# Granular: "Stochastic space-time and quantum theory" 

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#### Abstract

In an earlier paper[1], a stochastic model had been presented for the Planck-scale nature of space-time. From it, many features of quantum mechanics and relativity were derived. But as mathematical points have no extent, the stochastic manifold cannot be tessellated with points (if the points are independently mobile) and so a granular model is required. As grains have orientations as well as positions, spinors (or quaternians) are required to describe them, resulting in phenomena as described by the Dirac equation. We treat both space and time stochastically and thus require a new interpretation of time to prevent an object being in multiple places at the same time. As the grains do have a definite volume, a mechanism is required to create and annihilate grains (without leaving gaps in space-time) as the universe, or parts thereof, expands or contracts. Making the time coordinate complex provides a mechanism. From geometric considerations alone, both the General Relativity field equations (the master equations of Relativity) and the Schrödinger equation (the master equation of quantum mechanics) are produced. Finally, to preserve the constancy of the volume element even internal to a mass, we propose a rolled-up fifth-dimension which is non-zero only in the presence of mass or energy.


## I. INTRODUCTION

Although it is a remarkably reliable schema for describing phenomena in the small, quantum mechanics has conceptual problems; e.g. How can entanglement transfer information instantaneously (without violating relativity)? What is happening in the two-slit experiment? In what medium does the wave propagate? What is the wave function? What explains superposition? Can the two-slit experiment (at least in theory) be performed with macroscopic masses? Further problems arise when considering the instantaneous collapse of the wave function, as in the Einstein-Podolsky-Rosen paradox[2], or when treating macroscopic systems, as in the Schrödinger's cat or Wigner's Friend paradoxes[3]. And finally, quantum mechanics is not overly compatible with general relativity[3].

The aim of 'Stochastic space-time' is to introduce stochasticity into the structure of space-time itself, rather than into the properties of the particles in the space-time. This is a similar, geometrodynamic, approach to Nelson's groundbreaking model[4] that indeed has matter moving stochastically in the space-time

Stochasticity decreases as one approaches the mass. And, insofar as stochasticity correlates to entropy which establishes the arrow of time, the 'length' of that arrow is not constant throughout space-time.

Since the fluctuations are not in space-time but of space-time, and because points have no extent, there seemed to be no way to prevent events (points) migrating to the same point. Therefore tessellating space-time would be problematic. So a granular model of spacetime seemed necessary. Granular space-time models suffer from the problem that if the grains have a specific size, then the model cannot be Lorentz invariant. Accordingly,

[^0]we'll model grains (which we call 'venues' to distinguish them from point-like 'events'), as having constant volumes (rather than constant dimensions) and volumes are Lorentz invariant.

Further, whereas the only geometrical property of an event is its coordinate location, grains, having extent, can have different values of $\Delta \mathrm{x}, \Delta \mathrm{y}, \Delta \mathrm{z}$, and $\Delta \mathrm{t}$. And that allows an explanation of curvature within four dimensions (as opposed to explaining it by embedding the four dimensional space-time manifold in a five dimensional Euclidean space). And as long as the 4 -volume of the grains (which we call 'venues') is constant, we do not violate Lorentz invariance.

In order that we treat time in the same way as we treat space (and not to have particles appear at different places at the same time), we needed a new version of time, $\tau$-time, where ' $\tau$-Time Leaves No Tracks' (that is to say, in the sub-quantum domain, there is no 'history'). The model provides a 'meaning' of curvature as well as a (loose) derivation of the Schwarzschild metric without need for the General Relativity field equations. In order to tessellate the space-time manifold, it seems necessary to introduce a complex time with the imaginary component 'rolled-up' at the Planck scale. The implication is that our usual t-time is just a human construct, not actually intrinsic to space-time.

As we wish to treat time and space similarly, we propose fluctuations in time. In order that a particle not appear at different points in space at the same time, we found it necessary to introduce a new model for time where time as we know it is emergent from an analogous coordinate, tau-time, $\tau$,

## II. SUMMARY: "STOCHASTIC SPACE-TIME AND QUANTUM THEORY"

An (much) earlier paper[1] posited a stochastic spacetime. From the five statements (postulates) in that paper, the uncertainty principle, interference and the spread of a free particle were derived. The statements (described in detail in the earlier paper) are as follows:

## Statement 1. A Mach-like principle.

1.1. In the absence of mass, space-time becomes not flat, but stochastic.
1.2. The stochasticity is manifested in a stochastic metric $g_{\mu \nu}$.
1.3. The mass distribution determines not only the space-time geometry, but also the space-time stochasticity.
1.4. The more mass in the space-time, the less stochastic the space-time becomes.

## Statement 2, the contravariant observable theorem.

All measurements of dynamical variables correspond to contravariant components of tensors.

Note that when one makes an observation of a dynamical variable (e.g. position, momentum, etc.), the measurement is usually in the form of a reading of a meter (or meter-stick). It is only through a series of calculations that one can reduce the datum to, say, a displacement in a coordinate system. For this reduction to actually represent a measurement (in the sense of Margenau[5]) it must satisfy two requirements. It must be instantaneously repeatable with the same results, and it must be a quantity which can be used in expressions to derive physical results (i.e., it must be a physically "useful" quantity). It has been shown[1] that for Minkowski space, the derived "useful" quantity is contravariant.

## Statement 3. The metric probability postulate.

$P(x, t)=f(-g)$, where for a one particle system is the particle probability distribution. $f$ is a real-valued function and $g$ is the determinant of the metric tensor.

In the earlier paper, $P(x, t)=f(-\sqrt{g})$, but further analysis implied the current probability distribution. The arguments are as follows:
$P(x, t)=f(-g)$ can be justified by the following: Consider that there is given a sandy beach with one black grain among the white grains on the beach. If a number of observers on the beach had buckets of various sizes, and each of the observers filled one bucket with sand, one could ask the following: What is the probability that a particular bucket contained the black grain? The probability would be proportional to the volume of the bucket.

Consider now the invariant volume element $d V_{I}$ in Riemann geometry. One has that[6]
$d V_{I}=\sqrt{-|g|} d x^{1} d x^{2} d x^{3} d x^{4}$.
It seems reasonable then, to take $\sqrt{-|g|}$ as proportional to the probability density $\left(\Psi^{*} \Psi\right)$ for free space.

Consider again, the sandy beach. Let the black grain of sand be dropped onto the beach by an aircraft as it flies over the center of the beach. Now the location of the
grain is not random. The probability of finding the grain increases as one proceeds toward the center, so that in addition to the volume of the bucket there is also a term in the probability function which depends on the distance to the beach center. In general then, we would expect the probability function $P(x, t)$ to be $P(x, t)=A \sqrt{-|g|}$ where $A$ is a function whose value is proportional to the distance from the center of the beach.
(From here on, we'll represent the determinant of $g_{\mu \nu}$ by $g$ rather then by $|g|$.)

The arguments above apply to the three-dimensional volume element. But we left out the other determinant of the probability density, the speed of the particle (the faster the particle moves in a venue, the less likely it is to be there.) And therefore, the larger the $\Delta t$ the more likely the particle is to be found in the venue. So indeed (it seems as if ) it is the four-dimensional volume element that should be used.

The metric probability statement $P(x, t)=A \sqrt{-g}$, as it stands, has additional problems:

First, if one considers the 'particle in a box' solution, one has places in the box where the particle has zero probability of being. And if $P(x, t)=A \sqrt{-g}=0$, that means the determinant of the metric tensor is zero and there is a space-time singularity at that point. We address this problem by noting that the metric tensor is composed of the average, non-stochastic, background (Machian) metric $g_{\mu \nu}^{M}$ and the metric due to the Particle itself $g_{\mu \nu}^{P}$. We say then that the probability density is actually $P(x, t)=A\left(\sqrt{-g^{T}}-\sqrt{-g^{M}}\right)$ where $g^{T}$ is the determinant of the composite metric. In this case, $P(x, t)$ can be zero without either $g_{\mu \nu}^{T}$ or $g_{\mu \nu}^{P}$ being singular.

