Mathematical Journal X manuscript No.
(will be inserted by the editor)

Complex Container-based proof and explanation for Riemann hypothesis and two types of Gram points

John Ting

April 5, 2021

Abstract Intractable open problem in Number theory of Riemann hypothesis is the 1859-proposed conjecture that all nontrivial zeros in Riemann zeta function, or its proxy Dirichlet eta function, lie on the critical line denoted by parameter sigma = 1/2. Using correct and complete mathematical arguments, solving Riemann hypothesis and explaining closely related two types of Gram points is completed by deriving Dirichlet Sigma-Power Law which enables computing of required pseudo-zeroes that can all be converted to zeroes. By fully complying with fundamental concepts from Information-Complexity conservation and Complex Container, exact and inexact Dimensional analysis homogeneity will occur at appropriate times in total summation of all fractional exponent (1 - sigma) that is always twice present in this Law. The most significant meta-property to logically incorporate into our proof and explanation is that exact Dimensional analysis homogeneity always and only occurs when sigma = 1/2.

Keywords Dirichlet Sigma-Power Law · Pseudo-zeroes · Riemann hypothesis · Zeroes

Mathematics Subject Classification (2010) 11M26 · 11A41

Contents

Corresponding Author E-mail: jycting@hotmail.com (Alma Mater: University of Queensland, St Lucia QLD 4072, Australia)
Dental and Medical Surgery, 729 Albany Creek Road, Albany Creek QLD4035, Australia

1 Introduction

Gram[x=0,y=0] points, Gram[y=0] points and Gram[x=0] points are three types of Gram points (GP) dependently computed directly from Riemann zeta function (RζF or ζ(s)) [or its *proxy* Dirichlet eta function (DηF or η(s))]. Respectively, these three entities are based on Origin intercept points, x-axis intercept points and y-axis intercept points. Nontrivial zeros (NTZ) are synonymous with Gram[x=0,y=0] points. Prime and composite numbers are two other entities dependently computed, respectively, directly and indirectly from Sieve of Eratosthenes. We justify below how all these five well-defined entities will manifest Incompletely Predictable properties. For RζF, its equivalent Euler product formula using product over prime numbers [instead of summation over natural numbers] will faithfully represent this function. Consequently, all prime [and, by default, composite numbers] are intrinsically "encoded" in this function.

Regarded as primary spin-offs in peer-reviewed research paper[1] located in its entirety at URL http://www.ccsenet.org/journal/index.php/jmr/article/download/0/0/43951/46679, relevant mathematical arguments for solving Riemann hypothesis (viz, conjecture that all NTZ are located on $\sigma = \frac{1}{2}$ critical line of RζF) and explaining closely related two types of GP (viz, Gram[y=0] points and Gram[x=0] points) can inherently be explained to belong to our coined Mathematics for Incompletely Predictable problems. We adapt some of these mathematical arguments [but none of its other mathematical arguments on Polignac's and Twin prime conjectures] into this stand-alone paper. Considered as novel analytic tool, exact and inexact Dimensional analysis homogeneity when applied to derived Dirichlet Sigma-Power Law (DSPL) symbolize rigorous proof for Riemann hypothesis and precise explanation for two types of GP. Selected mathematical arguments or concepts from this paper are reiterated or reformulated to facilitate optimal understanding on them.

With full permission from its author, significant sections of the text in this paper are openly declared to be conveniently and largely copied *verbatim* from [1]. We justify our ethical conduct here to the scientific community by citing this vital action allows us to optimally compose our paper in accordance with acceptable publishing standard. We will minimally elaborate upon the profound statement "*With this one solution, we have proven five hundred theorems or more at once*". Regarded as secondary spin-offs arising out of solving Riemann hypothesis, this statement apply to many important theorems in Number theory (mostly on prime numbers) that rely on properties of RζF such as where trivial zeros and NTZ are / are not located. We will not include discussion on previously derived innovative *Fic-Fac Ratio*[1] in 2020 [which is regarded as tertiary spin-offs potentially serving as medical or epidemiological tool to help understand highly contagious SARS-CoV-2 causing COVID-19 and 2020 Coronavirus Pandemic resulting in unprecedented negative global health and economic impacts]. This Ratio (range: 0 - ∞) is roughly 'Inverse Accuracy' since it varies in opposite direction to Accuracy (range: 0 - 1); and it connects seemingly unrelated subject of Medicine with frontier Mathematics from Number theory.

Useful preliminary information: A variable e.g. n in f(n) = RζF, n_1 in f(n_1), and n_2 in f(n_2) represents a model state and may change during simulation. A parameter, e.g. σ and t in RζF, is normally a constant in a single simulation used to describe objects statically and is changed only when we need to adjust our model behavior. A single-variable function e.g. f(n_1) or multiple-variable function e.g. f(n_1, n_2) is a set of input-output pairs that follows a few particular rules. An expression usually contains number(s), parameter(s), variable(s) and operator(s). A particular function e.g. f(n_1) is an expression involving variable n_1 that is defined for interval [a,b]. An equation is an assertion that two expressions are equal from which one can determine a particular quantity. An algorithm is a precise step-by-step plan for a computational procedure that possibly begins with an input value and yields an output value in a finite number of steps. A complex algorithm e.g. for generating prime numbers is only defined at two end-points a,b (but not for interval [a,b] as it is not a function). A formula is a fact or a rule written with mathematical symbols, and usually connects two or more quantities with an equal to sign. The terms 'variable', 'parameter', 'function', 'algorithm', 'equation' and 'formula' could be loosely used in some situations of this paper. Colloquially, we insightfully employ "$\Delta x \longrightarrow 0$" expression to indicate continuous-type equations or functions and "$\Delta x \longrightarrow 1$" expression to indicate discrete-type equations or functions. Antiderivative $F(n)$ denotes the result obtained when performing integration on function $f(n)$; viz, $\int f(n)dn = F(n) + C$ with $F'(n) = f(n)$.

1.1 Full Compliance with Information-Complexity Conservation by Completely and Incompletely Predictable Entities

The word “number” [singular noun] or “numbers” [plural noun] in reference to prime and composite numbers, NTZ and two types of GP can interchangeably be replaced with the word “entity” [singular noun] or “entities” [plural noun]. We propose an innovative method to validly classify certain appropriately chosen equations or algorithms in two ways by using relevant locational properties of its output. This output consist of generated entities either from function-based equations or from algorithms. Our classification system is formalized by providing definitive definitions for Completely Predictable (CP) entities obtained from CP equations or algorithms, and Incompletely Predictable (IP) entities obtained from IP equations or algorithms. ‘Container’ is a useful analogy that metaphorically group CP entities and IP entities to be exclusively located in, respectively, ‘Simple Container’ and ‘Complex Container’.
Definitions for CP numbers and IP numbers: Respectively, using CP simple equation or algorithm, and IP complex equation or algorithm; a generated CP number, and a generated IP number, is locationally defined as a number whose position is *independently* determined by simple calculations without, and *dependently* determined by complex calculations with, needing to know related positions of all preceding numbers in neighborhood. Simple properties are inferred from a sentence such as “This simple equation or algorithm by itself will intrinsically incorporate actual location [and actual positions] of all CP numbers”. Solving CP problems with simple properties amendable to *simple* treatments using *usual* mathematical tools such as Calculus gives ‘Simple Elementary Fundamental Laws’-based solutions. Complex properties, or “meta-properties”, are inferred from a sentence such as “This complex equation or algorithm by itself will intrinsically incorporate actual location [but not actual positions] of all IP numbers”. Solving IP problems with complex properties amendable to *complex* treatments using *unusual* mathematical tools such as exact and inexact Dimensional analysis homogeneity, and Dimension (2x - N) system as well as using *usual* mathematical tools such as Calculus gives ‘Complex Elementary Fundamental Laws’-based solutions.

Classified as IP problems, solving and explaining our nominated open problems is intuitively perceived as burdened with “Supramaximal Complexity”. Prime numbers are defined as “All Natural numbers apart from 1 that are evenly divisible by itself and by 1” and composite numbers are defined as “All Natural numbers apart from 1 that are evenly divisible by numbers other than itself and 1”. We conduct [complex] exercise of solving IP problem involving prime and composite numbers by proving their gaps are always varying (see P-C Pairing). This is a mathematician’s paradigm of an ideal example for this type of problem and is in sharp contrast to solving CP problem endowed with “Supraminimal Complexity” as demonstrated by [simple] exercise of proving even and odd number gaps always equal to 2 (see E-O Pairing) whereby even number (**n**) is defined as “Any integer that can be divided exactly by 2 with last digit always being 0, 2, 4, 6 or 8” and odd number (**n**) is defined as “Any integer that cannot be divided exactly by 2 with last digit always being 1, 3, 5, 7 or 9”. Congruence $\mathbf{n} \equiv 0 \pmod 2$ holds for even **n** and congruence $\mathbf{n} \equiv 1 \pmod 2$ holds for odd **n**. Thus, ‘0’ is an even **n** when we consider all (non-negative) positive even and odd **n** obtained from whole numbers = 0, 1, 2, 3, 4, 5,.... For convenience, we shall only consider all positive even and odd **n** obtained from natural numbers = 1, 2, 3, 4, 5, 6,... in this paper with following implication: The phrase “all even numbers” is generally taken to denote 2, 4, 6, 8, 10, 12,...; viz, this phrase is equivalent to the expression “all even numbers 2, 4, 6, 8, 10, 12,... equate to all positive even numbers 0, 2, 4, 6, 8, 10, 12.... but with even number ‘0’ intentionally and conveniently ignored”.

E-O Pairing: For i = 1, 2, 3,..., ∞; let i^{th} Even and i^{th} Odd numbers = E_i and O_i, and i^{th} even and i^{th} odd number gaps = $eGap_i$ and $oGap_i$. The positions of E_i and O_i are CP and their independence from each other is shown below.

E_i	2		4		6		8		10		12	.....
$eGap_i$		2		2		2		2		2		2

We can precisely, easily and independently calculate E_5=2X5=10 and O_5=(2X5)-1=9.

O_i	1		3		5		7		9		11	.....
$oGap_i$		2		2		2		2		2		2

Finite calculations shown here depict even and odd number gaps [= 2] are constant but even and odd numbers are infinite in magnitude requiring an infinite number of calculations ("mathematical impasse") in order to show these gaps will always be constant and non-varying. Obtaining rigorous proof for this property then consist of recognizing it as CP problem which requires deriving a CP 'non-varying' equation for confirming all even and odd numbers will [intrinsically] contain simple property "All even and odd number gaps = [constant] 2". The [colloquially expressed-"$\Delta x \longrightarrow 1$"] discrete-type equation E_i = 2 X i and O_i = (2 X i) - 1 are, respectively, for computing even and odd numbers [as zero-dimensional points]. When converted into [colloquially expressed-"$\Delta x \longrightarrow 0$"] continuous-type equation E = 2 X i and O = (2 X i) - 1 for i = all real numbers ≥ 0 [as one-dimensional lines which include even number '0' when i = 0 in E = 2 X i], calculating gradient ΔE/Δi or ΔO/Δi (= 2) and differentiating dE/di or dO/di (= 2) is precisely equivalent to all even and odd number gaps = 2. Using here notation x [instead of i] to illustrate computation of Area under the Curve (AUC) over interval [a,b] for f(x) [instead of f(i)]: AUC ("precisely") = $\int_a^b f(x)dx$ [viz, definite integral]. AUC ("approximately") = $\lim_{n\to\infty}\sum_{i=1}^{n} \Delta x \cdot f(x_i)$ [viz, $\lim_{n\to\infty}$ Riemann sum] whereby i = 1 to n [and not i = 1 to ∞], $\Delta x = \frac{b-a}{n}$ and $x_i = a + \Delta x \cdot i$. Our precise AUC [straight line] between a and b [as two-dimensional area] is given by $\int_a^b (2i)di = [i^2 + C]_a^b$ = (b^2 - a^2) for even numbers and $\int_a^b (2i-1)di$ = $[i^2$ - i + C$]_a^b$ = (b^2 - b - a^2 + a) for odd numbers. These discrete-type equations [given by 'Even number' variable vs 'Integer' variable and 'Odd number' variable vs 'Integer' variable] represent two mutually exclusive 'Simple Containers' that contain or generate all even and odd numbers with knowing their **overall actual location [and their actual positions].**

P-C Pairing: For i = 1, 2, 3,..., ∞; let i^{th} Prime and i^{th} Composite numbers = P_i and C_i, and i^{th} prime and i^{th} composite number gaps = $pGap_i$ and $cGap_i$. The positions of P_i and C_i are IP and their dependence on each other is shown below.

P_i	2		3		5		7		11		13	.....
$pGap_i$		1		2		2		4		2		4

We can precisely, tediously and dependently compute P_6 = 13: 2 is 1^{st} prime number, 3 is 2^{nd} prime number, 4 is 1^{st} composite number, 5 is 3^{rd} prime number, 6 is 2^{nd} composite number, 7 is 4^{th} prime number, 8 is 3^{rd} composite number, 9 is 4^{th} composite number, 10 is 5^{th} composite number, 11 is 5^{th} prime number, 12 is 6^{th} composite number, and our desired 13 is 6^{th} prime number.

C_i	4		6		8		9		10		12	.....
$cGap_i$		2		2		1		1		2		2

Finite calculations shown here depict prime number gaps = 1, 2, 2, 4, 2, 4,... and composite number gaps = 2, 2, 1, 1, 2, 2,... are varying but prime and composite numbers are infinite in magnitude requiring an infinite number of calculations ("mathematical impasse") in order to show these gaps will always be varying. Obtaining rigorous proof for this property then consist of recognizing it as IP [and not CP] problem which requires deriving two IP ["$\Delta x \longrightarrow 1$"] 'varying' [and not 'non-varying'] discrete-type algorithms P_{i+1} = P_i + $pGap_i$ and C_{i+1} = C_i + $cGap_i$ for calculating all prime and composite numbers [as zero-dimensional points]. With P_1 = 2 and C_1 = 4, '1' is neither prime nor composite. For (arbitrarily) i = all real numbers $\geq$ 1, 'varying' continuous-type algorithm P_{i+1} = P_i + $pGap_i$ and C_{i+1} = C_i + $cGap_i$ [as one-dimensional lines] incorporate all prime and composite numbers. Corresponding AUC ['varying' line] between a and b [as two-dimensional area] can be metaphorically visualized as [mathematically invalid] $\int_a^b (P_i + pGap_i)di$ for prime numbers and $\int_a^b (C_i + cGap_i)di$ for composite numbers. Respectively, the two algorithms will [intrinsically] contain complex property "Apart from first prime gap = 1, all prime gaps consist of perpetually varying even numbers" and "All composite gaps consist of perpetually varying numbers constituted by 1 and 2". Note: The integration is strictly invalid as these algorithms (and not functions) are only defined at end-points a, b (and not defined in interval [a,b]). Given by 'Prime number' variable vs 'Prime gap' variable and 'Composite number' variable vs 'Composite gap' variable, they represent two mutually exclusive 'Complex Containers' that contain or generate all prime and composite numbers with knowing their **overall actual location [but not actual positions].**

Remark 1 We can validly embrace 'Complex Container' concept for discrete format versions ["$\Delta x \longrightarrow 1$"] D$\eta$F via zeroes [*proxy* for ["$\Delta x \longrightarrow 1$"] R$\zeta$F via zeroes] and ["$\Delta x \longrightarrow 1$"] simplified D$\eta$F (sim-D$\eta$F or sim-$\eta(s)$) via zeroes whereby these functions (and the derived continuous format version ["$\Delta x \longrightarrow 0$"] DSPL via pseudo-zeroes to zeroes conversion) are literally '$\sigma = \frac{1}{2}$ Complex Containers' containing three types of mutually exclusive entities constituted by (i) NTZ viz, Origin intercept points or Gram[x=0,y=0] points (G[x=0,y=0]P), (ii) x-axis intercept points viz, 'usual' Gram points ('usual' GP) or Gram[y=0] points (G[y=0]P) and (iii) y-axis intercept points viz, Gram[x=0] points (G[x=0]P). Here, the term Gram points [as zeroes] conveniently encompass (i), (ii) and (iii). The corollary is that '$\sigma \neq \frac{1}{2}$ Complex Containers' [e.g. for $\sigma = \frac{2}{5}$ or $\frac{3}{5}$] will never contain any of these listed entities but instead contain two types of mutually exclusive entities constituted by (i) virtual G[y=0]P and (ii) virtual G[x=0]P. Here, the term virtual Gram points [as virtual zeroes] conveniently encompass (i) and (ii). These mutually exclusive 'Complex Containers' contain or generate all relevant IP entities with knowing their **overall actual location [but not actual positions]**.

Corresponding [perpetually varying] GP gaps or virtual GP gaps [both being transcendental numbers] are defined as the difference between any two nominated [consecutive] $\sigma = \frac{1}{2}$ GP or $\sigma \neq \frac{1}{2}$ virtual GP. All NTZ in RζF are proposed by Riemann hypothesis to only be located at $\sigma = \frac{1}{2}$ (critical line) of this function. Remark 1 above connotes $\sigma = \frac{1}{2}$ and $\sigma \neq \frac{1}{2}$ situations represent two mutually exclusive 'Complex Containers'. In principle, the preceding two sentences when combined together should allow rigorous proof for Riemann hypothesis to materialize. This rigorous proof is successfully obtained by rigidly applying exact [denoting $\sigma = \frac{1}{2}$] and inexact [denoting $\sigma \neq \frac{1}{2}$] Dimensional analysis homogeneity to DSPL (as pseudo-zeroes to zeroes conversion) which is rigorously derived from DηF (as zeroes) [*proxy* for RζF (as zeroes)] via intermediate sim-DηF (as zeroes). An identical procedure is used to precisely explain two types of GP.

