

# Proof of Goldbach Conjecture

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## Proof of Goldbach Conjecture

### ABSTRACT

Objective:

Goldbach & Euler

Method:

Triangular lattice

Result:

The number of prime pair an even  $a$  can be written as is denoted as  $T(a)$ ,

$$T(a) \sim \left( \left( \frac{a}{2} \right) / 2 - \left( \frac{a}{2} \right) / \ln \left( \frac{a}{2} \right) \right) * \left( \left( \frac{a-1}{2} - \left( \frac{a-1}{2} \right) / \ln \left( \frac{a-1}{2} \right) \right) - \left( \frac{a/2-1}{2} - \left( \frac{a/2-1}{2} \right) / \ln \left( \frac{a/2-1}{2} \right) \right) \right) - \left( \frac{a/2-1}{2} - \left( \frac{a/2-1}{2} \right) / \ln \left( \frac{a/2-1}{2} \right) \right) * \left( \left( \frac{a-3}{2} - \left( \frac{a-3}{2} \right) / \ln \left( \frac{a-3}{2} \right) \right) - \left( \frac{a/2-2}{2} - \left( \frac{a/2-2}{2} \right) / \ln \left( \frac{a/2-2}{2} \right) \right) \right) * \left( \left( \frac{a/4}{2} - \left( \frac{a/4}{2} \right) / \ln \left( \frac{a/4}{2} \right) \right) * \left( \left( \frac{3*a/4}{2} - \left( \frac{3*a/4}{2} \right) / \ln \left( \frac{3*a/4}{2} \right) \right) - \left( \frac{a/2}{2} - \left( \frac{a/2}{2} \right) / \ln \left( \frac{a/2}{2} \right) \right) \right) \right) / \left( \left( \left( \frac{a/4}{2} - \left( \frac{a/4}{2} \right) / \ln \left( \frac{a/4}{2} \right) \right) * \left( \left( \frac{3*a/4}{2} - \left( \frac{3*a/4}{2} \right) / \ln \left( \frac{3*a/4}{2} \right) \right) - \left( \frac{a/2}{2} - \left( \frac{a/2}{2} \right) / \ln \left( \frac{a/2}{2} \right) \right) \right) \right) + \left( \left( \frac{a/2}{2} - \left( \frac{a/2}{2} \right) / \ln \left( \frac{a/2}{2} \right) \right) - \left( \frac{a/4}{2} - \left( \frac{a/4}{2} \right) / \ln \left( \frac{a/4}{2} \right) \right) \right) * \left( \left( \frac{a/2-a}{\ln(a)} - \left( \frac{3*a/4}{2} - \left( \frac{3*a/4}{2} \right) / \ln \left( \frac{3*a/4}{2} \right) \right) \right) \right) + \left( \left( \frac{a/2-1}{2} - \left( \frac{a/2-1}{2} \right) / \ln \left( \frac{a/2-1}{2} \right) \right) * \left( \left( \frac{a/2-1}{2} - \left( \frac{a/2-1}{2} \right) / \ln \left( \frac{a/2-1}{2} \right) \right) + 1 \right) / 2 - \left( \frac{a/2-2}{2} - \left( \frac{a/2-2}{2} \right) / \ln \left( \frac{a/2-2}{2} \right) \right) * \left( \left( \frac{a/2-2}{2} - \left( \frac{a/2-2}{2} \right) / \ln \left( \frac{a/2-2}{2} \right) \right) + 1 \right) / 2 + a / \ln(a) - a/4.$$

Conclusions:

If {1 is also a prime number} is true, then any even number greater than 0 can be written as the sum of two prime numbers.

If {1 is also a prime number} is false, then any even number greater than 4 can be written as the sum of two prime numbers.

Key words: Goldbach; Euler; even; prime.

## 1 Structure

### 1.1 Concept

Set of natural numbers is denoted as  $N$ ,  $N = \{n\}$ .

If one variable belongs to  $N$ , then it is denoted as  $n$ .

If two variables belong to  $N$ , then they are denoted as  $n_1$  and  $n_2$ .

Set of even numbers is denoted as  $A$ ,  $A = \{a | a = 2 * n\}$ .

If one variable belongs to  $A$ , then it is denoted as  $a$ .

If two variables belong to  $A$ , then they are denoted as  $a_1$  and  $a_2$ .

