SELF-CONSISTENT EM FIELD

WU SHENG-PING

Abstract. This article try to unified the four basic forces by Maxwell equations, the only experimental theory. Self-consistent Maxwell equation with the e-current coming from matter current is proposed, and is solved to electrons and the structures of particles and atomic nucleus. The static properties and decay are reasoned, all meet experimental data. The equation of general relativity sheerly with electromagnetic field is discussed as the base of this theory. In the end the conformation elementarily between this theory and QED and weak theory is discussed.

Contents

1. Unit Dimension of *sch*

A rebuilding of units and physical dimensions is needed. Time s is fundamental. We can define:
The unit of time: s (second)
The unit of length: cs (c is the velocity of light)

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The unit of energy: $\hbar/s$ (h is Plank constant)

The unit dielectric constant ϵ is

$$[\epsilon] = \frac{[Q]^2}{[E][L]} = \frac{[Q]^2}{\hbar c}$$

The unit of magnetic permeability μ is

$$[\mu] = \frac{[E][T]^2}{[Q]^2[L]} = \frac{\hbar}{c[Q]^2}$$

The unit of Q (charge) is defined as

$$c[\epsilon] = c[\mu] = 1$$

then

$$[Q] = K\sqrt{\hbar}, K = 1$$
$$[H] = [Q]/[L]^2 = [E]$$

Then

$$\sqrt{\hbar} = (1.0546 \times 10^{-34})^{1/2} C$$

C is charge's SI unit Coulomb.

For convenience, new base units by unit-free constants are defined,

$$c = 1, \hbar = 1, [Q] = \sqrt{\hbar} = [1]$$

then all physical units are power of second s^n, the units are reduced.

Define

$$UnitiveElectricalCharge : \sigma = \sqrt{\hbar}$$
$$\sigma = 1.03 \times 10^{-17} C \approx 64e$$
$$e_{/\sigma} = e/\sigma = 1.57 \times 10^{-2} \approx 1/64$$

The system is redefined and rebuilt as:

$$s \to C's : 1 = m_e/e =: \beta$$

$s \to Cs$ means the value of the redefined second as becomes C seconds. We always use this units system. If varying K more freedom is for the the system.

2. Inner Field of Electron

Try the self-consistent Maxwell equation for the inner electromagnetic (EM) field of electrons

$$\partial' \cdot \partial A_\nu = iA^*_\mu \partial_\nu A^\mu = J_\nu, \qquad \beta = 1, \sigma = 1 \tag{2.1}$$
$$\partial^\nu \cdot A_\nu = 0$$

with definition

$$(A^i) := (V, \mathbf{A}), (A_i) := (V, -\mathbf{A})$$
$$(J^i) = (\rho, \mathbf{J}), (J_i) = (\rho, -\mathbf{J})$$
$$\partial := (\partial_i) := (\partial_t, \partial_{x_1}, \partial_{x_2}, \partial_{x_3})$$
$$\partial' := (\partial^i) := (\partial_t, -\partial_{x_1}, -\partial_{x_2}, -\partial_{x_3})$$
$$g_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

We deduce by devising momentum to express e-current in a electron: the mass and charge have the same movement in electron. The equation 2.1 has symmetry:

$$cc.PT$$

3. General Electromagnetic Field

We find

$$(x', t') := (x, t - r)$$
$$\partial_x^2 - \partial_t^2 = \partial_{x'}^2 =: \nabla'^2$$

The following is the energy of a piece of field A:

$$\varepsilon := \frac{1}{2}(< E, D > + < H, B >) \tag{3.1}$$

The time-variant part is neglected, as a convention for energy calculation. If the field has Fourier transformation then the *field energy* becomes

$$\varepsilon = \frac{1}{2} < A_\nu | \partial_t^2 - \nabla^2 | A^\nu > \tag{3.2}$$

It's found that for a valid EM field

$$E' :=< A_\nu | i\beta\partial_t | A^\nu >=< A_\nu | \partial^\mu \partial_\mu | A^\nu > /2, \quad \sigma = 1, \beta = 1 \tag{3.3}$$

E' is called *interaction.* The magnetic interaction can't change the energy of the system. The *mechanical energy* is

$$I :=< A, J >= 2E'$$

4. Recursive Re-substitution Algorithm

The solution by recursive re-substitution (RRS) for the two sides of the equation is proposed. For the equation

$$\hat{P}' B = \hat{P} B$$

Its algorithm is that (It's approximate, the exact solution needs a rate on the start state in the re-substitution for the normalization condition)

$$\hat{P}'(\sum_{k \le n} B_k + B_{n+1}) = \hat{P} \sum_{k \le n} B_k$$

A function is initially set and is corrected by RRS of the equation 2.1. Here is the start state

$$A_i = A_r e^{-ikt}, \partial_\mu \partial^\mu A_i = 0$$

The fields' correction A_n with n degrees of A_i is called the n degrees correction.

