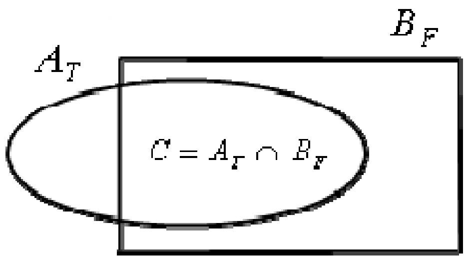

Figure 3. Graphical interpretation of the relationship $C = A_T \cap B_F$.

represent formal-logical errors.

<u>Remark</u>.

The symbols "$\cup$" and "$\cap$" are notations of the words "or" and "and", respectively. Therefore, the symbols "$\cup$" and "$\cap$" are not symbols of mathematical (quantitative, numerical) operations, because set is not a mathematical concept. Set is a formal-logical concept. In the formal-logical point of view, there is the only one correct operation in the set theory: decomposition (partition) of set in terms of the non-overlapping (non-intersecting) sets [41, 43].

The concepts “set” and “class” are identical. Division (decomposition) of a set into the non-overlapping (non-intersecting) classes represents the inverse operation with respect to the union of the non-overlapping (non-intersecting) classes. This implies that the standard operation "$\cup$" is wrong because the standard operation "$\cap$" is wrong [41, 43]. Thus, the correct use of the word “or” is that the word “or” must have the exclusive meaning: “given proposition or other proposition, but not both”. The correct use of the word “and” must have the following meaning: union of the non-intersecting (non-overlapping) classes. In this case, the symbol "$\cup$" (arises from the word “union”) can be used for notation of the word “and’.

3) Proposition has two important and connected aspects: aspect of truth and aspect of content (meaning). Truth does not exist without content (meaning); content (meaning) is useless without truth. Therefore, the proposition has two important features (properties): “truth” and “content (meaning)”. “Truth” is an essential feature (property) of the element a_n of the class A_T. The elements of the class A_T are independent elements. “False” is an essential feature (property) of the element b_m of the class B_F. The elements of the class B_F are independent elements. The concepts “proposition” and “property of proposition” are not identical.

(a) If one takes into account only the essential feature (property), “truth”, then one comes to the following uniquely correct relationship between true propositions:

(truth) is (truth),
i.e. *(truth) = (truth).*

But mathematical logic contains standard truth tables that contradict to formal logic. For example, the following common truth-functional operations

$$(false) \vee (truth) = (truth),$$
$$(truth) \vee (false) = (truth);$$
$$(false) \wedge (truth) = (false),$$
$$(truth) \wedge (false) = (false);$$
$$(false) \Rightarrow (truth) = (truth),$$
$$(truth) \Rightarrow (false) = (false)$$

represent absurd. The absurdity is that the contradictory properties (“truth” and “false”) of the elements a_n and b_m are in the left-hand sides of the common (united) relationships in mathematical logic. In addition, the symbols “$\neg$”, “$\vee$” , “$\wedge$”, “$\Rightarrow$”, “$\Leftrightarrow$”, etc. are not symbols of mathematical (quantitative) relationships. The symbols “$\neg$”, “$\vee$”, “$\wedge$”, “$\Rightarrow$”, “$\Leftrightarrow$”, etc. are symbols of qualitative relationships. Thus, common truth-functional operations in mathematical logic represent formal-logical errors.

(b) If one also takes into account the feature (property) “meaning content”, then one can establish logical (but not quantitative) relations between the elements (propositions) of the class A_T. The property (feature) “meaning content” is not subject to mathematical (quantitative) operations. Establishment of relations between propositions is a formal-logical problem. This problem is a solved problem in formal logic.

Formal-logical solution is the following statements: (i) proposition does not exist without grammatical (verbal) form of expression of thought; (ii) proposition is the logical content of a grammatical sentence; (iii) proposition is a system of concepts (i.e., connection of concepts) defined and expressed by words and grammatical sentences. Hence, if one removes concepts, then one removes propositions from formal logic. In this case, formal logic loses its scientific meaning, and science loses its correct methodological basis.

