# A new calculable Orbital-Model of the Atomic Nucleus based on a Platonic-Solid framework and based on constant Phi

Harry K. Hahn

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#### **Abstract :**

Crystallography indicates that the structure of the atomic nucleus must follow a crystal-like order. Quasicrystals and Atomic Clusters with a precise Icosahedral- and Dodecahedral structure indicate that the five Platonic Solids are the theoretical framework behind the design of the atomic nucleus. With my study I advance the hypothesis that the reference for the shell-structure of the atomic nucleus are the Platonic Solids. In my new model of the atomic nucleus I consider the central space diagonals of the Platonic Solids as the long axes of Proton - or Neutron Orbitals, which are similar to electron orbitals. Ten such Proton- or Neutron Orbitals form a complete dodecahedral orbital-structure (shell), which is the shell-type with the maximum number of protons or neutrons. An atomic nucleus therefore mainly consists of dodecahedral shaped shells. But stable Icosahedral- and Hexagonal-(cubic) shells also appear in certain elements. Constant PhI which directly appears in the geometry of the Dodecahedron and Icosahedron seems to be the fundamental constant that defines the structure of the atomic nucleus and the structure of the wave systems (orbitals) which form the atomic nucelus. Albert Einstein wrote in a letter that the true constants of an Universal Theory must be mathematical constants like Pi ( $\pi$ ) or e. My mathematical discovery described in chapter 5 shows that all irrational square roots of the natural numbers and even constant Pi ( $\pi$ ) can be expressed with algebraic terms that only contain constant Phi ( $\phi$ ) and 1 Therefore it is logical to assume that constant Phi, which also defines the structure of the Platonic Solids must be the fundamental constant that defines the structure of the atomic nucleus. Indication for the important role which the Dodecahedron plays in the structure of a matter also seems to come from the observation of the M87 black hole.

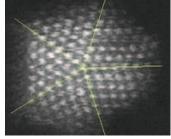
#### Please also see the following two studies :

The Black Hole in M87 (EHT2017) may provide evidence for a Poincare Dodecahedral Space Universe ( or alternative  $\rightarrow$  here )

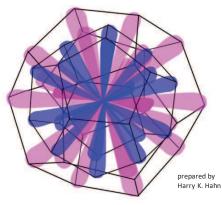
and

Genesis of an Universal Physical Theory based on constant Phi as considered by Albert Einstein



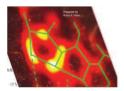


TEM-image of an Icosahedral Gold nanoparticle (Atomic Cluster).



#### A new Orbital-Model of the Atomic Nucleus

The image shows an atomic nucleus with two Shells, according to the new model. The two Shells are fully occupied with the maximum of 10 Orbitals. The 10 Orbitals of each shell represent the diagonals of a Dodecahedron which is the reference sructure for a full Shell. The Shells are slightly rotated to each other.



Brightness- and contrastenhanced image of the shadow of the M87 central black hole (EHT2017) which indicates a Dodecahedral structure of the gravitational singularity (grav. Maximum)

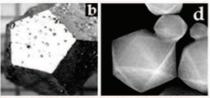


Fig 1: b.) Dodecahedral Quasicrystal Ho-Mg-Zn d.) Icosahedral Silica Quasicrystal (Nanocrystals)

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#### 1 A new model for the Atomic Nucleus based on the Platonic Solids

Beside evidence which comes from Quasicrystals, Atomic Clusters and from Crystallography, and indication that comes from the M87 central gravitational singularity, that the structure of matter seems to be based on a framework which refers to the Platonic Solids, there are also some pure logical

arguments which demand a new and redesigned model of the **atomic nucleus**.

No logical Model of the atomic core

From the logical point of view the current model of the atomic nucleus, a "disorderly heap of balls" can't be correct !

The ordered arrangement of atoms in crystal structures demands an ordered structure of the atomic nucleus, because there is a clear correlation between a crystal structure and the structure of the electronshells of the atoms which form the crystal structure. Further

the electron shell structure of an atom (element) is clearly influenced by the atomic nucleus as the Hyperfine-Structure of the spectral-lines of a certain element precisely indicates.

#### To the concept for the new Atomic Nucelus model :

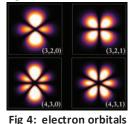
In the new model of the atomic nucleus I consider that the structure of the nucleus has clear similarities to the structure of the electron shells. This is logical from my point of view.

I also consider that a proton has an orbital-structure similar like an electron. And I consider that the mass of the Proton, which is 1836 times that of the electron, isn't a product of "hypothetical static quarks" but rather a result of the high frequency & energy of the wave that forms the proton orbital

 $\rightarrow$  (some indication that the "Quark-Theory" may be wrong:) see:Link1, Link2 **Definition of the Platonic Solid Orbitals** 

The Platonic Solids are the perfect 

Fig 3: 4f-electron orbital



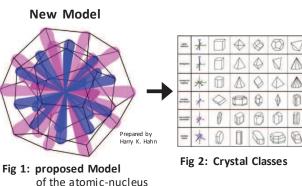
reference framework for an ordered structure of the atomic nucleus. The 5 Platonic Solids (PS) are regular convex uniform polyhedrons which consist of polygonal faces that are

identical in shape & size, and all angles are equal between the space diagonals which pass through the center of the PS (  $\rightarrow$  the diagonals between two vertices on opposite sides of the PS ).

In my model I consider the space diagonals between two vertices that pass through the center of the **PS** as the long axes of the Proton- or Neutron-Orbitals. Each Platonic Solid has a specific number of such space-diagonals and therefore a defined maximal number of Orbitals which can be occupied. For example the **Icosahedron** has **6** & the **Cube 4** such orbitals The **Dodecahedron** has the highest number of **10** such orbitals A full occupied "Dodecahedral Orbital-Structure" therefore is considered to be a "full Sub-Shell" in this nucleus model.

The inner sub-shells of most elements are considered to be full occupied "dodecahedral Sub-Shells". But there seem to

be also "hexagonal- and icosahedral sub shells" that occur in some elements. Because there are nearly always more Neutrons than Protons in an atomic nucleus, the Neutron is considered to be a "companion-Orbital" that belongs to each Proton. (see image). However it is also considered that "excess-neutrons" can form complete Neutron-Sub Shells