A second and more trenchant problem is that $P(x, t)=$ $A \sqrt{-g}$ describes the probability density for a test particle placed in a space-time with a given (average) metric due to a mass, with determinant $g$. What we want, however, is the probability of the particle (not the testparticle) due to the metric contribution of the particle itself. Related to this is that $P(x, t)=A \sqrt{-g}$ doesn't seem to replicate the probability distributions in quantum mechanics in that the probability distribution, $\Psi^{*} \Psi$, is the square of a quantity (assuring that the distribution is always positive). But the differential volume element, $d V=\sqrt{-g} d x d y d z d t$ is not the square of any obvious quantity. Further, $P(x, t)=k \sqrt{-g}$ is something of a dead end, as it gives $\Psi^{*} \Psi$ but no hint of what $\Psi$ itself might represent. It would be nice if the probability density were proportional to the square of the volume element rather than to the volume element itself. With that in mind we'll again look at the probability density. (Multiple researchers have agreed with Part A's $P(x, t)=A \sqrt{-g}$ and it is therefore with some trepidation that we consider that the probability density might be subject to revision.)

The initial idea was that, given a single particle, if space-time were filled with 3 -dimensional boxes (venues), then the probability of finding a particle in a box would be proportional to the relative volume of the box. That
was extended to consider the case where the particle was in motion. The probability density would then also depend on the relative speed of the particle. We will however, now argue that $P(x, t) \neq A \sqrt{-g}$, but instead $P(x, t)=-A g$ (essentially the square of the previous). But this will apply only when the quantum particle is measured (a contravariant measurement) in the laboratory frame. If however, one considers the situation cotemporally (i.e. covariantly) with the quantum particle, then $P(x, t)$ does equal $A \sqrt{-g}$, which is to say that the probability density is [co- or contra-variant] frame dependent.

There is another argument, but it assumes the model of time in Section IV (relating to a time-like fifth dimension we have called tau).

Consider a quantum particle at a $\tau$-time slice at, say, $\tau=$ now. And also consider a static quantum probability function (e.g. a particle in a well) at $\tau=$ now +1 . (That function is a result of the quantum particle's migrations in time and space,) Then if we take a negligible mass test particle at $\tau=$ now, it will have a probability of being found at a particular location at $\tau=$ now +1 equal to that static probability function. And that function is proportional to the volume element (the square root of minus the determinant of the metric tensor). But what we're interested in is general the probability function of the quantum particle as $\tau$ goes from now to now +1 . We are considering the probability function at $\tau+1$ as static. But it is the result of the migrations of the particle. At tau $=$ now, it would then be the same probability function. So, as we go from now to now plus one, we would need to multiply the two (equal) probability functions. This results in the function being proportional to the determinant of the metric tensor (not its square root). This is rather nice as it allows us to suggest that the volume element is proportional to $\Psi$ while the probability density is proportional to $\Psi^{*} \Psi$. Note that this result is due to a mass interacting with the gravitational field it itself has generated. (This is analogous to the quantum field theory case of a charge interacting with the electromagnetic field it itself has created.)

As yet another approach, consider the spread of probability due to the migration of venues. In the absence of a potential, the spread (due to Brownian-like motion) will be a binomial distribution in space (think of it at the moment, in a single dimension and time). But there is also the same binomial distribution in time. This, for example, expresses that the distant wings of the space distribution require a lot of time to get to them. The distribution then seems to require that we multiply the space distribution by the time distribution. The two distributions are the same so the result is the square of the binomial distribution. (The argument can be extended to the three spacial dimensions.) In the laboratory frame, time advances smoothly, which is to say that the time probability density distribution is a constant, so we do not get the square of the binomial distribution..

It seems then that there are both the distribution and
its square in play. It might be that the covariant representation, i.e. the distribution 'at' the particle, is the binomial while a distant observer, where time advances smoothly (not in the quantum system being observed), observes (i.e contravariant measurements) the square of the binomial distribution.

Note: As the probability density is not stochastic while the metric components are, that puts constraints on the metric tensor, i.e. the determinant of the metric tensor is constant while the metric components are not. So (stochastic) changes in one or more components are compensated by opposite changes in the others. This implies that while a venue is in constant flux, its dimensions continuously and unpredictably change while the venue maintains a constant volume. This also implies that the metric stochasticity is due to a single (and the same) random variable in each non-zero metric component (That variable will then drop out in the determinant.)

A note on the holometer experiment[7]: The experiment looks for space-time fluctuations at the Planck scale, and has found no fluctuations. (The experimenters though, suggested that perhaps some unknown symmetry masks fluctuations.) The quantum zero point energy uncertainty however, says that there are energy fluctuations which are equivalent to mass fluctuations which (from general relativity) generate stochastic metric tensor fluctuations. We (among others) identify the spacetime volume element as proportional to the quantum mechanical probability density which is not stochastic. The volume element is the determinant of the metric tensor. Again we have then, a stochastic metric tensor and a non-stochastic determinant of the metric tensor. This could happen if for any fluctuation of a metric tensor component, other components reverse the fluctuation. In the case of a local diagonal metric, the fluctuations of the space components could be counteracted by the time component. This could be that aforementioned unknown symmetry. We suggest then, that the null result of the holometer experiment supports our granular stochastic space-time model. (Craig Hogan, who came up with the Fermilab holometer experiment concurs (personal communication) that the metric model above could indeed possibly explain the holometer negative results.)

## Statement 4. the metric superposition postulate.

If at the position of a particle the metric due to a specific physical situation is $g_{\mu \nu}(1)$ and the metric due to a different physical situation is $g_{\mu \nu}(2)$ then the metric at the position of the particle due to the presence of both of the physical situations is $g_{\mu \nu}(3)$,
$g_{\mu \nu}(3)=\frac{1}{2}\left[g_{\mu \nu}(1)+g_{\mu \nu}(2)\right]$.
This is the case where the probabilities, $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$, of the two metrics are the same. In general though, Statement 4 becomes,

$$
g_{\mu \nu}(3)=P_{1} g_{\mu \nu}(1)+P_{2} g_{\mu \nu}(2)
$$

## Statement 5. The metric $\Psi$ postulate.

There exists a local complex diagonal coordinate system in which a component of the metric at the location
of the particle is the wave function $\Psi$.

## III. GRANULAR: SPACE-TIME

Vacuum energy fluctuations are stochastic, and so then is also the metric tensor, so stochastic space-time is nowhere integrable nor even continuous. So it is hard to see how point-like 'events' could tessellate space-time, since points take up no volume and therefore could migrate over one another. For those reasons, we propose that space-time time is granular. The grains (which we call 'venues' to distinguish them from 'events') cannot have constant dimensions as that would not be Lorentz invariant. Instead we posit constant volumes (which is Lorentz invariant). The venue dimensions however are at the Planck scale. (A previous paper[8] argued that the Planck length is the smallest possible length, the Planck time is the shortest possible time increment, and the Planck mass is the smallest mass not subject to quantum uncertainties so, for example, the two-slit experiment, even in principle, cannot be reproduced with, say, cannonballs.)

We could visualize a venue as a somewhat amorphous marshmallow. One can gently squeeze it. The dimensions will change, but the volume will stay more or less constant.


An event is specified by four parameters; $x, y, z, t$. But a venue has an orientation, requiring more parameters. We can specify the orientation as a rotation described by a quaternian, $\mathbf{q} \cdot \boldsymbol{q}=a+b i+c j+d k$ where $\mathrm{a}, \mathrm{b}, \mathrm{c}$, and d are real, and $i^{2}=j^{2}=k^{2}=i j k=-1$.

If we consider a pure imaginary quaternian $(a=0)$, it can be shown[9] that given a vector $\mathbf{v}$, we can rotate it around $\mathbf{q}$ by $e^{\boldsymbol{q} \theta} \boldsymbol{v} \boldsymbol{e}^{-\boldsymbol{q} \theta}$. This a rotation by $2 \theta$ (showing the 720 degree symmetry displayed by fermions).

Perhaps a more intuitive exposition can be given using a spinor (spinors are isomorphic to quaternians).

Again consider our marshmallow model of a venue. Take a pencil and stick it into one side of the marshmallow.


By moving the eraser end of the pencil (acting as a vector), we can change the orientation of the venue. But we can't attain every possible orientation. In order to achieve that, we need to rotate the pencil around its axis. The pencil is no longer a vector as vectors can't rotate around their axes. With our pencil, we can indicate that axis rotation by affixing a 'flag' to the eraser end.


What we have now is the real representation of a spinor,

consisting of four parameters as per the diagram[10]. However it does not show the 720 degree symmetry. (After a spinor rotates by 360 degrees, a binary variable switches from zero to one, A further 360 degree rotation returns the variable to zero.)

To replicate this, we need to consider the full complex form of a spinor. (We should note that if a venue is not interacting with any other venue, then the symmetry is 360 degrees.)