Net Area Values and Total Area Values: $\int f(n)dn = F(n) + C$ with $F'(n) = f(n)$. Consider a nominated function $f(n)$ for interval [a,b]. We define Net Area Value (NAV) calculated using its antiderivative $F(n)$ as the net difference between positive area value(s) [above horizontal x-axis] and negative area value(s) [below horizontal x-axis] in interval [a,b]; viz, NAV = all +ve value(s) + all -ve value(s). Again calculated using $F(n)$, we define Total Area Value (TAV) as the total sum of (absolute value) positive area value(s) [above horizontal x-axis] and (absolute value) negative area value(s) [below horizontal x-axis] in interval [a,b]; viz, TAV = all |+ve value(s)| + all |-ve value(s)|. Calculated NAV and TAV are precise using antiderivative $F(n)$ obtained from integration of $f(n)$ but will only be approximate when using the Riemann sum on $f(n)$.

Zeroes and Pseudo-zeroes: [We will progressively explain these two entities below.] There are three types of stationary points in a given periodic f(n) involving sine and/or cosine functions that could act as x-axis intercept points via three types of f(n)'s zeroes with corresponding three types of F(n)'s pseudo-zeroes: maximum points e.g. with f(n) or F(n) = sin n - 1; minimum points e.g. with f(n) or F(n) = sin n + 1; and points of inflection e.g. with f(n) or F(n) = sin n [**which also has Origin intercept point as a zero or pseudo-zero**]. A fourth type of f(n)'s zeroes and F(n)'s pseudo-zeroes consist of non-stationary points occurring e.g. with f(n) or F(n) = sin n + 0.5.

With (j - i) = (l - k) = 2π [viz, one Full cycle], let a given zero be located in f(n)'s interval [i,j] viz, i $<$ zero $<$ j; and its corresponding pseudo-zero be located in F(n)'s pseudo-interval [k,l] viz, k $<$ pseudo-zero $<$ l. For this zero and pseudo-zero characterized by either point of inflection or non-stationary point; both will comply with preserving positivity [going from (-ve) below x-axis to (+ve) above x-axis] as explained using the zero case [with the pseudo-zero case following similar lines of explanations]. This can be stated as follow for interval [i,j]: If j $>$ i, then computed f(j) $>$ computed f(i). In particular, the condition "If i $\geq$ 0, then computed f(i) $\geq$ 0" must not be present for these two particular types of zero to validly exist in interval [i,j]. With reversal of inequality signs, the converse situation for j $<$ zero $<$ i and corresponding l $<$ pseudo-zero $<$ k will be equally true in preserving negativity [going from (+ve) above x-axis to (-ve) below x-axis]. These are useful observed properties for zeroes and pseudo-zeroes.

Preservation or conservation of Net Area Value and Total Area Value: For f(n)'s interval [a,b] whereby a = initial zero and b = next zero, and F(n)'s pseudo interval [c,d] whereby c = initial pseudo-zero and d = next pseudo-zero; we show below how compliance with preservation or conservation of NAV and TAV will simultaneously occur in both f(n)'s zeroes and F(n)'s pseudo-zeroes given by their sine and/or cosine functions only when zero gap = (b - a) = pseudo-zero gap = (d - c) = 2π [viz, involving one Full cycle].

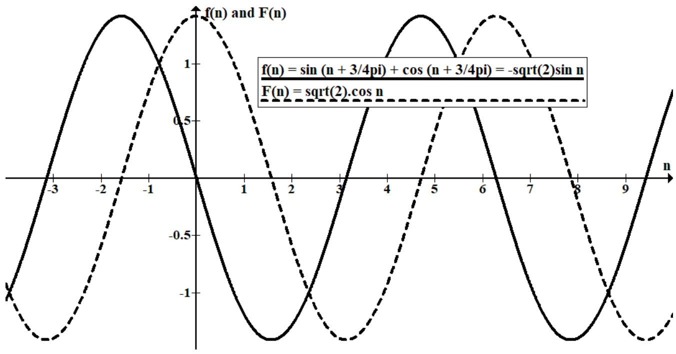

Fig. 1: Plot of $f(n) = sin(n+\frac{3}{4}\pi)+cos(n+\frac{3}{4}\pi) = -\sqrt{2}\sin n$ and $F(n) = \sqrt{2}\cos n$ [+ C].

Consider a (first) randomly chosen example of single-variable [simple] periodic sin and/or cosine function $f(n) = (\cos n)^{\frac{1}{3}} - (\sin n)^{\frac{1}{3}}$ which has zeroes and two individual exponents $\frac{1}{3}$ [with their sum $\frac{2}{3}$] being persistently fractional numbers. Observed characteristics of exponents from this first example [and second example below] as CP problems with perpetually present [intrinsic] simple properties could hint at possible invalidity of exact and inexact DA homogeneity as useful analytic tool used for IP problems. However, these are irrelevant (counter)examples since exact and inexact DA homogeneity in this research paper for our IP problems are perpetually present [intrinsic] complex properties that clearly do not refer to these CP recurring zeroes but instead validly refer to totally different IP recurring zeroes calculated as axes-intercept points in DηF and approximate NAV = 0 from Riemann sum interpretation of sim-DηF.

Derived complex properties of exact and inexact DA homogeneity in our IP problems may be assumed by some to be just associations and not proper explanations. We advocate this assumption to be incorrect since these perpetually present [intrinsic] complex properties are, nevertheless, valid properties fully supported by irrefutable [albeit convoluted] correct and complete set of mathematical arguments with proper explanations behind them based on, for instance, valid analysis on modulus of DηF, valid mathematical definition for NTZ, and valid compliance with preserving or conserving NAV = 0 condition.

Consider a (second) randomly chosen example of single-variable [simple] periodic sin and/or cosine function f(n) = (sin (n+$\frac{3}{4}\pi$))^1 + (cos (n+$\frac{3}{4}\pi$))^1 which has zeroes [**that also include the Origin intercept point as a zero**]. Depicted in Figure 1, this function is also equivalently simplified to f(n) = $-\sqrt{2}(\sin n)^1$ with non-negotiable trigonometric identity $\cos n - \sin n = \sqrt{2}\sin\left(n + \frac{3}{4}\pi\right)$ application. We note the individual or sum exponents of all involved sine and/or cosine terms is 1 or 2, being persistently whole numbers.

We compare and contrast the f(n)'s sine and cosine terms in the above two mentioned (unrelated) examples resulting in following two non-specific observations [without detailed discussion on their limited generalization to two other unrelated examples below]: [I] The dual sine and cosine f(n) with individual exponent = whole number 1 and sum exponent = whole number 2 in second example, which has zeroes, can only be **non-negotiably simplified (using trigonometric identity)** to be expressed in [solitary] sine term with exponent = whole number 1. [II] The dual sine and cosine f(n) with individual exponent = fractional number $\frac{1}{3}$ and sum exponent = fractional number $\frac{2}{3}$ in first example, which has zeroes, cannot be simplified further and remains expressed in [combined] sine and cosine terms. Two other unrelated examples: The f(n) = (sin n)^1 + (cos n)^1 which has zeroes and endowed with whole number 1 for its individual exponent with their sum = whole number 2 contradicts observation [I] as this function cannot be simplified further and remains expressed in [combined] sine and cosine terms. The f(n) = (sin n)$^{\frac{1}{2}}$ + (cos n)$^{\frac{1}{2}}$ [conveniently considered here as a different f(n) variety that *do not have zeroes*] endowed with two fractional exponent $\frac{1}{2}$ but with their sum = whole number 1, cannot be simplified further and remains expressed in [combined] sine and cosine terms – this case validly comply with observation [II] but sum of exponents being a whole number, and not a fractional number, could also be non-specifically interpreted to (partially) contradict observation [I].

Two useful points about f(n)'s Zeroes and F(n)'s Pseudo-zeroes involving sine and/or cosine terms; and exact and inexact Dimensional analysis homogeneity:
(I) With (Zeroes) = (Pseudo-zeroes) – ($\frac{\pi}{2}$) [given in terms of Full cycles] being valid in both CP problems and IP problems, the zeroes obtained from IP DηF via axes-intercept points and IP sim-DηF via approximate NAV = 0 are exactly related to the pseudo-zeroes obtained via precise NAV = 0 calculated using IP DSPL [the antiderivative for integration of sim-DηF]. **Summary of useful shorthand notations: DηF [as Zeroes], sim-DηF [as Zeroes], and DSPL [as Pseudo-zeroes to Zeroes conversion].**
(II) Parameter σ = (i) $\frac{1}{2}$ [viz, exact Dimensional analysis homogeneity for IP problem with full presence of Origin intercept points that **compulsorily involve trigonometric identity** $\cos n - \sin n$] and (ii) $\neq \frac{1}{2}$ [viz, inexact Dimensional analysis homogeneity for IP problem with full absence of Origin intercept points] situations are, respectively, the non-specific analogy of (i) CP f(n) from second example with individual / sum exponent = whole number [*ala* exact Dimensional analysis homogeneity for CP problem with full presence of Origin intercept point that **compulsorily involve trigonometric identity** $\cos n - \sin n$] and (ii) CP f(n) from first example with individual / sum exponent $\neq$ whole number [*ala* inexact Dimensional analysis homogeneity for CP problem with full absence of Origin intercept point]. ***Supplementary materials* in Conclusion include mathematical explanation why nontrivial zeros must inevitably exist in sim-DηF [as zeroes] uniquely and only at [fractional number] individual exponent $\sigma = \frac{1}{2}$ (viz, complying with exact Dimensional analysis homogeneity for IP problem based on [whole number] sum of two exponents = $2(1-\sigma) = \frac{1}{2} + \frac{1}{2} = 1$) in Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion].**

Full cycle-zeroes and Half cycle-zeroes of f(n) = $-\sqrt{2}\sin n$ from above second example are its recurring x-axis intercept points as seen in Figure 1. The term Full cycle symbolizes "non-varying CP full loop from 0π to 2π"; and "Half cycle" symbolizes "non-varying CP half loop from 0π to 1π or from 1π to 2π". From +ve to -ve; x-axis line is denoted by 0π to 1π, and y-axis line is denoted by $\frac{\pi}{2}$ to $\frac{3\pi}{2}$. In interval [0,2π], $f(n) = 0$ when n = 0π [Full cycle-zero], 1π [Half cycle-zero] and 2π [Full cycle-zero]. Ignore the 1π [Half cycle-zero] and conveniently name 0π [Full cycle-zero] and 2π [Full cycle-zero] as the initial zero and next zero. $F(n)$ is the antiderivative of integral $\int f(n)dn$ since $F'(n) = f(n)$. Precise NAV is given by $\int_0^{2\pi} f(n)dn = F(n)$ for the same interval [0,2π]. This NAV = 0 as $F(n) = [\sqrt{2}\cos n + C]_0^{2\pi} = (\sqrt{2}\cdot 1) - (\sqrt{2}\cdot 1) = 0$. In pseudo interval [$\frac{\pi}{2}$,$\frac{5\pi}{2}$] of $F(n)$, $\frac{\pi}{2}$ and $\frac{5\pi}{2}$ are its pseudo x-axis intercept points which we conveniently name here as initial pseudo-zero and next pseudo-zero. Both f(n)'s zero gap (initial zero minus next zero) and F(n)'s pseudo-zero gap (initial pseudo-zero minus next pseudo-zero) = 2π which resulted in f(n)'s interval gap and F(n)'s pseudo interval gap being also of identical magnitude 2π. As a direct consequence, calculated precise NAV = 0 will also apply to $F(n)$ on this pseudo interval with $(\sqrt{2}\cdot 0) - (\sqrt{2}\cdot 0) = 0$. Since NAV = $-\sqrt{2}$ + $+\sqrt{2}$ = 0 for (different) interval [0,π] and NAV = $-\sqrt{2}$ + $-\sqrt{2}$ = $-2\sqrt{2}$ for its corresponding pseudo-interval [$\frac{\pi}{2}$,$\frac{3\pi}{2}$], we observe in Figure 1 that NAV is not preserved or conserved when the interval and pseudo-interval do not involve a Full cycle. **Hence, NAV is preserved or conserved for corresponding CP f(n)'s zeroes and CP F(n)'s pseudo-zeroes only if the interval and pseudo-interval involve a Full cycle.**

With $f(n)$ and $F(n)$ involving [cofunctions] sine and cosine, we see in Figure 1 that calculated precise TAV $\neq$ 0 for given interval e.g. [0,$\frac{\pi}{2}$] with interval gap = $\frac{\pi}{2}$ when derived using $F(n) = \sqrt{2}\cos n + C$ [= $|\sqrt{2}\cdot 0| + |\sqrt{2}\cdot 1|$ = $\sqrt{2}$] will be identical in magnitude to corresponding pseudo-interval [$\frac{\pi}{2}$,π] with interval gap = $\frac{\pi}{2}$ when derived using $F(n) = \sqrt{2}\cos n + C$ [= $|\sqrt{2}\cdot -1| + |\sqrt{2}\cdot 0|$ = $\sqrt{2}$]. Similarly for one Full cycle with interval [0,2π] and corresponding pseudo-interval [$\frac{\pi}{2}$,$\frac{5\pi}{2}$], the calculated precise TAV = $4\sqrt{2}$ for both. **Hence, TAV is preserved or conserved for corresponding CP $f(n)$'s zeroes and CP $F(n)$'s pseudo-zeroes that involve any given interval and pseudo-interval.**

Always being $\frac{\pi}{2}$ out-of-phase with each other, sine and cosine are cofunctions with sin n = cos ($\frac{\pi}{2}$ - n), cos n = sin ($\frac{\pi}{2}$ - n), $\frac{d(\sin n)}{dn} = \cos n$, $\frac{d(\cos n)}{dn} = -\sin n$, $\int \sin n \cdot dn = -\cos n + C$ [= sin (n - $\frac{\pi}{2}$) + C] and $\int \cos n \cdot dn = \sin n + C$ [= cos (n - $\frac{\pi}{2}$) + C]. Last two integrals explain valid relation between f(n)'s zeroes and F(n)'s pseudo-zeroes when they involve sine and/or cosine terms viz, **f(n)'s CP Zeroes = F(n)'s CP Pseudo-zeroes – $\frac{\pi}{2}$**.

1.2 Valid application of both f(n)'s zeroes and F(n)'s pseudo-zeroes [when converted to zeroes] to represent Gram points and virtual Gram points

Whereby f(n) and F(n) have parameters σ and t; the above crucial findings are validly extrapolated to single-variable [complex] periodic (sine and cosine) functions: (i) f(n) DηF (*proxy* for RζF) = 0 to obtain zeroes, (ii) f(n) sim-DηF = 0 to obtain zeroes, and (iii) F(n) DSPL = 0 to obtain pseudo-zeroes [which can be converted to zeroes]. At $\sigma = \frac{1}{2}$ [critical line], the GP consisting of NTZ, G[y=0]P and G[x=0]P precisely correspond to t values for these f(n)'s zeroes. The t values for F(n)'s pseudo-zeroes can be used to calculate t values for f(n)'s zeroes since **f(n)'s IP Zeroes (t values) = F(n)'s IP Pseudo-zeroes (t values) – $\frac{\pi}{2}$**. In addition, NAV and TAV are preserved or conserved for corresponding IP f(n)'s zeroes and IP F(n)'s pseudo-zeroes since both f(n)'s [varying] zero gap (initial zero minus next zero) and F(n)'s [varying] pseudo-zero gap (initial pseudo-zero minus next pseudo-zero) is given by 2π [denotes one Full cycle traversed by parameter t].

At $\sigma \neq \frac{1}{2}$ [or non-critical lines], the virtual GP consisting of virtual G[y=0]P and virtual G[x=0]P precisely correspond to t values for these f(n)'s virtual zeroes. [Virtual NTZ do not exist.] The t values for F(n)'s virtual pseudo-zeroes can be used to calculate t values for f(n)'s virtual zeroes since **f(n)'s IP Virtual Zeroes (t values) = F(n)'s IP Virtual Pseudo-zeroes (t values) – $\frac{\pi}{2}$**. NAV and TAV will also be preserved or conserved for these corresponding IP f(n)'s virtual zeroes and IP F(n)'s virtual pseudo-zeroes since both f(n)'s virtual zero gap (initial virtual zero minus next virtual zero) and F(n)'s virtual pseudo-zero gap (initial virtual pseudo-zero minus next virtual pseudo-zero) is given by 2π [denotes one Full cycle traversed by parameter t].

Cartesian Coordinates (x,y) is related to Polar Coordinates (r,θ) with $r = \sqrt{x^2+y^2}$ and $\theta = tan^{-1}(\frac{y}{x})$. In anti-clockwise direction, it has four quadrants defined by the + or - of (x,y); viz, Quadrant I as (+,+), Quadrant II as (-,+), Quadrant III as (-,-), and Quadrant IV as (+,-). Figure 3 is the polar graph of $\zeta(\frac{1}{2}+\imath t)$ plotted along critical line for real values of t running from 0 to 34, horizontal axis: $Re\{\zeta(\frac{1}{2}+\imath t)\}$, and vertical axis: $Im\{\zeta(\frac{1}{2}+\imath t)\}$. NTZ are Origin intercept points or G[x=0,y=0]P. With 'gap' being synonymous with 'interval', NTZ gap is given by initial NTZ t-value minus next NTZ t-value. Running a Full cycle from 0π to 2π, size of each IP varying loop in Figure 3 is proportional to magnitude of its corresponding IP NTZ varying gap. We note the 2π here as observed in Figure 3 [on GP], Figure 4 [on virtual GP] and Figure 5 [on virtual GP] refers to IP varying loops transversed by parameter t with NTZ corresponding to t at Origin intercept (G[x=0,y=0]P); G[y=0]P and virtual G[y=0]P corresponding to t at x-axis intercept on x-axis' (+ve) 0π part and (-ve) 1π part; and G[x=0]P and virtual G[x=0]P corresponding to t at y-axis intercept on y-axis' (+ve) $\frac{\pi}{2}$ part and (-ve) $\frac{3\pi}{2}$ part. The virtual NTZ entity do not exist; viz, Origin intercept points do not occur in Figure 4 and Figure 5.