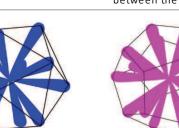
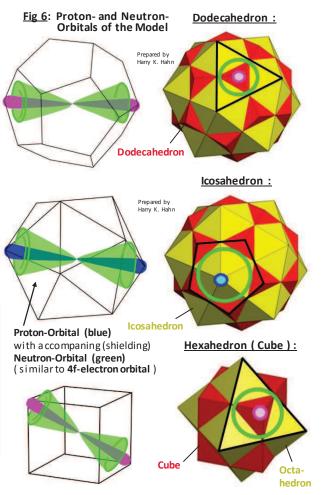
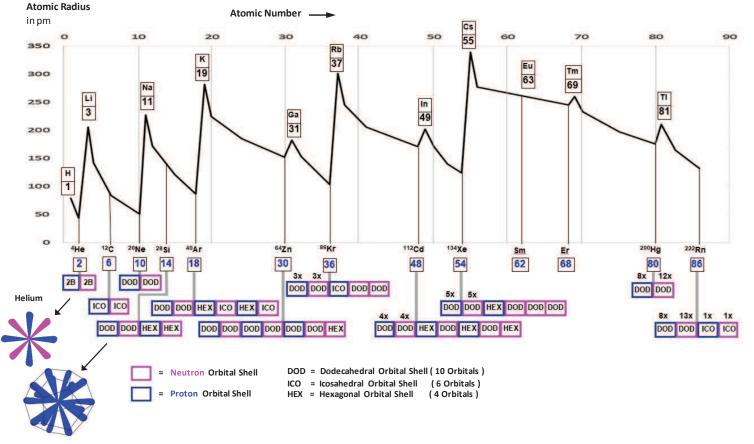


Fig 5: An icosahedral Sub-Shell with 6 Orbitals and an dodecahedral Sub-Schell with 10 Orbitals



→ Current Model !

Coulomb Coulomb Forces want to achieve equal distances between the Protons



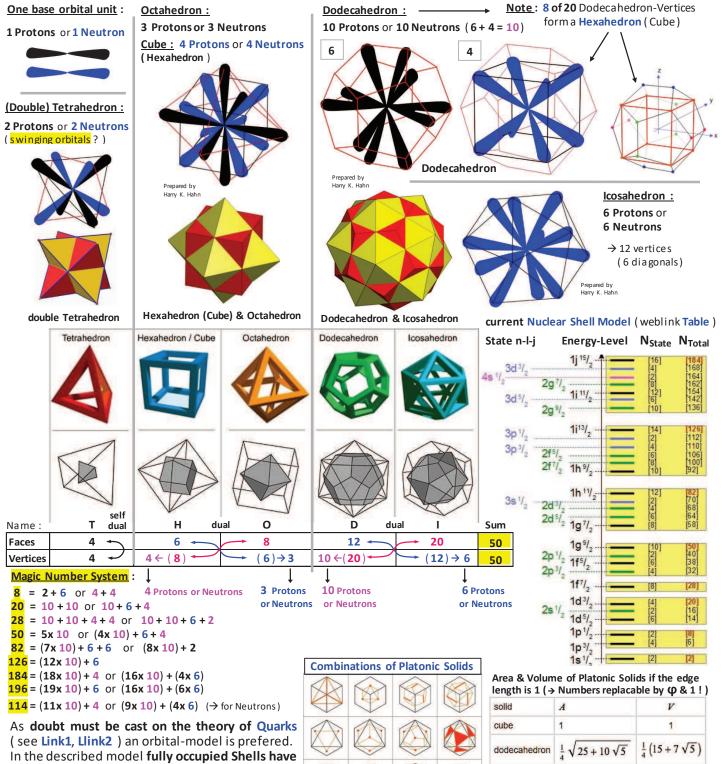
A new Atomic Nucleus Model based on a Platonic-Solids Framework :  $\rightarrow$  selected elements of the periodic table are described with the new Model in a schematic way  $\rightarrow$  see below

Dodecahedral Orbital Shell fully occupied  $\rightarrow$  10 Orbitals

## 1.1 A new Orbital-Model of the atomic nucleus defined by the diagonals of the Platonic Solids

The described new model of the **atomic nucleus** can also explain the **magic numbers** in nuclear physics : (2, 8, 20, 28, 50, 82, 126..) easily with a framework based on the Platonic Solids, (& based on constant Phi)

- 1.) In this model **1** Proton or **1** Neutron is represented by one orbital similar to p- or f-orbitals of Hydrogen
- 2.) The diagonals of the Platonic Solids represent & define the long axes of these Proton & Neutron-Orbitals
- 3.) The Platonic Solids represent Shells of the new nuclear shell model (NSM) (→ general info to NSM here) If a Shell is complete (→ each Shell tries to form a Dodecahedron !), then a new shell begins.
- 4.) All Proton- & Neutron-Orbitals can be precisely calculated with wave equations (e.g. Schrödinger WE)
- 5.) The defined nucleus orbitals influence the electron orbitals and cause in this way the Hyperfine-Structure
- 6.) Each Shell consists either of Proton-Orbitals accompanied by Neutron-Orbitals or of Neutron-Orbitals