Collecting the four parameters from the diagram above,
$a \equiv \sqrt{r} \cos (\theta / 2) e^{i(-\alpha-\phi) / 2}$
$b \equiv \sqrt{r} \sin (\theta / 2) e^{i(-\alpha+\phi) / 2}$
If we rotate the spinor around the y axis by $\theta$, for example, we obtain[10],
$\left(\frac{a^{\prime}}{b^{\prime}}\right)=\left(\frac{\cos (\theta / 2),-\sin (\vartheta / 2)}{\sin (\theta / 2), \cos (\theta / 2)}\right)\left(\frac{a}{b}\right)$ again exhibiting the 720 degree symmetry.

Note that the pencil represents a null vector.
Let us return to our marshmallows and see if we can provide a graphic justification for the 720 degree symmetry.

Consider a venue/marshmallow interacting with another via a band of force (analogous to lines of force, below). If the venue were not interacting, the rotational symmetry would be 360 degrees.


Now (rather like 'The Dirac Belt Trick'[11], we rotate a marshmallow first 360 degrees, and then a second 360 degrees. (Note: a spinning venue doesn't leave holes in space-time.)


As per the Dirac belt trick, we can, without any further rotations, we can undo the rotations (as shown).


If the 'belt' is interpreted as a band of force, we can go further. In the same way that two orthogonal beams of light can pass through each other, a section of the belt overlaying another section, can pass through the belt, resulting in a zero degree rotation. Imagine the Dirac Belt trick where one doesn't need to move the buckle over or under the belt, because the 'force bands' do that automatically (to go to a state of minimum energy.)

We propose that only venues holding mass can interact. Venues are very small so they can't hold much mass, certainly much less than any fundamental particle. But a large cluster of interacting venues can act as a single body, and the rotations thereof can be particle spin. Classically, a particle with spin would need to have a tangential velocity greater than the speed of light to give the observed spin angular momentum. But here, venues inside the cluster can rotate independently of the other venues, so one could obtain the required angular momentum without exceeding c. (This is similar to quark rotations inside a nucleon.)

So 720 degrees is equivalent to 0 degrees in a spinning venue (or venue cluster).

As pointed out above, the pencil is essentially a nullvector, so spinors exist on the light cone[10]. But, considering the stochastic nature of this model, the spinor can point either forward or backward (and on average move forward via zittebewegung/jittery-motion). This seems to require a model for 'time'.

## IV. TIME

As we are treating space stochastically, for covariance we would like to treat both space and time similarly. To do that, we then let the stochasticity apply to time as well as space. This leads to an obvious problem: If a venue contains mass, then migrations can position the mass so it appears at multiple positions in space at the same time. E.g. A venue containing mass could migrate one unit backward in time, then one unit forward in, say, x , then one unit forward in time, resulting in the mass
being at both ( $\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t}$ ) and ( $\mathrm{x}+1, \mathrm{y}, \mathrm{z}, \mathrm{t}$ ). Preventing this necessitated a change in how we view time.

First, let's consider the idea of the 'world-line'. Moving forward from the present, we are predicting the future. And with quantum uncertainties (as well as with the intervention of outside forces) that future cannot be predicted according to classical determinism. And if there is no completely deterministic trajectory going forward, then arguably neither is there one going backward in time. The world-line then, seems to have limited utility in quantum mechanics. Instead of a world-line, we consider a 'world-double-cone', with its apex at 'now' that widens as one moves forward or backward in time. So while quantum mechanics lets us probabilistically predict the future it also lets us probabilistically predict the past.

We suggest that for the quantum world, t is not the (real component of the) fourth dimension, and that $t$ is an emergent quantity, if not merely a human construct based on memory. The time coordinate, $t$, is a defined quantity in the laboratory frame whereas we suggest (below) another quantity, $\tau$ (tau-time) is appropriate in the quantum domain.

We'd like to treat the time dimension, t , in the same way as we treat spacial dimensions. But there is a big difference between a space and time coordinate: Consider the graphic below:


A particle (the black disk) starts at $\mathrm{x}=0$, then moves to $\mathrm{x}=1$, then 2 , then 3 . (We are considering space-time to be granular, hence the coordinate boxes.) There is a single instance of the particle.

But time is different:


A particle at rest is at $\mathrm{t}=0$, then moves to $\mathrm{t}=1$, etc. But when it goes from $\mathrm{t}=0$ to $\mathrm{t}=1$, it also remains at $\mathrm{t}=0$. There are now two instances of the particle, etc. In other words, a particle at a particular time is still there as time advances, and the particle is at the advanced time as well.

We define then, a new quantity, $\tau$ (tau-time), that acts much like the usual time, but in accord with the first graphic, above, i.e. when the particle advances in time, it erases the previous instance. That is to say, ' $\tau$-Time Leaves No Tracks'. Aside from fixing the problem of the same mass appearing at an enormous number of different locations at the same time, $\tau$ will be seen to provide a solution to the collapse of the wave-function problem.

As per above, we treat time and space similarly. And so we will consider diffusion in space as well as in time.

Consider the graph (of 10000 points) below. (The vertical and horizontal lines are artifacts of the graphing software.) The graph represents the path of a a single venue migrating in $x$ and also in $t$, both with a measure
of 0.5 , where the coordinate axes are laboratory x and laboratory t .

> Graph 'A'


And here is the graph with the same data as above, but where 'time moves forward' is taken into consideration).


In either graph, there is an immediate problem:
Consider what these graphs signify: At any given laboratory-time $t$, the same venue will (simultaneously) be at a very large number of x coordinates. If there were mass/energy at the venue, this would be very problematic as causality and conservation of mass would be violated.

This problem has been addressed by introducing $\tau$ (tau-time), and the ' $\tau$-Time Leaves no Tracks' idea.

We can still consider Graph 'A', but we'll interpret it differently: If we take any (horizontal) time ( $\tau$ ) as a 'now', A venue (containing a mass) stochastically flits forward and back in space, and forward and being stationary in time. So that at 'now' there is one and only one particle. But where it is cannot be predicted. However, the likelihood of the particle being at a particular $x(+/-d x)$ position is determined by the relative number of times the particle is at that position. In the case of Graph 'A', if we take as 'now' the $\tau$-time slice at -0.2 , for example, we find (by examining the data) the following probability curve:


This is analogous to $\Psi^{*} \Psi$. But the graph is a construct. It represents, but is not actually, the particle. When the particle is measured by, for example, being absorbed in a detector, it freezes (no longer moves stochastically). It no longer flits through time and space so the graph 'collapses' to the measured position. (that position is only determinable by the measurement.) This is analogous to the collapse of the wave function, but here (as the graph was merely a mathematical construct) there is no collapse problem.

There are a few points/speculations to be made about measurements. First, to be a true measurement, there must be a latch/flip-flop/memory so that the 'film' cannot be run backwards. As an example, consider the two slit experiment with electrons. If a measurement device is placed at a slit, there is no interference pattern. But when an electron goes through a slit, the orbital electrons in atoms of the wall of the slit will be distorted by the passage of the electron. This distortion is almost a measurement. But when the electron passes through the slit, the orbital electrons become un-distorted. The interference pattern is still produced because there is no latching of measurement information. A latch could be some mechanical contrivance, or even human (or nonhuman) memory. A fruit-fly observing at the slit will kill the interference pattern, but only for the fruit-fly. We think the process should be transitive; A human observing the fruit-fly's memory will cause the interference to be killed for the human as well. A measurement forges a connection between the thing being measured and the measurer-forcing them to have the same relative now. In the macro-world, virtually everything observes (via photons) everything else, forcing that macro-world (or a portion thereof) to have the same relative now. And measurements forces time to have tracks. Not that time is frozen, but looking back to a particular time will show uniquely what the world looked like at that time. E.g., if one were to do high-speed filming of particle 'tracks' in a cloud chamber, one would see the time-tracks.

Observation, a crucial part of a measurement, is conducted via photons. We speculate that all measurements are via photons (or, equivalently, by the electromagnetic field)?

The time leaves no tracks concept implies that there are multiple futures, and they all 'happen'. (This is somewhat redolent of the Everett many-world interpretation[12].) In this model (Granular: Stochastic

Space-time [G:SST]), an observation from the laboratory will select a particular future (making a track).

In the above, if the particle were in a potential well with perfectly reflecting walls, the above graph would (after a time) represent the probability density of finding the particle at a particular position in the well.

Again, the particle has always existed at only a single venue, but the venue migrations happen roughly at the rate of the Planck time, making the particle appear (in some sense) to be at multiple positions at a particular time. Further, (because of the properties of Wiener Processes) the particle appears to spread. If the particle were not constrained by the well, (because time is moving forward and back) the graph would evolve (spread) arbitrarily rapidly. In that case the curve would represent the relative probability density of finding the particle at a particular position. The curve then would represent DeBroglie's 'ghost waves that guide the particle'[13].
(The jagged lines in the graph, as opposed to a smooth curve, is an artifact of the binning algorithm in the software.)

