Prove Grimm's Conjecture by the Stepwise Formation of Consecutive Composite Number Points

Zhang Tianshu

Email:chinazhangtianshu@126.com;
xinshijizhang@hotmail.com.
Nanhai West Oil Administrative Bureau;
Zhanjiang city, Guangdong province, China

Abstract

Regard positive integers which have a common prime factor as a kind, then the positive half line of the number axis consists of infinitely many recurring line segments which have same permutations of χ kinds of integer points, where $\chi \geq 1$. So let us make use of a positive half line of the number axis, but marked only with integer points at the positive half line. After that, change gradually symbols of each kind of composite number points thereat according to the order of common prime factors from small to large, so as to form consecutive composite number points within limits that proven Bertrand's postulate restricts to prove Grimm's conjecture.

AMS subject classification: 11A41; 11P82; 11B99

Keywords: consecutive composite numbers; distinct prime factors; Bertrand's postulate; the number axis

1. Introduction

Grimm's conjecture named after Carl Albert Grimm. The conjecture states that if $n+1$, $n+2$, … $n+k$ are all composite numbers, then there are distinct primes p^i_j such that $p^i_j \mid (n+j)$, for $1 \leq j \leq k$; [1].

For example, for consecutive composite numbers between primes 523 and 541, take out one another's- distinct prime factors from them as follows.

Composite numbers:	524,	525,	526,	527,	528,	529,	530,
Decomposing prime factors:	$2^2\times131$,	$3\times5^2\times7$,	2×263,	17×31,	$2^4\times3\times11$,	23^2,	$2\times5\times53$,
Distinct prime factors:	131,	3,	263,	17,	11,	23,	53,

531,	532,	533,	534,	535,	536,	537,	538,	539,	540
$3^2\times59$,	$2^2\times7\times19$,	13×41,	$2\times3\times89$,	5×107,	$2^3\times67$,	3×179,	2×269,	$7^2\times11$,	$2^2\times3^3\times5$
59,	19,	41,	89,	107,	67,	179,	269,	7,	2

The conjecture was first published in *American Mathematical Monthly*, *76 (1969) 1126-1128*. Yet, it is still both unproved and un-negated a conjecture hitherto; [2].

2. Confirming length of consecutive composite numbers

In order to restrict the length of consecutive composite numbers, it is necessary to quote proven Bertrand's Postulate.

Bertrand's postulate, also called the Bertrand-Chebyshev theorem or Chebyshev's theorem, states that if $n>3$, there is always at least one prime p between n and $2n-2$. Equivalently, if $n>1$, then there is always at least one prime p such that $n<p<2n$; [3].

In other words, the length of consecutive composite numbers is shorter than the distance from n to $2n$.

3. Analyzing integer points concerned at the number axis

It is well known, each and every integer point at positive half line of the number axis expresses a positive integer, and that the length between every two adjacent integer points is equivalent.

First use the symbol "•" to denote an undetermined integer point, and that the symbol "•" will appear in the formulation or at the number axis.

At the beginning, the positive half line of the number axis is marked with infinitely many seriate symbols of "•" purely, and that adjacent two of "•" are equidistant, as listed below.

First Illustration

Whether symbols of integer points at the half line happen how changes, a form of address of the half line is invariable thereinafter.

Also, the symbol "•s" in the formulation expresses the plural of "•".

In addition, regard smallest prime *2* as *№1* prime, and regard prime P_χ as *№*χ prime where $\chi \geq 1$, similarly hereinafter. Then, prime *2* is written as P_1, and that the greater the value of χ, the greater is the prime P_χ.

Beyond that, let us regard positive integers which share prime factor P_χ as *№*χ kind of integers.

In reality, *№*χ kind of integers consists of infinitely many a product which multiplies each and every positive integer by P_χ.

As thus, there is a *№*χ kind's integer point within consecutive P_χ integer points at the half line.

Excepting P_χ as a prime point, others are all composite number points, so regard them as №χ kind of composite number points.

If a composite number contains at least two distinct prime factors, then it belongs to at least two kinds of composite numbers concurrently.

Since there are infinitely many primes, thus there are infinitely many kinds of composite numbers. Correspondingly there are infinitely many kinds of composite number points at the half line.

By now, what we need is to find a law that take out one another's-distinct prime factors from consecutive composite numbers, one for one.