Set of odd numbers is denoted as  $B$ ,  $B = \{b | b = 2 * n + 1\}$ .

If one variable belongs to  $B$ , then it is denoted as  $b$ .

If two variables belong to  $B$ , then they are denoted as  $b_1$  and  $b_2$ .

Set of odd composite numbers is denoted as  $C$

$C = \{c | c = (2 * n_1 + 1) * (2 * n_2 + 1), n_1 \text{ is not } 0 \text{ and } n_2 \text{ is not } 0.\}$

If one variable belongs to  $C$ , then it is denoted as  $c$ .

If two variables belong to  $C$ , then they are denoted as  $c_1$  and  $c_2$ .

Set of prime numbers is denoted as  $D$

If  $\{1 \text{ is also a prime number}\}$  is true,

then  $D = \{d | d \text{ belongs to } B \text{ and } d \text{ does not belong to } C\}$

If  $\{1 \text{ is also a prime number}\}$  is false,

then  $D = \{d | d \text{ belongs to } B \text{ and } d \text{ does not belong to } C, d \text{ is not } 1.\}$

If one variable belongs to  $D$ , then it is denoted as  $d$ .

If two variables belong to  $D$ , then they are denoted as  $d_1$  and  $d_2$ .

### 1.2 $N(a)$

$a = a/2 + a/2, a > 0$ .

If  $a/2$  belongs to  $A$ , then set  $a = [(a/2 - 1) - 2n] + [(a/2 + 1) + 2n]$

$n < (a - 2)/4, \text{Card}(n) = a/4$ .

$(a/2 + 1) - 2n$  is denoted as  $b_L, (a/2 + 1) + 2n$  is denoted as  $b_R$ .

If  $a/2$  belongs to  $B$ , then set  $a = (a/2 - 2n) + (a/2 + 2n)$

$n < a/4, \text{Card}(n) = (a + 2)/4$ .

$a/2 - 2n$  is denoted as  $b_L, a/2 + 2n = b_2$  is denoted as  $b_R$ .

$\text{Card}(n)$  is one function of  $a$ , it is denoted as  $N(a)$ .

If  $a_0$  is big, then  $N(a) \sim a/4$  when  $a > a_0$ .

Error is denoted as  $O(a), O(a) \sim 0$  when  $a > a_0$ .

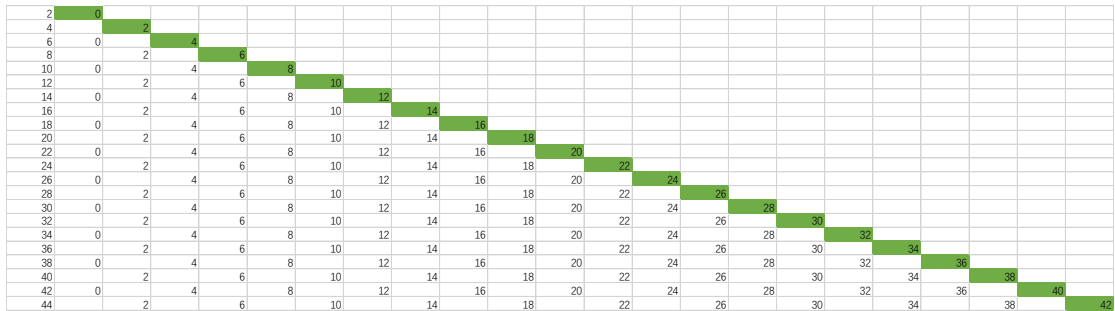
### 1.3 $e = b_R - b_L$

Set one increasing positive even sequence, it corresponds to  $\{a | a > 0\}$ .

