Design and Analysis of a Multiband Fractal Antenna for Applications in Cognitive Radio Technologies

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Abstract-Rapid development in wireless communication systems and an increase in the number of users of wireless devices is bound to result in spectrum shortage in the near future. The concept of Cognitive radio is envisaged to be a paradigm of new methodologies for achieving performance enhanced radio communication system through an efficient utilization of available spectrum. Research on antenna design is very critical for the implementation of cognitive radio. A special antenna is required in cognitive radio for sensing and communication purposes. This papers investigates the use of multiband fractal antennas for spectrum sensing application in cognitive radio units. The performance of a new fractal antenna design which generates four bands of operation in the range of 900-4000 MHz has also been studied. Through a thorough discussion on its return loss and radiation plots as well as other parameters such as gain and radiation efficiency, it is proved that the it is a promising antenna for future cognitive radio systems¹.

Keywords—Antennas; Fractal antenna; Cognitive radio; spectrum sensing.

I. INTRODUCTION

Wireless communications has seen an explosive growth due to an ever increasing market demand for advanced wireless devices. Due to this rapid development, data traffic is bound to double every year eventually causing saturation in dedicated spectrum. However, some studies have pointed out that spectrum occupancy is irregular [1]. While some frequencies are saturated, other frequencies such as those for TV broadcasting are almost empty. Thus, a better global quality of service can be ensured by allowing users of saturated frequencies to go into empty bands. This has prompted the Federal Communications Commission (FCC) open licensed spectrum bands to unlicensed users through the use of Cognitive radio (CR) and Software defined radio (SDR) technology. Ultimately, it is predicted that user migration from one band to the other will be permitted, thereby deregulating spectrum in total or in part.

Though SDR and CR are relatively new concepts in wireless communication, they are attracting the attention of a large number of researchers due to the new possibilities given by these systems. Presently various research communities have different definitions of CR and its unique defining features. However, it can be thought of as a transponder capable of sensing the operating environment and adjusting certain parameters to optimize its performance, thus maintaining its Quality of Service (QoS) above a certain threshold. This expectation of dynamic spectrum sensing and transmission will change the designing and operating methodology of radio systems, thus affecting antenna requirements in a number of applications significantly. Hence, an antenna will play an active role in system level performance improvement, thus becoming one of the important parts of a CR system.

This paper studies the application of fractal antennas in CR systems. In order to study various parameters associated with the antenna, the full-wave electromagnetic (EM) field simulator Ansoft HFSS software has been used. The paper is organized as follows. In Section II, detailed information about CR architecture has been given followed by Section III which gives a description of various fractal shapes and the associated antennas. Section IV describes the antenna design under study and its iterative performance analysis has been presented in Section V. Results and discussion has been given in Section VI followed by Conclusion in Section VII and then references².

II. COGNITIVE RADIO

According to the FCC, a cognitive radio is "a radio that can change its transmitter parameters based on the environment in which it operates". Thus, for more efficient communication and spectrum use, a CR should be able to recognize spectrum availability and reconfigure itself accordingly [2]. Users who own the channel are termed as "primary" users (PUs) while the unlicensed users are called "secondary" users (SUs). These SUs need to continuously monitor the activities of the PUs to find unused frequency bands that can be utilized without any interference to the licensed services. This procedure is called as spectrum sensing and the unutilized bands are called as spectrum holes (SHs).

Once a SH is found, the CR system should be able to adjust its parameters such as transmission power, carrier frequency, modulation strategy and transmission data rate, [4], so that the unused frequency bands can be used by the SUs for transmission. These SUs can utilize the spectrum as long as the Quality of Service (QoS) is not compromised. However, channel conditions can change rapidly and so a SU has to continuously monitor all the licensed bands and keep looking for SHs. Spectrum sensing, therefore, plays a very crucial role in Cognitive Radio systems.

 $^{{}^{1}}$ ©[2013]. This is the author's version of the work. It is posted here for your personal use. The definitive version was published in [24]

²For further reading on the author's work, readers can refer to [23], [24], [25], [26], [27], [28], [35]. For further reading on WLAN features, readers can refer to [29], [30], [31], [32], [33], [34]

SDR which offers promise to increase spectrum usage efficiencies to users in a wide variety of applications is seen as an enabling technology for CR. Just like CR, SDR does not represent any specific concept. Wireless Innovation Forum, [3], defines it as "a radio in which some or all the layer functions are software defined".