Sin and/or cosine f(n)'s IP (virtual) zeroes and F(n)'s IP (virtual) pseudo-zeroes:

Property I: At parameter $\sigma = \frac{1}{2}$, we denote zeroes to represent t-values for NTZ, G[y=0]P and G[x=0]P. $f(n)$ as DηF = 0 or sim-DηF = 0 gives rise to zeroes with varying zeroes gaps and $F(n)$ as DSPL = 0 gives rise to pseudo-zeroes with varying pseudo-zeroes gaps [whereby pseudo-zeroes can be converted to zeroes]. Since the corresponding zeroes gaps and pseudo-zeroes gaps are always identical in magnitude; NAV = 0 condition is validly preserved or conserved. [For simplicity, we will not discuss TAV $\neq$ 0 condition which is also validly preserved or conserved.] ***Property I* is usefully abbreviated as: NAV = 0 condition is validly preserved or conserved for f(n)'s IP zeroes and F(n)'s IP pseudo-zeroes. *Ditto* for f(n)'s IP virtual zeroes and F(n)'s IP virtual pseudo-zeroes at parameter $\sigma \neq \frac{1}{2}$ as previously explained above; viz, NAV = 0 condition is validly preserved or conserved for f(n)'s IP virtual zeroes and F(n)'s IP virtual pseudo-zeroes.**

Property II: At parameter $\sigma = \frac{1}{2}$, [IP t-values for NTZ, G[y=0]P and G[x=0]P obtained from $f(n)$] is equal to [IP (different) t-values for pseudo-NTZ, pseudo-G[y=0]P and pseudo-G[x=0]P obtained from $F(n)$] minus [Constant $\frac{\pi}{2}$]. ***Property II* is usefully abbreviated as: f(n)'s IP zeroes (t values) = F(n)'s IP pseudo-zeroes (t values) – $\frac{\pi}{2}$. *Ditto* for f(n)'s IP virtual zeroes and F(n)'s IP virtual pseudo-zeroes at parameter $\sigma \neq \frac{1}{2}$ as previously explained above; viz, f(n)'s IP virtual zeroes (t values) = F(n)'s IP virtual pseudo-zeroes (t values) – $\frac{\pi}{2}$.**

Deduction I: The x variable used in Riemann sum above in E-O Pairing is now replaced by the n variable used in sim-DηF. Conventionally, for a left [finite-interval] Riemann sum, i = 0, 1, 2, 3,..., n-1; and for a right [finite-interval] Riemann sum, i = 1, 2, 3, 4,..., n. Analogically, sim-DηF as a complex function with n = 1, 2, 3, 4,..., ∞ is now itself (*interpreted as*) a right [infinite-interval] Riemann sum given by $\sum_{i=1}^{n} \Delta n \cdot f(n_i)$ = $\sum_{n=1}^{\infty} \Delta n \cdot f(n)$ = $\sum_{n=1}^{\infty} f(n)$ since $\Delta n = 1$ in this function. As (i) $\int_{n=1}^{n=\infty} f(n)dn$ and (ii) $\sum_{n=1}^{\infty} f(n)$ are proportional; their f(n)'s zeroes derived from corresponding (i) DSPL [as pseudo-zeroes converted to zeroes] and (ii) sim-DηF [as zeroes] when interpreted as Riemann sum must agree whereby f(n) has parameters σ and t. Again, we note f(n)'s zeroes can be obtained from DSPL [viz, the antiderivative F(n)] because corresponding F(n)'s pseudo-zeroes (t values) = f(n)'s zeroes (t values) + $\frac{\pi}{2}$ whereby F(n) also has same parameters σ and t.

Deduction II: The f(n)-in-isolation from sim-DηF is usefully perceived to give rise to one-dimensional lines whereas its corresponding full function with summation $\sum_{n=1}^{\infty}$ [which can be interpreted as Riemann sum] give rise to approximate two-dimensional NAV; viz, sim-DηF is validly and usefully regarded as Riemann sum. Precise two-dimensional NAV are obtained with integration application to sim-DηF [as zeroes] viz, definite integral $\int_{1}^{\infty}$(sim-DηF)dn = antiderivative DSPL [as pseudo-zeroes converted to zeroes]. As opposed to (a) x-axis intercept, (b) y-axis intercept and (c) Origin intercept which are usefully depicted with vertical axis: $Im\{\eta(\sigma+\imath t)\}$ and horizontal axis: $Re\{\eta(\sigma+\imath t)\}$; their corresponding two-dimensional NAVs can be usefully depicted with vertical axis: (a) $Im\{\eta(\sigma+\imath t)\}$, (b) $Re\{\eta(\sigma+\imath t)\}$ and (c) $ReIm\{\eta(\sigma+\imath t)\}$, and horizontal axis: t.

Information-Complexity conservation. Formulae can consist of (1) equations e.g. CP two 'non-varying' discrete-type and two continuous-type equations to independently calculate and incorporate all even and odd numbers, and IP 'varying' discrete-type equations DηF [as zeroes] (*proxy* for RζF [as zeroes]) and sim-DηF [as zeroes] or 'varying' continuous-type equation DSPL [as pseudo-zeroes converted to zeroes] to dependently calculate all NTZ and two types of GP; or (2) algorithms e.g. Sieve of Eratosthenes giving rise to IP two 'varying' discrete-type algorithms to dependently compute all prime and composite numbers. Thus, a given formula is simply a Black Box generating Output (having qualitative-like structural 'Complexity') when supplied with Input (having quantitative-like data 'Information').

A set of correct and complete ["formulae-laden"] mathematical arguments depicted as lemmas, corollaries and propositions must fully comply with Information-Complexity conservation. Intuitively, this is synonymous with Information-based complexity and can be considered as a unique all-purpose [quantitative and qualitative] analytic tool used with Mathematics for Completely Predictable problems and Mathematics for Incompletely Predictable problems. Respectively, these problems can literally be perceived as "simple systems" containing well-defined CP entities such as even and odd numbers; and "complex systems" containing well-defined IP entities such as prime and composite numbers, NTZ and two types of GP.

Complying with Information-Complexity conservation by DηF (*proxy* for RζF) in a qualitative-like manner will always result in maximum three types of axes-intercepts (viz, x-axis, y-axis and Origin intercepts as three types of GP) occurring at $\sigma = \frac{1}{2}$ and minimum two types of axes-intercepts (viz, x-axis and y-axis intercepts as two types of virtual GP) occurring at $\sigma \neq \frac{1}{2}$. Complying with Information-Complexity conservation, preservation or conservation of approximate NAV = 0 using sim-DηF [as (virtual) zeroes] in a quantitative-like manner occur at $\sigma = \frac{1}{2}$ as three types of GP, and occur at $\sigma \neq \frac{1}{2}$ as two types of virtual GP. Corresponding preservation or conservation of precise NAV = 0 using DSPL [as (virtual) pseudo-zeroes to (virtual) zeroes conversion] occur at $\sigma = \frac{1}{2}$ as three types of GP, and occur at $\sigma \neq \frac{1}{2}$ as two types of virtual GP.

The following are **two key concepts from Information-Complexity conservation**:

(I) Relevant end-product Laws [with obtained pseudo-zeroes converted to zeroes], equations or algorithms (i) will generate or incorporate CP entities such as even and odd numbers with simple properties consistently manifested by these entities; and (ii) will generate or incorporate IP entities such as prime and composite numbers, NTZ and two types of GP with complex properties consistently manifested by these entities.

(II) In principle, [A] CP 'non-varying' two discrete-type and two continuous-type equations to independently compute and incorporate all even and odd numbers, [B] IP two 'varying' discrete-type algorithms to dependently compute all prime and composite numbers, and [C] IP 'varying' discrete-type equations DηF [as zeroes] (*proxy* for RζF [as zeroes]) and sim-DηF [as zeroes] to dependently compute all NTZ and two types of GP could, respectively and correspondingly, be derived in a reverse-engineered manner from [A] two 'non-varying' discrete-type equations when language-expressed [in one combined table] using Dimension (2x - N) system[1], [B] two 'varying' discrete-type algorithms when language-expressed [in one combined table] using Dimension (2x - N) system[1], and [C] 'varying' continuous-type equation DSPL [as pseudo-zeroes converted to zeroes].

Our mathematical-formated and geometrical-formated treatises containing pure and applied mathematics in relevant "Mathematics for Incompletely Predictable problems" encompass new signatory ideas that will overcome insurmontable difficulties present in many previously attempted mathematical proofs for nominated open problem of Riemann hypothesis and explanations for its closely related two types of Gram points. *These difficulties are advocated to inevitably arise simply because of failure during previous attempts to specifically treat Riemann zeta function (or its proxy Dirichlet eta function) as unique mathematical object for full intrinsic analysis on its derived* ***de novo*** *complex properties*. As an intractable open problem in Number theory, Riemann hypothesis historically belongs to one of USD $1,000,000 Millennium Prize Problems in field of mathematics that were identified by Clay Mathematics Institute at turn of new millennium on May 24, 2000.

1.3 Open Problems in Number Theory with Incompletely Predictable Entities

["$\Delta x \longrightarrow 0$"] DSPL is the continuous format version of discrete format ["$\Delta x \longrightarrow 1$"] D$\eta$F (*proxy* for ["$\Delta x \longrightarrow 1$"] R$\zeta$F) and ["$\Delta x \longrightarrow 1$"] sim-D$\eta$F. Colloquially speaking, they enable either quantitative-like calculations on NAV or qualitative-like computations on axes-intercept points. As previously explained, these actions will result in the desired zeroes and pseudo-zeroes with Zeroes (t values) = Pseudo-zeroes (t values) – $\frac{\pi}{2}$. We take note, firstly, the unique situation of Origin intercept points (NTZ) obtained via approximate NAV = 0 giving zeroes t-values or via precise NAV = 0 giving pseudo-zeroes [which can be converted to zeroes] t-values will only occur when $\sigma = \frac{1}{2}$; and, secondly, the critical line of Riemann zeta function (or its *proxy* Dirichlet eta function) is also uniquely denoted by $\sigma = \frac{1}{2}$ whereby, in Riemann hypothesis, all NTZ are conjectured to be located.

The Number '1' is neither prime nor composite. ["$\Delta x \longrightarrow 1$"] Sieve of Eratosthenes is a simple ancient algorithm for finding all prime numbers up to any given limit by iteratively marking as composite (i.e., not prime) the multiples of each prime, starting with first prime number 2. Multiples of a given prime are generated as a sequence of numbers starting from that prime, with constant difference between them equal to that prime. Dimension (2x - N) system can *dependently* generate all prime and composite numbers [and "extrapolated" Number '1' which is uniquely represented by Dimension (2x - 2)] whereas Sieve of Eratosthenes directly and indirectly give rise to prime and composite numbers (but not Number '1'). In using the unique Dimension (2x - N) system with N = 2x - ΣPC$_x$-Gap [see [1] for full details] and x = all integers commencing from 1; Dimension (2x - N) when fully expanded is numerically just equal to ΣPC$_x$-Gap since Dimension (2x - N) = 2x - 2x + ΣPC$_x$-Gap = ΣPC$_x$-Gap.

To be complete, definition for the above system is fully explained using two examples for position x = 31 and 32. For i and x $\in$ **N** [as per data from Table 5 in [1]]; ΣPC$_x$-Gap = ΣPC$_{x-1}$-Gap + Gap value at $\mathbf{P}_{i-1}$ or Gap value at $\mathbf{C}_{i-1}$ whereby (i) $\mathbf{P}_i$ or $\mathbf{C}_i$ at position x is determined by whether relevant x value belongs to a **P** or **C**, and (ii) both ΣPC$_1$-Gap and ΣPC$_2$-Gap = 0. Example, for position x = 31: 31 is **P** (**P11**). Desired Gap value at **P10** = 2. Thus ΣPC$_{31}$-Gap (55) = ΣPC$_{30}$-Gap (53) + Gap value at **P10** (2). Example, for position x = 32: 32 is **C** (**C20**). Desired Gap value at **C19** = 2. Thus ΣPC$_{32}$-Gap (57) = ΣPC$_{31}$-Gap (55) + Gap value at **C20** (2).

For i = natural numbers, ["$\Delta x \longrightarrow 1$"] equations E$_i$ = 2 X i and O$_i$ = (2 X i) - 1 *independently* give rise to all even and odd numbers (which are obtained from natural numbers thus excluding even number '0'). For i = real numbers ≥ 0, ["$\Delta x \longrightarrow 0$"] equations E = 2 X i and O = (2 X i) - 1 *independently* incorporates all even and odd numbers (including even number '0' at i = 0). Then, Origin intercept (0,0) will only occur in equation E = 2 X i when Output E = 0 is uniquely generated by Input i = 0. Dimension (2x - N) system can *independently* generate all even and odd numbers [including the ("zeroth") even number '0' which is uniquely represented by Dimension (2x - 0) obtained by incorporating x = 0]. In using the unique Dimension (2x - N) system with N = 2x - ΣEO$_x$-Gap [see [1] for full details] and x = all integers commencing from 1 [with even number '0'

Table 1: Two options to solve open problems in Number theory

Riemann zeta function	Sieve of Eratosthenes
$\downarrow$ [*Path A option*] $\downarrow$	$\downarrow$ [*Path A option*] $\downarrow$
Nontrivial zeros and two types of Gram points	Prime and Composite numbers
$\uparrow$ [*Path B option*] $\uparrow$	$\uparrow$ [*Path B option*] $\uparrow$
Dirichlet Sigma-Power Laws	**Dimension (2x - N), N = 2x - ΣPC$_x$-Gap**

arbitrarily excluded]; Dimension (2x - N) when fully expanded is numerically just equal to ΣEO$_x$-Gap since Dimension (2x - N) = 2x - 2x + ΣEO$_x$-Gap = ΣEO$_x$-Gap.

To be complete, definition for the above system is fully explained using two examples for position x = 31 and 32. For i and x $\in$ **N** [as per data from Table 6 in [1]]; ΣEO$_x$-Gap = ΣEO$_{x-1}$-Gap + Gap value at E$_{i-1}$ or Gap value at O$_{i-1}$ whereby (i) E$_i$ or O$_i$ at position x is determined by whether relevant x value belongs to **E** or **O**, and (ii) both ΣEO$_1$-Gap and ΣEO$_2$-Gap = 0. Example, for position x = 31: 31 is **O** (O16). Our desired Gap value at O15 = 2. Thus ΣEO$_{31}$-Gap (58) = ΣEO$_{30}$-Gap (56) + Gap value at O15 (2). Example, for position x = 32: 32 is **E** (E16). Our desired Gap value at E15 = 2. Thus ΣEO$_{32}$-Gap (60) = ΣEO$_{31}$-Gap (58) + Gap value at E15 (2).

To solve Riemann hypothesis, Polignac's and Twin prime conjectures (and explain two types of Gram points) while fully complying with Information-Complexity conservation; we could theoretically follow Path A or Path B as schematically depicted in Table 1. Both options require Mathematics for IP problems. Our utilized Path B option involves deriving DSPL and using Dimension (2x - N) system.

Riemann hypothesis (1859) proposed all NTZ in Riemann zeta function to be located on its critical line. Defined as IP problem is essential to correctly prove this hypothesis. All of infinite magnitude, NTZ when geometrically depicted as corresponding Origin intercept points together with two types of Gram points when geometrically depicted as corresponding x- and y-axes intercept points explicitly confirm they intrinsically form relevant component of point-intersections in these functions at $\sigma = \frac{1}{2}$. The equivalence of axes-intercept points are precise NAV = 0 [as pseudo-zeroes which can be converted to zeroes] calculated using DSPL and approximate NAV = 0 [as zeroes] calculated using Riemann sum when sim-DηF is interpreted as such. Defined as IP problems is essential for these explanations to be correct.

Remark 2 Mathematics for Incompletely Predictable problems equates to *sine qua non* (correctly) classifying problems involving Incompletely Predictable entities as Incompletely Predictable problems. This is achieved by incorporating certain identifiable (non-negotiable) mathematical steps with this procedure ultimately enabling us to rigorously prove or precisely explain our nominated open problems in Number theory.

Refined information on Incompletely Predictable entities of Gram and virtual Gram points: These entities all of infinite magnitude are *dependently* calculated using *complex equation* Riemann zeta function, $\zeta(s)$, or its *proxy* Dirichlet eta function, $\eta(s)$, in critical strip (denoted by $0 < \sigma < 1$) thus forming the relevant component of point-intersections. In Figure 3, Gram[y=0], Gram[x=0] and Gram[x=0,y=0] points are, respectively, geometrical x-axis, y-axis and Origin intercepts at critical line (denoted by $\sigma = \frac{1}{2}$). Gram[y=0] and Gram[x=0,y=0] points are, respectively, synonymous with traditional *'Gram points'* and *nontrivial zeros*. In Figures 4 and 5, virtual Gram[y=0] and virtual Gram[x=0] points are, respectively, geometrical x-axis and y-axis intercepts at non-critical lines (denoted by $\sigma \neq \frac{1}{2}$). Virtual Gram[x=0,y=0] points do not exist.