For this purpose, the author will proceed from the stepwise formation of consecutive composite number points at the half line.

In this connection, it is necessary to define symbols of composite number point and prime point in advance.

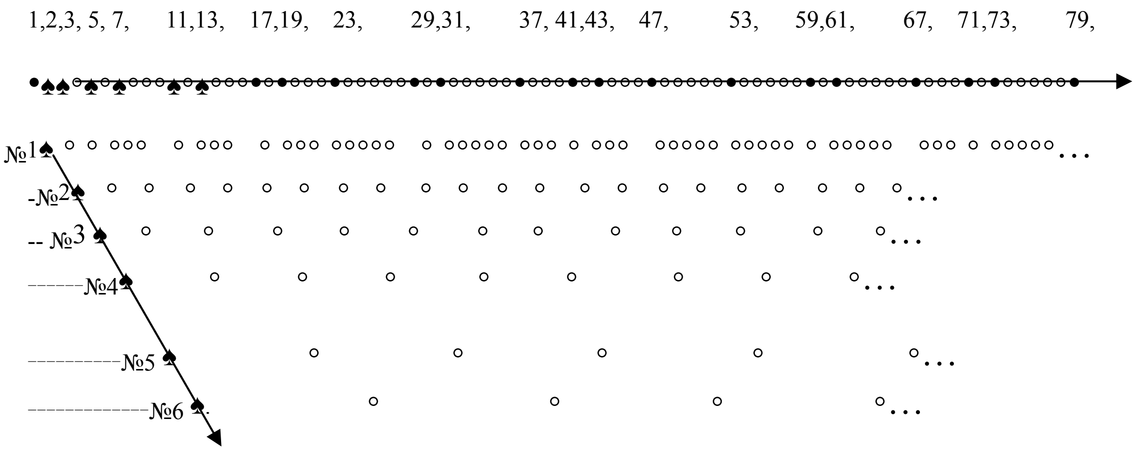
If an integer point • at the half line is defined as a composite number point, then change the symbol "◦" for the symbol "•". In addition to this, the symbol "◦s" in the formulation expresses the plural of "◦".

If an integer point "•" at the half line is defined as a prime point, then change the symbol "♠" for the symbol "•".

Afterwards, change one of ◦s for each of •s at places of №χ kind of composite number points according to the order of χ from small to large.

Then permutations *inter* χ kinds of integer points always assume infinitely many recurring forms to same pattern at the half line, irrespective of their respective prime / composite attributes and integer points amongst them.

Seriatim decompose *№*χ kind of integer points at the half line according to the order of χ from small to large, see also the following illustration.



Second Illustration

Excepting which P_2 adjoins two points of №_1 kind of integer points, any point of any kind of integer points adjoins two integer points that belong not to a kind of integer points, at the half line.

At two line segments of the half line, according to the proper order from left to right, if any two *№k* points of χ kinds of integer points belong to same a kind, then the χ kinds of integer points are possessed of same permutations at two such line segments, where $k\geq 1$ and $\chi\geq 1$.

At the half line, let us regard equilong shortest line segments which are possessed of same permutations *inter* χ kinds of integer points as recurring line segments of permutations of the χ kinds of integer points, and use the character string "$RLS_{\text{№}1\sim\text{№}\chi}$" to express a recurring line segment of the χ kinds of integer points, where $\chi\geq 1$. In addition to this, "$RLSS_{\text{№}1\sim\text{№}\chi}$" are the plural of "$RLS_{\text{№}1\sim\text{№}\chi}$".

Generally speaking, *№1* $RLS_{№1\sim№\chi}$ begins with the integer point *1*.

There are $\prod P_\chi$ integer points per $RLS_{№1\sim№\chi}$, where $\prod P_\chi = P_1P_2 \ldots P_\chi$, $\chi \geq 1$.

Thus it can be seen, a $RLS_{№1\sim№\chi}$ consists of consecutive P_χ $RLSS_{№1\sim№(\chi-1)}$, and that they link one by one.

Therefore, permutations *inter* χ kinds of integer points at each of seriate $RLSS_{№1\sim№\chi}$ are just the same, irrespective of integer points amongst them.

Number the ordinals of integer points at each of seriate $RLSS_{№1\sim№\chi}$, such that from left to right each of integer points at each of seriate $RLSS_{№1\sim№\chi}$ is marked with from small to great a natural number ≥ 1.

