Hidden Circle's Duel in a Family of Projectile Paths And The Burning Point of The Spherical Mirror

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In an earlier publication in this journal[1], we showed the presence of a hidden circle in a family of projectile paths shot from a point with a common value of speed and at different angels to the horizontal. We described the interesting properties associated with it. In this paper we describe the hidden circle's duel in a family of ideal projectile paths whose apexes lie on the horizontal. We call this circle, the duel circle The center of this circle is the burning point of the spherical mirror, defined by the revolution of this circle around its vertical axis. We highlight the interesting properties of this family of projectiles. For our purpose here, we consider projectiles shot from points on the horizontal with speeds obtained from free fall from points on the fourth and third quadrants of a circle in the vertical plane. In this method, we don't use vectors or calculus or Newton's laws of motion.

Description of the method and analysis

Projectile paths

Let us consider an arbitrary circle in the vertical plane with vertical diameter AB (see Fig. 1), Draw the tangent (horizontal) through the lower end, B of the diameter, Locate points A_i on the fourth quadrant of the circle. From points A_i drop particles to fall freely with uniform acceleration. Locate mirrors at points m_i on the horizontal vertically below A_i inclined to the horizontal at 45°. The mirrors reflect the particles so that their motion immediately after the (perfectly elastic collision) is along the horizontal, away from B. As the particles leave the mirrors after reflection to move along the horizontal with a constant speed acquired in the free fall from the points on the circle to the points on the horizontal, they are subjected to a

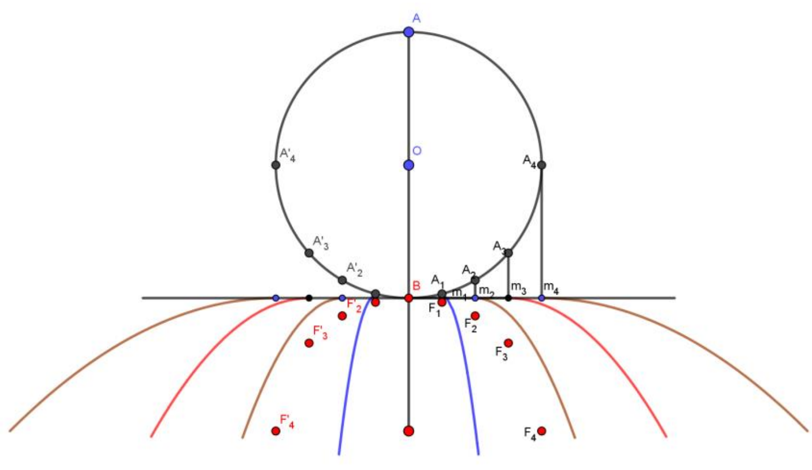

Fig. 1. Parabolic paths of projectiles reflected by mirrors at m_i on the horizontal with speeds acquired in free fall, with uniform acceleration, from points A_i on the fourth quadrant of the circle on to the horizontal.

uniform acceleration in the vertically downward direction. These two motions give a resultant parabolic motion. As a result, the particles move as projectiles. They move along parabolic paths (Fig. 1).

The method of obtaining the parabolic paths is described below

Description of the method of obtaining the parabolic paths

The points of reflection of A_i in the horizontal give the foci of the parabolas. The horizontals through A_i give the respective directrices. Thus, for example, we take A_3 reflect it in the horizontal through B. The point obtained is marked F_3. The horizontal through A_3 gives the directrix D_3 (not shown in the figure). Using A_3 as focus and D_3 directrix we draw a parabola (red). We repeat this procedure for each A_i. We show only semi parabolas.

In a similar way we take points on the third quadrant of the circle A'_i. Drop particles from A'_i to fall freely along the vertical with uniform acceleration. When they reach the horizontal, they are reflected (in an elastic collision) by mirrors located at m'_i on the horizontal vertically below A'_i. The particles then move with a constant speed in the horizontal direction away from B, and uniform acceleration in the vertically downward direction. These two motions give a resultant parabolic motion. The foci of these parabolic paths are reflections F'_i of points A'_i, in the horizontal through B. The horizontal through A'_i gives the directrix D_i (not shown in the figure). Using F'_i as focus and D_i directrix we draw the parabola. We show only semi parabolas.

The Hidden circle in the family of these parabolic paths

We join all the foci. We find the locus is a circle (Fig. 2).