Refined information on Incompletely Predictable entities of prime and composite numbers: These entities all of infinite magnitude are *dependently* computed (respectively) directly and indirectly using *complex algorithm* Sieve of Eratosthenes. Denote $\mathbb{C}$ to be uncountable complex numbers, **R** to be uncountable real numbers, **Q** to be countable rational numbers or roots [of non-zero polynomials], **R–Q** to be uncountable irrational numbers, **A** to be countable algebraic numbers, **R–A** to be uncountable transcendental numbers, **Z** to be countable integers, **W** to be countable whole numbers, **N** to be countable natural numbers, **E** to be countable even numbers, **O** to be countable odd numbers, **P** to be countable prime numbers, and **C** to be countable composite numbers. **A** are $\mathbb{C}$ (including **R**) that are countable rational or irrational roots. (i) Set **N** = Set **E** + Set **O**, (ii) Set **N** = Set **P** + Set **C** + Number '1', (iii) Set **A** = Set **Q** + Set **irrational roots**, and (iv) Set **N** $\subset$ Set **W** $\subset$ Set

Z ⊂ Set **Q** ⊂ Set **R** ⊂ Set ℂ. Then Set **R–Q** = Set **irrational roots** + Set **R–A**. Note: With **E** and **O** obtained from **N**, we did not include '0' as **E** in above discussion.

Cardinality of a given set: With increasing size, arbitrary Set **X** can be countable finite set (CFS), countable infinite set (CIS) or uncountable infinite set (UIS). Cardinality of Set **X**, |**X**|, measures "number of elements" in Set **X**. E.g. Set **negative Gram[y=0] point** has CFS of negative Gram[y=0] point with |**negative Gram[y=0] point**| = 1, Set **even P** has CFS of even **P** with |**even P**| = 1, Set **N** has CIS of **N** with |**N**| = $\aleph_0$, and Set **R** has UIS of **R** with |**R**| = $\mathfrak{c}$ (cardinality of the continuum).

We compare and contrast CP entities (obeying Simple Elementary Fundamental Laws) against IP entities (obeying Complex Elementary Fundamental Laws) using examples:
(I) **E** are CP entities constituted by CIS of **Q** 2, 4, 6, 8, 10, 12.... Note: These are (positive) **E** derived from **N** which then do not include '0' as even number.
(II) **O** are CP entities constituted by CIS of **Q** 1, 3, 5, 7, 9, 11....
(III) **P** are IP entities constituted by CIS of **Q** 2, 3, 5, 7, 11, 13....
(IV) **C** are IP entities constituted by CIS of **Q** 4, 6, 8, 9, 10, 12....
(V) With values traditionally given by parameter t, nontrivial zeros in Riemann zeta function are IP entities constituted by CIS of **R–A** [rounded off to six decimal places]: 14.134725, 21.022040, 25.010858, 30.424876, 32.935062, 37.586178,....
(VI) Traditional 'Gram points' (or Gram[y=0] points) are x-axis intercepts with choice of index 'n' for 'Gram points' historically chosen such that first 'Gram point' [by convention at n = 0] corresponds to the t value which is larger than (first) nontrivial zero located at t = 14.134725. 'Gram points' are IP entities constituted by CIS of **R–A** [rounded off to six decimal places] with the first six given at n = -3, t = 0; at n = -2, t = 3.436218; at n = -1, t = 9.666908; at n = 0, t = 17.845599; at n = 1, t = 23.170282; at n = 2, t = 27.670182. We will not calculate any values for Gram[x=0] points.

Denoted by parameter t; nontrivial zeros, 'Gram points' and Gram[x=0] points all belong to well-defined CIS of **R–A** which will twice obey the relevant location definition [in CIS of **R–A** themselves and in CIS of numerical digits after decimal point of each **R–A**]. First and only negative 'Gram point' (at n = -3) is obtained by substituting CP t = 0 resulting in $\zeta(\frac{1}{2} + \imath t) = \zeta(\frac{1}{2})$ = -1.4603545, a **R–A** number [rounded off to seven decimal places] calculated as a limit similar to limit for Euler-Mascheroni constant or Euler gamma with its precise (1^{st}) position only determined by computing positions of all preceding (nil) 'Gram point' in this case. '0' and '1' are special numbers being neither **P** nor **C** as they represent nothingness (zero) and wholeness (one). In this setting, the idea of having factors for '0' and '1' is meaningless. All entities derived from well-defined simple/complex algorithms or equations are "dual numbers" as they can be simultaneously depicted as CP and IP numbers. For instance, **Q** '2' as **P** (and **E**), '97' as **P** (and **O**), '98' as **C** (and **E**), '99' as **C** (and **O**); CP '0' values in x=0, y=0 and simultaneous x=0, y=0 associated with various IP t values in ζ(s).

Proposed by German mathematician Bernhard Riemann (September 17, 1826 – July 20, 1866) in 1859, Riemann hypothesis is mathematical statement on Riemann zeta function, $\zeta(s)$ [or its *proxy* Dirichlet eta function, $\eta(s)$] that critical line denoted by $\sigma = \frac{1}{2}$ contains complete Set **nontrivial zeros** with |**nontrivial zeros**| = $\aleph_0$. Alternatively, this hypothesis is geometrical statement on $\zeta(s)$ [or its *proxy* $\eta(s)$] that generated curves at $\sigma = \frac{1}{2}$ contain complete Set **Origin intercepts** with |**Origin intercepts**| = $\aleph_0$.

Remark 3 Confirming first 10,000,000,000,000 nontrivial zeros location on critical line implies but does not prove Riemann hypothesis to be true.

Locations of first 10,000,000,000,000 nontrivial zeros on critical line have previously been computed to be correct. Hardy[2], and with Littlewood[3], showed infinite nontrivial zeros on critical line (denoted by $\sigma = \frac{1}{2}$) by considering moments of certain functions related to $\zeta(s)$. This discovery cannot constitute rigorous proof for Riemann hypothesis because they have not exclude theoretical existence of nontrivial zeros located away from this line (when $\sigma \neq \frac{1}{2}$). Furthermore, it is literally a mathematical impossibility ("mathematical impasse") to be able to computationally check [in a complete and successful manner] the locations of all infinitely many nontrivial zeros to correctly lie on critical line.

2 Riemann Zeta Function and its *proxy* Dirichlet Eta Function

An L-function consists of a Dirichlet series with a functional equation and an Euler product. Examples of L-functions come from modular forms, elliptic curves, number fields, and Dirichlet characters, as well as more generally from automorphic forms, algebraic varieties, and Artin representations. They form an integrated component of 'L-functions and Modular Forms Database' (LMFDB) with far-reaching implications. In perspective, $\zeta(s)$, being the simplest example of an L-function, is a function of complex variable s (= $\sigma \pm \imath t$) that analytically continues sum of infinite series $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$. The common convention is to write s as $\sigma + \imath t$ with $\imath = \sqrt{-1}$, and with σ and t real. Valid for $\sigma > 0$, we write $\zeta(s)$ as $Re\{\zeta(s)\}+\imath Im\{\zeta(s)\}$ and note that $\zeta(\sigma + \imath t)$ when $0 < t < +\infty$ is the complex conjugate of $\zeta(\sigma - \imath t)$ when $-\infty < t < 0$.

Also known as alternating zeta function, $\eta(s)$ must act as *proxy* for $\zeta(s)$ in critical strip (viz. $0 < \sigma < 1$) containing critical line (viz. $\sigma = \frac{1}{2}$) because $\zeta(s)$ only converges when $\sigma > 1$. This implies $\zeta(s)$ is undefined to left of this region in critical strip which then requires $\eta(s)$ representation instead. They are related to each other as $\zeta(s) = \gamma \cdot \eta(s)$ with proportionality factor $\gamma = \frac{1}{(1-2^{1-s})}$ and $\eta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} = \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \cdots$.

$$\begin{aligned}
\zeta(s) &= \sum_{n=1}^{\infty} \frac{1}{n^s} \qquad (2.1)\\
&= \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots \\
&= \Pi_{p\ prime} \frac{1}{(1-p^{-s})} \\
&= \frac{1}{(1-2^{-s})} \cdot \frac{1}{(1-3^{-s})} \cdot \frac{1}{(1-5^{-s})} \cdot \frac{1}{(1-7^{-s})} \cdot \frac{1}{(1-11^{-s})} \cdots \frac{1}{(1-p^{-s})} \cdots
\end{aligned}$$

Eq. (2.1) is defined for only $1 < \sigma < \infty$ region where $\zeta(s)$ is absolutely convergent with no zeros located here. In Eq. (2.1), equivalent Euler product formula with product over prime numbers [instead of summation over natural numbers] also represents $\zeta(s) \implies$ all prime and, by default, composite numbers are (intrinsically) "encoded" in $\zeta(s)$. This observation represents a strong reason to conveniently combine proofs for Riemann hypothesis, Polignac's and Twin prime conjectures as was done in the previous 2020 research paper[1].

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \cdot \Gamma(1-s) \cdot \zeta(1-s) \qquad (2.2)$$

With $\sigma = \frac{1}{2}$ as symmetry line of reflection, Eq. (2.2) is Riemann's functional equation valid for $-\infty < \sigma < \infty$. It can be used to find all trivial zeros on horizontal line at $\imath t$ = 0 occurring when σ = -2, -4, -6, -8, -10,..., ∞ whereby $\zeta(s) = 0$ because factor $\sin(\frac{\pi s}{2})$ vanishes. Γ is gamma function, an extension of factorial function [a product function denoted by ! notation whereby n! = $n(n-1)(n-2)\ldots(n-(n-1))$] with its argument shifted down by 1, to real and complex numbers. That is, if n is a positive integer, $\Gamma(n) = (n-1)!$

$$\begin{aligned}
\zeta(s) &= \frac{1}{(1-2^{1-s})} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} \qquad (2.3)\\
&= \frac{1}{(1-2^{1-s})} \left(\frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \cdots\right)
\end{aligned}$$

Eq. (2.3) is defined for all $\sigma > 0$ values except for simple pole at σ = 1. As alluded to above, ζ(s) without $\frac{1}{(1-2^{1-s})}$ viz. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s}$ is η(s). It is a holomorphic function of s defined by analytic continuation and is

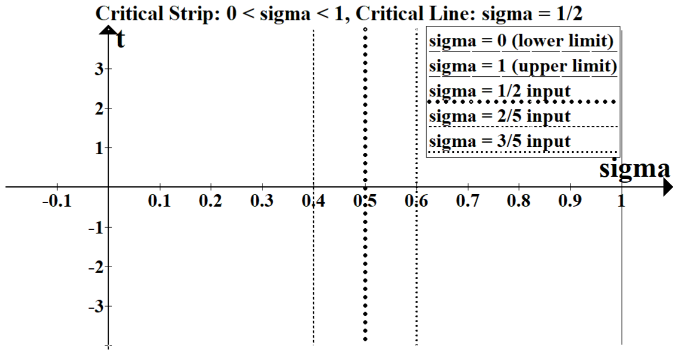

Fig. 2: INPUT for $\sigma = \frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{5}$. $\zeta(s)$ has countable infinite set (CIS) of Completely Predictable trivial zeros at σ = all negative even numbers and [proposed] CIS of Incompletely Predictable nontrivial zeros at σ = $\frac{1}{2}$ for various t values.

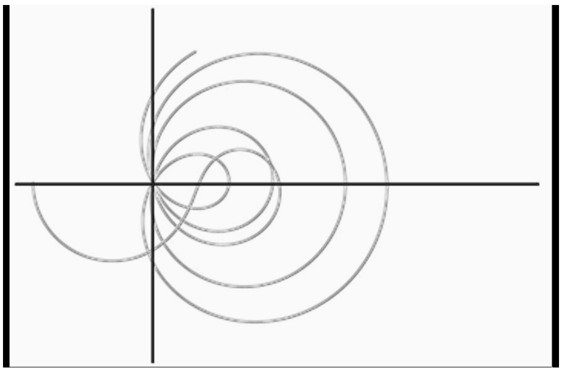

Fig. 3: OUTPUT for $\sigma = \frac{1}{2}$ as Gram points. Figures 3 represents schematically depicted polar graph of $\zeta(\frac{1}{2} + \imath t)$ plotted along critical line for real values of t running from 0 to 34, horizontal axis: $Re\{\zeta(\frac{1}{2} + \imath t)\}$, and vertical axis: $Im\{\zeta(\frac{1}{2} + \imath t)\}$. There are presence of Origin intercept points which are totally absent in Figures 4 and 5 [with identical axes definitions but, respectively, adjusted to $\sigma = \frac{2}{5}$ and $\sigma = \frac{3}{5}$]

mathematically defined at σ = 1 whereby analogous trivial zeros with presence for η(s) [but not for ζ(s)] on vertical straight line σ = 1 are found at $s = 1 \pm \imath \frac{2\pi k}{\ln(2)}$ where k = 1, 2, 3, 4,$\ldots$, ∞.

Figure 2 depict complex variable s (= $\sigma \pm \imath t$) as INPUT with x-axis denoting real part Re$\{$s$\}$ associated with σ, and y-axis denoting imaginary part Im$\{$s$\}$ associated with t. Figures 3, 4 and 5 respectively depict $\zeta(s)$ as OUTPUT for real values of t running from 0 to 34 at $\sigma = \frac{1}{2}$ (critical line), $\sigma = \frac{2}{5}$ (non-critical line), and $\sigma = \frac{3}{5}$ (non-critical line) with x-axis denoting real part Re$\{\zeta(s)\}$ and y-axis denoting imaginary part Im$\{\zeta(s)\}$. There are infinite types-of-spirals (loops) possibilities associated with each σ value arising from all infinite σ values in critical strip. Mathematically proving all nontrivial zeros location on critical line as denoted by solitary $\sigma = \frac{1}{2}$ value equates to geometrically proving all Origin intercepts occurrence at solitary σ = $\frac{1}{2}$ value. Both result in rigorous proof for Riemann hypothesis.

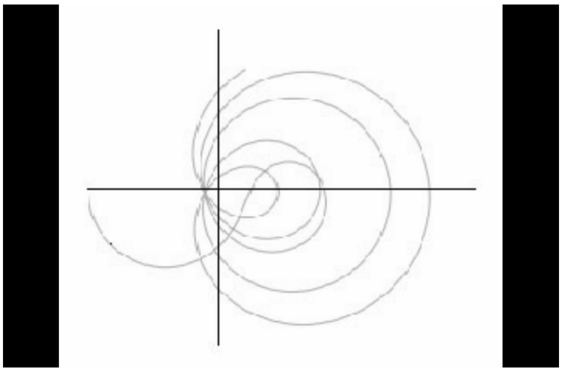

Fig. 4: OUTPUT for $\sigma = \frac{2}{5}$ as virtual Gram points. Incompletely Predictable loops are shifted to the left of Origin with horizontal axis: $Re\{\zeta(\frac{2}{5}+\imath t)\}$, and vertical axis: $Im\{\zeta(\frac{2}{5}+\imath t)\}$. There are total absence of Origin intercept points.

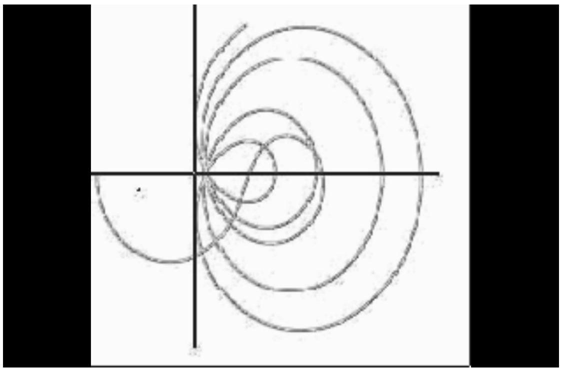

Fig. 5: OUTPUT for $\sigma = \frac{3}{5}$ as virtual Gram points with horizontal axis: $Re\{\zeta(\frac{3}{5}+\imath t)\}$, and vertical axis: $Im\{\zeta(\frac{3}{5}+\imath t)\}$. Incompletely Predictable loops are shifted to the right of Origin. There are total absence of Origin intercept points.

3 Equations for Riemann Hypothesis and Two Types of Gram Points

Calculations using Dirichlet eta function, $\eta(s)$ [*proxy* for Riemann zeta function, $\zeta(s)$] in discrete (summation) format for all σ values result in infinitely many equations **[and all Gram points entities at $\sigma = \frac{1}{2}$ as zeroes t-values axes-intercept points solutions]** for $0 < \sigma < 1$ critical strip region of interest with n = 1, 2, 3, 4, 5,. . . , ∞ as discrete integer number values. These equations geometrically represent entire plane of critical strip, thus (at least) allowing our proposed proof to be of a complete nature. We note sim-$\eta(s)$ in discrete (summation) format is obtained by applying Euler formula to $\eta(s)$ and when interpreted as Riemann sum, it gives rise to **approximate Net Area Value = 0 solutions to obtain all Gram points entities at $\sigma = \frac{1}{2}$ [as zeroes t-values]** with n = 1, 2, 3, 4, 5,. . . , ∞ as discrete integer number values. Dirichlet Sigma-Power Law is the antiderivative derived from solving the improper integrals of sim-$\eta(s)$ (lower limit a = 1 and upper limit b = ∞) whereby n = 1 to ∞ are continuous real number values. This Law will have its [multiple] +ve (above x-axis) and -ve (below x-axis) numerical **precise Net Area Value = 0 solutions to obtain all Gram points entities at $\sigma = \frac{1}{2}$ [as pseudo-zeroes t-values which are converted to zeroes t-values]** successfully computed – see Proposition 1.2 in Appendix A for greater details.