By Statement 1.4, the particle location becomes less stochastic as mass increases. There is a point where the stochasticity ceases. At that point, (since it is not migrating back and forth through $\tau$-time), one can use the usual t-time. So, we consider t-time (and also causality) to be an emergent quantity. In the rest of this paper, when we do not reference history, we will simply use $t$ instead of $\tau$. Also, as a result of measurement, when the above graph 'collapses', the time is fixed, so measurement causes time to 'leave tracks' causing $\tau$ to become $t$.

Now we can revisit Statement 3: The metric probability postulate, $P(x, t)=-A g$.


Consider the above diagram. (Note: time increases downward.)

A particle is placed at the apex (more accurately, we place a venue at the apex). It migrates stochastically until it ends up in one of the numbered bins. A typical migration path is shown in the diagram. If we repeat the process a large number of times, the number of particles in each bin (of the x coordinate) can be described by a binomial distribution centered on bin 5 . That binomial distribution can represent the probability density of finding a particle in a particular bin at laboratory time 10 (the bottom of the pegboard). As we increase the number of time steps, the distribution will flatten, until it is
essentially flat. At that point the size of the invariant volume element $d V_{I}=\sqrt{-|g|} d x^{1} d x^{2} d x^{3} d x^{4}$ determines the probability density. And that is the original version of Statement 3.

In the diagram above, the bin underlines represent different volume elements at different regions of the space. A way of determining the probability densities is to count the number of possible migration paths to a particular region. (The space-time is discrete so the number is countable.) The probability densities are proportional to those numbers.

Consider the typical path shown above. We can represent it as follows:

| $\mathrm{t}:$ | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{x}:$ | +1 | -1 | -1 | +1 | -1 | +1 | +1 | -1 | -1 | -1 |

Time increases by one unit at each transition.
At this point, the original version of Statement 3 still applies.

But now let there also be diffusion in time, as well as in space. We can repeat the pegboard analogy, but consider the venue time rather than venue mass. We will of course obtain the same probability distribution as previously. The distribution will represent the probability density of the quantum time having the value of the laboratory time at laboratory time equal to 10 (the bottom of the pegboard).

The peak of the distribution curve corresponds to the peak on the previous curve. Accordingly, the probability density for x to be at 5 (the middle of the pegboard) must be that previous distribution value at $x=5$ times the quantum time distribution at $x=5$. But the two distributions are the same, so the probability density is the square of the distribution. The same argument applies for any point along the base (x coordinate), i.e. the probability density for $x$ is the square of the binomial distribution.

So, again considering the invariant volume element, this means that the probability densities are the square of the values of the original Series 3 values, i,e, $P(x, t)=$ $-A|g|$.

## Complex Time

'Time' can be considered to have two characteristics: a coordinate $(\tau)$ from minus to plus infinity (or from the big bang to some end of time), and a sequencer ( $v$ ), an ordering schema as described by H. Reichenbach[14]. determining the direction and 'speed' of time. We'll consider the sequencer function to be described by the imaginary (rolled-up) component of time (the phase).

The imaginary component acts much like a separate (time-like) fifth dimension. This is vaguely similar to the idea that there is a fifth dimension which is mass, as proposed by Mashhoon \& Wesson[16] and the Space-Time-Matter consortium[17].

We define Total (complex) time T .
$\mathrm{T}=\tau+\mathrm{i} v$.
$\tau$ is the 'Time Leaves no Tracks' version of $t$.
$v$ is the imaginary component of time. It is rolled-up at the Planck scale so in the macroworld T is indistinguishable from $\tau$.
(Note: 'complex time' is not an entirely new idea, e.g. S. Hawking[18].)

Letting $\tau$ and $v$ be represented by a real and imaginary coordinate axis, we define Time-length (duration), $\mathrm{T}_{\mathrm{d}}=$ $\sqrt{\tau^{2}+(i v)^{2}}$.

A property of time is that it (usually) advances. As $v$ is a component of time, we assume it advances as well. But $v$ is rolled-up, so, as it continuously advances, it continuously reaches a maximum and rolls over to zero. We represent this as a frequency.

Masreliez[19] and Mukhopadhyay[20] among others have suggested that a mass oscillates at its Compton frequency, (and without such oscillation, there would be no DeBroglie wave, or indeed a $\Psi$ ). We accept that suggestion. The Compton frequency, $f_{c}$, is defined as $f_{c}=\frac{m c^{2}}{h} H z$.

We first convert Hz to cycles/Planck time.

$$
\frac{f_{c}}{\sqrt{\frac{h G}{c^{5}}}}=\frac{m c^{2}}{h}
$$

Now we'll convert m from kilograms to Planck mass, $m_{p}$.

$$
\frac{f_{c}}{\sqrt{\frac{h G}{c^{5}}}}=\frac{m c^{2}}{h} \sqrt{\frac{h c}{G}}
$$

Simplifying, we have $f_{c}=m_{p}$.
This says that if the mass in a venue is zero, (from the viewpoint of the laboratory observer) the $v$ time does not advance (which allows the creation/annihilation mechanism to work). The more mass in a venue, the more 'rapidly' $v$ advances until at a maximum venue mass of one Planck mass, the frequency has increased to one cycle per Planck time. And in that latter case, every Planck time advances $v$ to the same angular point, which is then indistinguishable from a frequency of zero. In short then, we associate mass with a frequency (the Compton frequency) of the imaginary time component.

## V. THE DIFFUSION AND SCHRÖDINGER EQUATIONS

Now that we've introduced complex time, we'll use the non-relativistic diffusion equation to derive the nonrelativistic wave equation (the Schrödinger equation).

The G:SST model is essentially a description of diffusion of space-time. As such, one might think that the diffusion equation, $\frac{\partial \varphi}{\partial t}=D \frac{\partial^{2} \varphi}{\partial x^{2}}$ would be part of that description. This, the one-dimensional diffusion equation, is easy to derive.

First consider the 'flux' , j , (in the x direction) of a quantity through a section perpendicular to x (per unit area and per unit time). We ignore the bulk motion of the carrier (assume fluid). And let $\varphi$ be the 'concentration' of the quantity.

We can see that $\frac{\partial \varphi}{\partial t}=-\frac{\partial j}{\partial x}$.

We can also see (Fick's first law [21]) that $j=-D \frac{\partial \varphi}{\partial x}$ where D is the Diffusivity coefficient.
( D is a proportionality factor between the diffusion flux and the gradient in the concentration of the diffusing substance. The higher the levels of diffusivity of a certain substance to another, the faster the diffusion rate by both of the substances. It is given the unit of length squared per unit time.)

So $\frac{\partial \varphi}{\partial t}=\frac{\partial}{\partial x}\left(D \frac{\partial \varphi}{\partial x}\right)$. And if D is constant, that yields the above diffusion equation,

$$
\frac{\partial \varphi}{\partial t}=D \frac{\partial^{2} \varphi}{\partial x^{2}} .
$$

The equation is very similar to the (potential free) one dimensional Schrödinger equation, but with several significant differences: The 'diffusion constant' in the Schrödinger equation is complex, and the interpretations of the solutions differ. In the Diffusion equation, the solution, $\varphi$, (the concentration) can trivially be interpreted as a probability density (of a test particle having diffused to another position in space), whereas with the Schrödinger equation, it is the square of the solution that corresponds to the probability density. And significantly, the diffusion equation, while it describes diffusion in space, does not describe diffusion in time, nor does it consider the effect of the rolled-up imaginary time component.

We'll attempt now to include diffusion in time to see how that effects the solution of the diffusion equation.

We start by observing that the probability of a particle starting at $x_{0}=t_{0}=0$ arriving at a point $x=x_{1}$ is proportional to the number of ways the particle in a fixed number of steps, $n$, (corresponding to n time increments in the laboratory frame) can arrive at point $x_{1}$. And we'll calculate it from the 'laboratory' frame where time is granular but not stochastic. As an example, consider the following diagram. The pegboard represents where a 'particle' will land when 'dropped' from the top of the pegboard.


First we consider the cases where there is no time diffusion.