 $\sqrt{3}$ 

 $\frac{1}{4}\sqrt{3}$ 

 $\frac{1}{4}\sqrt{3}$ 

icosahedron

octahedron

tetrahedron

 $\frac{5}{12}(3+\sqrt{5})$ 

 $\frac{1}{3}\sqrt{2}$ 

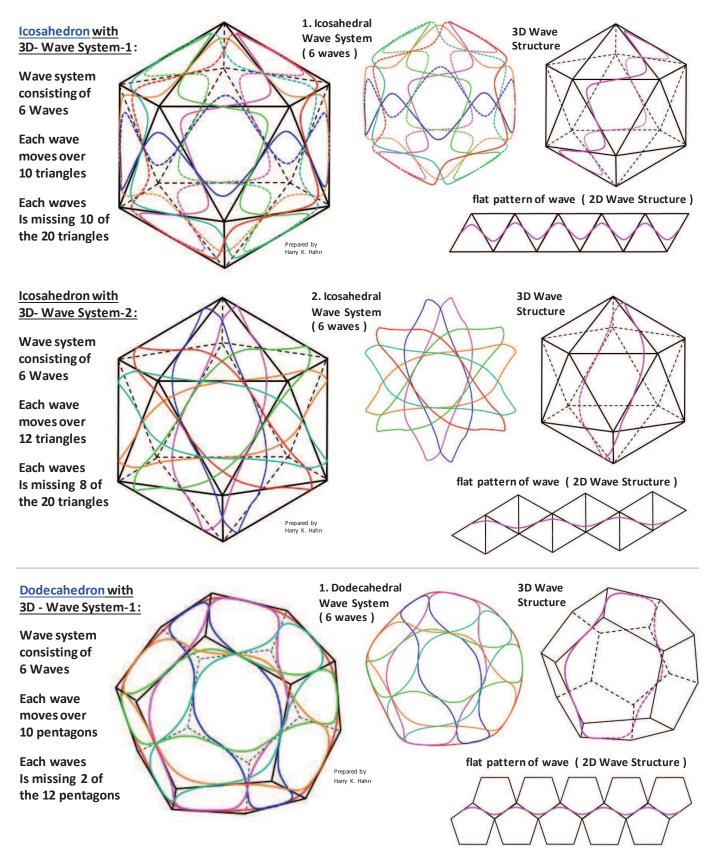
 $\frac{1}{12}\sqrt{2}$ 

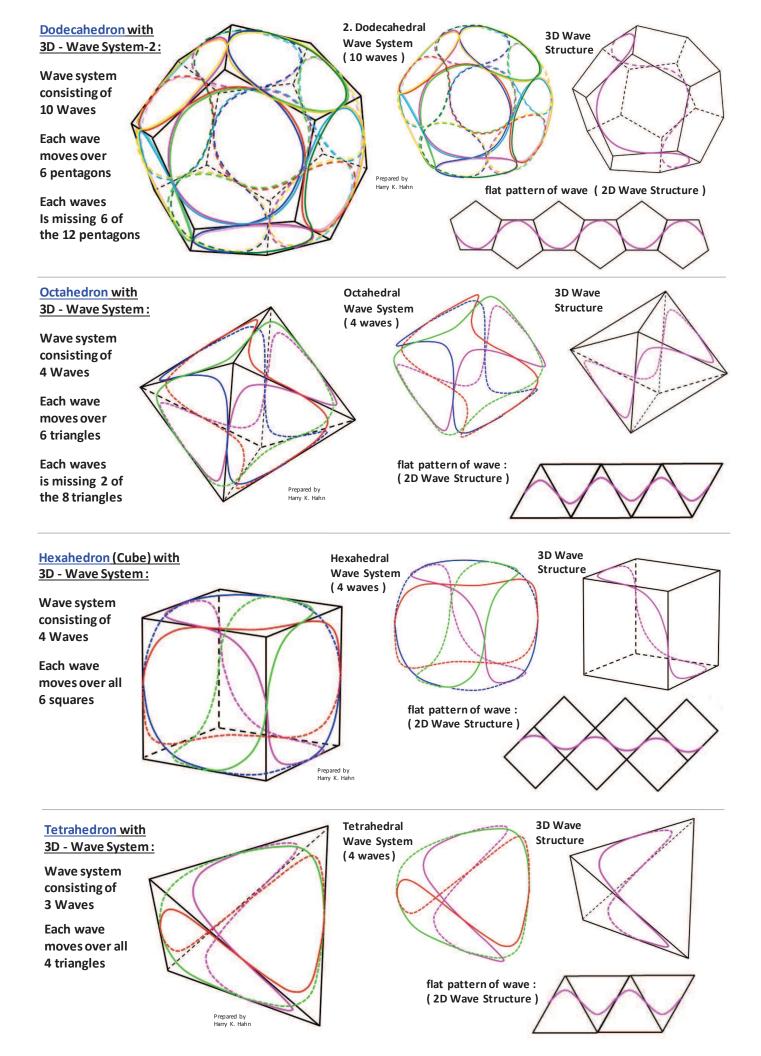
a Dodecahedral-Shell-Structure. This model can explain dodecahedral gravitational singularities as EHT2017 which must be a result of a principle Dodecahedral atomic nucleus structure

a principle Dodecahedral atomic nucleus structure <u>Note</u> : All Platonic Solids are only based on  $\phi \& 1$ !

## 1.2 The Platonic Solids also provide the framework for Platonic-Solid Shell-Wave-Systems

Beside an Orbital Model for the **atomic nucleus** that is based on the diagonals of the **Platonic Solids**, there is also a **system of "Platonic-Solid Shell-Waves**" thinkable.  $\rightarrow$  see the following images below ! In such a model the shown "**Platonic-Solid Shell-Waves**" may directly represent Protons or Neutrons. Or such "Platonic-Solid Shell-Waves" may have the function to stabilize Proton- & Neutron-Orbitals along the diagonals of the Platonic Solids (framework) as described on the previous page. In this case they may be high-energetic electromagnetic waves. <u>Note:</u> The existence of icosahedral atomic clusters and of icosahedral-& dodecahedral Quasicrystals indicates that outer atomic nucleus shells of certain elements (e.g. Gold) may have icosahedral- or dodecahedral structures which influence the shape of their electron shell ( $\rightarrow$  Hyperfine-Structure)





## 1.3 Development of a Reference-Framework for the Atomic Nucleus based on Platonic Solids

The image on the right shows an example how a reference framework for a nuclear shell model based on Platonic Solids can be developed. The Platonic Solids can be inscribed into each other, or superimposed in different combinations, but geometrically precisely defined in their orientation.

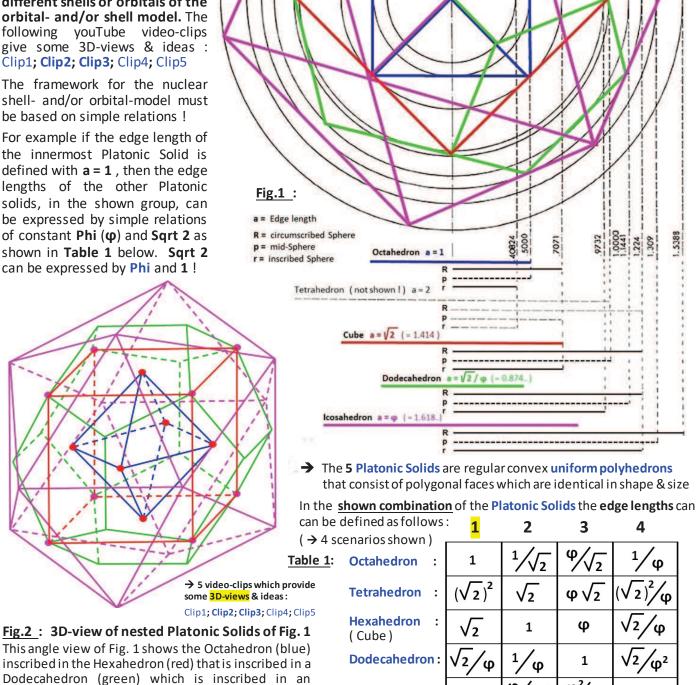
There are various shell-models possible. The shown shell-system is only an example. Shell-systems with many more layers (shells) can be defined ! The Inscribed **Mid-Spheres** Spheres, and **Circumscribed Spheres of the** Platonic Solids can here be used as inner- and outer limits of different shells or orbitals of the orbital- and/or shell model. The following youTube video-clips give some 3D-views & ideas : Clip1; Clip2; Clip3; Clip4; Clip5

The framework for the nuclear shell- and/or orbital-model must be based on simple relations !

For example if the edge length of the innermost Platonic Solid is defined with **a** = 1, then the edge lengths of the other Platonic solids, in the shown group, can be expressed by simple relations of constant Phi (φ) and Sqrt 2 as shown in Table 1 below. Sort 2 can be expressed by Phi and 1 !

Icosahedron (pink). The Icosahedron forms the outer

shell of this nested Platonic Solids group.