As mentioned in [5], many of the proposed dynamic spectrum sharing approaches (DSS) can be broadly classified into two categories viz. open sharing and hierarchical sharing. In open sharing, all users can simultaneously access the spectrum. There are, however, some limits on the transmit signal. In the hierarchical sharing model, licensed users are assumed to act sporadically in the frequency bands owned by them. As long as the QoS is not compromised, PUs may allow or lease their unutilized channels to the SUs. Three main paradigms have been considered in literature in this category.

They are spectrum underlay, interweaving and overlay. In spectrum underlay, SUs may transmit simultaneously with PUs. But they should keep their transmission power below an interference margin or noise floor tolerated by the PUs. In spectrum interweave, SUs use appropriate mechanisms to locate SHs for their use. Thus, they should determine when and where they may transmit. On the other hand, cognitive users in spectrum overlay system already know the primary message. They use sophisticated implementation techniques to reduce the interference at the primary and secondary receivers. In short, they are allowed to utilize the SHs while avoiding or limiting collisions with primary transmission. The underlay and overlay approaches in the hierarchical model are illustrated in Fig. 1(a) and Fig. 1(b) respectively.

The development of spectrum sensing and spectrum sharing has facilitated the application of CR in many areas such as TV white spaces, cellular networks, military usage etc.

III. COGNITIVE RADIO ARCHITECTURE

A basic design of CR consists of an antenna system with a direct connection to an analogue to digital convertor [2]. The digitised data goes into a processor which contains all the previously done processing information. The added advantage is that the processor would have the ability to reconfigure itself to change the standard that is being used by the radio, thus giving the system a great flexibility essential to handle an increasing level of traffic. In case of SDR, mixed chip architecture allows such a realization but with a compromised performance. The research in this paper has been primarily on CR systems in which spectrum allocation related decision is made locally by individual terminals rather than central base stations. When SHs are used by detection at the terminal, the system can be divided into two categories [3]:

 As shown in fig. 2, transmission of signal and monitoring of spectrum is done continuously in parallel. In this category, systems have been proposed in which two antennas are used- a wideband antenna connected to a receiver capable of spectrum sensing and a directional antenna feeding a frequency agile front end capable of tuning to the selected band. In sensing unit, the use of Ultrawideband antenna has also been previously studied [4].



Fig. 1. (a) Underlay spectrum sharing approach (b) Overlay spectrum sharing approach

2) A single channel is used for both spectrum sensing and transmission of information as can be seen from fig. 3. Two thresholds are used in this case [5]. Spectrum sensing is triggered when the link quality falls below the first threshold and appropriate channels are detected. As the link quality falls below the second threshold, the system is reconfigured.

In both the categories, CR would have two modes of operation: sensing mode and transmission mode. Thus, its basic RF structure would comprise of a sensing antenna and a reconfigurable transmit receive antenna as shown in a generic cognitive radio workflow diagram given in fig. 4.

In sensing mode, the radio would take a decision on the channel conditions or the quality of service. In order to do this, it would take a series of measurements by using the sensing antenna. These could be measurements of noise or interference levels in the band of operation of the sensing antenna. This data would be used to make a decision about channel availability. Once an unutilized band, also known as spectrum hole (SH), has been detected, the reconfigurable antenna adjusts its parameters so that it may transmit in the detected SH. Thereafter, the transmission unit begins sending data to distant users.

CR has been suggested for the bands in the range of 30 MHz to 5.9 GHz, though a more realistic range would be from 0.4 to 2.5 GHz.



Fig. 2. CR architecture with parallel sensing and communication [2]



Fig. 3. CR architecture with combined sensing and communication [2]



Fig. 4. A generic cognitive radio workflow diagram

IV. FRACTAL ANTENNA CONCEPTS AND DESIGN DETAILS

A. Use of fractal shape in antenna design

The term fractal which means broken or irregular fragments was first defined by Benoit Mandelbrot in 1975 to represent a family of complex shapes that possess an inherent self-affinity and self-similarity in their geometrical structure. A self-affine set is a contraction which reduces an image by different factors horizontally and vertically [6] whereas a self-similar is one that consists of scaled down copies of itself *i.e.* a contraction which reduces an image by same factors horizontally and vertically [6]. Due to these properties, fractals have infinite complexity and detail. As long as you are zooming in on the right location, their complexity and detail remain the same no matter how far you zoom-in.

Many ideal fractal shapes are constructed by applying an iterative algorithm an infinite number of times on an initial structure called as the "generator" which gets replicated at different directions, scales and positions. Different antennas have been developed by using fractal shaped patches. Some of the commonly used fractals and their generator shapes are summarised in table I. Fractal shapes are popular among antenna designers due their ability to generate a multiband performance. Also, the number of bands generally increases with the number of iterations performed.