5. Solution of Electron

Firstly

$$\nabla^2 \phi = -k^2 \phi$$

is solved. Exactly, it's solved in spherical coordinate

$$-k^2 = \nabla^2 = \frac{1}{r^2} \partial_r (r^2 \partial_r) + \frac{1}{r^2 \sin\theta} \partial_\theta (\sin\theta \partial_\theta) + \frac{1}{r^2 \sin^2\theta} (\partial_\varphi)^2$$

Its solution is

$$\Omega(x) := \phi = j_l(r) Y_{lm}(\theta, \varphi), k^2 = 1$$
$$j_1(r) = \frac{\sin(r)}{r^2} - \frac{\cos(r)}{r}$$

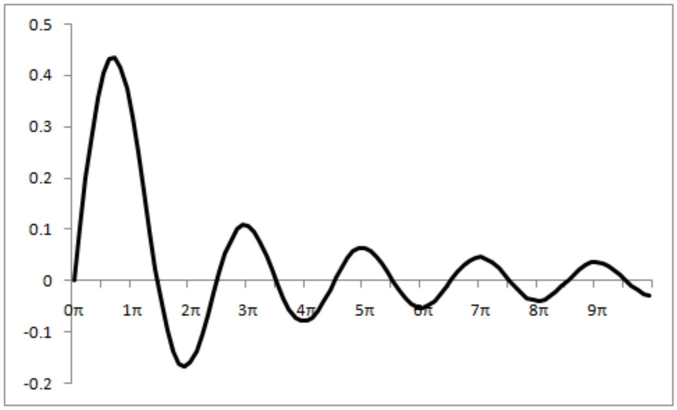

FIGURE 1. The function of j_1

In fact, it has to be re-defined in truncated form (T-form) and in process of limit

$$\Omega^\beta(x) = \Omega(x)(h(r) - h(r - \beta^{-1})), \beta \to 0$$

We use the following definition

$$\Omega(kx) := \sum_{\hat{k}} F(\hat{k}) e^{-ik\hat{k}x}, \hat{k} = \frac{\mathbf{k}}{|\mathbf{k}|}$$

$$< k\Omega(kx)|k\Omega(kx) >= 1$$

There are calculations:

$$\int_{t=a}^{b} dt f(t) = \int_{kt=a}^{b} d(kt) f(kt)$$

$$\Omega^*(kx) * \Omega(kx) =< \Omega(kx)|\Omega(kx) > \frac{\delta^3(x)}{\delta^3(0)} = |k|^{-3} < \Omega(x)|\Omega(x) > \frac{\delta^3(x)}{\delta^3(0)}$$

$$\nabla'^2 e^{-ikr'}\Omega(kx') = -2k^2 e^{-ikr'}\Omega(kx')$$

$$(\partial_t^2 - \nabla^2)u = -\nabla'^2 u = \delta(x')\delta(t') = \delta(x)\delta(t)$$

$$u := \frac{\delta(t-r)}{4\pi r},$$

For the objected function $\Omega(kx) * \Omega(x)$:

$$\nabla^2 = -\sum_{\mathbf{k}} (k^2 e^{-i\mathbf{k}x} * + e^{-i\hat{\mathbf{k}}x} *)$$