Then, there is a $№(\chi+1)$ kind's integer point within $P_{\chi+1}$ integer points which share an ordinal at $P_{\chi+1}$ $RLSS_{№1\sim№\chi}$ of a $RLS_{№1\sim№\chi+1}$.

Of course, there is a $№(\chi+1)$ kind's composite number point within $P_{\chi+1}$ integer points which share an ordinal at $P_{\chi+1}$ $RLSS_{№1\sim№\chi}$ of each of seriate $RLSS_{№1\sim№\chi+1}$ on the right side of *№1* $RLS_{№1\sim№\chi+1}$.

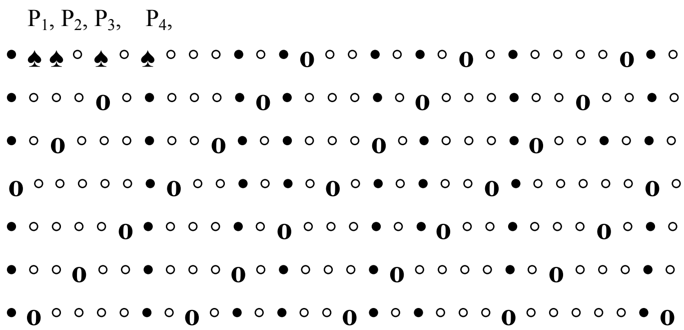
Pursuant to the order of χ from small to large to change symbols of $№\chi$ kind's composite number points, if every prime factor of a $№\chi$ kind's composite number is not less than P_χ, then regard the $№\chi$ kind's composite number point as a newly-added $№\chi$ kind's composite number point.

If there is at least a prime factor of a $№\chi$ kind's composite number is less than P_χ, then the $№\chi$ kind's composite number point is regarded as a repetitive composite number point.

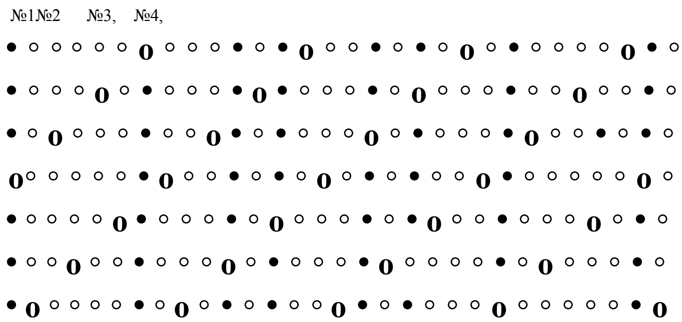
Prime points P_1, $P_2 \ldots P_{\chi-1}$ and P_χ exist at *№1* $RLS_{№1\sim№\chi}$. Yet, there are χ

composite number points on ordinals of P_1, P_2, P_3... and P_χ at each of seriate $RLSS_{№1\sim№\chi}$ on the right side of $№1\ RLS_{№1\sim№\chi}$, thus, $№1\ RLS_{№1\sim№\chi}$ is a special $RLS_{№1\sim№\chi}$.

For convenience to observe, $P_{\chi+1}\ RLSS_{№1\sim№\chi}$ of a $RLS_{№1\sim№\chi+1}$ can be folded at an illustration, such as *№1*, *№2*, *№3* and *№4* kinds of integer points at $P_4\ RLSS_{№1\sim№3}$ of *№1* and $№n\ RLS_{№1\sim№4}$ are listed below.



$№1\ RLS_{№1\sim№4}$



$№n\ RLS_{№1\sim№4}$ where $n \geq 2$

Third Illustration

Annotation: each of $_{№1,\ №2}$ and $_{№3}$ kind's composite number points is expressed by ◦, in addition, each of $_{№4}$ kind's composite number points is expressed by **o**.

4. The formation of consecutive composite number points

The way of doing things is based on aforementioned conceptions concerned. After change symbols at places of $№\chi$ kind's composite number points at the half line, to form consecutive composite number points stepwise along from small to large values which χ be endowed with.

(**1**) After change ◦s for •s at places of $№1$ kind of composite number points, there is one ◦ at each of $RLSS_{№1}$ on the right side of $№1$ $RLS_{№1}$.

Then, there is n_1 ◦ between two adjacent •s, where $n_1=0, 1$.