The jagged line in the above diagram represents a typical path of the ball at the top of the diagram dropped down on the 'pegboard'. If many balls are dropped, the balls will fall into bins as above, and their numbers in each bin will result in a binomial distribution. (The Binomial distribution is equivalent to the Gaussian distri-
bution when the number of axis points is large.)
In the diagram, we can consider the height the time axis (increasing downward) and the horizontal the x axis. The top ball then is initially at $\mathrm{t}=0, \mathrm{x}=5$. As it falls, at each time interval (when it encounters one of the pegs), it can move either one $x$ unit to the left or right.

We can represent the typical path above as follows:

| $\mathrm{t}:$ | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{x}:$ | +1 | -1 | +1 | +1 | -1 | +1 | -1 | -1 | +1 | -1 |

Again, summing over all possible paths (with time increasing one unit per point of direction determination), the probability (here, the binomial) distribution results.

Now if we consider also diffusion in time, the top row of the table will no longer be all +1 . The numbers in the top row will have the same number of variations as in the second row. And, as before, we us assume isotropy, i.e., the distributions are the same for $\mathrm{x}, \mathrm{y}, \mathrm{z}$, and t . And so the number of paths will be the square of the number of paths of the non-time-diffusion paths. And so the probabilities of (the balls being in a particular bin) in the time diffusion paths will be the square of the number in the non-time-diffusion paths. In other words, the solution, $\varphi$, of the diffusion equation represents the square root of the probability density. (In our Brownian motion model where only one direction at a time migrates, the square property holds over three dimensions and not just over the one-dimension case above.)

Now, having included time-diffusion in the (interpretation of) the diffusion equation, we turn our attention to including the possible effects of complex time.

First regarding $\frac{\partial \varphi}{\partial t}=D \frac{\partial^{2} \varphi}{\partial x^{2}}$, Nettel in 'Wave Physics'[22] says: "If we are to have a solution to a first order differential equation, that solution will have to be an exponential function rather than a trigonometric one. Moreover, to avoid having the solution go to infinity or be exponentially damped as t goes to either plus or minus infinity but rather to get waves, the exponent in the solution will have to be imaginary. As the reader can easily check (if we include i in the equation), we get the solution $\psi(x, t)=e^{i(k x-\omega t)}$."

The diffusion equation is for diffusion in 3 -space (although we've interpreted it as a diffusion also in time). But how do we include the rolled-up, imaginary time. Taking guidance from the above, we will again introduce another coordinate axis, an imaginary-time axis perpendicular to the real-time axis.

Imaginary time is rolled-up. It's coordinate then continues to increase, rolling around to zero, etc. This gives a complex frequency $e^{i v}$, where we can let $v$ be $k x-\omega \tau$.

So, now having an imaginary axis for $v$-time, we'll again define a 'total time' T which will be the combination of $\tau$-time and $v$-time,

$$
\underset{\partial T}{\mathrm{~T}}=\tau+\mathrm{i} v .
$$

$\frac{\partial T}{\partial t}=1$ and $\frac{\partial T}{\partial v}=i$
We have then, $\frac{\partial \Psi}{d T}=\frac{\partial \Psi}{\partial \tau} * \frac{\partial \tau}{\partial T} * \frac{\partial \Psi}{\partial v} * \frac{\partial v}{\partial T}$.
Substituting gives, $\frac{\partial \Psi}{\partial T}=\frac{\partial \Psi}{\partial \tau} * \frac{\partial \Psi}{\partial v} *(-i)$.

What can we say about $\frac{\partial \Psi}{\partial v}$ ?
As $v$ is periodic, then so to is $\frac{\partial \Psi}{\partial v}$, and the period is at the Planck scale.

Now the Diffusion equation, though working in the macro (and to some extent) quantum domains, might not be expected to work at the Planck scale. We 'blur' the time in the Diffusion equation (i.e., take over a short (but not too short) time, then $\frac{\partial \Psi}{\partial v}$ will average out to a constant. And as the solution depends on details of the physical situation, that constant, here called $k$, is (at least most all of the time) not zero.

We will use the 'total time' T , rather than $\tau$ in the equation. So now, we have, $\frac{\partial \Psi}{\partial T}=i k \frac{\partial \Psi}{\partial \tau}$ and the Diffusion equation becomes, $-i k^{-1} \frac{\partial \Psi}{\partial T}=D \frac{\partial^{2} \varphi}{\partial x^{2}}$

Of course, we could have just used Schrödinger's argument (about needing i for there to be waves) to arrive at his equation. but once we have included i , in $i \frac{\partial \varphi}{\partial t}$ the equation, though very useful, is unphysical. The above argument is intended to provide a physical (i.e. geometric) description.

## VI. CREATION AND ANNIHILATION OF VENUES

An issue with a space-time of granules with constant volumes rather than with points is how to handle an expanding or contracting space-time or region of spacetime. We need a mechanism to create and annihilate empty venues (venues not containing mass) without leaving gaps in the space-time manifold. We suggest the following mechanism:

As a part of the universe contracts, a venue's 4 -volume must also contract. Since the contraction is a 'time' process, we suggest that the venue contraction is in the (real) time component. To keep the volume element constant, as the real time component contracts, the imaginary time component expands. (We've previously shown that in the absence of mass u-time is static.)

The following diagrams show coordinates $\tau$ and $\cup$ (the imaginary time component), and a rectangular solid representing a venue.


Again, as the $\tau$ coordinate of the venue contracts, to preserve the volume, $u$ must expand. At some point, the contraction coordinate, $\tau$, approaches zero while $u$, approaches its circumference.


At the point where the contraction reaches zero, the $u$ component 'rolls over' to zero. The volume is then zero and the venue blinks out of existence.

Creation of venues is similar: When a mass-less venue expands, it increases the real time coordinate. The imaginary time component decreases to compensate. The
imaginary time component rolls over to give a high value to the component. The venue volume is then is far too high. The venue then splits, giving each new venue half the original real time component and half the original imaginary time component. As the expansion increases, the imaginary time components decrease as the venues' real time component increase, moving the two new venues towards equilibrium.

At no point then, is the space-time manifold not fully tessellated.

## VII. VENUES IN SPACE-TIME

A venue has, as previously postulated, a constant volume. Far from a mass, the volume is symmetric, its dimensions being (arguably) the smallest possible, which we take to be the Planck length (and time). So a venue has a volume of a Planck length Planck-length cubed times a Planck-time. And since the venues migrate across the space-time, we can ask the distance difference between a venue before and after a migration (resulting from the exchange of two adjacent venues). We postulate that this is the minimal possible distance. If there were no migrations that distance would be zero. We take it that the distance is still zero, but in Minkowski space. In other words, a migration's change of 4-position equals zero. So, migrations occur on the forward or backward light cone (as we noted earlier, spinors live on the light cone).

In differential geometry, Loveridge[23] has pointed out that the Ricci tensor governs the evolution of a small volume element (i.e. $\sqrt{-g}$ ) as it travels along a geodesic.

Following Loveridge, assume a very small spherical volume of dust $o$ centered on point $x^{\mu}(0)$ moving along a direction $T^{\mu} .\left(T^{\mu} \equiv \frac{d x^{\mu}}{d \tau}\right)$. One has that $\frac{D^{2}}{d \tau^{2}} o-\frac{D_{f l a t}^{2}}{d \tau^{2}} o=$ $-o R_{\mu \nu} T^{\mu} T^{\nu}$, where $D$ is the covariant derivative along the path. The equation applies for both three and four dimensional volumes. The reason for subtracting the second term is that the choice of coordinates could give an apparent (not intrinsic) change of volume.

In Special Relativity, the Ricci tensor is zero. Which means that the volume element, $\sqrt{-g}$, is invariant. (For Special Relativity, this is easy to see: In a Lorentz transform, as the length shrinks, time expands to leave the volume unchanged.) In General Relativity, in empty space-time, while the Riemann tensor is not zero, the Ricci tensor is. So, in empty space (i.e. exterior to a mass), the volume element is also invariant, i.e. $R_{\mu \nu}=0$. This expression is the Einstein field equations where the stress-energy tensor is identically zero (which was obtained merely by requiring that the volume element is constant).

In a mass though, the Ricci tensor is not zero so the volume element is not constant. Furthermore, our stochastic space-time quantum mechanics model postulates that the volume element (squared) is not constant
and is proportional to $\Psi^{*} \Psi$. In order that the volume element does remain constant, we postulate (a 5 -D volume and) a fifth dimension that is zero except in a mass. This is a proposal similer to that made by the Space-time-matter Consortium[17]. (Were the volume element not constant, it is hard to see how to tessellate spacetime.) So that there is no measurable effect on 'distances', we require that this fifth dimension be rolledup at the Planck scale. This seems consistent with the Kaluza-Klein formalism[24] to bring electromagnetism into geometry.