And

$$\int_I dy < w(y/2+x)|-4\nabla'^2|w(-y/2+x) >= \int_I dy < \nabla' w(y/2+x)|4\nabla' w(-y/2+x) >$$

$$= -\int_I dy < (\nabla w)(y/2+x)|(\nabla w)(-y/2+x) >$$

6. Electrons

It's the start electron function for the RRS of the equation 2.1:

$$A_i^\nu(k) := i\partial^\nu \Omega(kx)e^{-ikt}, k > 0$$

which meet the covariant condition

$$\partial_\nu A_i^\nu = 0 \tag{6.1}$$

Some states are defined as the core of the electron, which's the start function A_i for the RRS of the equation 2.1 to get the whole electron function e:

$$e_r^+ : A_i(x,t), \quad e_l^- : A_i(x,-t)$$
$$e_l^+ : A_i(-x,t), \qquad e_r^- : A_i(-x,-t)$$

r, l is the direction of spin of electron. The following are anti-electrons that obey the equation 2.1,

$$-e := e^*(-x,-t)$$

The anti-operation reverses the energy and the world-line of electron. e_c is the general symbol of electrons. We find

$$< e^*|e >= 0$$
$$< e^*(x)|e^\pm(-x) >= 0$$

That's to say it's no coupling between all anti-electron and normal electron.

Using the equation 2.1, the electron function is normalized with charge as

$$| < A_i^\mu|i\partial_t|A_{i\mu} > | = e, \quad \beta = 1, \sigma = 1$$
$$|k_e| \approx m_e, [A_i] = [Q/L]$$

The MDM of electron is calculated as the second degree proximation

$$-\mathbf{r} \times \partial \cdot \partial' A/2$$
$$\mu_z =< A_{i\nu}| - i\partial_\varphi|A_i^\nu > /2$$
$$= \frac{Q_e}{2m_e}$$

The spin is

$$S_z = \mu_z k_e/e = 1/2$$

The correction in RRS of the equation 2.1 is calculated as

$$A - A_i = \frac{u * (A_i^* \cdot i\partial A_i)}{1 - u * (((-i\partial A_i^*)\cdot)^* \cdot + A_i^* \cdot i\partial)}, \quad k_e = 1, \beta = 1$$
$$= \frac{(A_i^* \cdot i\partial A_i) * u}{1 - \cdot i\partial(A_i - A_i^*)/2 * u}$$
$$A = A \cdot h(t)$$

We find the Lorentz gauge:

$$\partial_\nu e^\nu = 0$$

The function of e_r^+ is decoupled with e_l^+

$$\beta < (e_r^+)^\nu|i\partial_t|(e_l^+)_\nu >= \beta < (e_r^+)^\nu|\beta|(e_l^+)_\nu >= 0$$

The following is the increment of the energy ε on the coupling of e_r^+, e_r^-, mainly between A_{2-n} and A_{2+n}

$$\varepsilon_e =< e_r^+|i\partial_t, \beta|e_r^- > + < e_r^-|i\partial_t, \beta|e_r^+ >, \quad k_e = 1, \beta = 1$$

$$\approx -2e^4\beta = -\frac{1}{1.66\times 10^{-16}s}$$

The calculation is unit dimension analysis. Only in measure $k_e = 1$, it's obtained the correct number.

The following is the increment of the energy ε on the coupling of e_r^+, e_l^-, mainly between A_{4-n} and A_{4+n}.

$$\varepsilon_x =< e_r^+|i\partial_t, \beta|e_l^- > + < e_l^-|i\partial_t, \beta|e_r^+ >$$
$$\approx -\frac{1}{2}e^8_{/\sigma}\beta = -\frac{1}{1.08\times 10^{-8}s}$$

7. System of Electrons

The the movement of a electron to make an EM field:

$$A := f *_3 \sum_i e_i = N\sum_{x'} f(x',t')\delta(x-x',t-t') *_4 \sum_i e_i(x,t)|_{t'=t}$$

with the particle number normalization:

$$< f|f >= 1, \quad \sigma = 1, \beta = 1$$

The following are naked stable particles:

particle	*electron*	*photon*	*neutino*
notation	e_r^+	γ_r	ν_r
structure	e_r^+	$(e_r^+ + e_r^-)$	$(e_r^+ + e_l^-)$

The following is the system of particle x with the initial state

$$A_0 := \sum_v e_x^v * E_v$$
$$E_v := \sum_c n_{cv}e_c$$
$$e_x^v = e_x, e_x(-\varphi) =: e'_x$$

The *condition for the general EM field* is

$$\frac{1}{2}(\partial_t^2 - \nabla^2)A = i\beta\partial_t A, \quad \sigma = 1, \beta = 1 \tag{7.1}$$