Icosahedron

φ

PSF-1: Top-view of a group of nested Platonic Solids

φ²

 $\sqrt{2}$ 

1

# 2 Constant Phi $(\phi)$ defines the structure of Matter

Constant PhI ( $\varphi$ ) seems to be the fundamental constant that defines the scattering (distribution) of particles and waves in the universe, and the base unit (number) 1 seems to represent a base energy/wave element like the quantum of electromagnetic action  $\rightarrow$  the Planck Constant h that is defined by Energy / Frequency The Mathematical Physical Triangle (MPT) and the Square Root Spiral (SRS) represent a starting point for an Universal Mathematical- & Physical Theory. To understand the meaning of the MPT & SRS for Physics we must understand the physical meaning of the most irrational constant PhI ( $\varphi$ ) and the meaning of the base unit 1 !

Please note that the **MPT**, the **SRS** and in all probability all existing **irrational constants** can be represented by transparent constructions out of **base unit** (**number**) **1** and the most **simple**  $\varphi = \frac{1}{1 + 1}$ 

infinite continued fraction : Constant PhI ( $\phi$ ) that is only based on Number 1 — which essentially is the set-up for a universal theory Albert Einstein was looking for !

 $\varphi = \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$ 

In Physics the socalled **Continued Fraction Method (MCF)** was developed for solutions of integral equations of the **Quantum Scattering Theory**, which describe particle-collisions and –scattering of particles and waves. For example the **MCF** is used for the **Fermi-Dirac Function**, which describes the macroscopic properties of a system consisting of **Fermions** ( $\rightarrow$  particles which form matter : quarks, leptons and baryons : ( e.g. electrons, protons ).  $\rightarrow$  see weblinks to some exemplary studies which use the MCF :

Continued fraction representation of the Fermi Dirac function for large scale electronic structure calculations Application of matrix valued integral continued fractions to the spectral problems on periodic graphs (see also: Algebra of waves) The MCF for electron (positron)-atom scattering and The MCF in the theory of slow electron scattering by molecules

On the other hand there is clear indication that constant Phi ( $\phi$ ) also defines the distribution of matter in the astronomical-scale ! For example the ratios of orbital periods of Solar-planetary and Exo-planetary systems show a preference for Fibonacci-Number ratios, which are defined by constant PhI ( $\phi$ ).  $\rightarrow$  See the Study 5 in Chapter 8.5. And if the M87 gravitational singularity (EHT2017) indeed indicates a dodecahedral structure then we even have a proof that PhI ( $\phi$ ) defines the distribution of matter in very extreme gravitational fields.

The Volume of the Dodecahedron and Icosahedron expressed only with constant  $\phi$  and 1 !

$$V_{\text{Dod.}} = \frac{a^{3}}{4} (15 + 7\sqrt{5}) \rightarrow V_{\text{Dod.}} = \frac{\phi^{4} (\phi^{4} - 1)}{\phi^{4} - \phi^{2} + 1} ; \text{ for } a = 1 \text{ (edge)}$$

$$V_{\text{Ico.}} = \frac{5a^{3}}{12} (3 + \sqrt{5}) \rightarrow V_{\text{Ico.}} = \frac{\phi^{2} (\phi^{4} - 1)^{2}}{(\phi^{4} - \phi^{2} + 1) (\phi^{4} + 1)} = \frac{Fig_{1:} \text{ b.} \text{ Dodecahedral Quasicrystal Ho-Mg-Zn}}{(b_{1:}b_{1:}b_{2:$$

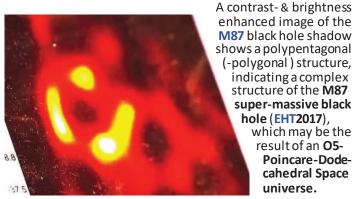
→ Three possible ways to find the Universal Theory and uncover the true geometry of our Universe :

1.) Through advances in Number Theory.  $\rightarrow$  By applying a new mathematics based only on Phi and 1

2.) By fully understanding the Physics & Mathematics of Quasicrystals, Atomic-Clusters & Atomic-Nuclei

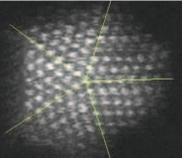
3.) By high-resolution astronomical observations of gravitational singularities, like the one in M87

The M87 black hole has a complex (dodecahedral) geometry



This image is from the section of the documentation **Black Hole Hunters** which shows how the algorithms calculate the first image of the M87 black hole shadow. <u>Note :</u> only the left (bright) ring was presented to the public ! The rest of the structure was ignored ! Icosahedral Atom-Clusters indicate an icosahedral structure of the atom's electron shell & atomic-core

Elements which are chemically inert often form clusters with an icosahedral shape. The shell structure of such Icosahedral clusters is defined by the electron configuration of the whole cluster, that is a consequence of the electron shell of the single atom, which again is a result of the atom-core-structure ! Image: MacKay cluster made of Gold atoms.



the single atom, which TEM-image of an **Icosahedral Gold** again is **a result of the nanoparticle**. A variety of nano**atom-core-structure**! structures assume icosahedral form Image: MacKay cluster made of Gold atoms.  $( e.g. condensing Argon- \& metal atoms ) \rightarrow see : Icosahedral Twins$  $( \rightarrow see also : Superatom )$ 

## 3 Quasicrystals will help us to find the true geometry & dimensionality of Matter

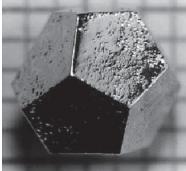
Here on Earth we can find out the true geometry and dimensionality of space-time and matter by fully understanding Quasicrystals and quasicrystalline structures from the mathematical - and physical point of view. Quasicrystals indicate that space-time and/or matter (energy) is based on higher geometrical dimensions of at least 4 maybe even 5 or 6! According to the classical Crystallographic Restriction Theorem crystals can only possess two-, three-, four- and six-fold rotational symmetries. However in 1984 Quasicrystals with five-fold symmetry were discovered ! Two types of Quasicrystals exist: 1.) Polygonal Quasicrystals which have one axis of 8-, 10- or 12-fold local symmetry.

2.) Icosahedral Quasicrystals. These Quasicrystals are aperiodic in all directions. Similar to atomic clusters ( $\rightarrow$  previous page) the electron shell structure (electron configuration) of the whole quasicrystal defines ist structure -> see : Scanning tunnelling microscopy reveals a quasiperiodic order in the electronic wave functions For Quasicrystals at least 5 linearly independent vectors are necessary in order to assign integer indices to the diffraction intensities of quasicrystals. We need 5 indices for polygonal quasicrystals and 6 indices for icosahedral quasicrystals. The necessary n vectors span a nD-reciprocal space in which a structure can be built that produces a diffraction pattern as observed for quasicrystals. In higher-dimensional space we can describe a quasiperiodic structure as a periodic one.