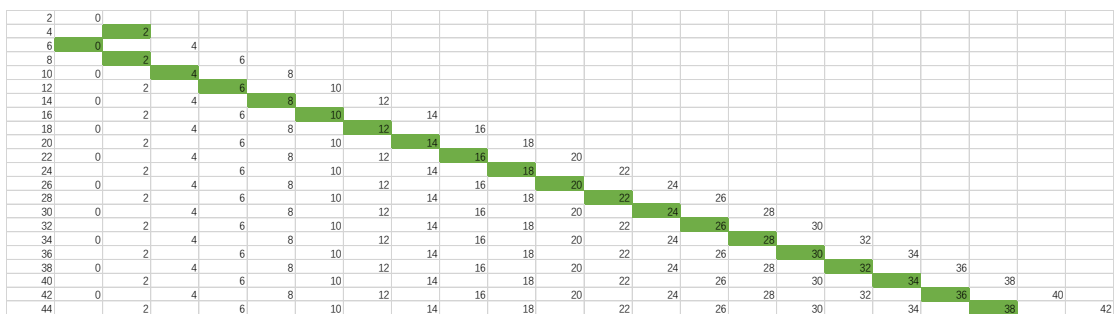


e is one function of a when g is invariable, any odd composite number in (0, a) corresponds to one cell in {L=a}. Equation is  $e=|(a-g)-g|$ ,  $a>g$ .

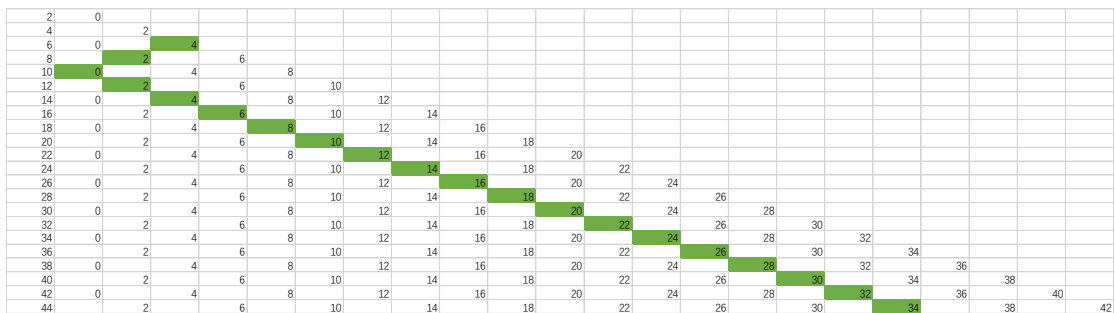
$e=|(a-1)-1|$ ,  $a>1$ .



$e=|(a-3)-3|$ ,  $a>3$ .



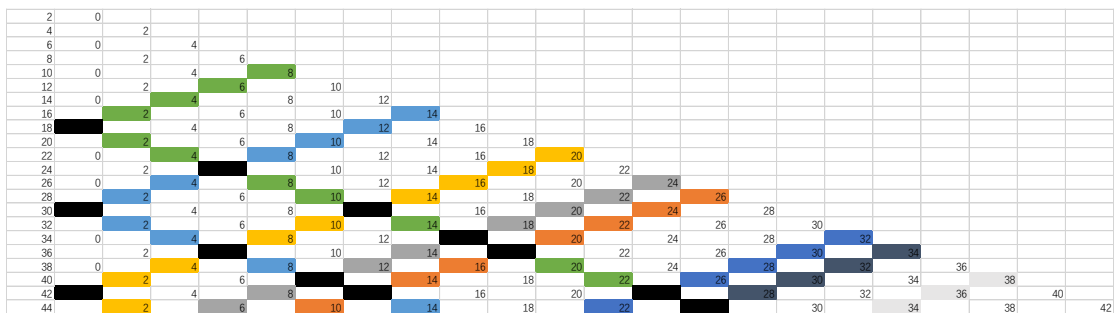
$e=|(a-5)-5|$ ,  $a>5$ .



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1.5  $U(a)-T(a)=S(a)-N(a)$

If bL or bR belongs to C, then color the cell.



If bL and bR belong to C, then color it black.



If  $b_L$  and  $b_R$  belong to  $C$ , then the cell is denoted as  $(c_L, c_R)$ .

$X = \{(c_L, c_R) | c_L \text{ and } c_R \text{ belong to } (0, a/2-1]\}$ ;

$\text{Card}(c_L, c_R)$  is denoted as  $X(a)$ ,  $X(a) = S(a/2-1) * (S(a/2-1)+1)/2$ .

$Y = \{(c_L, c_R) | c_L \text{ belongs to } (0, a/2] \text{ and } c_R \text{ belongs to } [a/2, a-1]\}$ ;

$\text{Card}(c_L, c_R)$  is denoted as  $Y(a)$ ,  $Y(a) = S(a/2) * (S(a-1) - S(a/2-1))$ .