The inner field of electron subjects to this equation (at $k_e = -1$), in linear functional meaning. Hence, for A_0

$$\partial^\nu\partial_\nu e_x^v = 0, \quad k_x^2 = 1$$

The singularity at grid-origin makes something unusual. Then

$$e_x := k_x^2(x+iy)\frac{j_1(k_x r)}{k_x r}e^{-ik_x t}$$

which's solution is spherical Bessel function like Ω. We find in this case

$$< e_x * E_v|i\partial_t|e_x * E_v >\approx Q_x$$
$$n\frac{e^2}{m_e} = n\frac{[Q]e/e_{/\sigma}}{k_x} \approx Q_x, \quad n := \sum_{vc} n_{vc}^2, \quad k_x = 1$$
$$k_x \approx \frac{nm_e}{Q_x}, \quad \sigma = 1, \beta = 1$$

Their initial MDM are

$$\mu_z =< A_{0\nu}| - i\partial_\varphi|A_0^\nu > /2$$

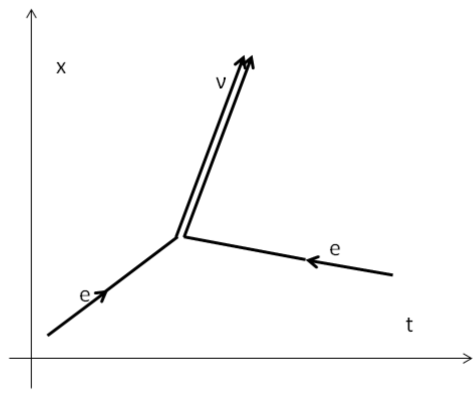

FIGURE 2. neutrino radiation

$$= \sum_{vc} n_{cv}^2 < e_x^v * e_{cv} | - i\partial_\varphi | e_x^v * e_{cv} > /2 \approx \sum_{vc} \frac{\mu_{zvc} n_{cv}^2}{k_x/m_e}$$

Its orbital angular momenta are neglected for their little amplitudes. The spins of the particle are uncoupling between the electrons of same state.

8. Muon

The initial of muon is

$$\mu_r^- : e_\mu * (e_r^- - e_r^- - e_l^+), \quad e_\mu = e_x^*(k_x = m_\mu)$$

μ is approximately with mass $3m_e/e_{/\sigma} = 3 \times 64 m_e$ [3.2][1] (The data in bracket is experimental by the referenced lab), spin S_e (electron spin), MDM $\mu_B m_e/k_\mu$.

The main channel of decay is

$$\mu_r^- \to e_r^- - \nu_l, \quad e_r^- \to -e_l^+ + \nu_l$$

$$e_\mu * e_r^- - e_\mu * \nu_l \to e_\mu * e_r^- - L(\delta(x,t)) *_4 \nu_l$$

$$L(e(x,t)) = L(\delta(x)\delta(t)) *_4 e$$

L is Lorentz transformation. The main EM emission is

$$\varepsilon_\mu = - < e_\mu^* * e_\mu(-x) | e_r^{-*} * i\beta\partial_t e_l^+(-x) > - < e_\mu^* * e_\mu(-x) | e_l^{+*} * i\beta\partial_t e_r^-(-x) >$$

$$\varepsilon_\mu := \frac{\varepsilon_x m_e}{k_\mu} = -\frac{1}{2.18 \times 10^{-6} s} \quad [2.1970 \times 10^{-6} s][1], \quad k_\mu = 1$$

It's the variation of E' (by reference to 3.3), in fact the self interaction of the two charges of neutrinos counteracted. And

$$[\frac{-k_\mu}{|-k_\mu|}]_{k_\mu=-1} \cdot \frac{1}{|k_e|} = \frac{k_\mu}{|k_\mu|} \cdot \frac{1}{|k_e|} = \frac{k_e}{|k_e|}, k_e = -1, 1$$

9. Pion

The initial of pion perhaps is

$$\pi_r^- : e_\pi * (e_l^+ - e_l^+) + e'_\pi * e_r^-$$

It's approximately with mass $3 \times 64 m_e$ [4.2][1], spin S_e, MDM $\mu_B m_e / k_{\pi^-}$.

Decay Channels:

$$\pi_r^- \to -e_l^+ + \nu_l, \quad e_l^+ \to -e_r^- + \nu_l$$

The mean life approximately is

$$-\varepsilon_x/2 = \frac{1}{2.2 \times 10^{-8} s} \quad [(2.603 \times 10^{-8} s][1]$$

The precise result is calculated with successive decays.