Consequently, use of the symbols “$\neg$”, “$\vee$”, “$\wedge$”, “$\Rightarrow$”, “$\Leftrightarrow$”, etc. instead of words is meaningless, useless, fruitless, unsuccessful attempt of the junction of formal logic and mathematics. The main formal-logical error in mathematical logic is that propositions are replaced by the properties: “truth” and “false”.

Thus, the junction (unification) of formal logic and pure mathematics is impossible, because formal logic has no quantitative aspect, and pure mathematics has no qualitative aspect. Mathematics (as a quantitative science) cannot exist in formal logic; formal logic cannot be squeezed into mathematics.

5. Discussion

1. Why are scientists wrong? My 40 years experience of critical analysis of the foundations of theoretical physics and mathematics [1-48] shows that the main causes are as follows:
(a) firstly, the haste and immaturity of thinking intrinsic (proper, inherent) to youth;
(b) secondly, the unwillingness and inability (inefficiency, disability) of the scientist to find the correct methodological basis and criterion of truth;
(c) thirdly, the reluctance of a scientist to admit (to acknowledge) the existence of errors in science.

2. The essence of mathematics is that mathematics is a special science that does not rely on the correct methodological basis: the unity of formal logic and rational dialectics [1-48]. The unity of formal logic and rational dialectics represents the correct criterion of truth. Therefore, mathematics does not contain the dialectical concept “measure as the unity of qualitative and quantitative determinacy (aspects)”. Mathematics has the quantitative aspect but not the qualitative aspect. This means that mathematics does not contain the criterion of truth and the methodological basis [1-48]. Mathematical thinking (reasoning) ignores practice. Therefore, mathematical thinking (reasoning) is narrow, limited thinking (reasoning).

3. The essence of formal logic is that formal logic is a general science of the laws of correct thinking. Therefore, formal logic has the qualitative aspect (meaning content) but not the quantitative aspect. The unity of formal logic and rational dialectics is the correct criterion of truth and the correct methodological basis of science.

4. Junction (unification) of formal logic and mathematics is impossible as it is impossible to join philosophy (in particular, dialectics) and mathematics. The explanation is the fact that the essence of formal logic and mathematics are different. Therefore, mathematical logic is a gross methodological error. The desire of mathematicians to substantiate the junction of formal logic and mathematics is meaningless, useless, fruitless, unsuccessful effort.

Conclusion

A critical analysis of the starting point of mathematical (symbolic) logic within the correct methodological basis (i.e., the unity of formal logic and rational dialectics) leads to the following statements:

(1) Pure mathematics operates on mathematical quantity. Pure mathematics and mathematical (symbolic) logic ignore the correct methodological basis (i.e., criterion of truth) of science.

(2) Mathematics and mathematical (symbolic) logic abolishes (deletes) essence of formal logic: concept and proposition (as a logical form of verbal expression (statement) of thought). But thought without words does not exist. If concepts and propositions as meaning contents are removed from consideration, then formal logic is destroyed.

(3) Mathematical (symbolic) logic replaces proposition by the property of proposition: “truth” or “false”. But the concepts “proposition” and “property of proposition” are not identical. In this case, the formal logic is destroyed. In addition, the concepts “proposition” and “property of proposition” do not represent mathematical quantities, and the concepts cannot be in mathematical formalism.

(4) The starting point of mathematical (symbolic) logic represents several symbols that connect (join, unite) the words "truth" and "false" in truth tables. But the words "truth" and "false" do not represent mathematical quantities, and the words "truth" and "false" cannot be in mathematical formalism. The symbols are erroneously called propositional connectives. The symbols denote the words "negation", "and", "or", "if, then", "if and only if", etc.

(5) The symbols of common truth-functional operations are not mathematical symbols (i.e., symbols of mathematical operations). The definition of symbols is based on set theory. But set theory is an erroneous theory. Therefore, the use of these symbols in mathematical (quantitative) expressions is a gross mistake.

(6) The essences of formal logic and mathematics are different. The qualitative aspect (i.e., meaning content) is the essence of formal logic; the quantitative (i.e., numerical) aspect is the essence of mathematics. Since formal logic has no quantitative aspect, and mathematics has no qualitative aspect, the union (join, conjunction, combination) of formal logic and mathematics is a gross methodological error.

Thus, mathematical logic is a thoughtless, absurd theory in science.

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