We explore now whether the model might possibly reproduce the Schwarzschild metric (without formally solving $R_{\mu \nu}=0$ ). [Note that we have already reproduced the metric by requiring a constant volume element.]

A mass generates 'curvature', that is to say, a deformation of venues. While to a distant observer the venues are deformed to be spatially concentrated around the mass, to the venues near the mass there is no observable evidence of such concentration as the space-time itself is 'deformed' (by way of the venues) so any 'observer' in a venue would be unaware of the deformation.

We'll introduce another variable: 'Indeterminacy', the probability that a migration will actually happen.

As with 'Measure', Indeterminacy is implemented with a 'coin flip'. And we'll suggest that outside of a mass, the Indeterminacy decreases with decreasing distance from the mass/energy (i.e. space becomes more determinate as one approaches a mass). It will be seen that 'Measure mainly influences quantum effects while Indeterminacy influences relativistic effects.

The space-time Indeterminacy decreases as one approaches a mass. But this is under-specified; masses can have different densities, so we wouldn't expect the Indeterminacy to necessarily vanish at the surface of a mass. We suggest however, that venues can migrate into a mass until, at some point the Indeterminacy vanishes. Yet we do not want masses to be pulled apart by the space-time so we'll posit that migrations of adjacent venues each containing mass must stay adjacent. And in that case, one could consider each of those venues having zero Indeterminancy compared to the others.

We'd expect that at some distance, Rs, from the center of the mass, the venues, would be trapped, i.e. unable to migrate away. This is highly suggestive of the event horizon of the Schwarzschild solution. We'll assume Rs (the Indeterminacy radius) and the Schwarzschild radius are the same.

Consider space-time with a single spherical mass $m$ with an Indeterminacy radius Rs. As one increases the number of coin flips towards infinity, the time interval decreases to an infinitesimal, dt. For a granular spacetime though, the number of coin flips isn't infinite and the time interval, though small, isn't infinitesimal. Once again, Indeterminacy is the probability of, given that the venue is at a position with that Indeterminacy, the venue migrates from that position at the next coin flip.

Since migrations slow as venues approach a mass, in-
determinacy then, expresses the slowdown in time and the compression of space as the venue approaches Rs. [As we'll be frequently employing Indeterminacy, we'll represent it by the letter 'u' (from the German word for indeterminacy, Unbestimmtheit)].

As a venue migrates in towards $R_{s}$, u decreases. The probability density of the venue being at a particular radial distance, $r$, therefore, increases. This results in venues piling up as they approach $R_{s}$. But as the venues 'tile' space-time, the only way they can pile up is by way of curvature (i.e. squishing in the radial dimension and compensating by lengthening in the real-time $\tau$ dimension): To a distant observer, the venues would decrease in size and migrate more slowly which is to say time would slow down.

Recalling (see Statement 2) that the contravariant distance to a black hole is $\int_{0}^{\bar{r}} d r=\bar{r}$, while the covariant distance is $\int_{0}^{\bar{r}} d\left(\frac{r}{1-2 \mathrm{Gm} / r}\right)=\infty$, we (in Cartesian coordinates) associate the contravariant distance with the number of Planck lengths from the observer to the point of observation and the covariant distance with the number of venues from the observer to the point of observation.

This implies that local to the particle, space-time is not stochastic. And there, a deterministic Lagrangian can be defined. That 'local to the particle space-time' coordinate system is covariant (as it is moving with the particle). From another coordinate frame (e.g. the laboratory frame) measurements on that local frame are subject to the intervening stochasticity, and because of that stochasticity, the measurements are also stochastic, and the measurements are contravariant, as can be seen by the raising of the covariant coordinates by the stochastic metric tensor).]

Now, near $\mathrm{r}=R_{s}$, space-time becomes Q-classical (no quantum effects, as opposed here to R-classical: no general relativity effects) so a metric makes some sense. Since the Measures (bias in the coin flips) are presumed not to be a function of location, we take the simplifying assumption that the metric tensor does not depend on the Measures, but only on the Indeterminacy, u. And, for the moment, we'll ignore how a venue migrates in a mass (when $R_{s}$ is less than the mass radius).

Since for a mass, we have spherical symmetry, we can let, $d s^{2}=-f(u) d t^{2}+g(u) d r^{2}+r^{2} d \Omega^{2}$ where f and g are two (to be determined) functions of u , and $d \Omega^{2} \equiv$ $d \theta^{2}+\sin ^{2}(\theta) d \varphi^{2}$ is the metric of a 2-dimensional sphere. Consider $\mathrm{f}(\mathrm{u})$ and $\mathrm{g}(\mathrm{u})$. We wish dt to lengthen and dr to shorten as $u$ decreases. ds can be thought of as the time element in the frame of the venue. So, for example, as $u$ goes to zero, a big change in $t$ will result in a small change of $s$, and a small change in $r$ results in a large change in s. The simplest implementation of the above suggests that $f(u)$ is just $u$ itself and $g(u)$ is $u^{-1}$ i.e. $d s^{2}=-u d t^{2}+u^{-1} d r^{2}+r^{2} d \Omega^{2}$.

Now, as to $u$, note that,
at $\mathrm{r}=$ infinity $\mathrm{u}=1$,
at $\mathrm{r}=$ Rs $: \mathrm{u}=0$, and
for $\mathrm{r}<$ Rs : u can become unphysical $(\mathrm{u}<0)$.

The simplest expression for u satisfying the above is, $u=\left(1-\frac{R_{s}}{r}\right)$ which gives us
$d s^{2}=-\left(1-\frac{R_{s}}{r}\right) d t^{2}+\left(1-\frac{R_{s}}{r}\right)^{-1} d r^{2}+r^{2} d \Omega^{2}$
We have of course, as described earlier, equated the Schwarzschild radius with the Indeterminacy radius.

This is the result Karl Schwarzschild derived from the General Relativity field equations. One can easily go a bit further by noting that $R_{s}$ can only be a function of the mass, and finding a product of mass with some physical constants to give a quantity with dimensions of length suggests $R_{s}=\frac{k G m}{c^{2}}$ where k is a constant. So we now have (setting units so that $\mathrm{c}=1$ ),

$$
d s^{2}=-\left(1-\frac{k G m_{s}}{r}\right) d t^{2}+\left(1-\frac{k G m_{s}}{r}\right)^{-1} d r^{2}+r^{2} d \Omega^{2}
$$

We still need to determine the value of the constant, k. But this is known territory. $R_{s}$ was derived (by Karl Schwarzschild and others) by requiring the metric to reproduce the Newtonian result at large values of $r$ and small values of mass, and we need not reproduce the derivation(s) here.

## VIII. THE CONSTANCY OF THE SPEED OF LIGHT

We consider that the Granular: Stochastic Space-time (G:SST) theory is (or can be made to be) a super-set of the Lorentz Aether Theory (LAT) where the aether is space-time itself (specifically, the 'grains'/venues making up the space-time). By doing so, we can appropriate the LAT derivation of the constancy of the speed of light. (We feel that any theory of space-time should contain an explanation of that constancy.)

As is widely known[25], the Michelson-Morley experiment failed to find the Lorentz aether, thus seemingly invalidating the Lorentz Theory[26]. Less widely known perhaps, is that the second version of Lorentz's theory (with H. Poincairé as second author) reproduced Einstein's Special Relativity (ESR) so well that there is no experimental way to decide between the two theories[26]. The second LAT theory differs from the first in that it posits that the aether is partially dragged along with a moving body in the aether. This is akin to frame dragging (e.g. the Lense-Thirring effect) in the Kerr Metric[27]. We will posit frame dragging in G:SST as well, i.e. the dragging along of venues by a moving object. (Note that the Kerr metric itself 'breaks' the continuity space-time. If it didn't, the frame dragging would 'wind-up' space-time, and it doesn't[28]. One might take this as an argument for a discrete space-time such as in G:SST.)

Although LAT derives the constancy of the speed of light whereas ESR takes it as a given, there are objections to LAT:

There is an 'aether', the makeup of which is not specified.

There is a privileged, albeit unobservable, reference frame where the aether is at rest (isotropic).

The (constant) velocity of light results from electromagnetic interactions with waves (and matter), and not from properties of space-time.