The mathematics which describes quasicrystals indicates that matter (energy) has higher embedded dimensions ! Nicolaas G. de Brujin showed that aperiodic quasicrystal-like Penrose Tilings can be viewed as 2-dimensional slices of five-dimensional hypercubic structures. The study of Penrose Tilings is important for understanding Quasicrystals. (->Simulation of 2D-quasicrystals : Quasicrystalline Bose-Einstein-Condensate provides a glimpse of physics in higher dimensions)

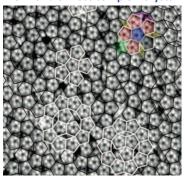


Electron Microscope image of a real Decagonal Quasicrystal (Weblink)



Dodecahedral HoMgZn quasicrystal A Quasicrystal is a structure that is ordered but not periodic, it lacks translational symmetry.

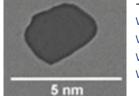
In a simulation scientists showed that the golden ratio (Phi) governs the interaction of the atoms. See : The World's most complex Crystal



A special Two-dimensional quasicrystal made from self-assembling organic molecules which form pentagons, stars, boats and rhombi → see : Bizarre organic Quasicrystal

Interesting is the fact that the production of Quasicrystals in the lab is difficult and tricky, requiring precise temperatures and strange conditions including a vacuum and an Argonatmosphere, see Weblink 4 ( $\rightarrow$  Argon is an inert noble gas that forms Icosahedral clusters during condensation)

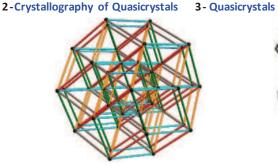
Natural (mineral) guasicrystals only seem to form under extreme high-pressure and temperature conditions  $\rightarrow$  see right image  $\rightarrow$ 

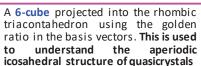


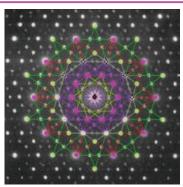
 $\rightarrow$  see : Weblink 1 Weblink 2 Weblink 3 Weblink4

**Extraterrestrial Quasicrystal** 

A grain of stishovite that only occurs at ultrahigh pressures ( ≥10 Gpa ), contains an Icosahedrite Quasicrystal Three studies about the Mathematics of Quasicrystals: inclusion Al63Cu24Fe13



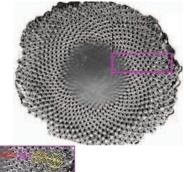




А **5-cube** as an orthographic projection into 2D using Petrie polygon basis vectors overlaid on the diffractogram from an icosahedral Ho-Mg-Zn quasicrystal



Four-Dimensional Iron-Oxid Fe<sub>4</sub>O<sub>5</sub> below 150K this iron-oxid goes through an unusual phase transition related to a formation of chargedensity waves-which lead to a "four-dimensional crytal structure" It formed at very high temperatures and very high pressure hundreds of kilometres below Earth's surface.



Sunflower capitulum : The Fibonacci number spirals (Phyllotactic-pattern) indicate an icosahedral quasicrystal structure, probably caused by the large icosahedral Water-Cluster (H2O)100 or  $(H_2O)_{280} \rightarrow$  see my following Study

## 4 There is clear evidence that the structure of matter (the atomic nucleus) is related to the Platonic Solids

The Platonic Solids **Icosahedron & Dodecahedron** are closely related to **constant Phi** ( $\phi$ ). The Icosahedron plays an important role in the structure of extremely stable atomic- and molecular- clusters. And together with the Dodecahedron the Icosahedron can **describe quasi-crystalline structures with five-fold symmetry**.

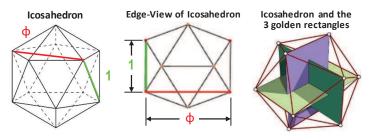
The Dodecahedron also seems to play an important role in the large scale structure of the universe, especially in the formation of very strong gravitational maxima (supermassive black holes) like the one in **M87**.

The **Icosahedron** & **Dodecahedron** form Quasicrystals that can only be described in "higher-dimensional" space Therefore these Platonic Solids must play a crucial role in unifying Quantum Mechanics with General Relativity

#### The Icosahedron :

#### The Icosahedron is No. 5 of the five Platonic Solids

With 20 faces it has the most faces of the five Platonic Solids. The faces are equilateral triangles Further the Icosahedron has 30 edges and 12 vertices. It has the Schläfli symbol {3,5}



**<u>Fig 1</u>**: shows how the **Golden Ratio** ( $\rightarrow$  the ratio of constant **Phi** ( $\phi$ ) and 1) is built into the Geometry of the **Icosahedron** 

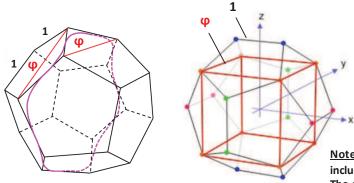
Formulas for the Icosahedron in reference to the sphere :

If the edge length of the Icosahedron is <i>a</i>
the radius of a <u>circumscribed sphere</u> <b>r</b> <sub>u</sub> is : $r_u = \frac{a}{4} \cdot \sqrt{10 + 2 \cdot \sqrt{5}}$
and the radius of an <u>inscribed sphere</u> <b>r</b> i is : $r_i = \frac{a}{12} \cdot \sqrt{3} \cdot (3 + \sqrt{5})$
while the <u>mid-radius</u> <b>r</b> k ( mid of edges ) is : $r_k = rac{a}{4} \cdot \left(1 + \sqrt{5} ight)$

## The Dodecahedron :

#### The Dodecahedron is No. 4 of the five Platonic Solids

With 20 vertices It has the most vertices of all Platonic Solids. The 12 faces of the Dodecahedron are regular Pentagons. Further it has 30 edges, and its Schläfli symbol is {3,5}



**Fig 3**: shows how **constant Phi** ( $\phi$ ) and **1** are built into the Geometry of the **Dodecahedron** and the **Pentagon** 

#### Dodecahedral & Icosahedral Quasicrystals :

In the higher-dimensional space we can describe a quasi-periodic structure as a periodic one. The actual guasiperiodic structure in the 3D-physical space can then be obtained by appropriate projection/section techniques. Thus it is enough to define a single unit cell of the nD-structure. The contents of that nD-unit cell consists of "hyperatoms" (occupation domains, ..) in analogy to the atoms in a normal unit cell. This enables us to describe the whole quasicrystal structure with a finite set of parameters. If we described it in 3Dspace only, we needed thousands of atoms to obtain a representative volume segment of the whole structure as well as all parameters that go with it (eg. thousands of positions).  $\rightarrow$  see : weblink

→ Image Source

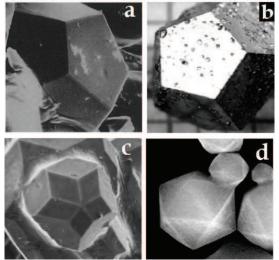
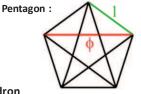


Fig 2 : a.) Dodecahedral Quasicrystal Al65Cu20Fe15 b.) Dodecahedral Quasicrystal Ho-Mg-Zn c.) Quasicrystal Al-Cu-Fe with Triacontahedron shape

d.) Icosahedral Silica Quasicrystal (Nanocrystals)



<u>Note</u>: The Dodecahedron includes the Cube structure ! The cube-edges cross all 12 pentagons

## The Dodecahedron :

A regular Dodecahedron or pentagonal dodecahedron is a dodecahedron that is regular, which is composed of twelve regular pentagonal faces (Pentagons) three meeting at each vertex

The Dodecahedron has 12 faces, 20 vertices, 30 edges, and 160 diagonals (60 face diagonals, 100 space diagonals).