Notice to Readers: In this paper, we will not invoke the additional use of "inequations" mentioned in [1] as their use is simply a luxury which is not essential when providing rigorous proof for Riemann hypothesis and precise explanations for two types of Gram points. Prerequisite lemmas, corollaries and propositions on $\eta(s)$ with associated extensive Calculus to derive necessary equations corresponding to relevant Dirichlet Sigma-Power Laws [as pseudo-zeroes to zeroes conversion] for nontrivial zeros (Riemann hypothesis) and two types of Gram points are, respectively, outlined in Appendix A and Appendix B. These derived equations – same ones as outlined in Appendix A and Appendix B – are given below in full by [additionally] incorporating constant of integration 'C' whereby 'C' is inconsistently used in Appendix A and Appendix B:

I. Gram[x=0,y=0] points (or nontrivial zeros) for Riemann hypothesis obtained via pseudo-zeroes to zeroes conversion

$$\begin{aligned}\frac{1}{2\left(t^2+(\sigma-1)^2\right)}\cdot\Big[&(2n)^{1-\sigma}((t+\sigma-1)\sin(t\ln(2n))+(t-\sigma+1)\\ &\cdot\cos(t\ln(2n)))-(2n-1)^{1-\sigma}((t+\sigma-1)\\ &\cdot\sin(t\ln(2n-1))+(t-\sigma+1)\cos(t\ln(2n-1)))+C\Big]_1^\infty=0\end{aligned} \tag{3.1}$$

II. Gram[y=0] points (or 'usual' Gram points) obtained via pseudo-zeroes to zeroes conversion

$$-\frac{1}{2\left(t^2+(\sigma-1)^2\right)}\cdot\Big[(2n)^{1-\sigma}((\sigma-1)\sin(t\ln(2n))+t\cos(t\ln(2n)))-$$

$$(2n-1)^{1-\sigma}((\sigma-1)\sin(t\ln(2n-1))+t\cos(t\ln(2n-1)))+C\Big]_1^\infty=0 \tag{3.2}$$

III. Gram[x=0] points obtained via pseudo-zeroes to zeroes conversion

$$\frac{1}{2\left(t^2+(\sigma-1)^2\right)}\cdot\Big[(2n)^{1-\sigma}(t\sin(t\ln(2n))-(\sigma-1)\cos(t\ln(2n)))-$$

$$(2n-1)^{1-\sigma}(t\sin(t\ln(2n-1))-(\sigma-1)\cos(t\ln(2n-1)))+C\Big]_1^\infty=0 \tag{3.3}$$

Critical line is denoted by $\sigma = \frac{1}{2}$ and is conjectured in Riemannn hypothesis to be the unique location for all nontrivial zeros of Riemann zeta function. For nontrivial zeros, (1) its mathematical definition: ζ(s) = 0 or η(s) = 0 or sim-η(s) = 0, and (2) its geometrical definition: Origin intercept points (Gram[x=0,y=0] points) [as zeroes] or precise NAV = 0 [as pseudo-zeroes converted to zeroes] in Dirichlet Sigma-Power Law or approximate NAV = 0 [as zeroes] in sim-η(s) when interpreted as Riemann sum will only be uniquely valid when $\sigma = \frac{1}{2}$. Thus, presence of Origin intercept points only occur when $\sigma = \frac{1}{2}$; and absence of Origin intercept points only occur when $\sigma \neq \frac{1}{2}$. Also supported by arguments involving modulus of $\eta(s)$ in Lemma 1.1 from Appendix A and fully complying with Information-Complexity conservation, all nontrivial zeros when $\sigma = \frac{1}{2}$ are deduced to exist at (i) ζ(s) or η(s) or sim-η(s) = 0 as Origin intercept points [viz, zeroes], (ii) Dirichlet Sigma-Power Law = 0 as precise NAV = 0 [viz, pseudo-zeroes converted to zeroes], or (iii) Riemann sum interpretation of sim-η(s) = 0 as approximate NAV = 0 [viz, zeroes].

From subsection 3.1, exact Dimensional analysis (DA) homogeneity at $\sigma = \frac{1}{2}$ denotes $\sum$(all fractional exponents) as $2(1-\sigma)$ uniquely equates to ["exact"] whole number '1'; and inexact DA homogeneity at $\sigma \neq \frac{1}{2}$ denotes $\sum$(all fractional exponents) as $2(1-\sigma)$ uniquely equates to ["inexact"] fractional number '$\neq$1'. As will also be subsequently accomplished in Appendix A and Appendix B, we leave it as a simple exercise here for readers to confirm all above equations for substituted $\sigma = \frac{1}{2}$ and $\sigma \neq \frac{1}{2}$ [e.g. for $\sigma = \frac{2}{5}$ and $\frac{3}{5}$] values will, respectively, fully comply with exact and inexact DA homogeneity. We logically deduce this exercise will definitively equate to substantiating rigorous proof for Riemann hypothesis and providing precise explanations for two types of Gram points. In effect, original Dirichlet eta function [as zeroes], *proxy* for Riemann zeta function [as zeroes], is dependently treated as an unique mathematical object with direct application of Riemann integral to simplified Dirichlet eta function [as zeroes] to derive relevant Dirichlet Sigma-Power Laws [as pseudo-zeroes converted to zeroes]. Key complex properties or behaviors as exact and inexact Dimensional analysis homogeneity [that can uniquely represent mutually exclusive sets of Gram points and virtual Gram points] are elucidated from subsequent analysis of these Laws. We emphasize here that, strictly speaking, Dirichlet Sigma-Power Laws can ultimately only be directly derived from Dirichlet eta function and not Riemann zeta function with the later function not converging, and is thus undefined, in the critical strip ($0 < \sigma < 1$) of interest [whereby the critical line ($\sigma = \frac{1}{2}$) is located].

3.1 Exact and Inexact Dimensional Analysis Homogeneity for Equations

Respectively for ‘base quantities’ such as *length*, *mass* and *time*; their fundamental SI ‘units of measurement’ meter (m) is defined as distance travelled by light in vacuum for time interval 1/299 792 458 s with speed of light c = 299,792,458 ms^{-1}, kilogram (kg) is defined by taking fixed numerical value Planck constant h to be 6.626 070 15 X 10^{-34} Joules·second (Js) [whereby Js is equal to $\text{kgm}^2\text{s}^{-1}$] and second (s) is defined in terms of ΔvCs = $\Delta(^{133}\text{Cs})_{hfs}$ = 9,192,631,770 s^{-1}. Derived SI units such as J and ms^{-1} respectively represent ‘base quantities’ *energy* and *velocity*. The word ‘dimension’ is commonly used to indicate all those mentioned ‘units of measurement’ in well-defined equations.

Dimensional analysis (DA) is an analytic tool with DA homogeneity and non-homogeneity (respectively) denoting valid and invalid equation occurring when ‘units of measurements’ for ‘base quantities’ are “balanced” and “unbalanced” across both sides of the equation. E.g. equation 2 m + 3 m = 5 m is valid and equation 2 m + 3 kg = 5 ‘m·kg’ is invalid (respectively) manifesting DA homogeneity and non-homogeneity.

Remark 4 We can validly apply exact and inexact Dimensional analysis homogeneity to certain well-defined equations.

Let (2n) and (2n-1) be ‘base quantities’ in our derived versions of [continuous-format] Dirichlet Sigma-Power Laws formatted in simplest forms as equations. E.g. DA on exponent $\frac{1}{2}$ in $(2n)^{\frac{1}{2}}$ when depicted in simplest form is desirable for our purpose but DA on exponent $\frac{1}{4}$ in equivalent $(2^2n^2)^{\frac{1}{4}}$ *not* depicted in simplest form is undesirable for our purpose. Fractional exponents as ‘units of measurement’ given by $(1-\sigma)$ for equations when $\sigma = \frac{1}{2}$ coincide with exact DA homogeneity[1]; and $(1-\sigma)$ for equations when $\sigma \neq \frac{1}{2}$ coincide with inexact DA homogeneity[2]. Respectively, exact DA homogeneity at $\sigma = \frac{1}{2}$ denotes $\sum$(all fractional exponents) as $2(1-\sigma)$ equates to [“exact”] whole number ‘1’; and inexact DA homogeneity at $\sigma \neq \frac{1}{2}$ denotes $\sum$(all fractional exponents) as $2(1-\sigma)$ equates to [“inexact”] fractional number ‘$\neq$1’ [Range: $0 < 2(1-\sigma) < 1$ and $1 < 2(1-\sigma) < 2$]. Note: *For calculations involving $2(1-\sigma)$ or $2(-\sigma)$ below, it is inconsequential whether σ values in these fractional exponents are depicted in simplest form or not in simplest form.*

Footnote 1, 2: (i) Exact and (ii) inexact DA homogeneity is applicable to Dirichlet Sigma-Power Laws as equations to calculate precise Net Area Values = 0 for (i) $\sigma = \frac{1}{2}$ (critical line) Gram points (given as pseudo-zeroes t-values which can be converted to zeroes t-values) and for (ii) $\sigma \neq \frac{1}{2}$ (non-critical lines) virtual Gram points (given as virtual pseudo-zeroes t-values which can be converted to virtual zeroes t-values). *Law of Continuity* is a heuristic principle *whatever succeed for the finite, also succeed for the infinite*. These Laws which inherently manifest themselves on finite and infinite time scale should “succeed for the finite, also succeed for the infinite”.

Additional comments and deductions: Performing exact and inexact Dimensional analysis homogeneity on versions of [discrete-format] simplified Dirichlet eta functions is equally valid. Again, (2n) and (2n-1) are ‘base quantities’. Fractional exponents as ‘units of measurement’ are now given by $(-\sigma)$. Respectively, exact DA homogeneity at $\sigma = \frac{1}{2}$ denotes $\sum$(all fractional exponents) as $2(-\sigma)$ equates to [“exact”] (negative) whole number ‘-1’; and inexact DA homogeneity at $\sigma \neq \frac{1}{2}$ denotes $\sum$(all fractional exponents) as $2(-\sigma)$ equates to [“inexact”] (negative) fractional number ‘$\neq$-1’ [Range: -2 < $2(-\sigma)$ < -1 and -1 < $2(-\sigma)$ < 0]. Geometrically, computation with at $\sigma = \frac{1}{2}$ (critical line) using simplified Dirichlet eta function [when interpreted as Riemann sum] will give rise to approximate Net Area Value = 0 condition. This condition enable obtaining results of relevant zeroes t-values and virtual zeroes t-values which, respectively, represent all Gram points and virtual Gram points.

Dirichlet eta function, $\eta(s)$, at s = $\sigma + \imath$t with $\imath$ = $\sqrt{-1}$, σ and t real is valid for $\sigma > 0$. Here, $\eta(s)$ = $Re\{\eta(s)\}$+$\imath Im\{\eta(s)\}$. Then, $\eta(\sigma + \imath t)$ when $0 < t < +\infty$ is the complex conjugate of $\eta(\sigma - \imath t)$ when $-\infty < t < 0$ [which is also valid for $\sigma > 0$]. Given as [identical] $\pm$t values; CIS nontrivial zeros or Gram[x=0,y=0] points (Origin intercepts) occurring when $\eta(s)$ [as zeroes] = $\zeta(s)$ [as zeroes] = simplified Dirichlet eta function = 0 [as zeroes] with calculated CIS of approximate Net Area Value = 0 is equivalent to Dirichlet Sigma-Power Law = 0 [as pseudo-zeroes which can be converted to zeroes] with calculated CIS of precise Net Area Value = 0. This situation which uniquely occur only when $\sigma = \frac{1}{2}$ can essentially represent Riemann hypothesis.

3.2 Summary of Rigorous Proof for Riemann Hypothesis

Outline of proof for Riemann hypothesis. To simultaneously satisfy two mutually inclusive conditions: I. *With rigid manifestation of exact DA homogeneity*, Set **nontrivial zeros** with |**nontrivial zeros**| = $\aleph_0$ is located on critical line (viz. $\sigma = \frac{1}{2}$) when $2(1-\sigma)$ as $\sum$(all fractional exponents) = whole number '1' in Dirichlet Sigma-Power Law[3] [as pseudo-zeroes which are converted to zeroes]. II. *With rigid manifestation of inexact DA homogeneity*, Set **nontrivial zeros** with |**nontrivial zeros**| = $\aleph_0$ is not located on non-critical lines (viz. $\sigma \neq \frac{1}{2}$) when $2(1-\sigma)$ as $\sum$(all fractional exponents) = fractional number '$\neq$1' in Dirichlet Sigma-Power Law[3] [as virtual pseudo-zeroes which are converted to virtual zeroes].

Footnote 3: Ultimately derived from $\eta(s)$ [*proxy* for $\zeta(s)$], this Law [as 'Complex Elementary Fundamental Laws'-based solution results in (virtual) pseudo-zeroes which are converted to (virtual) zeroes] symbolizes end-product proof on Riemann hypothesis.

Riemann hypothesis mathematical foot-prints. Six identifiable steps to prove Riemann hypothesis: *Step 1* Use $\eta(s)$, *proxy* for $\zeta(s)$, in critical strip. *Step 2* Apply Euler formula to $\eta(s)$. *Step 3* Obtain simplified Dirichlet eta function which intrinsically incorporates *actual location [but not actual positions]* of all nontrivial zeros[4]. *Step 4* Apply Riemann integral to simplified Dirichlet eta function in discrete (summation) format results in continuous (integral) format. *Step 5* Obtain its antiderivative Dirichlet Sigma-Power Law [as pseudo-zeroes which are converted to zeroes] which also intrinsically incorporates *actual location [but not actual positions]* of all nontrivial zeros. *Step 6* Confirm exact DA homogeneity or inexact DA homogeneity for $\sum$(all fractional exponents) in this Law to, respectively, validate presence of nontrivial zeros or absence of nontrivial zeros.

Footnote 4: Respectively, Gram[y=0] points, Gram[x=0] points and nontrivial zeros are Incompletely Predictable entities with actual positions determined by setting $\sum Im\{\eta(s)\} = 0$, $\sum Re\{\eta(s)\} = 0$ and $\sum ReIm\{\eta(s)\} = Re\{\eta(s)\} + Im\{\eta(s)\} = 0$ to *dependently* calculate relevant positions of all preceding entities in neighborhood. Respectively, actual location of Gram[y=0] points, Gram[x=0] points and nontrivial zeros; and virtual Gram[y=0] points, virtual Gram[x=0] points and "absent" nontrivial zeros occur precisely at $\sigma = \frac{1}{2}$; and $\sigma \neq \frac{1}{2}$. Euler formula is commonly stated as $e^{\imath x} = \cos x + \imath \cdot \sin x$. *Step 2* is linked to *Step 3* since simplified Dirichlet eta function is obtained by applying Euler formula to $\eta(s)$ whereby $\zeta(s) = \gamma \cdot \eta(s) = \gamma \cdot [Re\{\eta(s)\} + \imath \cdot \mathrm{Im}\{\eta(s)\}]$. Proportionality factor $\gamma = \dfrac{1}{(1-2^{1-s})}$, $Re\{\eta(s)\} = \sum_{n=1}^{\infty}((2n-1)^{-\sigma}\cos(t\ln(2n-1)) - (2n)^{-\sigma}\cos(t\ln(2n)))$ and $Im\{\eta(s)\} = \sum_{n=1}^{\infty}((2n)^{-\sigma}\sin(t\ln(2n)) - (2n-1)^{-\sigma}\sin(t\ln(2n-1)))$. Complex number s in critical strip is designated by s = $\sigma + \imath t$ for $0 < t < +\infty$ and s = $\sigma - \imath t$ for $-\infty < t < 0$. *Step 4* is linked to *Step 5* since applying Riemann integral to simplified Dirichlet eta function will give rise to Dirichlet Sigma-Power Law.

Overall Proof for Riemann Hypothesis given by Theorem Riemann I – IV. Our elementary proof for Riemann hypothesis is now summarized in an overall manner by Theorem Riemann I – IV. For completeness and clarification of this proof, we supply the following underlying important (simple) mathematical arguments.

For $0 < \sigma < 1$, then $0 < 2(1-\sigma) < 2$. The only whole number between 0 and 2 is '1' which coincide with $\sigma = \frac{1}{2}$. When $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$, then [correspondingly] $0 < 2(1-\sigma) < 1$ and $1 < 2(1-\sigma) < 2$.

Legend: $\mathbf{R}$ = all real numbers. For $0 < \sigma < 1$, σ consist of $0 < \mathbf{R} < 1$. For $0 < 2(1-\sigma) < 2$, $2(1-\sigma)$ must (respectively) consist of $0 < \mathbf{R} < 2$. An important caveat is that previously used phrases such as "$\sum$(all fractional exponents) = whole number '1' / fractional number '$\neq$1'", although not incorrect *per se*, should respectively be replaced by "$\sum$(all real exponents) = whole number '1' / real number '$\neq$1'" for complete accurracy. We additionally note that as whole numbers $\subset$ real numbers, we could also validly depict this phrase as "$\sum$(all real exponents) = real number '1' / real number '$\neq$1'". We apply this caveat to Theorem Riemann I – IV.

Theorem Riemann I. Derived from Dirichlet eta function (*proxy* for Riemann zeta function), simplified Dirichlet eta function will exclusively contain *de novo* property for actual location [but not actual positions] of all nontrivial zeros.

Proof. We logically advocate the phrase "actual location [but not actual positions] of all nontrivial zeros" can validly be shortened to "actual location of all nontrivial zeros" which is also used in Theorem Riemann II, III and IV below. Theorem Riemann I essentially equates to Lemma 1.1 except without mentioning Euler formula application to Dirichlet eta function as required in the derivation of simplified Dirichlet eta function [which contains *de novo* property for elucidating "actual location of all nontrivial zeros"]. *The proof for Theorem Riemann I is now complete as it successfully incorporates proof for Lemma 1.1*□.

Theorem Riemann II. Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion) [antiderivative in continuous (integral) format], which is derived from simplified Dirichlet eta function (as zeroes) [in discrete (summation) format], will exclusively manifest exact DA homogeneity only when real number exponent $\sigma = \frac{1}{2}$.