G:SST can address these issues: As for the makeup of the aether, G:SST says the aether is the space-time itself. And in 1922, Einstein himself said essentially the same thing.
[Note: Einstein (translation)-'Recapitulating, we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an ether. According to the general theory of relativity, space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this ether may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it"]

A privileged reference frame is also not an issue in G:SST. The stochastic nature of space-time makes it impossible to define a global rest frame. But we can consider a local privileged reference frame where the correlation region (the region where we can consider a background privileged frame) is large compared to the region where we are doing experiments. And the Unruh effect[29] implies that Lorentz frames are privileged.

The constancy of the speed of light not a result of the properties of space-time, can be addressed as well. While there is nothing wrong with the LAT derivation of the constancy, we can give a qualitative geometrical model as an alternate way of thinking about the constancy:

We suggest that frame-dragging occurs whenever a mass (non-zero rest mass) moves through space-time. Photons, as their rest mass is zero, move without framedragging. This (as we will see) allows an argument showing the constancy of $c$.


Consider an object (here, the black circle) moving at high speed in the direction of the arrow. The object moves through the venues (here represented by the white rectangles). But due to venue frame dragging at high velocities, the venues are pushed ahead of the moving object. But venues are constant in volume, and the only way that they can 'pile up' is by contracting in the direction of motion (and expanding in other dimensions).

The object must move through these venues. If we increase the object's speed, the contraction also increases. To an external observer (making contravariant observations), the objects increase in velocity decreases until it stops completely (where the venue dimension in the direction of motion goes to zero). This establishes that a mass has a limiting velocity.

We have postulated then, that a particle with nonzero rest mass drags along (empty) venues as it moves

Photons, having zero rest mass, do not drag venues.
So, if a particle moving with respect to the local privileged reference frame emits a photon, the photon does initially travel with a velocity of c plus the velocity of the particle. But the particle is dragging venues. As the venue contracts in the direction of motion, since its volume is constant, it expands in the time dimension. And this makes the time a photon takes to pass through the venue constant. The photon has more venues to pass through than it would have if the particle were not moving. Because of the additional distance (i.e. number of venues) the photon needs to travel, its speed at the detector, would be a constant, which is to say c.

If the detector were extremely close to the emitter (on the order of Planck lengths) one would measure a value of the velocity greater than c. This length scale is too small to measure so the velocity greater than c is unobservable.

The G:SST model violates Galilean Relativity in that motion is not (in this model) relative. LAT violates it as well. This is allowed (in both cases) by having a privileged reference frame.

With G:SST then, there is a new phenomenon at play: 'Velocity Induced Frame-dragging. So, in addition to frame-dragging being generated by (a rotating) mass (or acceleration), it is also generated by an object's linear motion in the space-time aether. One way of perhaps justifying this is to consider the conservation of energy, as the sum of potential and kinetic energy. The former is gravity dependent while the other is motion dependent. Since gravity yields curvature, perhaps velocity does as well. Potential then, could be considered a result of Mach's Principle. (Linear frame-dragging, though little known, was discussed by Einstein in around 1920.[30, 31])
[Frame-dragging has much in common with curvature, specifically Schwarzschild curvature. We might therefore expect the metric tensors to be similar. Indeed, without doing any calculations, we can guess at a metric for the moving object. Consider the $g_{11}$ (the radial component of the Schwarzschild metric) $\left(1-\frac{2 G m}{r c^{2}}\right)^{-1}$. The velocity induced model is not a function of mass, so m and G are unlikely to be in $g_{11}$. However, note that $G m / r c^{2}$ has units of $v^{2} / c^{2}$, so we might expect $g_{11}$ to be $\left(1-k \frac{v^{2}}{c^{2}}\right)^{-1}$ where k is a constant. We would expect a (coordinate) singularity to occur when $v=c$, so that would make $k=1$. A similar argument can be made for $g_{00}$, the time component.]

## IX. GEOMETRIC PROPERTIES OF MASS

The G:SST model associates mass with a non-zero fifth dimension, moving the concept of mass into geometry.

The principal function of mass is (in the model) the stabilization of space-time, i.e, one would like the fluctuations in/of space-time not to rip apart masses. In particular, a mass causes adjacent mass-containing venues, because of stabilization, to act as a single larger venue (which is why E (mass)=hf works).

In empty space, in particular, the venues' dimension coordinates fluctuate (and this is required for the creation and annihilation of empty venues). The fluctuating mass can be associated with vacuum energy fluctuations and metric tensor fluctuations. The idea of metric tensor fluctuations was the initial idea behind the stochastic space-time theory (the precursor paper).

In (the usual interpretation of) General Relativity, mass causes 'curvature'. But what is curvature? Arguably, it is merely an artifact of describing space-time with one too few dimensions. For example, if a (two dimensional) ant were wandering on the surface of a sphere, he could measure curvature and determine that his environment was non-Euclidean. A three-dimensional (ignoring time) being would say the space(-time) was Euclidean and the ant was not able to see that third dimension. G. 't. Hooft has made a similar argument, as has J. Beichler. (And, of course, 'Campbell's Embedding Theorem'[32] states that any n-dimensional Riemannian manifold can be embedded locally in an $n+1$-dimensional Ricci-flat manifold.) But for us, rather than using a full extra dimension to explain curvature, we describe curvature as an artifact of the dimensional contraction or expansion of venue coordinate values. A feature of this granular model is that some phenomena attributed to the large-scale structure of space-time (e.g curvature) can be explained by extremely small scale phenomena (e.g. compression and expansion of venue dimensions).

Graphically, the left image below represents the traditional 'gravity well' curvature representation, and the right image is the left image but looking directly down from above.


The right image is still a three-dimensional representation. But, in the G:SST interpretation, it is a twodimensional image, the 'depth' being due to the compression of venues. We have reduced space-time by one dimension (without losing information) and since there is no curvature, the space-time (arguably) is flat. (This is a kind of 'holographic principle'.)

Note: As previously noted, for the motion of a mass, the mass doesn't become 'fuzzy', but (because of migration external to the mass) its location does begin to blur as the mass decreases below the Planck mass. This results in an effectively larger mass diameter.

A quantum particle apparently spreads. So, in some sense, the mass is effectively spread through the spacetime. And the field equations act on the spread mass. And (since inside a mass, the Ricci tensor is not zero) the
space-time near a quantum particle has a non-constant real 4 -volume element.

In short then, there is relationship between a particle's mass and its radius; the higher the mass, the shorter the radius.

While Newton and Einstein described the action of gravity, a mechanism for gravity was not provided. G:SST though, does suggest a mechanism: When venues are near a large mass (from outside the mass), their 3volumes are compressed. The constancy of the volume is maintained by a corresponding expansion of the time coordinate.

The space compression continues when a venue is interior to the mass. Here the fifth dimension important and is non-zero. As one approaches The Schwarzschild radius, (at least one of) the venue's dimensions approaches zero. As it reaches zero, the venue thus annihilates. A venue then comes in to take its place. So there is a continuous stream of venues approaching the Schwarzschild radius and annihilating. Venue creation in the space-time at large makes up for the loss of venues. If a venue holds a test mass, it will fall in toward the surface of the large mass. (This is largely because of the mass-free venues 'in front' of that venue.) The speed (as a function of the radial distance from the center from the large mass) can easily be calculated (and the result of the calculation compared with the Newtonian result):

Consider a spherical shell at some distance from a mass (M). As the venues at the shell migrate in toward the mass, the number of venues at the shell do not change, so as the shell's radius changes the venues must compress in the two dimensions perpendicular to the radial direction. To keep the volume constant, the real time component (the imaginary-time component is too small in scale to have any effect here) must expand as the square of a coordinate perpendicular to r (because of the two space coordinate compressions). Venues are being annihilated at a constant rate. So if one uses a stopwatch to monitor how fast a venue (containing a test particle) is falling toward the surface of the mass, it will appear to go faster as it approaches because of the slowing of the stopwatch. So the distance covered by the falling venue will go as the square of the rate of the slowing of time. This is to say that, $v^{2}=\frac{\text { constant }}{r}$. There is also a contraction of the venue in the r direction, but that is a relativistic effect that we will ignore for the moment. The velocity equation from Newtonian physics is, $v^{2}=\frac{2 G M}{r}$. The constant above is (related to) the rate at which venues are annihilated, so we can associate that rate with 2 GM which gives a connection of G:SST to 'physics'. Further, since the Newtonian description of an object falling under the influence of gravity is a conversion of potential to kinetic energy, the G:SST derivation of the fall of gravity provides the link to kinetic and potential energy. More importantly though, it might explain the concept of energy in terms of G:SST.