$Z = \{(c_L, c_R) | c_L \text{ and } c_R \text{ belong to } Y, b_L + b_R \text{ belongs to } (0, a]\}$ ;

$\text{Card}(c_L, c_R)$  is denoted as  $Z(a)$ ,  $Z(a) = H(a) * Y(a)$ .

## 2.1 $H(a) \sim H(a-2)$

Maximum error is denoted as  $O_r(a)$ ,  $O_r(a) \sim 0$  when  $a > a_0$ .

$M = \{(c_L, c_R) | c_L + c_R \text{ belongs to } (0, a]\}$ ,  $\text{Card}(c_L, c_R)$  is denoted as  $M(a)$ .

$M(a) = X(a) + Y(a)$ ,  $U(a) = M(a) - M(a-2)$ .

Let  $T(a) = 0$ ,  $U(a) = S(a) - N(a)$ .

$H(a) \sim (S(a) - N(a) - X(a) + X(a-2)) / (Y(a) - Y(a-2))$

$H(a) \sim (a/4 - a/\ln(a) - ((a/2-1)/2 - (a/2-1)/\ln(a/2-1)) * ((a/2-1)/2 - (a/2-1)/\ln(a/2-1) + 1)/2 + ((a/2-2)/2 - (a/2-2)/\ln(a/2-2)) * ((a/2-2)/2 - (a/2-2)/\ln(a/2-2) + 1)/2 / (((a/2)/2 - (a/2)/\ln(a/2)) * (((a-1)/2 - (a-1)/\ln(a-1)) - ((a/2-1)/2 - (a/2-1)/\ln(a/2-1))) - ((a/2-1)/2 - (a/2-1)/\ln(a/2-1)) * (((a-3)/2 - (a-3)/\ln(a-3)) - ((a/2-2)/2 - (a/2-2)/\ln(a/2-2)))$

Let  $T(a) = 1$ ,  $U(a) = S(a) - N(a) + 1$ .

$H(a) \sim (S(a) - N(a) + 1 - X(a) + X(a-2)) / (Y(a) - Y(a-2))$

$H(a) \sim (a/4 - a/\ln(a) + 1 - ((a/2-1)/2 - (a/2-1)/\ln(a/2-1)) * ((a/2-1)/2 - (a/2-1)/\ln(a/2-1) + 1)/2 + ((a/2-2)/2 - (a/2-2)/\ln(a/2-2)) * ((a/2-2)/2 - (a/2-2)/\ln(a/2-2) + 1)/2 / (((a/2)/2 - (a/2)/\ln(a/2)) * (((a-1)/2 - (a-1)/\ln(a-1)) - ((a/2-1)/2 - (a/2-1)/\ln(a/2-1))) - ((a/2-1)/2 - (a/2-1)/\ln(a/2-1)) * (((a-3)/2 - (a-3)/\ln(a-3)) - ((a/2-2)/2 - (a/2-2)/\ln(a/2-2)))$

## 2.2 $H(a) \sim J(a) / (J(a) + K(a))$ , $a > 0$ .

Maximum error is denoted as  $O_0(a)$ ,  $O_0(a) \sim (W(a) - (J(a) + K(a))) / W(a) \sim 1/2$ .

$J = \{(c_L, c_R) | c_L \text{ belongs to } (0, a/4] \text{ and } c_R \text{ belongs to } (a/2, 3*a/4]\}$ ;

$\text{Card}(c_L, c_R)$  is denoted as  $J(a)$ ,  $J(a) = S(a/4) * (S(3*a/4) - S(a/2))$ .

$K = \{(c_L, c_R) | c_L \text{ belongs to } (a/4, a/2] \text{ and } c_R \text{ belongs to } (3*a/4, a]\}$ ;

$\text{Card}(c_L, c_R)$  is denoted as  $K(a)$ ,  $K(a) = (S(a/2) - S(a/4)) * (S(a) - S(3*a/4))$ .