It is represented by the Schläfli symbol {5,3}.

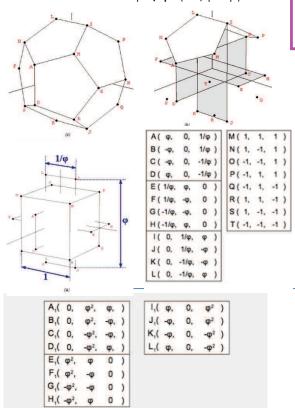
If the edge length of a regular dodecahedron is a , the radius of a circumscribed sphere ru (one that touches the regular dodecahedron at

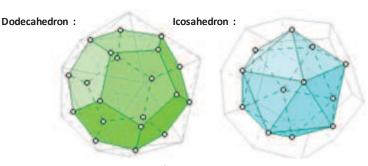
all vertices) is : and the radius of an inscribed sphere **r**i (tangent to each of the regular dodecahedron's faces) is :

while the midradius **r**m, which touches the middle of each edge, is :

#### Please have a look at the following websides $\rightarrow$ The golden ratio Phi ( $\varphi$ ) in Platonic Solids : $\rightarrow$ Phi\_sacred\_Solids

The Dodecahedron in cartesian coordinates : The vertices of the dodeca hedron obtained from the cube and three orthogonal Golden Rectangles with the side relationship  $1/\phi^2$  (=  $2/\phi$  :  $2\phi$ )







Connecting the centers of adjacent faces of the dodecahedron results in an icosahedron, and connecting the centers of the icosahedron faces results in a dodecahedron.

Weblink to more formulas of the dodecahedron see: polyhedra dodecahedron

$$\begin{aligned} r_u &= a \frac{\sqrt{3}}{4} \left( 1 + \sqrt{5} \right) \quad \Rightarrow \quad r_u = a \\ r_i &= a \frac{1}{2} \sqrt{\frac{5}{2} + \frac{11}{10} \sqrt{5}} \quad \Rightarrow \quad r_i = a \\ r_m &= a \frac{1}{4} \left( 3 + \sqrt{5} \right) \quad \Rightarrow \quad r_m = a \end{aligned}$$

these quantules can also  
be expressed as :  
$$r_u = a \frac{\sqrt{3}}{2} \phi$$
$$r_i = a \frac{\phi^2}{2\sqrt{3-\phi}}$$
$$r_m = a \frac{\phi^2}{2}$$

Those quantities can also

where  $\phi$  is the golden ratio.

The Dodecahedron has geometric relations to the other four **Platonic Solids** (see also image above  $\rightarrow$  dual of the Icosahedron):

By connecting selected vertices of the dodecahedron, it is possible to form a Tetrahedron or a Cube. By connecting midpoints of certain edges, it is possible to form an Octahedron :



The small stellated Dodecahedron contains three powers of  $\phi$ 

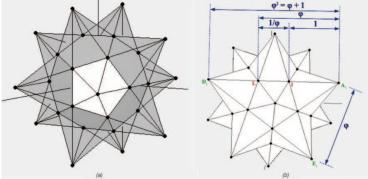


Figure 6: The small stellated dodecahedron contains three powers of the Golden Ratio. This can be cle Its outer vertices define an icosahedron whose edge length ( $A_1E_1$ ) is in a proportion 1. $\phi^2$  with the edge of the original dodecahedron (IL).

Table 4: The cartesian coordinates of the small stellated dodecahedron. The vertices of the original dodecahedron (Table 3), though visible, are not vertices of the stellation.

## 5 The important connection between Mathematics & Physics - The Mathematical Physical Triangle (MPT)

The following study regarding Special Relativity provided an **important algebraic term** for cathetus **u** of the **MPT**, which makes it possible to base Number Theory and Geometry ( and Physics ! ) on a transparent construction out of **Number 1**, as **Albert Einstein** was hoping for ! :

→ "Phase spaces in Special Relativity : Towards eliminating gravitational singularities" - by Peter Danenhower

See Weblink: <u>https://arxiv.org/pdf/0706.2043.pdf</u>

This study uses phase spaces in special relativity by expanding Minkowski Space to model the physical world.

The phase spaces developed in this study indicate that graviational singularities can be eliminated !

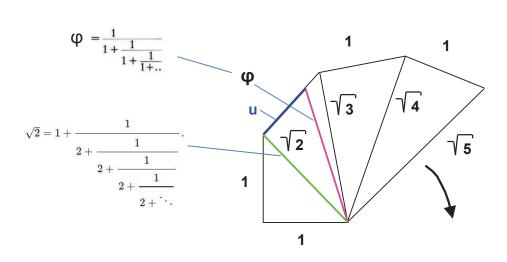
The key mathematical idea used in this study is the inclusion of a complex phase factor, such as,  $e^{i\Phi}$  in the Lorentz transformation, and to use both the proper time and the proper mass as parameters. Additional a simple (invariant) parameter, the "energy to length" ratio, defined by  $c^4/G$  was used for any spherical region of space-time-matter.

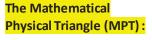
This study may show a way forward to combine General Relativity with Quantum Mechanics.

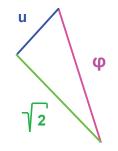
The Mathematical Physical Triangle and the start section of the Square Root Spiral, together with a more profound (number theoretical- and geometrical) analysis of the Square Root Spiral will help with this task :

The first right triangles of the Square Root Spiral, which are only defined by **constant φ and 1**, not only define the complex structure of the square root spiral, but also the Platonic Solids, and they also form the base of Number Theory, Geometry and Physics as well !

## The start of the Square Root Spiral is shown with the constant $\phi$ drawn in :







With the help of the constants in this right triangle we will find an important starting point on the way to the universal theory:

From the right triangle  $\mathbf{\Phi}$ , square root of 2 & u follows :

 $\varphi^2 = (\sqrt{2})^2 + u^2$ ; application of the Pythagorean theorem

→ 
$$u = \sqrt{\phi^2 - 2} = 0,786151377....$$
; we can calculate this value of u with the calculator

I did some research in the internet with Google, and I found a study where the constant u was expressed with an algebraic term ! With the help of this algebraic term it was possible to find interesting new properties of constant  $\phi$ 

→ See next page !