Puente et al. described in [7] the behaviour of, at the extent known by the authors, the first fractal multiband antennathe Sierpinski monopole antenna. In [8], this fractal antenna was proved to have a multiband behaviour with the help of both experimental and numerical results. Following this, a technique to suppress the effects of higher order modes and improve the multiband behaviour of the Sierpinski patch was introduced in [9]. Recently, some researchers have used a modified two iteration Sierpinski patch and a slotted ground plane to enhance the bandwidth performance [10]. The modified Sierpinski antenna has two operational bands which cover the GSM/DCS/PCS/IMT-2000/ISM/satellite DMB services. Puente et. al. were the first to study a fractal 2-D tree antenna generated by electrochemical deposition and reported a multi-band behaviour with a denser band distribution than the Sierpinski [11].

The deterministic fractal tree, a promising fractal geometry, has been successfully used in the design of an antenna whose experimental multiband behaviour has been studied in [12]. Simulated and measured results on thin-wire ternary-tree fractal antennas have also been studied in [14]. In [15], twodimensional (2-D) and three-dimensional (3-D) fractal trees have been reported to have the space-filling properties. The miniaturization techniques for 3-D fractal tree structures based on end-loaded dipoles [16], and their application for the design of reconfigurable antenna [17], have also been stated.

In [18], the hexagonal fractal antenna has been found to possess multiband behaviour similar to the Sierpinski gasket antenna. Also, it has been stated that the antenna allows flexibility in matching multiband operations in which a larger frequency separation is required. Unlike the Sierpinski gasket antenna resonant frequencies which repeat with a factor of two, the hexagonal fractal antenna resonant frequencies repeat with a factor of three.

TABLE I. COMMON FRACTAL SHAPES AND THEIR GENERATORS

Fractal name	Generator		
Sierpinski gasket	Equilateral triangle		
Fractal Trees	Tree		
Koch snowflake	Equilateral triangle		
Ring fractal	Circular ring		
Hexagonal fractal	solid hexagon		

B. Iterated function Systems

The iterated function systems are very important for generating a wide variety of useful fractal structures [19], [20]. They are based on the application of a series of affine transformations, w, defined by the matrix equation:

$$w\begin{pmatrix} x\\ y \end{pmatrix} = \begin{pmatrix} a & b\\ c & d \end{pmatrix} \begin{pmatrix} x\\ y \end{pmatrix} + \begin{pmatrix} e\\ f \end{pmatrix}$$

This can also be represented by the following equation:

$$w(x, y) = (ax + by + e, cx + dy + f)$$

where a, b, c, d, e and f are real numbers.

Now let A be the initial geometry and w_1, w_2, \dots, w_N be a set of affine linear transformations. The new geometry produced by applying the set of transformations to the original geometry, A, and collecting the results from $w_1(A)$, $w_2(A), \dots, w_N(A)$, can be represented by

$$W(A) = \bigcup_{n=1}^{N} w_n(A)$$

Here W is known as the Hutchinson operator. W can be repeatedly applied to to the previous geometry to obtain fractal geometry. IFS have proven to be a very powerful tool for fractal antenna designers since they provide a general framework for the description, classification and manipulation of fractals.

C. Fractal Antenna Design

Fractal antennas generate a multi-band performance and can be used for spectrum sensing in cognitive radio units [21]. An antenna design using an E-shape fractal patch has been proposed in [22]. Various stages in the generation of the E-shaped fractal have been shown in fig. 5 starting with a solid E shape as the generator. The higher stages of the antenna are generated in accordance with the following formula:

$$n_k = \frac{L_{Bk}}{L_{Sk} - L_{Bk}}$$

where k is the iteration number, L_{Bk} is the width of the subtracted rectangle and L_{Sk} is the width of the main rectangle. The value of n differs from one fractal iteration to the other. The complete antenna has been given in fig. 6 while various dimensions of the antenna have been listed out in table II. As the number of iterations increase, E-shapes of smaller sizes are subtracted from the patch generated in the previous iteration, thus decreasing the surface area of the patch.