$J(a) / (J(a) + K(a)) \sim (((a/4)/2 - (a/4)/\ln(a/4)) * (((3*a/4)/2 - (3*a/4)/\ln(3*a/4)) - ((a/2)/2 - (a/2)/\ln(a/2))) / (((a/4)/2 - (a/4)/\ln(a/4)) * (((3*a/4)/2 - (3*a/4)/\ln(3*a/4)) - ((a/2)/2 - (a/2)/\ln(a/2))) + (((a/2)/2 - (a/2)/\ln(a/2)) - ((a/4)/2 - (a/4)/\ln(a/4)) * ((a/2 - a/\ln(a)) - ((3*a/4)/2 - (3*a/4)/\ln(3*a/4))))$

### 2.2.1 $H(a) \sim (J(a) + p_1 + p_2) / (J(a) + K(a) + p_1 + p_2 + q_1 + q_2)$ , $a > 8$ .

Maximum error is denoted as  $O_2(a)$ ,  $O_2(a) \sim 1/4$ .

$P_1 = \{(c_L, c_R) | c_L \text{ belongs to } (0, a/8] \text{ and } c_R \text{ belongs to } (3*a/4, 7*a/8]\}$ ;

$\text{Card}(c_L, c_R)$  is denoted as  $p_1$ ,  $p_1 = S(a/8) * (S(7*a/8) - S(3*a/4))$ .

$P_2 = \{(c_L, c_R) | c_L \text{ belongs to } (a/4, 3*a/8] \text{ and } c_R \text{ belongs to } (a/2, 5*a/8]\}$ ;

$\text{Card}(c_L, c_R)$  is denoted as  $p_2$ ,  $p_2 = (S(3*a/8) - S(a/4)) * (S(5*a/8) - S(a/2))$ .

$Q1 = \{(cL, cR) | cL \text{ belongs to } (a/8, a/4] \text{ and } cR \text{ belongs to } (7*a/8, a]\};$   
 $\text{Card}(cL, cR)$  is denoted as  $q1$ ,  $q1 = (S(a/4) - S(a/8)) * (S(a) - S(7*a/8))$ .  
 $Q2 = \{(cL, cR) | cL \text{ belongs to } (3*a/8, a/2] \text{ and } cR \text{ belongs to } (5*a/8, 3*a/4]\};$   
 $\text{Card}(cL, cR)$  is denoted as  $q2$ ,  $q2 = (S(a/2) - S(3*a/8)) * (S(3*a/4) - S(5*a/8))$ .

2.2.2  $H(a) \sim (J(a) + p1 + \dots + p6) / (J(a) + K(a) + p1 + \dots + p6 + q1 + \dots + q6)$ ,  $a > 24$ .

Maximum error is denoted as  $O6(a)$ ,  $O6(a) \sim 1/8$ .

$P3 = \{(cL, cR) | cL \text{ belongs to } (0, a/16] \text{ and } cR \text{ belongs to } (7*a/8, 15*a/16]\};$

$\text{Card}(cL, cR)$  is denoted as  $p3$ ,  $p3 = S(a/16) * (S(15*a/16) - S(7*a/8))$ .

$P4 = \{(cL, cR) | cL \text{ belongs to } (a/8, 3*a/16] \text{ and } cR \text{ belongs to } (3*a/4, 13*a/16]\};$

$\text{Card}(cL, cR)$  is denoted as  $p4$ ,  $p4 = (S(3*a/16) - S(a/8)) * (S(13*a/16) - S(3*a/4))$ .

$P5 = \{(cL, cR) | cL \text{ belongs to } (a/4, 5*a/16] \text{ and } cR \text{ belongs to } (5*a/8, 11*a/16]\};$

$\text{Card}(cL, cR)$  is denoted as  $p5$ ,  $p5 = (S(5*a/16) - S(a/4)) * (S(11*a/16) - S(5*a/8))$ .

$P6 = \{(cL, cR) | cL \text{ belongs to } (3*a/8, 7*a/16] \text{ and } cR \text{ belongs to } (a/2, 9*a/16]\};$

$\text{Card}(cL, cR)$  is denoted as  $p6$ ,  $p6 = (S(7*a/16) - S(3*a/8)) * (S(9*a/16) - S(a/2))$ .

$Q3 = \{(cL, cR) | cL \text{ belongs to } (a/16, a/8] \text{ and } cR \text{ belongs to } (15*a/16, a]\};$

$\text{Card}(cL, cR)$  is denoted as  $q3$ ,  $q3 = (S(a/8) - S(a/16)) * (S(a) - S(15*a/16))$ .