#### PHASE SPACES IN SPECIAL RELATIVITY : TOWARDS ELIMINATING GRAVITATIONAL SINGULARITIES

#### from PETER DANENHOWER > see weblink : https://arxiv.org/pdf/0706.2043.pdf

Abstract : This paper shows one way to construct phase spaces in special relativity by expanding Minkowski Space. These spaces appear to indicate that we can dispense with gravitational singularities. The key mathematical ideas in the present approach are to include a complex phase factor, such as,  $e^{i\Phi}$  in the Lorentz transformation and to use both the proper time and the proper mass as parameters. To develop the most general case, a complex parameter  $\sigma = s + im$ , is introduced, where s is the proper time, and m is the proper mass, and  $\sigma$  and  $\sigma / |\sigma|$  are used to parameterize the position of a particle (or reference frame) in space-time-matter phase space. A new reference variable, u = m/r, is needed (in addition to velocity), and assumed to be bounded by 0 and  $c^2/G = 1$ , in geometrized units. Several results are derived: The equation  $E = mc^2$  apparently needs to be modified to  $E^2 = (s^2 c^{10})/G^2 + m^2 c^4$ , but a simpler (invariant) parameter is the "energy to length" ratio, which is  $c^4/G$  for any spherical region of space-time-matter. The generalized " momentum vector" becomes completely "masslike" for  $\mathbf{u} \approx 0.7861...$ , which we think indicates the existence of a maximal gravity field. Thus, gravitational singularities do not occur. Instead, as  $u \rightarrow 1$  matter is apparently simply crushed into free space. In the last section of this paper we attempt some further generalizations of the phase space ideas developed in this paper.

Extract from page 11 of the study (equation 4.9) :

$$\mathbf{P} = \frac{\left[(\sqrt{1-u^2}-u^2)+i(u\sqrt{1-u^2}+u)\right]}{\sqrt{1+u^2}}\gamma < 1, v > 0$$

In this form the real and imaginary part of  ${\bf P}$  have a very interesting property, namely, if

(4.10)  $u = \frac{\sqrt{2\sqrt{5}-2}}{2} \approx 0.786151377... = U$ , then the real part of **P** is zero, and the imaginary part takes its maximum value (= 1).

I think it makes sense to argue that when the real part of  $\mathbf{P} = 0$ ,  $\mathbf{P}$  is entirely "mass like", which we could understand to be representative of the state of space-time-matter for which the maximal gravity field occurs. In this picture gravity is understood to be the propensity of space-time-matter to become completely mass like. The more mass-like a region of space-time-matter is, then the stronger the external gravity field. Thus, within the discussion of this paper, I think the only reasonable interpretation of the existence of the special value of **u** given in equation 4.10 is that there is a maximal gravity field at this value of **u**. It is important to observe that the value of **u** considered above, substantially exceeds the value of **u** for a typical neutron star ( $\approx 0.1 - 0.2$ ).

Thus, I think the maximal gravity field concept can be used to explain all of the experimental evidence for enormous gravity fields.

→ Now we can equate the two algebraic terms which represent the same constant ! :

$$\sqrt{\phi^2 - 2} = \frac{\sqrt{2\sqrt{5} - 2}}{2}$$
; we square both sides

$$\rightarrow$$
 4 $\phi^2$  - 8 = 2 $\sqrt{5}$  - 2 ; and transform

$$\varphi^2 = \frac{\sqrt{5} + 3}{2}$$
; (1) we solve for  $\varphi^2$ 

$$\sqrt{5} = 2\phi^2 - 3$$
; (2) we solve for  $\sqrt{5}$ 

→ Now we use the following right triangle :

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2$$
; Pythagorean theorem  
6 =  $(2\phi^2 - 3)^2 + 1$ ; we replace  $\sqrt{5}$  by (2)

$$\Rightarrow 3 = \frac{\varphi^4 + 1}{\varphi^2} (3) \Rightarrow \sqrt{3} = \sqrt{\frac{\varphi^4 + 1}{\varphi^2}} (4)$$

$$\Rightarrow \text{ square root 3 expressed by (0 and 1)}$$

With the other right triangles of the square root spiral we can calculate all square roots of the natural numbers expressed only by  $\varphi$  and 1: (see Appendix 1 of study !)

See also this document :

→ Space-Time-Matter

$$2 = \frac{\phi^{4} \cdot \phi^{2} + 1}{\phi^{2}} (5) \text{ and } \sqrt{2} = \sqrt{\frac{\phi^{4} \cdot \phi^{2} + 1}{\phi^{2}}} (6)$$

$$\sqrt{5} = 2\phi^{2} \cdot \frac{\phi^{4} + 1}{\phi^{2}} \quad \Rightarrow \quad \sqrt{5} = \frac{\phi^{4} \cdot 1}{\phi^{2}} \quad ; \quad (7)$$

$$6 = \frac{\phi^{8} \cdot \phi^{4} + 1}{\phi^{4}} (8) \text{ and } \sqrt{6} = \sqrt{\frac{\phi^{8} \cdot \phi^{4} + 1}{\phi^{4}}} (9)$$

$$7 = \frac{\phi^{8} + 1}{\phi^{4}} (10) \Rightarrow \sqrt{7} = \sqrt{\frac{\phi^{8} + 1}{\phi^{4}}} (11)$$

$$\varphi^{8} + \phi^{4} + 1 (10) = \sqrt{7} = \sqrt{\frac{\phi^{8} + \phi^{4} + 1}{\phi^{4}}} (11)$$

$$8 = \frac{\phi^4}{\phi^4} (12) \quad \forall 8 = \sqrt{\phi^4} (13)$$

$$10 = \frac{\phi^8 + 3\phi^4 + 1}{\phi^4} (14) \quad \sqrt{10} = \sqrt{\phi^8 + 3\phi^4 + 1} (15)$$

$$10 = \frac{\phi^4}{\phi^4} (14) \quad \forall 10 = \sqrt{\phi^4} (15)$$

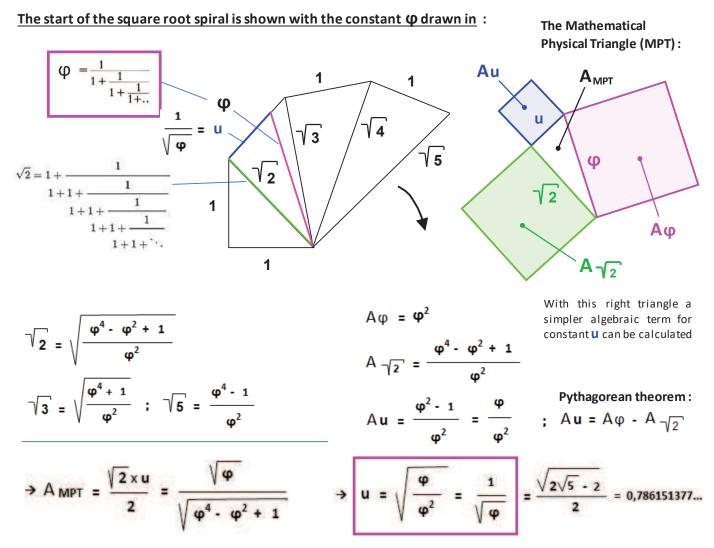
11 = 
$$\frac{\phi^{\circ} + 4\phi^{\circ} + 1}{\phi^4}$$
 (16)  $\sqrt{11} = \sqrt{\frac{\phi^{\circ} + 4\phi^{\circ} + 1}{\phi^4}}$  (17)