Proof. Proposition 1.2 refers to rigorous derivation of Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion) [which contains *de novo* property for elucidating "actual location of all nontrivial zeros"] from simplified Dirichlet eta function (as zeroes). Proposition 1.3 refers to unique manifestation of exact DA homogeneity in Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion) when real number exponent $\sigma = \frac{1}{2}$. Therefore Theorem Riemann II successfully incorporate the proofs from Propositions 1.2 and 1.3. *The proof for Theorem Riemann II is now complete*□.

Theorem Riemann III. Real number exponent σ [$= \frac{1}{2}$] parameter, being part of real number exponent (1 - σ) in Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion) that satisfy exact DA homogeneity, is identical to real number exponent σ parameter mentioned in Riemann hypothesis which propose σ to also have exclusive value of $\frac{1}{2}$ (representing critical line) for "actual location of all nontrivial zeros", thus confirming this hypothesis to be true with full support and clarification provided by Theorem Riemann IV.

Proof. Since s = $\sigma \pm \imath t$, complete set of nontrivial zeros which is defined by $\eta(s) = 0$ is exclusively associated with one (and only one) particular $\eta(\sigma \pm \imath t) = 0$ value solution, and by default one (and only one) particular σ [conjecturally] $= \frac{1}{2}$ value solution. When performing exact DA homogeneity on Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion) [which contains *de novo* property to elucidate "actual location of all nontrivial zeros"], the expression "If real number exponent σ parameter has exclusively $\frac{1}{2}$ value, only then will exact DA homogeneity be satisfied" implies one (and only one) possible mathematical solution. Theorem Riemann III reflect Theorem Riemann II on presence of exact DA homogeneity for $\sigma = \frac{1}{2}$ in Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion). Consider three defining reasons: (i) σ parameter is intrinsically present in both Riemann zeta function and Dirichlet eta function, (ii) σ [$= \frac{1}{2}$] parameter is used to denote critical line of Riemann zeta function as part of the original Riemann hypothesis [whereby all nontrivial zeros are conjectured to be located on critical line], and (iii) Dirichlet Sigma-Power Law [with converting its pseudo-zeroes to zeroes to obtain nontrivial zeros] containing σ parameter is ultimately derived from Dirichlet eta function, which is *proxy* for Riemann zeta function. Then this Law definitely has identical σ parameter that is referred to by Riemann hypothesis. *The proof for Theorem Riemann III is now complete as we have simultaneous confirmation of (i) solitary $\sigma = \frac{1}{2}$ value in Dirichlet Sigma-Power Law [with converting its pseudo-zeroes to zeroes to obtain nontrivial zeros] satisfying exact DA homogeneity and (ii) critical line defined by solitary $\sigma = \frac{1}{2}$ value being the logically deduced "actual location [but with no request to determine actual positions] of all nontrivial zeros" as was proposed in original Riemann hypothesis*□.

Theorem Riemann IV. Condition 1. All $\sigma \neq \frac{1}{2}$ values representing (infinitely many) non-critical lines, viz. $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$, will exclusively not contain "actual location of all nontrivial zeros" [and manifest *de novo* inexact DA homogeneity in Dirichlet Sigma-Power Law (as virtual pseudo-zeroes to virtual zeroes conversion with non-existent virtual nontrivial zeros)], together with Condition 2. One (and only one) $\sigma = \frac{1}{2}$ value representing (solitary) critical line will exclusively contain "actual location of all nontrivial zeros" [and manifest *de novo* exact DA homogeneity in Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion to obtain all nontrivial zeros)], now confirm Riemann hypothesis to be true when these two mutually inclusive conditions are met.

Proof. Condition 2 Theorem Riemann IV simply reflect proof from Theorem Riemann III [which also incorporates Proposition 1.3 as alluded to by Theorem Riemann II] for "actual location of all nontrivial zeros" to exclusively be on critical line (given by $\sigma = \frac{1}{2}$ value) with manifesting *de novo* exact DA homogeneity $\sum$(all real number exponents) = real number '1' for this Law [with converting its pseudo-zeroes to zeroes to obtain all nontrivial zeros]. *The proof for Condition 2 Theorem Riemann IV is now complete*□. Corollary 1.4 confirms *de novo* inexact DA homogeneity in this Law (as virtual pseudo-zeroes to virtual zeroes conversion with non-existent virtual nontrivial zeros) [manifested as $\sum$(all real number exponents) = real number '$\neq 1$'] for all $\sigma \neq \frac{1}{2}$

values (non-critical lines) which are exclusively not associated with "actual location of all nontrivial zeros". This can also be rigorously confirmed by further applying inclusion-exclusion principle: Exclusive presence of nontrivial zeros on critical line for Condition 2 Theorem Riemann IV implies exclusive absence of nontrivial zeros on non-critical lines for Condition 1 Theorem Riemann IV. *The proof for Condition 1 Theorem Riemann IV is now complete*□.

We logically deduce that explicit mathematical explanations why presence and absence of nontrivial zeros should (respectively) coincide precisely with $\sigma = \frac{1}{2}$ and $\sigma \neq \frac{1}{2}$ [which are literally the meta-properties ('overall' *complex properties*)] will require "complex" or convoluted mathematical arguments. Attempting to provide explicit mathematical explanation with "simple" mathematical arguments would intuitively mean nontrivial zeros have to be (incorrectly and impossibly) treated as Completely Predictable entities. These meta-properties are: Gram points equate to "Presence of three entities (i) nontrivial zeros, (ii) Gram[y=0] points and (iii) Gram[x=0] points that coincide precisely with $\sigma = \frac{1}{2}$"; and virtual Gram points equate to "Presence of two entities (i) virtual Gram[y=0] points and (ii) virtual Gram[x=0] points that coincide precisely with $\sigma \neq \frac{1}{2}$".

4 Conclusions

We envisage two *mutually exclusive* groups of entities: [totally] Unpredictable entities and [totally] Predictable entities. The first group dubbed Type I entities or Completely Unpredictable entities can arise as [totally] random physical processes in nature e.g. radioactive decay is a stochastic (random) process occurring at level of single atoms. According to Quantum theory, it is impossible to predict when a particular atom will decay regardless of how long the atom has existed. For a collection of atoms, expected decay rate is characterized in terms of their measured decay constants or half-lives. The second group is constituted by two subgroups: dubbed Type II entities or Completely Predictable entities e.g. Even-Odd number pairing and dubbed Type III entities or Incompletely Predictable entities e.g. Prime-Composite number pairing. Intuitively, every single mathematical argument from complete set of mathematical arguments required to fully solve a given Incompletely Predictable problem (containing *dependent* types of Incompletely Predictable entities) must be correct obeying Mathematics for Completely Predictable problems. Then Mathematics for Incompletely Predictable problems is literally the mathematical framework for describing complex properties present in these entities.

Harnassed properties: (1) Nontrivial zeros and two types of Gram points are [*dependently*] derived from "Axes intercept relationship interface" using Riemann zeta function, or its *proxy* Dirichlet eta function; and (2) Prime and composite numbers are [*dependently*] derived from "Numerical relationship interface" using Sieve of Eratosthenes. Using prime gaps as analogy, there are (for instance) "nontrivial zeros gaps" between two consecutive nontrivial zeros with these gaps being Incompletely Predictable entities. Prime number theorem describes asymptotic distribution of prime numbers among positive integers by formalizing intuitive idea that prime numbers become less common as they become larger through precisely quantifying rate at which this occurs using probability. An important secondary spin-off arising out of solving Riemann hypothesis result in absolute and full delineation of prime number theorem. This theorem relates to prime counting function which is usually denoted by $\pi(x)$ with $\pi(x)$ = number of prime numbers $\leq$ x. In other words, solving Riemann hypothesis is instrumental in proving efficacy of techniques that estimate $\pi(x)$ efficiently. This confirm "best possible" bound for error ("smallest possible" error) of prime number theorem.

In mathematics, logarithmic integral function or integral logarithm li(x) is a special function. Relevant to problems of physics with number theoretic significance, it occurs in prime number theorem as an estimate of $\pi(x)$ whereby its form is defined so that li(2) = 0; viz. li(x) $\equiv \int_2^x \frac{du}{\ln u}$ = li(x) - li(2). There are less accurate ways of estimating $\pi(x)$ such as conjectured by Gauss and Legendre at end of 18th century. This is approximately x/ln$x$ in the sense $\lim_{x\to\infty} \frac{\pi(x)}{x/\ln x} = 1$. Skewes' number is any of several extremely large numbers used by South African mathematician Stanley Skewes as upper bounds for smallest natural number x for which li(x)<$\pi(x)$. These bounds have since been improved by others: there is a crossing near $e^{727.95133}$ but it is not known whether this is the smallest. John Edensor Littlewood who was Skewes' research supervisor proved in 1914[4] that there is such a [first] number; and found that sign of difference $\pi(x)$ - li(x) changes infinitely often. This refute all prior numerical evidence that seem to suggest li(x) was always $>$ $\pi(x)$. The key point is [100%

accurate] perfect $\pi(x)$ mathematical tool being "wrapped around" by [less-than-100% accurate] approximate li(x) mathematical tool infinitely often via this 'sign of difference' changes meant that li(x) is the most efficient approximate mathematical tool. Contrast this with "crude" x/ln$x$ approximate mathematical tool where studied values diverge away from $\pi(x)$ at increasingly greater rate for larger range of prime numbers.

Critical line of Riemann zeta function is denoted by $\sigma = \frac{1}{2}$ whereby all nontrivial zeros are proposed to be located in the 1859 Riemann hypothesis. Treated as Incompletely Predictable problems, we gave a relatively elementary proof on Riemann hypothesis while also explaining the existence of three types of Gram points and two types of virtual Gram points by analyzing the complex (meta-) properties of relevant Dirichlet Sigma-Power Laws viz. (1) exact DA homogeneity [occurring only once when $\sigma = \frac{1}{2}$] in these Laws with ability to convert their obtained pseudo-zeroes to zeroes in order to obtain nontrivial zeros (Origin intercept points or Gram[x=0,y=0] points) as one type of Gram points and two other closely related types of Gram points [since f(n)'s IP zeroes (t values) = F(n)'s IP pseudo-zeroes (t values) – $\frac{\pi}{2}$]; and (2) inexact DA homogeneity [occurring infinitely many times when $\sigma \neq \frac{1}{2}$] in these Laws with ability to convert their obtained virtual pseudo-zeroes to virtual zeroes in order to obtain two types of virtual Gram points [since f(n)'s IP virtual zeroes (t values) = F(n)'s IP virtual pseudo-zeroes (t values) – $\frac{\pi}{2}$].

4.1 **Supplementary materials:**

Completely Predictable even numbers with regular intervals of 'non-varying' even gaps of 2 as worked example for E-O Pairing: Approximate area (given as Riemann sum) from $\lim_{n\to\infty}\sum_{i=0}^{n} \Delta x \cdot (2i)$ = $\lim_{n\to\infty}\sum_{i=0}^{n}(2i)$ [as Δx = 1] = Precise area from $\int_0^n (2i)di = [\text{i}^2{+}C]_0^n$ = (n^2 - 0^2). The zero at i = 0 [which mathematically equates to i = 0 to 0] can always be identically given by (i) the Riemann sum $\sum_{i=0}^{0}(2i) = (2X0)+(2X0) = 0$ [zero area] and (ii) the solved integral (antiderivative) $\int_0^0 (2i)di = (0^2 \text{ - } 0^2) = 0$ [zero area]. We further note for i = 0 to (say) 3 viz, first four [whole] even numbers E_0 = 0, E_1 = 2, E_2 = 4 and E_3 = 6 [with sum total 0 + 2 + 4 + 6 = 12]; then the perpetual phenomenon (Approximate area) $\sum_{i=0}^{3}(2i) = \sum_{i=0}^{1}(2i)+\sum_{i=2}^{3}(2i) = (2X0)+(2X1)+(2X2)+(2X3) = 12$ that overestimate and is non-varyingly related to (Precise area) $\int_0^3 (2i)di$ = $\int_0^1 (2i)di+\int_1^2 (2i)di+\int_2^3 (2i)di = [\text{i}^2{+}C]_0^3$ = $(3^2 \text{ - } 0^2) = 9$ will be validly observed.

Incompletely Predictable prime numbers with irregular intervals of 'varying' prime gaps as worked example for P-C Pairing: Approximate area (given as Riemann sum) from $\lim_{n\to\infty}\sum_{i=1}^{n} \Delta x \cdot (P_i$ + pGap$_i$)=$\lim_{n\to\infty}\sum_{i=1}^{n}(P_i$ + pGap$_i$)[as Δx = 1] = Precise area from $\int_1^n (P_i$ + pGap$_i)di$. We further note for i = 1 to (say) 4 viz, first four prime numbers 2, 3, 5 and 7 [with sum total 2 + 3 + 5 + 7 = 17]; then the perpetual phenomenon (Approximate area) $\sum_{i=1}^{4}(P_i$ + pGap$_i$) = $\sum_{i=1}^{2}(P_i$ + pGap$_i$) + $\sum_{i=3}^{4}(P_i$ + pGap$_i$) = (2+1) + (3+2) + (5+2) + (7+4) = 26 that [arbitrarily] underestimate and is varyingly related to (Precise area) $\int_1^4 (P_i$ + pGap$_i$)di = $\int_1^2 (P_i$ + pGap$_i$)di + $\int_2^3 (P_i$ + pGap$_i$)di + $\int_3^4 (P_i$ + pGap$_i$)di = $\int_1^2 (3+2)+(2+1)$di + $\int_2^3 (5+2)+(3+2)$di + $\int_3^4 (7+4)+(5+2)$di = $[8i+C]_1^2+[12i+C]_2^3$ + $[18\text{i} + \text{C}]_3^4$ = 8 + 12 + 18 = 38 can only be metaphorically, but without true mathematical validity, observed. This is because our algorithm, using variable i and given as P_{i+1} = P_i + pGap$_i$, is only defined at end-points a, b. It is not a function and is thus not defined in interval [a,b].

Eq. (A.3) is the only version of simplified Dirichlet eta function (as zeroes) that can uniquely and non-negotiably incorporate trigonometry identity $\cos(n) - \sin(n) = \sqrt{2}\sin\left(n + \frac{3}{4}\pi\right)$ application and mathematically allow / explain Origin intercept points (nontrivial zeros or Gram[x=0,y=0] points) to exist when $\sigma = \frac{1}{2}$ [with sum exponents = $2(-\sigma)$ = -1 being a (negative) whole number]. This trigonometry identity only involve sine and cosine terms with exponent being whole number 1 and not fractional number. Thus, we can only comply with exact Dimensional analysis homogeneity for this Incompletely Predictable problem with sum exponents = $2(1-\sigma)$ = 1 being also a whole number for derived Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion). Eq. (A.3) is reproduced here:

$$\sum_{n=1}^{\infty}(2n)^{-\sigma}\sqrt{2}\sin(t\ln(2n)+\frac{3}{4}\pi) - \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\sqrt{2}\sin(t\ln(2n-1)+\frac{3}{4}\pi) = 0.$$

It can also be equivalently written as $\sum_{n=1}^{\infty} -(2n)^{-\sigma}(\sin(t\ln(2n)) - \cos(t\ln(2n)))$ – $\sum_{n=1}^{\infty} -(2n-1)^{-\sigma}(\sin(t\ln(2n-1)) - \cos(t\ln(2n-1))) = 0$ that contains both sine and cosine terms. This will stop unsuspecting readers from incorrectly treating this function [without $\sqrt{2}$ and $\frac{3}{4}\pi$ constants] as $\sum_{n=1}^{\infty}(2n)^{-\sigma}\sin(t\ln(2n) - \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\sin(t\ln(2n-1) = 0$. Serendipitously, this last equation is precisely the simplified Dirichlet eta function for Gram[y=0] points.

Just as dE/di = d(2i)/di = (constant) 2 mentioned in E-O Pairing for even numbers has its perpetually valid [intrinsic] simple property to precisely indicate even number gaps = 2; so are presence of exact and inexact Dimensional analysis (DA) homogeneity in simplified Dirichlet eta function [as zeroes] or Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion] has its perpetually valid [intrinsic] complex (meta-) property to precisely differentiate between Gram points and virtual Gram points.

The word 'Dimension' or 'Dimensional' in DA is traditionally, conveniently and arbitrarily used to indicate [but not used to define] analysis on 'units of measurement' [such as kg, m and, advocated by us, exponents involving σ and '1'] for corresponding 'base quantities' [such as mass, length and, advocated by us, (2n), (2n-1) and Dimension (2x - N) = Dimension $(2x - N)^1$]. Here, the two possible scenarios are [mathematically valid] DA homogeneity and [mathematically invalid] DA non-homogeneity. But the word Dimension is an English word with nil acceptance by anyone that it can even remotely indicate or resemble 'units of measurement' such as kg or m instead of exponents. We advocate use of [mathematically valid] exact and inexact DA homogeneity which are mathematically fully defined in the very specific context of Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion] and simplified Dirichlet eta function [as zeroes] is also conveniently and arbitrarily correct; viz. mathematically, this action is justifiably correct.

Declaration of interests This work was supported by the author's January 20, 2020 private research grant of AUS $5,000 from Mrs. Connie Hayes and Mr. Colin Webb. He received AUS $3,250 and AUS $510 reimbursements from Q-Pharm for participating in EyeGene Shingles trial and Biosimilar Study on G-CSF commencing on March 10, 2020 and February 10, 2021.

Acknowledgment Many thanks to Managing Editor and Referee who found errors in an earlier submission. Dedicated to author's daughter Jelena born 13 weeks early on May 14, 2012 and all front-line health workers globally fighting against the deadly 2020 Coronavirus Pandemic.