To summarize: The volume, V , of a sphere is $\frac{4 \pi r^{3}}{3}$. $\frac{d V}{d r}=4 \pi r^{2}$. But venue volume is invariant. So as the $3-$
volume, V, decreases proportionally to $r^{2}$, the real-time component must increase proportionally to $r^{2}$. However the $r$ coordinate of a venue decreases as the venue approaches a mass, and the decrease is non-linear. So far from a mass, in the Newtonian domain, a venue's r coordinate is essentially constant. So that leaves to increase proportionally to $r^{2}$.

Venue creation to replenish the venues lost to annihilation will happen over a large region of space-time. But one would expect that the closer one is to the mass responsible for the annihilation, the higher the rate of venue creation. The venues thus created would then cause a very small deviation from the inverse square law, and also from the General Relativity predictions. Because of the increase of venue creation as one approaches a mass, one would expect the deviation to be most evident with a test mass in a highly elliptical orbit around the mass responsible for the annihilations.

Note: Carlo Rovelli states that relativity's the slowing of time is the source of gravity[33]. G:SST says the annihilation of venues is the source of gravity and the rate of fall due to gravity is determined by the slowing of time.

## X. HIGHLY SPEULATIVE NOTES ON QUIDDITY, ENTANGLEMENT, AND THE TWO-SLIT EXPERIMENT

## A. Information \& Quiddity : Pilot-waves \& Entanglement

There are two forms of information at play: one of which is restricted to travel at no greater than the speed of light and the other (e.g. collapse of the wave function, entanglement and the like) not so restricted. These are very different processes, and so using the word 'information' for the both of them is confusing. We'll reserve 'information' for the first case, and 'quiddity' for quantum information. (Quiddity means the inherent nature or essence of something. And the first three letters, qui, make it easy to remember QUantum Information.)

Information is carried by photons or mass (energy). Quiddity, as it provides instantaneous signal transport (and therefore can also travel backward in time), can not be carried by energy. In Granular Stochastic Spacetime theory then, what can carry quiddity? The only thing left is empty venues. While a venue has an invariant 5 -dimensional volume, it can vary in its individual dimensions. As described earlier, the 4-dimensional volume is related to the probability density, $\Psi^{*} \Psi$. So that probability density is a type of quiddity.

## Entanglement

Bell's theorem[34] requires that to have entanglement, we must abandon 'objective reality' and/or 'locality'. Dropping locality means that things separated in space can influence each other instantaneously. Dropping objective reality means that a physical state isn't defined until it is measured (e.g. is the cat dead or alive?).

Weak measurement experiments[35-37] building on the work of Yakir Aharonov and Lev Vaidman[38] strongly suggests that there is objective reality in quantum mechanics [38, 39] (in contradiction to the Copenhagen interpretation of quantum mechanics). By objective reality, we mean a particle does have a path (blurred somewhat by space-time fluctuations) regardless of whether it is being observed or not.

We're left then, with non-locality. G:SST is non-local. The issue, of course, is how to have non-locality whilst not violating Einstein's prohibition of information traveling faster than light. We slightly re-interpret that prohibition by positing that it is energy (as opposed to information) that can't travel faster than light.

The usual way of thinking about entanglement is that a measurement the state of one of the entangled particles forces the state of the other. An alternate approach (which we adopt) says that both particles are synchronously flitting (at the Planck time scale) between their allowed states. When one particle's state is measured, it freezes the flitting of it and also the other particle.

The mechanism for the freezing is unknown, but possibly something like the following:

Empty venues carry no energy, and so can migrate through space-time arbitrarily rapidly. The hope then is that we can find a way that empty venues can carry quiddity. We suggest though, that through some unknown mechanism (which is why this is a suggestion and not a theory) that a number of empty venues can be bound together can migrate collectively through spacetime (e.g. spiraling through space-time) they would then carry a more complex quiddity. This is rather like the DeBrogli 'ghost waves'. So, for instance, two created entangled particle would carry this quiddity with them as they spread out (as a link between them). And through another unknown mechanism, a measurement of one particle forces the state of the other and then dissolves the link.

Entanglement is a process seeming to require superposition plus faster than light quiddity, and G:SST provides for both.

The aim in the previous was not to provide a theory/mechanism for entanglement, but to argue that Stochastic Granular space-time Theory allows for it.

## B. The Delayed-choice Two-slit Experiment



The diagram shows the 'delayed choice two-slit experiment': A low-intensity source directs electrons to a box containing two slits (slit 1 and slit 2). The beam inten-
sity is such that there is only one electron traveling in the box at any time. As expected, an interference pattern is gradually produced on the screen at the back of the box. If a particle detector is introduced at slit 2 to determine which slit an electron passed through, then there will be no interference produced. One can arrange that the detector is optionally turned on only when the electron has passed by slit 1 . If the detector is on at that point, then again, there will be no interference pattern produced. So it seems that when the electron gets to slit 2 and finds that the detector is on, it goes back in time to tell the electron to go through, or not go through slit-1.

How does the Granular Stochastic Space-time model possibly explain this?

First, we introduce the concept of an 'ephemeral' measurement: An electron has an associated electromagnetic field. As it goes through a slit, that field will interact with the electrons in the wall of the box at the slit. The box electrons then can tell if an electron has passed through a slit. And this could be considered a measurement; the box electrons could be considered a particle detector. But the interference pattern still occurs in this case. The difference is that the box electrons measurements are ephemeral; After the moving electron passes through the slit, the box electrons return to their undisturbed state, retaining no 'memory' of the measurement. The measurement is not preserved. The film can be run backward and it would be a valid physical situation. For there to be a true measurement then, there must be a mechanism to 'remember' the measurement - a latch or flip-flop of sorts. And that would mean the film could not be run backward. We regard measurement then, as a breaking of time-reversal symmetry. In the macro-world, everything is a measurement of sorts (viewing a scene gives an estimate of positions, etc.) and hence we can't run macro-world scenes backwards.

With quiddity (in this case, the pilot wave) able to move superluminaly as well as to move backward in time, there isn't much to explain. The pilot wave precedes the electron going into the box. The pilot wave determines the probability of the electron being found at any point in the box at any time. If (at any time) the detector is switched on, that would change the geometry and hence the wave (at all points, future and past). The electron would continue its motion, catching up with the revised pilot wave and then moving accordingly. (This is much like the mechanism of entanglement).

## XI. DISCUSSION

The object of the present model is to provide a conceptual geometrical basis for quantum mechanics-to show that the 'quantum weirdness' can be explained in terms of the behavior of space-time.

General relativity is a theory relating the large scale structure of space-time to the masses in it. Similarly, the granular stochastic space-time model relates the micro-
structure of space-time to the behavior of masses at the quantum level. One says for general relativity, mass tells space how to bend, and space tells mass how to move. And in G:SST, we say mass tells space how to jell, and space tells mass how to jiggle. The model is neither one of quantum mechanics nor General Relativity. It requires both theories in its development.

Having recognized that quantum mechanics is merely an operational calculus, and also having observed that general relativity is a true theory of nature with both an operational calculus and a Weltanschauung, we have attempted to generate quantum mechanics from the structure of space-time. As a starting point we have used a version of Mach's principle where in the absence of mass, space-time is not flat, but undefined (or more exactly, not well defined) such that $P_{\Theta}\left(g_{\mu \nu}\right)=-A\left|g_{\mu v}\right|$ (where A is a constant) is, at a given point $\Theta$, the probability distribution for $g_{\mu \nu}$ (in the Copenhagen sense[43]) .

The model, as it stands, subtends both the Bohm-DeBroglie and the Copenhagen interpretations (Bohmenhagen): At the extreme sub-quantum (Planck length/time) scale, there is objective reality independent of measurement. And it is Bohmian.

At the quantum scale the Copenhagen interpretation is effectively the case as states are changing much faster than can be measured.

In the model, particles move (in an indeterminate manner) due to the space-time fluctuations exterior to the
particle (similar to the way a Brownian Motion pollen grain moves). But unlike with Brownian motion, time (as well as space) fluctuates.

This is a global approach to quantum theory. It should be noted that there are two logically distinct approaches to conventional quantum mechanics: a local, and a global formulation. The local formalism relies on the existence of a differential equation (such as the Schrödinger equation) describing the physical situation (e.g. the wave function of the particle) at each point in space-time. The existence of this equation is operationally very convenient. On the other hand, the global formulation (or path formulation, if you will) is rather like the Feynman path formalism for quantum mechanics[44], which requires the enumeration of the "action" over these paths. This formalism is logically very simple, but operationally it is exceptionally complex. Our approach is a local formalism. Statement $3, P(x, t)=-A g$, is local and provides the basis for the further development of stochastic spacetime quantum theory.

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