$Q4 = \{(cL, cR) | cL \text{ belongs to } (3*a/16, a/4] \text{ and } cR \text{ belongs to } (13*a/16, 7*a/8]\};$

$\text{Card}(cL, cR)$  is denoted as  $q4$ ,  $q4 = (S(a/4) - S(3*a/16)) * (S(7*a/8) - S(13*a/16))$ .

$Q5 = \{(cL, cR) | cL \text{ belongs to } (5*a/16, 3*a/8] \text{ and } cR \text{ belongs to } (11*a/16, 3*a/4]\};$

$\text{Card}(cL, cR)$  is denoted as  $q5$ ,  $q5 = (S(3*a/8) - S(5*a/16)) * (S(3*a/4) - S(11*a/16))$ .

$Q6 = \{(cL, cR) | cL \text{ belongs to } (7*a/16, a/2] \text{ and } cR \text{ belongs to } (9*a/16, 5*a/8]\};$

$\text{Card}(cL, cR)$  is denoted as  $q6$ ,  $q6 = (S(a/2) - S(7*a/16)) * (S(5*a/8) - S(9*a/16))$ .

2.2.3  $H(a) \sim (J(a) + p1 + \dots + p\alpha) / (J(a) + K(a) + p1 + \dots + p\alpha + q1 + \dots + q\alpha)$ ,  $\alpha < N(a)$ .

Maximum error is denoted as  $O\alpha(a)$ ,  $O\alpha(a) \sim 1/(\alpha+2)$ .

$\alpha = 2^\beta - 2$ ,  $\beta$  belongs to  $N$  and  $\beta > 0$ .

Let  $\beta = [\ln(a/4)/\ln(2)]$ ,  $O\alpha(a) \sim 0$  when  $a > a0$ .

$H(a) \sim (J(a) + p1 + \dots + p14) / (J(a) + K(a) + p1 + \dots + p14 + q1 + \dots + q14)$

Maximum error is denoted as  $O14(a)$ ,  $O14(a) \sim 1/16$ .

$H(a) \sim (J(a) + p1 + \dots + p30) / (J(a) + K(a) + p1 + \dots + p30 + q1 + \dots + q30)$

Maximum error is denoted as  $O30(a)$ ,  $O30(a) \sim 1/32$ .

$H(a) \sim (J(a) + p1 + \dots + p62) / (J(a) + K(a) + p1 + \dots + p62 + q1 + \dots + q62)$

Maximum error is denoted as  $O62(a)$ ,  $O62(a) \sim 1/64$ .

$H(a) \sim (J(a) + p1 + \dots + p126) / (J(a) + K(a) + p1 + \dots + p126 + q1 + \dots + q126)$

Maximum error is denoted as  $O126(a)$ ,  $O126(a) \sim 1/128$ .

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### 2.3 Conclusions

$S(a) = Ch * (a/2 - a/\ln(a))$ ,  $Ch \sim 1$  when  $a > a0$ .

Error of  $S(a) \sim a/2 - a/\ln(a)$  is denoted as  $Oe(a)$ ,  $Oe(a) \sim 0$  when  $a > a0$ .

(1)  $H(a) \sim (J(a) + p1 + \dots + p\alpha) / (J(a) + K(a) + p1 + \dots + p\alpha + q1 + \dots + q\alpha)$ ,  $\alpha = 2^\beta - 2$ .



(2) If  $T(a)=0$ , then  $H(a)\sim(S(a)-N(a)-X(a)+X(a-2))/(Y(a)-Y(a-2))$

But,

$$(J(a)+p1+\dots+p\alpha)/(J(a)+K(a)+p1+\dots+p\alpha+q1+\dots+q\alpha)>(a/4-a/\ln(a)-((a/2-1)/2-(a/2-1)/\ln(a/2-1))*((a/2-1)/2-(a/2-1)/\ln(a/2-1)+1)/2+((a/2-2)/2-(a/2-2)/\ln(a/2-2))*((a/2-2)/2-(a/2-2)/\ln(a/2-2)+1)/2)/(((a/2)/2-(a/2)/\ln(a/2))*(((a-1)/2-(a-1)/\ln(a-1))-((a/2-1)/2-(a/2-1)/\ln(a/2-1)))-((a/2-1)/2-(a/2-1)/\ln(a/2-1))*(((a-3)/2-(a-3)/\ln(a-3))-((a/2-2)/2-(a/2-2)/\ln(a/2-2))))).$$

So,  $T(a)>0$  when  $a>a1$ .