References

1. Ting, J.Y.C. (2020). Mathematical Modelling of COVID-19 and Solving Riemann Hypothesis, Polignac's and Twin Prime Conjectures Using Novel Fic-Fac Ratio With Manifestations of Chaos-Fractal Phenomena. *J. Math. Res., 12*(6) 1-49. https://doi.org/10.5539/jmr.v12n6p1

2. Hardy, G. H. (1914). Sur les Zeros de la Fonction $\zeta(s)$ de Riemann. *C. R. Acad. Sci. Paris, 158,* 1012-1014. JFM 45.0716.04 Reprinted in (Borwein et al., 2008)
3. Hardy, G. H., & Littlewood, J. E. (1921). The zeros of Riemann's zeta-function on the critical line. *Math. Z., 10* (3-4), 283-317. http://dx.doi:10.1007/BF01211614
4. Littlewood, J.E. (1914). Sur la distribution des nombres premiers. *Comptes Rendus de l'Acad. Sci. Paris, 158*, 1869-1872.

A Prerequisite Lemma, Corollary and Propositions on Riemann Hypothesis

Lemma 1.1. Simplified Dirichlet eta function is derived directly from Dirichlet eta function with Euler formula application and it will intrinsically incorporate actual location [but not actual positions] of all nontrivial zeros.

Proof. Denote complex number ($\mathbb{C}$) as z = x + $\imath$·y. Then z = Re(z) + $\imath$·Im(z) with Re(z) = x and Im(z) = y; modulus of z, |z| = $\sqrt{Re(z)^2+Im(z)^2}$ = $\sqrt{x^2+y^2}$; and $|z|^2 = x^2+y^2$.

Euler formula is commonly stated as $e^{\imath x} = \cos x + \imath\cdot\sin x$. Euler identity (where $x=\pi$) is $e^{\imath\pi} = \cos\pi + \imath\cdot\sin\pi = -1+0$ [or stated as $e^{\imath\pi}+1=0$]. The n^s of $\zeta(s)$ is expanded to $n^s = n^{(\sigma+\imath t)} = n^{\sigma}\mathrm{e}^{t\ln(n)\cdot\imath}$ since $n^t = e^{t\ln(n)}$. Apply Euler formula to n^s result in $n^s = n^{\sigma}(\cos(t\ln(n)) + \imath\cdot\sin(t\ln(n)))$. This is written in trigonometric form [designated by short-hand notation $n^s(Euler)$] whereby n^{σ} is modulus and $t\ln(n)$ is polar angle (argument).

Apply $n^s(Euler)$ to Eq. (2.1). Then $\zeta(s)$ =Re$\{\zeta(s)\}+\imath\cdot$Im$\{\zeta(s)\}$ with $Re\{\zeta(s)\} = \sum_{n=1}^{\infty} n^{-\sigma}\cos(t\ln(n))$ and $Im\{\zeta(s)\} = -\sum_{n=1}^{\infty} n^{-\sigma}\sin(t\ln(n))$. As $\zeta(s)$ in Eq. (2.1) is absolutely convergent only when σ>1 where zeros never occur, we will not carry out further treatment here on this equation.

Apply $n^s(Euler)$ to Eq. (2.3). Then $\eta(s) = \gamma^{-1}\cdot\zeta(s) = Re\{\eta(s)\}+\imath\cdot$Im$\{\eta(s)\}$ whereby $Re\{\eta(s)\}$ = $\sum_{n=1}^{\infty}((2n-1)^{-\sigma}\cos(t\ln(2n-1))-(2n)^{-\sigma}\cos(t\ln(2n)))$ and $Im\{\eta(s)\} = -\sum_{n=1}^{\infty}((2n-1)^{-\sigma}\sin(t\ln(2n-1))-(2n)^{-\sigma}\sin(t\ln(2n)))$ $=\sum_{n=1}^{\infty}((2n)^{-\sigma}\sin(t\ln(2n))-(2n-1)^{-\sigma}\sin(t\ln(2n-1)))$ with proportionality factor $\gamma = \frac{1}{(1-2^{1-s})}$.

Complex number s in critical strip is designated by s = $\sigma+\imath t$ for $0<t<+\infty$ and s = $\sigma-\imath t$ for $-\infty<t<0$. Nontrivial zeros equating to $\zeta(s)$ = 0 give rise to our desired $\eta(s)$ = 0. Modulus of $\eta(s)$, $|\eta(s)|$, is defined as $\sqrt{(Re\{\eta(s)\})^2+(Im\{\eta(s)\})^2}$ with $|\eta(s)|^2 = (Re\{\eta(s)\})^2+(Im\{\eta(s)\})^2$. Mathematically $|\eta(s)| = |\eta(s)|^2 = 0$ is an unique condition giving rise to $\eta(s)$ = 0 occurring only when Re$\{\eta(s)\}$ = Im$\{\eta(s)\}$ = 0 as any non-zero values for Re$\{\eta(s)\}$ and/or Im$\{\eta(s)\}$ will always result in $|\eta(s)|$ and $|\eta(s)|^2$ having non-zero values. Important implication is that sum of $Re\{\eta(s)\}$ and $Im\{\eta(s)\}$ equating to zero [given by Eq. (A.1)] must always hold when $|\eta(s)| = |\eta(s)|^2$=0 and consequently $\eta(s)$=0.

$$\sum ReIm\{\eta(s)\} = Re\{\eta(s)\}+Im\{\eta(s)\} = 0 \tag{A.1}$$

Advocating for existence of theoretical s values leading to non-zero values in Re$\{\eta(s)\}$ and Im$\{\eta(s)\}$ depicted as possibility +Re$\{\eta(s)\}$ = -Im$\{\eta(s)\}$ or -Re$\{\eta(s)\}$ = +Im$\{\eta(s)\}$ could, in principle, satisfy Eq. (A.1). In reality, the reverse implication is not necessarily true as these s values will not result in $|\eta(s)| = |\eta(s)|^2$ = 0. In any event, we need not consider these two possibilities since solving Riemann hypothesis involves nontrivial zeros [which are rigidly defined by $\eta(s)$ = 0] with non-zero values in Re$\{\eta(s)\}$ and/or Im$\{\eta(s)\}$ not compatible with $\eta(s)$ = 0. Note that $\eta(s)$ = 0 (uniquely) occurring once when $\sigma=\frac{1}{2}$ as Gram points [viz, zeroes] and (non-uniquely) occurring infinitely often when $\sigma\neq\frac{1}{2}$ as virtual Gram points [viz, virtual zeroes] will always happen at appropriate times.

While fully complying with Information-Complexity conservation, preservation of quantitative NAV = 0 [(uniquely) occurring once when $\sigma=\frac{1}{2}$ as zeroes and (non-uniquely) occurring infinitely often when $\sigma\neq\frac{1}{2}$ as virtual zeroes] will always happen at appropriate times for Dirichlet Sigma-Power Law [as (virtual) pseudo-zeroes to (virtual) zeroes conversion] and simplified Dirichlet eta function [as (virtual) zeroes] when interpreted as Riemann sum. Again with direct connection to Riemann hypothesis through the common presence of parameter σ, critical line (denoted by $\sigma=\frac{1}{2}$) is inevitably and logically conjectured to also be uniquely associated with presence of exact DA homogeneity (occurring only when $\sigma=\frac{1}{2}$) in this Law [as pseudo-zeroes to zeroes conversion obtained via precise NAV = 0] and Riemann sum [as zeroes obtained via approximate NAV = 0].

Eq. (A.1) is intrinsically incorporated into Dirichlet Sigma-Power Law [ultimately derived from Dirichlet eta function (*proxy* for Riemann zeta function)] since Eq. (A.1) can literally be taken to constitute an intermediate or common step for deriving this Law with this situation also simultaneously satisfying three conditions: I. The $\eta(s)$ = 0 [as zeroes] definition for nontrivial zeros [conjectured to be located at $\sigma=\frac{1}{2}$ critical line] equates to Eq. (A.1), II. Precise NAV = 0 situation in Dirichlet Sigma-Power Law = 0 [as pseudo-zeroes to zeroes conversion] only occurs when $\sigma=\frac{1}{2}$, and III. [logically, as was originally proposed on the Riemann zeta function in Riemann hypothesis] "All nontrivial zeros must (consequently) be located on critical line of Riemann zeta function which is uniquely denoted only by $\sigma=\frac{1}{2}$".

Apply trigonometry identity $\cos(n)-\sin(n) = \sqrt{2}\sin\left(n+\frac{3}{4}\pi\right)$ to $Re\{\eta(s)\}+Im\{\eta(s)\}$ to get Eq. (A.2) with terms in last line built by mixture of terms from $Re\{\eta(s)\}$ and $Im\{\eta(s)\}$.

$$\sum ReIm\{\eta(s)\} = \sum_{n=1}^{\infty}[(2n-1)^{-\sigma}\cos(t\ln(2n-1)) - (2n-1)^{-\sigma}\sin(t\ln(2n-1)) - (2n)^{-\sigma}\cos(t\ln(2n)) + (2n)^{-\sigma}\sin(t\ln(2n))]$$

$$= \sum_{n=1}^{\infty}[(2n-1)^{-\sigma}\sqrt{2}\sin(t\ln(2n-1)+\frac{3}{4}\pi) - (2n)^{-\sigma}\sqrt{2}\sin(t\ln(2n)+\frac{3}{4}\pi)] \quad \text{(A.2)}$$

When depicted in terms of Eq. (A.1), Eq. (A.2) becomes

$$\sum_{n=1}^{\infty}(2n)^{-\sigma}\sqrt{2}\sin(t\ln(2n)+\frac{3}{4}\pi) = \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\sqrt{2}\sin(t\ln(2n-1)+\frac{3}{4}\pi)$$

$$\sum_{n=1}^{\infty}(2n)^{-\sigma}\sqrt{2}\sin(t\ln(2n)+\frac{3}{4}\pi) - \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\sqrt{2}\sin(t\ln(2n-1)+\frac{3}{4}\pi) = 0 \quad \text{(A.3)}$$

Eq. (A.3) can also be expanded as $\sum_{n=1}^{\infty} -(2n)^{-\sigma}(\sin(t\ln(2n)) - \cos(t\ln(2n))) - \sum_{n=1}^{\infty} -(2n-1)^{-\sigma}(\sin(t\ln(2n-1)) - \cos(t\ln(2n-1))) = 0$ which contains both sine and cosine terms. $\eta(s)$ calculations for all σ values result in infinitely many of these type of equations for $0<\sigma<1$ critical strip region of interest with n = 1, 2, 3, 4, 5,…, ∞ as discrete integer number values. All these equations will geometrically represent entire plane of critical strip, thus (at least) allowing our proposed proof to be of a complete nature.

Eq. (A.3), which is our simplified Dirichlet eta function [with trigonometry identity $\cos(x) - \sin(x)$ being incorporated], is derived directly from Dirichlet eta function and it will intrinsically incorporate *actual location [but not actual positions]* of all nontrivial zeros. *The proof is now complete for Lemma 1.1*□.

Proposition 1.2. Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion] representing continuous (integral) format and given as antiderivative can be derived directly from simplified Dirichlet eta function [as zeroes] in discrete (summation) format with Riemann integral application. Note: This Law [as pseudo-zeroes to zeroes conversion] representing continuous (integral) format refers to end-product obtained from "key step of converting Dirichlet eta function [as zeroes], *proxy* for Riemann zeta function [as zeroes], into its continuous format version".

Proof. In Calculus, integration is reverse process of differentiation viewed geometrically as the Area enclosed by curve of function and x-axis in a given interval. Apply definite integral I between limits (or points) a and b is to compute its value when $\Delta x \longrightarrow 0$, i.e. $I = \lim_{\Delta x\longrightarrow 0}\sum_{i=1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$. This is Riemann integral of function f(x) in interval [a, b] where a<b. Apply Riemann integral to simplified Dirichlet eta function [as zeroes] in ["$\Delta x \longrightarrow 1$"] discrete (summation) format which intrinsically incorporates *actual location [but not actual positions]* of all nontrivial zeros criterion in order to obtain Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion] in ["$\Delta x \longrightarrow 0$"] continuous (integral) format with the later validly representing the former. Then Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion] will also fullfil this criterion. Due to resemblance to power law functions in σ from s = $\sigma + \imath t$ being exponent of a power function n^{σ}, logarithm scale use, and harmonic ζ(s) series connection in Zipf's law; we elect to call this Law by its given name. A characteristic of this "discrete function" or "continuous Law" is the exact formula expression in usual mathematical language $y = f(x_1)$ format description for a single-variable function. The variable is n obtained from $(2n)$ and $(2n-1)$ as 'base quantities' with parameters σ and t. Thus, $y = f(n)$ with discrete n = 1, 2, 3, 4, 5,…, ∞ or continuous n = 1 to ∞ whereby -∞ < t < +∞ and 0 < σ < 1 are the two parameters.

A proper integral is a definite integral which has neither limit a or b infinite and from which the integrand does not approach infinity at any point in the range of integration. An improper integral is a definite integral that has either or both limits a and b infinite or an integrand that approaches infinity at one or more points in the range of integration.

Involving sine and/or cosine functions, [multiple] +ve (above x-axis) and -ve (below x-axis) numerical Net Area Value = 0 solutions can be successfully computed for simplified Dirichlet eta function [as zeroes] when interpreted as Riemann sum and Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion]. Here, Dirichlet Sigma-Power Law (antiderivative) is the solution to improper integral (with lower limit a = 1 and upper limit b = ∞) obtained from [validly] applying Riemann integral to simplified Dirichlet eta function. All relevant antiderivatives in this paper are derived from improper integrals with format $\int_1^{\infty} f(n)dn$ based on Eqs. (A.3), (B.2) and (B.4). Example for Eq. (A.3), involved improper integrals are from $\int_1^{\infty}(2n)^{-\sigma}\sqrt{2}\sin(t\ln(2n)+\frac{3}{4}\pi)dn$ - $\int_1^{\infty}(2n-1)^{-\sigma}\sqrt{2}\sin(t\ln(2n-1)+\frac{3}{4}\pi)dn$ = 0. These improper integrals are seen to involve [periodic] sine and/or cosine function between limits 1 and ∞. Each improper integral can be validly expanded as $\int_{n=1}^{n=2} f(n)dn$ + $\int_{n=2}^{n=3} f(n)dn$ + $\int_{n=3}^{n=4} f(n)dn$ +...+ $\int_{n=\infty-1}^{n=\infty} f(n)dn$ which, for all sufficiently large n as n⟶ ∞, will manifest *divergence by oscillation* (viz. for all sufficiently large n as n⟶ ∞, this cummulative total will not diverge in a particular direction to a solitary well-defined limit value such as sin π/2 = 1 or less well-defined limit value such as +∞).

With steps of manual integration shown using indefinite integrals [for simplicity], we solve definite integral based on T1 (first term) with $(2n)$ parameter in Eq. (A.3):

$$\int_1^{\infty}\frac{2^{\frac{1}{2}-\sigma}\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{n^{\sigma}}dn = \int_1^{\infty} -\frac{\sin(t\ln(2n)) - \cos(t\ln(2n))}{2^{\sigma}n^{\sigma}}dn.$$

We deduce the remaining two important integrals located in Proposition 2.2 to be "variations" of this particular integral here for nontrivial zero (with Right Hand Side term above involving both sine and cosine functions); viz, the integral with its term involving only sine function (for Gram[y=0] points) and the integral with its term involving only cosine function (for Gram[x=0] points). We check all derived antiderivatives to be correct using computer algebra system Maxima in this paper.

Simplifying and applying linearity, we obtain $2^{\frac{1}{2}-\sigma}\int \frac{\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{n^{\sigma}}\mathrm{d}n$.

Now solving $\int \frac{\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{n^{\sigma}}\mathrm{d}n$. Substitute $u = t\ln(2n)+\frac{3\pi}{4} \longrightarrow \mathrm{d}n = \frac{n}{t}\mathrm{d}u$,

use $n^{1-\sigma} = \mathrm{e}^{\frac{(1-\sigma)\left(u-t\ln(2)-\frac{3\pi}{4}\right)}{t}} = \frac{\mathrm{e}^{\frac{(\sigma-1)(4t\ln(2)+3\pi)}{4t}}}{t}\int \mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)\,\mathrm{d}u$.

Now solving $\int \mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)\,\mathrm{d}u$. We integrate by parts twice in a row: $\int \mathtt{f}\mathtt{g}' = \mathtt{f}\mathtt{g} - \int \mathtt{f}'\mathtt{g}$.