It is obvious that confirmed $№1$ kind's composite number points are all newly- added composite number points, because they first appear at the half line, clearly their smallest prime factor is p_1.

(**2**) After successively change ◦s for •s at places of $№2$ kind of composite number points, confirmed $№2$ kind's composite number points monogamously coincide with one • and one ◦ of $№1$ kind's composite number points at each of seriate $RLSS_{№1\sim№2}$ on the right side of $№1$ $RLS_{№1\sim№2}$, since there is a $№2$ kind's composite number point within P_2 integer points which share an ordinal at P_2 $RLSS_{№1}$ of a $RLS_{№1\sim№2}$.

In them, $№2$ kind's composite number points which coincide with ◦s monogamously, all are repetitive composite number points.

Yet each and every newly-added $№2$ kind's composite number point coincides with one • to constitute consecutive composite number points.

After pass this change, there are n_2 ◦s between two adjacent •s, where $n_2=0, 1$ and 3. For the case of $n_2=0,$ it appears only at $№1$ $RLS_{№1\sim№2},$ since p_1 and p_2 are adjacent primes, similarly hereinafter.

Then, smallest prime factor of newly-added $№2$ kind's composite numbers is p_2.

(**3**) After successively change ◦s for •s at places of $№3$ kind of composite number points, confirmed *№3* kind's composite number points monogamously coincide with one •, two ◦s of *№1* kind's composite number points, one ◦ of *№2* kind's composite number points, and one ◦ of *№1* plus *№2* kind's composite number points at each of seriate $RLSS_{№1\sim№3}$ on the right side of *№1* $RLS_{№1\sim№3}$, since there is a *№3* kind's composite number point within P_3 integer points which share an ordinal at P_3 $RLSS_{№1\sim№2}$ of a $RLS_{№1\sim№3}$.

In them, *№3* kind's composite number points which coincide with ◦s monogamously, all are repetitive composite number points.

Yet, each and every newly-added *№3* kind's composite number point coincides with one • to constitute consecutive composite number points.

After pass this change, there are n_3 ◦s between two adjacent •s, where $n_2=0, 1, 3$ and 5.

Then, smallest prime factor of newly-added *№3* kind's composite numbers is p_3.

And so on and so forth …

On balance, after successively change ◦s for •s at places of *№*χ kind of composite number points, where $\chi>1$, on the one hand, continue to there is every form of original string of consecutive composite number points.

On the other hand, each form of string of consecutive composite number points increased at least a composite number point or connect each other's-adjacent strings of consecutive composite number points into a longer string of consecutive composite number points at each of seriate $RLSS_{№1\sim№\chi}$, where $\chi \geq 2$.

After change ◦s for •s at places of $\sum №X$ kinds of composite number points, where $X=1, 2, 3 \ldots \chi$, smallest prime factor of newly-added $№\chi$ kind's composite numbers is p_χ, so every string of consecutive composite number points on the left side of P_χ^2 is fixed. In other words, every such string of consecutive composite numbers$\leq P_\chi^2$ adds no longer a composite number after changes via $\sum №X$ kinds.

5. There is at least a prime between P_χ^α and $P_\chi^{\alpha+1}$

By now, be necessary to judge whether or not P_χ^α and $P_\chi^{\alpha+1}$ coexist in consecutive composite numbers, because take out two of P_χ from them.

As stated, the length of consecutive composite numbers is shorter than the distance from n to $2n$, so let $n=P_\chi^\alpha$, then there is a prime between P_χ^α and $2P_\chi^\alpha$. Since it has $2P_\chi^\alpha < P_\chi^{\alpha+1}$except for $P_\chi=2$ and $\alpha=1$, so there is a prime between P_χ^α and $P_\chi^{\alpha+1}$. That is to say, P_χ^α and $P_\chi^{\alpha+1}$ exist not in a string of consecutive composite numbers.

Therefore, we are unable to take out two identical greatest prime factors from any kind of newly-added composite numbers within a string of consecutive composite numbers.

In other words, there are not two such composite numbers which have two identical greatest prime factors in any kind of newly-added composite numbers within a string of consecutive composite numbers.

6. A method that found out one another's -distinct primes

As has been mentioned, let us first take out a greatest or only prime factor from each of consecutive composite numbers for the moment.