The geometry of the antenna upto the third iteration with



Fig. 5. Various stages in the generation of the E-shape fractal

TABLE II. ANTENNA DESIGN DETAILS

Material	FR4
Dielectric constant	4.4
Loss Tangent	0.02
Substrate Thickness	1 mm
Centre frequencies	900 MHz, 1.9 GHz, 2.5 GHz, 3.7 GHz



Fig. 6. Design of the fractal antenna

optimized dimensions has been shown in fig. 6 and the specifications have been given in table II. A direct-connected probe has been used in order to achieve a good impedance matching. The fractal antenna consists of a ground plane of size $L_g \times W_g$ and a radiator of size $L_{S1} \times W_{S1}$. An FR4 substrate with a dielectric constant of 4.4, loss tangent of 0.02 and height 1 mm along with an air-filled layer or foam of thickness 5 mm and relative permittivity 1, sandwiched between the substrate and the ground plane, have been used in the design. The main purpose of the air-filled layer is to achieve multiband characteristics [22].

V. RESULTS AND DISCUSSION

The radiation pattern and the return loss of the antenna have been shown in fig. 7 and fig. 8 respectively. The patch shape after the third iteration has been considered for these

 TABLE III.
 SUMMARY VALUES OF IMPORTANT PARAMETERS

 ASSOCIATED WITH THE ANTENNA

$\mathbf{L}_{\mathbf{g}}$	19	W_{g}	17	L_{S1}	15
W_{S1}	13	L_{B1}	5	W_{B1}	1.5
h	0.1	$\mathbf{h}_{\mathbf{a}}$	0.5		

 TABLE IV.
 SUMMARY VALUES OF IMPORTANT PARAMETERS

 ASSOCIATED WITH THE ANTENNA

Band	1	2	3	4
Centre Frequency (MHz)	945	1945	2470	3690
Bandwidths (MHz)	250	400	260	370
Gain (dBi)	5.6-5.8	5.8-6.5	6-6.5	5.4-5.7
Radiation Efficiency (%)	95-98	90-94	90-93	89-92

plots. From the return loss plot in fig. 8 it can be seen that the antenna has centre frequencies at 945, 1945, 2470 and 3690 MHz with wide bandwidths of 250 MHz (820-1070 MHz, 26%), 400 MHz (1750-2150 MHz, 20%), and 260 MHz (2340-2600 MHz,10%) respectively. For the first band with centre frequency 945 MHz, the gain is about 5.6-5.8 dBi while the radiation efficiency is ranged from 95%-98%. It satisfactorily covers the GSM850/900 band of operation. The second band around 1945 MHz the gain varies from 5.8-6.5 dBi whereas the radiation efficiency is around 90-94% and it covers the GSM1800/1900/UMTS operation band. The antenna gain varies from 6-6.5 dBi for the third band with centre frequency 2470MHz while the radiation efficiency varies from 90-93%. Also, it covers the LTE2300/2500 operation band. The fourth band with centre frequency 3690 MHz, the bandwidth is 370 MHz (3550-3920 MHz, 10%) while the gain varies between 5.4-5.7 dBi and its radiation efficiency ranges from 89-92%.

The centre frequencies and bandwidths of the resonant bands along with the corresponding gains and radiation efficiencies have been summarized in table IV. The antenna has a high gain and efficiency in these frequency bands. As a result, the antenna can be used for spectrum sensing in these bands. Also, an added advantage of using this design for cognitive radio sensing applications is that it will not pick up interfering signals from other bands and so no additional filters will be needed at the processor side of the CR architecture.

VI. CONCLUSION

In this paper, the use of E-fractal antenna for spectrum sensing in cognitive radio systems has been studied. The E-fractal antenna design presented in this paper has resonant frequencies at 945, 1945, and 2470 MHz with wide bandwidths of 250 MHz (820-1070 MHz, 26%), 400 MHz (1750-2150 MHz, 20%), and 260 MHz (2340-2600 MHz, 10%) to cover GSM850/900, GSM1800/1900/UMTS, and LTE2300/2500, respectively. The antenna performance has been studied with the help of return loss, radiation pattern and gain plots at these frequencies. Also, a detailed parametric study has been performed to study the performance of the antenna in various iterations and also effect of varying the air-gap between the substrate and the ground plane. From the above performance analysis, it can be clearly concluded that the antenna is a promising design for cognitive radio sensing applications.

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Fig. 7. Radiation pattern: (a) H-plane pattern at 945 MHz (b) E-plane pattern at 945 MHz (c) H-plane pattern at 1.94 GHz (d) E-plane pattern at 1.94 GHz (e) H-plane pattern at 2.47 GHz (f) E-plane pattern at 2.47 GHz (g) H-plane pattern at 3.70 GHz (h) E-plane pattern at 3.70 GHz



Fig. 8. Return Loss for the fractal antenna

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