10. Pion Neutral

The initial of pion neutral is perhaps like

$$\pi^0 : e_{\pi^0} * (e_r^+ + e_l^+) + e'_{\pi^0} * (e_r^- + e_l^-)$$

It's with mass approximately $4 \times 64 m_e$ [4.2][1], zero spin, and zero MDM. It's the main decay mode as

$$\pi^0 \to \gamma_r + \gamma_l$$

The loss of energy is

$$-2\varepsilon_e = \frac{1}{8.3 \times 10^{-17} s} \quad [8.4 \times 10^{-17} s][1]$$

11. Tauon

The initial of tauon maybe

$$\tau_r^- : e'_\tau * (n e_l^- - n e_l^-) + e_\tau * (e_r^- - e_r^+ - e_r^-)$$

Its mass approximately $53 \times 64 m_e$ [54][1] ($n = 5$), MDM $\mu_B m_e / k_\tau$, spin S_e. It has decay mode with a couple of neutrinos counteracted

$$e_\tau * (e_r^- - e_r^+ - e_r^-) \to e_\tau * e_r^- - \gamma_r$$

The main EM emission is

$$\frac{\varepsilon_e m_e}{k_\tau} = -\frac{1}{5.5 \times 10^{-13} s} \quad [2.91 \times 10^{-13} s; B.R. : 0.17][1]$$

Perhaps, it's a mixture with distinct coefficients n.

12. Proton

The initial of proton may be like

$$p_r^+ : e_p * (-4 e_r^+ - 3 e_r^-) + e'_p * (-2 e_r^-), \quad e_p = e_x(k_x = m_p)$$

The mass is $29 \times 64 m_e$ [29][1] that's very close to the real mass. The MDM is calculated as $3\mu_N$, spin is S_e. The proton thus designed is eternal.

13. Neutron

Neutron is the atom of a proton and a muon

$$n = (p_r^+, \mu^-)$$

The muon take the first track, with the decay process

$$\Phi * \mu^- = \Phi * e_\mu * (e_r^- - e_r^- - e_l^+) \to \Phi * e_\mu * e_r^- - e_\mu * \nu_l$$

By the equation 7.1 and the inner energy of muon counteracted:

$$i\partial_t \Phi + \frac{1}{2}\nabla^2 \Phi = -\frac{\alpha'}{r}\Phi, \quad \beta = 1, m_\mu = 1 \tag{13.1}$$

It's resolved to

$$\Phi = N e^{-r/r_0} e^{-iE_1 t}$$

$$E_1 = -\frac{1}{2}c^2\alpha'^2 = -\frac{1}{2}c^2\alpha^2(\frac{\beta^2}{m_\mu^2}) = -13.6eV \cdot 3^{-2}$$

$$\alpha = \frac{e^2}{4\pi\epsilon\hbar c} \approx 1/137$$

It's approximately the decayed EM energy of muon in the track that

$$\varepsilon_n = \frac{-E_1}{m_\mu} e^3_{/\sigma} \varepsilon_x = -\frac{1}{936s}$$

14. Atomic Nucleus

We can find the equation for the sum field of Z' ones of protons: Φ and the sum field of n ones of muons: ϕ

$$\frac{1}{2}\partial_t^2 \Phi - i\partial_t \Phi + \frac{1}{2}\nabla^2 \Phi = (Z'+1)\frac{\alpha'}{r} * \Phi - n\frac{\alpha'}{r} * \phi$$

$$\frac{1}{2}\partial_t^2 \phi + iz\partial_t \phi + \frac{1}{2}\nabla^2 \phi = -Z'\frac{\alpha'}{r} * \Phi + (n-1)\frac{\alpha'}{r} * \phi$$

The more numbers on protons' interaction is from

$$< \Phi * p | \Phi * i\beta\partial_t p > - < \Phi * p | \Phi * \partial^\nu \partial_\nu p > /2$$

Make

$$t' = Ct : -\frac{1}{2C^2}E^2 - \frac{1}{C}E = -\frac{1}{2}E^2 + zE$$

to fit

$$\frac{1}{2}\partial_{t'}^2 \phi - i\partial_{t'} \phi + \frac{1}{2}\nabla^2 \phi = -Z'\frac{\alpha'}{r} * \Phi + (n-1)\frac{\alpha'}{r} * \phi$$