First time: $\mathtt{f} = \sin(u)$, $\mathtt{g}' = \mathrm{e}^{\frac{(1-\sigma)u}{t}}$

Then $\mathtt{f}' = \boxed{\cos(u)}$, $\mathtt{g} = \boxed{\frac{(1-\sigma)t\mathrm{e}^{\frac{(1-\sigma)u}{t}}}{\sigma^2-2\sigma+1}}$:

$$=\frac{(1-\sigma)t\mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)}{\sigma^2-2\sigma+1} - \int \frac{(1-\sigma)t\mathrm{e}^{\frac{(1-\sigma)u}{t}}\cos(u)}{\sigma^2-2\sigma+1}\,\mathrm{d}u$$

Second time: $\mathtt{f} = \boxed{\cos(u)}$, $\mathtt{g}' = \boxed{\frac{(1-\sigma)t\mathrm{e}^{\frac{(1-\sigma)u}{t}}}{\sigma^2-2\sigma+1}}$

Then $\mathtt{f}' = -\sin(u)$, $\mathtt{g} = \frac{t^2\mathrm{e}^{\frac{(1-\sigma)u}{t}}}{\sigma^2-2\sigma+1}$:

$$= \frac{(1-\sigma)t\mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)}{\sigma^2-2\sigma+1} - \left(\frac{t^2\mathrm{e}^{\frac{(1-\sigma)u}{t}}\cos(u)}{\sigma^2-2\sigma+1} - \int -\frac{t^2\mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)}{\sigma^2-2\sigma+1}\,\mathrm{d}u\right)$$

Apply linearity:

$$= \frac{(1-\sigma)t\mathrm{e}^{\frac{1-\sigma u}{t}}\sin(u)}{\sigma^2-2\sigma+1} - \left(\frac{t^2\mathrm{e}^{\frac{(1-\sigma)u}{t}}\cos(u)}{\sigma^2-2\sigma+1} + \frac{t^2}{\sigma^2-2\sigma+1}\int \mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)\,\mathrm{d}u\right)$$

As integral $\int \mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)\,\mathrm{d}u$ appears again on Right Hand Side, we solve for it:

$$= \frac{\frac{(1-\sigma)\mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)}{t} - \mathrm{e}^{\frac{(1-\sigma)u}{t}}\cos(u)}{\frac{\sigma^2-2\sigma+1}{t^2}+1}$$

Plug in solved integrals: $\frac{\mathrm{e}^{\frac{(\sigma-1)(4t\ln(2)+3\pi)}{4t}}}{t}\int \mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)\,\mathrm{d}u$

$$= \frac{\mathrm{e}^{\frac{(\sigma-1)(4t\ln(2)+3\pi)}{4t}}\left(\frac{(1-\sigma)\mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)}{t} - \mathrm{e}^{\frac{(1-\sigma)u}{t}}\cos(u)\right)}{\left(\frac{\sigma^2-2\sigma+1}{t^2}+1\right)t}$$

Undo substitution $u = t\ln(2n)+\frac{3\pi}{4}$ and simplifying:

$$= \frac{\mathrm{e}^{\frac{(\sigma-1)(4t\ln(2)+3\pi)}{4t}}\left(\frac{(1-\sigma)\mathrm{e}^{\frac{(1-\sigma)\left(t\ln(2n)+\frac{3\pi}{4}\right)}{t}}\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{t} - \mathrm{e}^{\frac{(1-\sigma)\left(t\ln(2n)+\frac{3\pi}{4}\right)}{t}}\cos\left(t\ln(2n)+\frac{3\pi}{4}\right)\right)}{\left(\frac{\sigma^2-2\sigma+1}{t^2}+1\right)t}$$

Plug in solved integrals: $2^{\frac{1}{2}-\sigma}\int \frac{\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{n^{\sigma}}\,\mathrm{d}n$

$$= \frac{2^{\frac{1}{2}-\sigma}\mathrm{e}^{\frac{(\sigma-1)(4t\ln(2)+3\pi)}{4t}}\left(\frac{(1-\sigma)\mathrm{e}^{\frac{(1-\sigma)\left(t\ln(2n)+\frac{3\pi}{4}\right)}{t}}\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{t} - \mathrm{e}^{\frac{(1-\sigma)\left(t\ln(2n)+\frac{3\pi}{4}\right)}{t}}\cos\left(t\ln(2n)+\frac{3\pi}{4}\right)\right)}{\left(\frac{\sigma^2-2\sigma+1}{t^2}+1\right)t}$$

By rewriting and simplifying, $\int_1^{\infty} \frac{2^{\frac{1}{2}-\sigma}\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{n^{\sigma}}dn$ is finally solved as

$$\left[\frac{(2n)^{1-\sigma}\left((t+\sigma-1)\sin(t\ln(2n))+(t-\sigma+1)\cos(t\ln(2n))\right)}{2\left(t^2+(\sigma-1)^2\right)}+C\right]_1^{\infty} \tag{A.4}$$

For T2 (second term) with $(2n-1)$ parameter in Eq. (A.3), Eq. (A.4) equates to

$$\left[\frac{(2n-1)^{1-\sigma}\left((t+\sigma-1)\sin\left(t\ln\left(2n-1\right)\right)+\left(t-\sigma+1\right)\cos\left(t\ln\left(2n-1\right)\right)\right)}{2\left(t^2+(\sigma-1)^2\right)}+C\right]_1^\infty \tag{A.5}$$

Without incorporating constant of integration 'C', Dirichlet Sigma-Power Law as equation derived from Eq. (A.3) is given by:

$$\begin{aligned}\frac{1}{2\left(t^2+(\sigma-1)^2\right)}\cdot\big[&(2n)^{1-\sigma}((t+\sigma-1)\sin(t\ln(2n))+(t-\sigma+1)\\&\cdot\cos(t\ln(2n)))-(2n-1)^{1-\sigma}((t+\sigma-1)\\&\cdot\sin(t\ln(2n-1))+(t-\sigma+1)\cos(t\ln(2n-1)))\big]_1^\infty=0\end{aligned} \tag{A.6}$$

Intended derivation of Dirichlet Sigma-Power Law [with intrinsic ability for pseudo-zeroes to zeroes conversion] as equation has been successful. *The proof is now complete for Proposition 1.2*□.

Proposition 1.3. Exact Dimensional analysis homogeneity at $\sigma=\frac{1}{2}$ in Dirichlet Sigma-Power Law [pseudo-zeroes to zeroes conversion] as equation is indicated by ∑(all fractional exponents) = whole number '1'.

Proof. Without incorporating constant of integration 'C', Dirichlet Sigma-Power Law as equation for $\sigma=\frac{1}{2}$ value is given by:

$$\begin{aligned}\frac{1}{2t^2+\frac{1}{2}}\cdot\Big[&(2n)^{\frac{1}{2}}\left(\left(t-\tfrac{1}{2}\right)\sin(t\ln(2n))+\left(t+\tfrac{1}{2}\right)\cos(t\ln(2n))\right)-\\&(2n-1)^{\frac{1}{2}}\left(\left(t-\frac{1}{2}\right)\sin(t\ln(2n-1))+\left(t+\frac{1}{2}\right)\cos(t\ln(2n-1))\right)\Big]_1^\infty=0\end{aligned} \tag{A.7}$$

Evaluation of definite integrals Eq. (A.7), Eq. (B.7) and Eq. (B.8) using limit as n → +∞ for $0<t<+\infty$ enable countless computations resulting in t values for (respectively) CIS nontrivial zeros, CIS Gram[y=0] and CIS Gram[x=0] points [as pseudo-zeroes to zeroes conversion]. Larger n values used for computations will correspond to increasing accuracy of these entities (which are all transcendental numbers). Complying with Information-Complexity conservation, preservation or conservation of quantitative Net Area Value = 0 when $\sigma=\frac{1}{2}$ will always happen at appropriate times for Eq. (A.7), Eq. (B.7) and Eq. (B.8). Otherwise, preservation or conservation of quantitative Net Area Value = 0 when $\sigma\neq\frac{1}{2}$ will always happen at appropriate times for Eq. (A.8), Eq. (B.9) and Eq. (B.10), respectively, enabling countless computations resulting in t values for CIS virtual Gram[y=0] and CIS virtual Gram[x=0] points [as virtual pseudo-zeroes to virtual zeroes conversion] with absent (virtual) nontrivial zeros.

∑(all fractional exponents) as $2(1-\sigma)$ = whole number '1' for Eq. (A.7). This finding signify presence of complete set nontrivial zeros [as pseudo-zeroes to zeroes conversion] for Eq. (A.7). *The proof is now complete for Proposition 1.3*□.

Corollary 1.4. Inexact Dimensional analysis homogeneity at $\sigma\neq\frac{1}{2}$ [illustrated using $\sigma=\frac{2}{5}$] in Dirichlet Sigma-Power Law [virtual pseudo-zeroes to virtual zeroes conversion] as equation is indicated by ∑(all fractional exponents) = fractional number '≠1'.

Proof. Without incorporating constant of integration 'C', Dirichlet Sigma-Power Law as equation for $\sigma=\frac{2}{5}$ value is given by:

$$\begin{aligned}\frac{1}{2t^2+\frac{18}{25}}\cdot\Big[&(2n)^{\frac{3}{5}}\left(\left(t-\tfrac{3}{5}\right)\sin(t\ln(2n))+\left(t+\tfrac{3}{5}\right)\cos(t\ln(2n))\right)-\\&(2n-1)^{\frac{3}{5}}\left(\left(t-\frac{3}{5}\right)\sin(t\ln(2n-1))+\left(t+\frac{3}{5}\right)\cos(t\ln(2n-1))\right)\Big]_1^\infty=0\end{aligned} \tag{A.8}$$

∑(all fractional exponents) as $2(1-\sigma)$ = fractional number '≠1' for Eq. (A.8). This finding signify absence of complete set nontrivial zeros [as virtual pseudo-zeroes to virtual zeroes conversion] for Eq. (A.8). *The proof is now complete for Corollary 1.4*□.

B Prerequisite Lemma, Corollary and Propositions on Two Types of Gram Points

For Gram[y=0] & Gram[x=0] points (and corresponding virtual Gram[y=0] & virtual Gram[x=0] points with totally different values), we apply a parallel procedure carried out on nontrivial zeros but only depict abbreviated treatments and discussions.

Lemma 2.1. Simplified Gram[y=0] and Gram[x=0] points-Dirichlet eta functions are derived directly from Dirichlet eta function with Euler formula application and (respectively) they will intrinsically incorporate actual location [but not actual positions] of all Gram[y=0] and Gram[x=0] points.

Proof. For Gram[y=0] points, the equivalent of Eq. (A.1) and Eq. (A.3) are respectively given by Eq. (B.1) and Eq. (B.2) below.

$$\sum ReIm\{\eta(s)\}=Re\{\eta(s)\}+0,\text{ or simply }\mathrm{Im}\{\eta(s)\}=0 \tag{B.1}$$

$$\begin{aligned}\sum_{n=1}^{\infty}(2n)^{-\sigma}\sin(t\ln(2n))&=\sum_{n=1}^{\infty}(2n-1)^{-\sigma}\sin(t\ln(2n-1))\\\sum_{n=1}^{\infty}(2n)^{-\sigma}\sin(t\ln(2n))&-\sum_{n=1}^{\infty}(2n-1)^{-\sigma}\sin(t\ln(2n-1))=0\end{aligned} \tag{B.2}$$

For Gram[x=0] points, the equivalent of Eq. (A.1) and Eq. (A.3) are respectively given by Eq. (B.3) and Eq. (B.4) below.

$$\sum ReIm\{\eta(s)\} = 0 + Im\{\eta(s)\}, \text{ or simply } \mathrm{Re}\{\eta(s)\} = 0 \tag{B.3}$$

$$\sum_{n=1}^{\infty}(2n)^{-\sigma}\cos(t\ln(2n)) = \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\cos(t\ln(2n-1))$$

$$\sum_{n=1}^{\infty}(2n)^{-\sigma}\cos(t\ln(2n)) - \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\cos(t\ln(2n-1)) = 0 \tag{B.4}$$

Eq. (B.2) and Eq. (B.4) being the simplified Gram[y=0] and Gram[x=0] points-Dirichlet eta functions derived directly from $\eta(s)$ will intrinsically incorporate *actual location [but not actual positions]* of (respectively) all Gram[y=0] and Gram[x=0] points. *The proof is now complete for Lemma 2.1*□.

Proposition 2.2. Gram[y=0] and Gram[x=0] points-Dirichlet Sigma-Power Laws [as pseudo-zeroes to zeroes conversion] representing continuous (integral) format and given as antiderivatives are derived directly from simplified Gram[y=0] and Gram[x=0] points-Dirichlet eta functions [as zeroes] in discrete (summation) format with Riemann integral application. Note: This Law [as pseudo-zeroes to zeroes conversion] representing continuous (integral) format refers to end-product obtained from "key step of converting Dirichlet eta function [as zeroes], *proxy* for Riemann zeta function [as zeroes], into its continuous format version".

Proof. Antiderivatives below using (2n) parameter help obtain all subsequent equations: first one for Gram[y=0] points and second one for Gram[x=0] points.

$$\int_1^{\infty}(2n)^{-\sigma}\sin(t\ln(2n))dn = \left[-\frac{(2n)^{1-\sigma}((\sigma-1)\sin(t\ln(2n))+t\cos(t\ln(2n)))}{2\left(t^2+(\sigma-1)^2\right)}+C\right]_1^{\infty}$$

$$\int_1^{\infty}(2n)^{-\sigma}\cos(t\ln(2n))dn = \left[\frac{(2n)^{1-\sigma}(t\sin(t\ln(2n))-(\sigma-1)\cos(t\ln(2n)))}{2\left(t^2+(\sigma-1)^2\right)}+C\right]_1^{\infty}$$

For Gram[y=0] points-Dirichlet Sigma-Power Law as equation [which is the equivalent of Eq. (A.6)], it is given by Eq. (B.5) [without incorporating constant of integration 'C'].

$$-\frac{1}{2\left(t^2+(\sigma-1)^2\right)}\cdot\Big[(2n)^{1-\sigma}((\sigma-1)\sin(t\ln(2n))+t\cos(t\ln(2n)))-$$
$$(2n-1)^{1-\sigma}((\sigma-1)\sin(t\ln(2n-1))+t\cos(t\ln(2n-1)))\Big]_1^{\infty} = 0 \tag{B.5}$$

For Gram[x=0] points-Dirichlet Sigma-Power Law as equation [which is the equivalent of Eq. (A.6)], it is given by Eq. (B.6) [without incorporating constant of integration 'C'].

$$\frac{1}{2\left(t^2+(\sigma-1)^2\right)}\cdot\Big[(2n)^{1-\sigma}(t\sin(t\ln(2n))-(\sigma-1)\cos(t\ln(2n)))-$$
$$(2n-1)^{1-\sigma}(t\sin(t\ln(2n-1))-(\sigma-1)\cos(t\ln(2n-1)))\Big]_1^{\infty} = 0 \tag{B.6}$$

Intended derivation of Gram[y=0] and Gram[x=0] points-Dirichlet Sigma-Power Laws [both with intrinsic ability for pseudo-zeroes to zeroes conversion] as equations is successful. *The proof is now complete for Proposition 2.2*□.

Proposition 2.3. Exact Dimensional analysis homogeneity at $\sigma = \frac{1}{2}$ in Gram[y=0] and Gram[x=0] points-Dirichlet Sigma-Power Laws [pseudo-zeroes to zeroes conversion] as equations are indicated by ∑(all fractional exponents) = whole number '1'.

Proof. Without incorporating constant of integration 'C', Gram[y=0] points-Dirichlet Sigma-Power Law as equation for $\sigma = \frac{1}{2}$ value is given by:

$$-\frac{1}{2t^2+\frac{1}{2}}\cdot[(2n)^{\frac{1}{2}}\left(t\cos(t\ln(2n))-\frac{1}{2}\sin(t\ln(2n))\right)-$$
$$(2n-1)^{\frac{1}{2}}\left(t\cos(t\ln(2n-1))-\frac{1}{2}\sin(t\ln(2n-1))\right)\Big]_1^{\infty} = 0 \tag{B.7}$$

Without incorporating constant of integration 'C', Gram[x=0] points-Dirichlet Sigma-Power Law as equation for $\sigma = \frac{1}{2}$ value is given by:

$$\frac{1}{2t^2+\frac{1}{2}}\cdot[(2n)^{\frac{1}{2}}\left(t\sin(t\ln(2n))+\tfrac{1}{2}\cos(t\ln(2n))\right)-$$
$$(2n-1)^{\frac{1}{2}}\left(t\sin(t\ln(2n-1))+\frac{1}{2}\cos(t\ln(2n-1))\right)\Big]_1^{\infty} = 0 \tag{B.8}$$

∑(all fractional exponents) as $2(1-\sigma)$ = whole number '1' for Eqs. (B.7) and (B.8). These findings signify presence of complete sets Gram[y=0] points for Eq. (B.7) and Gram[x=0] points for Eq. (B.8) [both as pseudo-zeroes to zeroes conversion]. *The proof is now complete for Proposition 2.3*□.

Corollary 2.4. Inexact Dimensional analysis homogeneity at $\sigma \neq \frac{1}{2}$ [illustrated using $\sigma = \frac{2}{5}$] in Gram[y=0] and Gram[x=0] points-Dirichlet Sigma-Power Laws [virtual pseudo-zeroes to virtual zeroes conversion] as equations are indicated by ∑(all fractional exponents) = fractional number '≠1'.

Proof. Without incorporating constant of integration 'C', Gram[y=0] points-Dirichlet Sigma-Power Law as equation for $\sigma = \frac{2}{5}$ value is given by:

$$-\frac{1}{2t^2+\frac{18}{25}}\cdot\left[(2n)^{\frac{3}{5}}\left(t\cos(t\ln(2n))-\frac{3}{5}\sin(t\ln(2n))\right)-\right.$$
$$\left.(2n-1)^{\frac{3}{5}}\left(t\cos(t\ln(2n-1))-\frac{3}{5}\sin(t\ln(2n-1))\right)\right]_1^\infty=0 \tag{B.9}$$

Without incorporating constant of integration 'C', Gram[x=0] points-Dirichlet Sigma-Power Law as equation for $\sigma=\frac{2}{5}$ value is given by:

$$\frac{1}{2t^2+\frac{18}{25}}\cdot\left[(2n)^{\frac{3}{5}}\left(t\sin(t\ln(2n))+\frac{3}{5}\cos(t\ln(2n))\right)-\right.$$
$$\left.(2n-1)^{\frac{3}{5}}\left(t\sin(t\ln(2n-1))+\frac{3}{5}\cos(t\ln(2n-1))\right)\right]_1^\infty=0 \tag{B.10}$$

$\sum$(all fractional exponents) as $2(1-\sigma)$ = fractional number '$\neq 1$' for Eqs. (B.9) and (B.10). These findings signify presence of complete sets virtual Gram[y=0] points for Eq. (B.9) and virtual Gram[x=0] points for Eq. (B.10) [both as virtual pseudo-zeroes to virtual zeroes conversion]. *The proof is now complete for Corollary 2.4*□.