Error analysis endorse  $a1(\text{minim})=0$ , appendix.

Any  $T(a)>0$ ,  $a>0$ .

Conclusion: If {1 is also a prime number} is true, then any even number greater than 0 can be written as the sum of two prime numbers.

(3) If  $T(a)=1$ , then  $H(a)\sim(S(a)-N(a)+1-X(a)+X(a-2))/(Y(a)-Y(a-2))$

But,

$$(J(a)+p1+\dots+p\alpha)/(J(a)+K(a)+p1+\dots+p\alpha+q1+\dots+q\alpha)>(a/4-a/\ln(a)+1-((a/2-1)/2-(a/2-1)/\ln(a/2-1))*((a/2-1)/2-(a/2-1)/\ln(a/2-1)+1)/2+((a/2-2)/2-(a/2-2)/\ln(a/2-2))*((a/2-2)/2-(a/2-2)/\ln(a/2-2)+1)/2)/(((a/2)/2-(a/2)/\ln(a/2))*(((a-1)/2-(a-1)/\ln(a-1))-((a/2-1)/2-(a/2-1)/\ln(a/2-1)))-((a/2-1)/2-(a/2-1)/\ln(a/2-1))*(((a-3)/2-(a-3)/\ln(a-3))-((a/2-2)/2-(a/2-2)/\ln(a/2-2))))).$$

So,  $T(a)>1$  when  $a>a2$ .

Error analysis endorse  $a2(\text{minim})=4$ , appendix.

Any  $T(a)>1$ ,  $a>4$ .

Conclusion: If {1 is also a prime number} is false, then any even number greater than 4 can be written as the sum of two prime numbers.

## 2.4 $T(a)$

The number of prime pair an even  $a$  can be written as is denoted as  $T(a)$ ,

$$T(a)\sim(Y(a)-Y(a-2))*J(a)/(J(a)+K(a)+X(a)-X(a-2)-S(a)+N(a));$$

$$T(a)\sim(((a/2)/2-(a/2)/\ln(a/2))*(((a-1)/2-(a-1)/\ln(a-1))-((a/2-1)/2-(a/2-1)/\ln(a/2-1)))-((a/2-1)/2-(a/2-1)/\ln(a/2-1))*(((a-3)/2-(a-3)/\ln(a-3))-((a/2-2)/2-(a/2-2)/\ln(a/2-2))))*((a/4)/2-(a/4)/\ln(a/4))*(((3*a/4)/2-(3*a/4)/\ln(3*a/4))-((a/2)/2-(a/2)/\ln(a/2)))/(((a/4)/2-(a/4)/\ln(a/4))*(((3*a/4)/2-(3*a/4)/\ln(3*a/4))-((a/2)/2-(a/2)/\ln(a/2)))+(((a/2)/2-(a/2)/\ln(a/2))-((a/4)/2-(a/4)/\ln(a/4))*((a/2-a/\ln(a))-((3*a/4)/2-(3*a/4)/\ln(3*a/4))))+((a/2-1)/2-(a/2-1)/\ln(a/2-1))*((a/2-1)/2-(a/2-1)/\ln(a/2-1)+1)/2-((a/2-2)/2-(a/2-2)/\ln(a/2-2))*((a/2-2)/2-(a/2-2)/\ln(a/2-2)+1)/2+a/\ln(a)-a/4.$$

## 2.5 New Conjecture

$$J(a)/(J(a)+K(a))\sim(J(a)+p1+\dots+p\alpha)/(J(a)+K(a)+p1+\dots+p\alpha+q1+\dots+q\alpha)$$

APPENDIX

Function	If	Error
$N(a)=a/4,$ $a/2$ belongs to A; $N(a)=(a+2)/4,$ $a/2$ belongs to B. $a>0$	$N(a)\sim a/4$	$O(a)$
$H(a)=Z(a)/Y(a)$	$H(a)\sim H(a-2)$	$O_r(a)$
$H(a)=Z(a)/Y(a)$	$H(a)\sim (J(a)+p_1+\dots+p_\alpha)/(J(a)+K(a)+p_1+\dots+p_\alpha+q_1+\dots+q_\alpha)$	$O_\alpha(a)$
$S(a)=a/2-I(a)$	$S(a)\sim a/2-a/\ln(a)$	$O_e(a)$