Define

$$\Phi' e^{-iEt} = \Phi, \phi' e^{-iEt} = \phi$$

$$\zeta = \Phi' + \eta\phi'$$

$$(Z'+1) - \eta Z' = -n/\eta + n - 1 =: N$$

$$\eta = \frac{(Z'-n+2) \pm \sqrt{(Z'-n+2)^2 + 4Z'n}}{2Z'}$$

then

$$-(E^2/2 + E)\nabla^2\zeta + \frac{1}{2}\nabla^4\zeta + 4\pi\alpha' N\zeta = 0$$

$$\nabla^2 = (E^2/2 + E) - \sqrt{(E^2/2+E)^2 - 8\pi\alpha' N} = -k^2$$

$$\zeta = j_l(kr) Y_{lm}(\theta, \varphi)$$

and

$$k^2 = 1$$

to fulfil

$$|\lim_{r\to 0}\frac{\partial}{\partial r}j_l(kr)| << \infty, \lim_{r\to\infty} j_l(kr) = 0$$

So that

$$E = -1 + \sqrt{-8\pi\alpha' N}, \alpha' = (\frac{\beta}{m_p})^2\alpha$$

$$N = \frac{1}{2}((Z'+n) - \sqrt{(Z'+n)^2 + 4(Z'-n) + 4})$$

$$\approx -\chi, \quad \chi := \frac{Z'-n}{Z'+n}$$

We find

$$\chi = 1/3: \quad E + 1 \approx 8.0MeV$$

It's noticed that the gross interaction is least (zero) when

$$\eta \approx -1/2, 1$$

which means most stable nucleus is of the same protons (Z) and neutrons (n) approximately.

15. Basic Results for Interaction

The left interactive in decay is

$$W(t) = \Gamma e^{-\Gamma t}, \Gamma = 1, \beta = 1 \tag{15.1}$$

$$\Gamma = \frac{1}{2} < A_\nu|\partial^\mu\partial_\mu|A^\nu > |_\infty^{t=0}$$

This' because a particle disappears stochasticly. It leads to the result between decay life and EM emission or the interactive potential.

The distribution shape of decay can be explain as

$$e^{-\Gamma t/2}e_x * \sum_i e_i \approx \Omega_x * \sum_i e_i \cdot e^{-\Gamma t/2 - ik_x t}, 0 < t < \Delta$$

It's the real wave of the particle x near the initial time and expanded in that time span

$$\approx \Omega_x * \sum_i e_i \cdot \int_{-\infty}^{\infty} dk \frac{Ce^{-ikt}}{k - k_x - i\Gamma/2}$$

16. Grand Unification

The General Theory of Relativity is

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G T_{ij}/c^4 \tag{16.1}$$

Firstly the unit second is redefined as S to simplify the equation 16.1

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}$$

Then

$$R_{ij} - \frac{1}{2}Rg_{ij} = F^*_{i\mu}F_j{}^\mu - g_{ij}F^*_{\mu\nu}F^{\mu\nu}/4$$

We observe that the co-variant curvature is

$$R_{ij} = F^*_{i\mu}F_j{}^\mu + g_{ij}F^*_{\mu\nu}F^{\mu\nu}/8$$

17. Conclusion

Fortunately, this model explained all the effects in the known world: strong, weak and electromagnetic effects, and even subclassify them further if not being to add new ones. In this model the only field is electromagnetic field, and this stands for the philosophical that the unified world is from an unique source, all that depend on a simple hypothesis: the current of matter in a system can be devised to analysis the e-charge current.

My description of particles is compatible with QED elementarily (The expression of current in the equation 2.1 is same to the one of QED), and only contributes to it with theory of consonance state in fact. In some way, the electron function is a good promotion for the experimental models of proton and electron that went up very early.

Underlining my calculations a fact is that the electron has the same phase (electron consonance), which the BIG BANG theory would explain, all electrons are generated in the same time and place, the same source.

References

[1] K. Nakamura et al. (Particle Data Group), JPG 37, 075021 (2010) (URL: http://pdg.lbl.gov)

E-mail address: `hiyaho@126.com`

Tianmen, Hubei Province, The People's Republic of China. Postcode: 431700