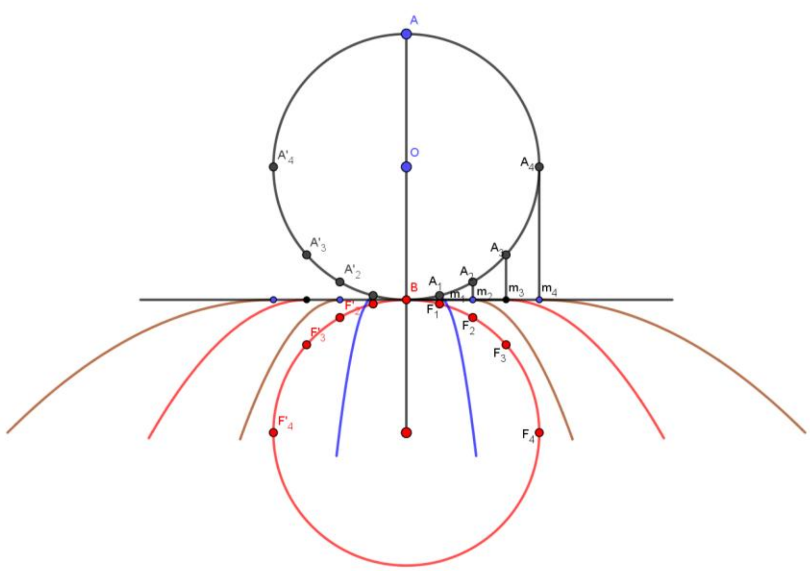

Fig. 2. Figure shows the hidden circle in the family of the parabolic paths of the projectiles.

The circle shown in the red color is the hidden circle. This is the dual of the hidden circle we showed in our earlier paper[1]. This circle has many interesting properties listed below.

1. This circle is the resultant motion of two motions with equal accelerations one along the horizontal and the other along vertically downward direction.

2. It has a uniformly accelerated tangential motion and a constant speed (zero acceleration) motion along the radial direction. This is interesting because it contrasts with the conventional uniform circular motion we study. In the conventional uniform circular motion, the motion is uniformly accelerated along the radial direction and has a constant speed motion along the tangential direction.

3. The light rays parallel to the axes of the parabolas are reflected by the parabolas to pass through their foci. (See Fig. 3).

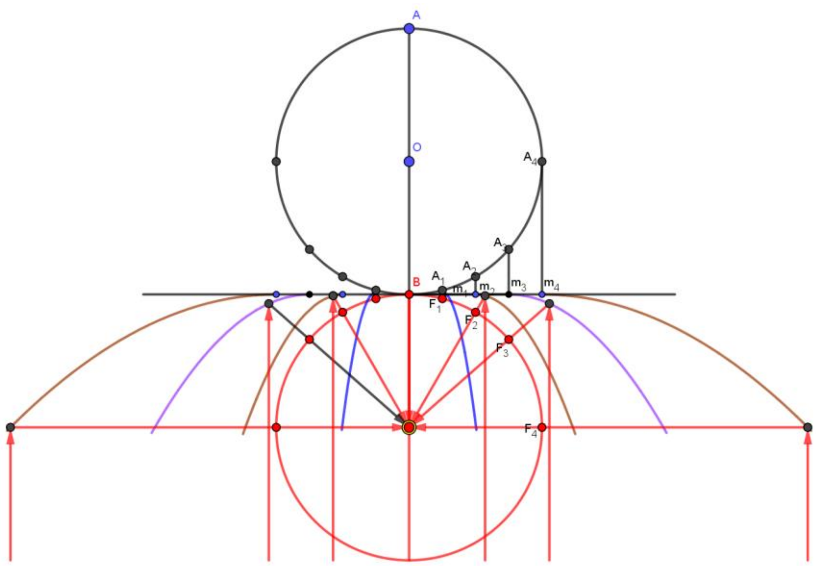

Fig. 3. Light rays incident parallel to the axes of the parabolas are reflected to pass through the respective foci. These reflected rays intersect at the center of the red circle.