After that, check all prime factors which are taken out, whether or not there are identical two in them.

If there are two identical prime factors which are taken out, then first retain one from the composite number like P_χ^a.

Then again, further take out only or smallest prime factor except for P_χ from another composite number concerned to replace P_χ from itself.

For example, for consecutive composite numbers: 24, 25, 26, 27 and 28, i.e. $2^3\times3$, 5^2, 2×13, 3^3 and $2^2\times7$, orderly take out a greatest prime factor from each of them: 3, 5, 13, 3 and 7.

Since there are identical two of 3 in these greatest prime factors which are taken out, so first retain 3 from 3^3.

Then again, further take out 2 from $2^3\times3$ to replace 3 from $2^3\times3$.

Thus it can be seen, newly-added composite numbers reach up to P_χ^a, just can take out two of identical greatest prime factor P_χ from the string of consecutive composite numbers.

Therefore, we can not take out three identical greatest prime factors from a string of consecutive composite numbers, since the author has proved that P_χ^a and P_χ^{a+1} exist not in a string of consecutive composite numbers.

As previously mentioned, it is unable to take out two identical greatest prime factors from any kind of newly-added composite numbers within a string of consecutive composite numbers, thus if two composite numbers within a string of consecutive composite numbers contain an identical greatest prime factor, then they contain not another identical prime factor. One another's- distinct prime factors which are monogamously taken out from a string of consecutive composite numbers are exactly satisfactory distinct primes for the string of the consecutive composite numbers.

The aforesaid way of doing thing is the very the law we need to look for.

So far, the author has found the feasible method to get distinct primes, such that consecutive composite numbers are divided integrally by the distinct primes, one for one.

7. Illustrate with an example

In order to impress upon readers concerning preceding way of doing the thing, might as well again give an example, i.e. as for consecutive composite numbers between primes 1327 and 1361.

First, orderly decompose consecutive composite numbers between primes 1327 and 1361 into prime factors: $1328=2^4\times83$, $1329=3\times443$, $1330=2\times5\times7\times19$, $1331=11^3$, $1332=2^2\times3^2\times37$, $1333=31\times43$, $1334=2\times23\times29$, $1335=3\times5\times89$, $1336=2^3\times167$, $1337=7\times191$, $1338=2\times3\times223$, $1339=13\times103$, $1340=2^2\times5\times67$, $1341=3^2\times149$, $1342=2\times11\times61$, $1343=17\times79$, $1344=2^6\times3\times7$, $1345=5\times269$, $1346=2\times673$, $1347=3\times449$, $1348=2^2\times337$, $1349=19\times71$, $1350=2\times3^3\times5^2$, $1351=7\times193$, $1352=2^3\times13^2$, $1353=3\times11\times41$, $1354=2\times677$, $1355=5\times271$,

$1356=2^2\times3\times113$, $1357=23\times59$, $1358=2\times7\times97$, $1359=3^2\times151$ and $1360=2^4\times5\times17$.

Pursuant to the preceding way of doing thing, orderly take out greatest prime factor from each of the string of consecutive composite numbers : 83, 443, 19, 11, 37, 43, 29, 89, 167, 191, 223, 103, 67, 149, 61, 79, 7, 269, 673, 449, 337, 71, 5, 193, 13, 41, 677, 271, 113, 59, 97, 151 and 17.

In order to watch the above-listed distinct primes conveniently, be necessary to arrange renewedly them according to the order from small to large: 5, 7, 11, 13, 17, 19, 29, 37, 41, 43, 59, 61, 67, 71, 79, 83, 89, 97, 103, 113, 149, 151, 167, 191, 193, 223, 269, 271, 337, 443, 449, 673 and 677.

It is observed that any two primes in the string of primes are not alike.

Hereto, the author has proved that consecutive composite numbers can be divided integrally by one another's-distinct primes, one for one.

The proof was thus brought to a close. As a consequence, Grimm's conjecture is tenable.

References

[1] Wolfram Mathworld, Grimm's conjecture, URL: http://mathworld.wolfram.com/GrimmsConjecture.html

[2] WIKIPEDIA, Grimm's conjecture, URL: *https://en.wikipedia.org/wiki/Grimm%27s_conjecture*

[3]Wolfram Mathworld, Bertrand's postulate, URL: *http://mathworld.wolfram.com/BertrandsPostulate.html*