4. All such reflected rays intersect at the center of the duel (red) circle.

5. Similarly, the light rays parallel to the axis of the duel circle incident on the circle, get reflected to pass through the center of the circle. The chord from the upper end, B of the vertical diameter to the point of incidence acts as the surface of reflection. The chord from the lower end, B’ of the vertical diameter to the point of incidence acts as the normal to the surface of reflection at the point of incidence. (See Fig. 4). A spherical mirror constructed by the rotation of the circle about the vertical axis, reflects sun’s rays which are parallel to the axis when the mirror faces the sun, to pass through the center of the spherical mirror. Thus, a spherical mirror concentrates sun’s rays at

its center enabling very high temperatures to be reached at the center. Therefore, this point of the mirror is known as the *burning point* of the spherical mirror.

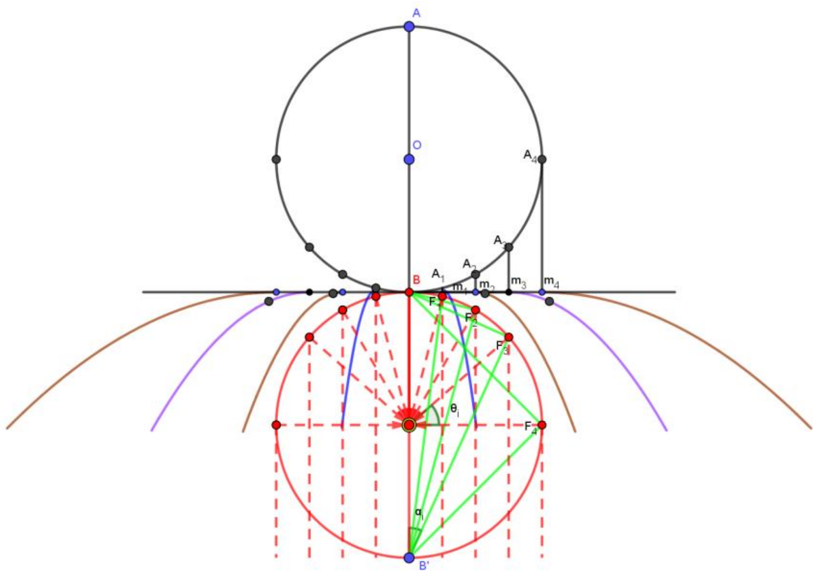

Fig. 4. The light rays incident on the duel circle parallel to its axis pass through its center after reflection. These reflected rays intersect at the center of the duel (red) circle.

This result again contrasts with the conventional result that spherical mirrors do not have well defined burning point[2,5]. Our result is in harmony with the result of Euclid[2,3].

6. The hidden circle is the locus of focii of the family of parabolas whose vertices lie on a straight line.

   This result again contrasts with the result of the circle in ref. 1. There, the vertices of the family of the parabolas lie on an ellipse.

If we take the origin at the center of the circle (red) the equation of the circle is given by $x^2 + y2 = r^2$. The apexes of the parabolas are located at $[x_i, (r - y_i)]$. The foci of the parabolas lie at points (x_i , y_i) on the duel circle.

The following from Galileo[6] appeared to me to give some useful background. It appears just before the projectile motion propositions are dealt with. I quote it from Galileo[6].

“Sagr. Allow me please, to interrupt in order that I may point out the beautiful agreement between this thoughtof the Author and the views of Plato concerning the origin of the various uniform speeds with which the heavenly bodies revolve, The later chanced upon the idea that a body could not pass from rest to any given speed and maintain it uniformly except by passing through all the degrees of speed intermediate between the given speed and rest. Plato thought that God, after having created the heavenly bodies, assigned them the proper and uniform speeds with which they were forever to revolve; and that He made them start from rest and move over definite distances under a natural and rectilinear acceleration such as governs the motion of terrestrial bodies. He added that once these bodies had gained their proper and permanent speed, their rectilinear motion converted into a circular one, the only motion capable of maintaining uniformity, a motion in which the body revolves without either receding from or approaching its desired goal. This conception is truly worthy of Plato; and it is to be all the more highly prized since its underlying principles remained hidden until discovered by our Author who removed from them the mask and poetical dress and set forth the idea in correct historical perspective. In view of the fact that astronomical science furnishes us such complete information concerning the size of the planetary orbits, the distances of these bodies from their centers of revolution, and their velocities, I cannot help thinking that our Author (to whom this idea of Plato was not unknown) had some curiosity to discover whether or not a definite "sublimity" might be assigned to each planet, such that, if it were to start from rest at this particular height and to fall with naturally accelerated motion along a straight line, and were later to change the speed thus acquired into uniform motion, the size of its orbit and it period of revolution would be those actually observed."

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