 $12 = \frac{\phi^8 + 5\phi^4 + 1}{\phi^4} (18) \quad \sqrt{12} = \sqrt{\frac{\phi^8 + 5\phi^4 + 1}{\phi^4}} (19)$ 

# Important for Mathematics & Physics : -> The Mathematical Physical Triangle (MPT) defined by Phi

Albert Einstein was looking for a universal theory based only on irrational constants like Pi ( $\pi$ ) & e, which are transparent constructions out of number 1 (see chapter 6), and which all have values based on the logical base of the complete theory. I.V.Volovich said that Albert Einstein tried to reduce all physics to algebraic geometry. This means the reduction of physics to Number Theory.

The Mathematical Physical Triangle (MPT) and the Square Root Spiral open the door to an Universal Theory ! With the algebraic term found by Peter Danenhower, all irrational square roots of natural numbers, constants like Pi & e, and Platonic Solids can be expressed with constant  $\varphi$  (Phi) and 1!  $\rightarrow$  all transparent constructions of 1!



From the above shown equations ( $\rightarrow$  see last two pages ) I have realized a general rule for all natural numbers > 10 :

<u>Note</u> :  $\rightarrow$  The expression (3+n) in the rule can be replaced by <u>products and / or sums</u> of the equations (3) to (13)

$$\Rightarrow (10+n) = \frac{\phi^8 + (3+n)\phi^4 + 1}{\phi^4} (20) \text{ and } \sqrt{(10+n)} = \sqrt{\frac{\phi^8 + (3+n)\phi^4 + 1}{\phi^4}} (30)$$
With this general formula we can express all natural numbers  $\ge 10$  and their square roots only with  $\phi$  and 1!
This statement is also valid for all rationals (fractions) and their square roots. This is a quite interesting discovery !!

Again to Viete's formula from 1593 :

 $\rightarrow$  It is also possible to derive from Viète's formula a related formula for  $\pi$  that still involves nested square roots of two, but uses only one multiplication :

$$\pi = \frac{2}{\sqrt{2}} \frac{2}{\sqrt{2+\sqrt{2}}} \frac{2}{\sqrt{2+\sqrt{2}+\sqrt{2}}} \cdots \qquad \pi = \lim_{k \to \infty} 2^k \sqrt{2-\sqrt{2+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}}}_{k \text{ square roots}}$$

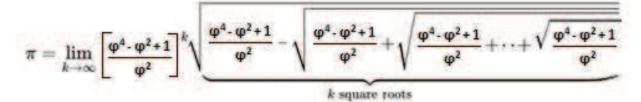
If we replace the number 2 in the above shown formulas by the found equation (5) where number 2 can be expressed by constant  $\varphi$  and 1, then we can express constant Pi ( $\pi$ ) also by only using the constant  $\varphi$  and 1!

Replace Number 2 in the above shown formulas with this term.

$$\Rightarrow \quad 2 = \frac{\phi^4 + 1}{\phi^2} - 1 \quad \Rightarrow \quad 2 = \frac{\phi^4 - \phi^2 + 1}{\phi^2} \quad (5) \text{ and } \sqrt{2} = \sqrt{\frac{\phi^4 - \phi^2 + 1}{\phi^2}} \quad (6)$$

It becomes clear that the irrationality of Pi ( $\pi$ ) is also only based on the constant  $\varphi$  and 1, in the same way as the irrationality of all irrational square roots, is only based on constant  $\varphi \& 1 !$  Numbers don't exist! Only  $\varphi \& 1$  exist!

Constant Pi ( $\pi$ ) can now be expressed in this way, by only using constant  $\phi$  and 1 :



It becomes clear that the irrationality of Pi ( $\pi$ ) is also only based on the constant  $\varphi$  and 1, in the same way as the irrationality of all irrational square roots, is only based on constant  $\varphi \& 1$ !

Numbers don't seem to exist! Natural Numbers, their square roots and irrational transcendental constants like Pi ( $\pi$ ) can be expressed by only using constant  $\varphi$  and 1 !!

This is an interesting discovery because it allows to describe most (maybe all) geometrical objects only with  $\varphi \& 1$ !

The result of this discovery may lead to a new base of number theory. Not numbers like 1, 2, 3,.... and constants like Pi ( $\pi$ ) etc. are the base of number theory ! Only the constant  $\varphi$  and the base unit 1 (which shouldn't be considered as a number) form the base of mathematics and geometry. This will certainly also have an impact on physics !

And constant  $\varphi$  and the base unit 1 must be considered as fundamental "Matter (energy) structure constants" of the real physical world ! With constant  $\varphi$  and 1 all geometrical objects including the Platonic Solids can be expressed !

There probably isn't something like a base unit if we consider a "wave model" as the base of physics and if we see the universe as one oscillating unit. In the universe everyting is connected with everything. see : Quantum Entanglement

# 6 Referring to my discovery regarding constant Phi ( $m \phi$ ) , I have defined these 12 Conjectures :

# <u>Conjectures</u> : ( $\rightarrow$ you can call them Harry K. Hahn's conjectures )

1.) All Natural Numbers and their square roots can be expressed (calculated) by only using the mathematical constant Phi (golden mean = 1.618..) and number 1. This statement is also valid for all rationals (fractions) and their square roots

2.) All existing irrational numbers seem to be constructions out of Phi and 1. For example the irrational transcendental constant Pi (3.1415926....) can also be expressed by only using Phi and 1 !

3.) Phi and 1 are the base units of Mathematics ! Numbers and number-systems don't exist ! They are manmade and therefore can be eliminated. In principle Mathematical Science can be carried out by only using Phi and 1, as base units.

4.) All geometrical objects, including the Platonic Solids can also be described by only using constant Phi and 1. Because all natural numbers, their square roots, rationals (fractions) and probably all irrational and all transcendental numbers too, can be expressed by only using Phi and 1.

5.) Point 4.) leads me to the conclusion that in the physical world the geometries of all possible crystal -lattice-structures are fundamentally based on Phi and 1. The more fundamental the lattice the simpler it can be expressed by Phi and 1.

6.) Point 4.) 5.) & 7.) lead me to the conclusion that on the molecular- and atomic-level, as well as on the macroscopic (cosmic) level the distribution and structure of matter (=energy), is fundamentally based on constant Phi and 1.

 $\rightarrow$  Therefore Constant Phi ( $\phi$ ) must be a fundamental "Energy (Matter) Structure Constant"

Because at the beginning of the universe (Big-Bang) matter formed out of pure energy, the property to form matter with defined structure must be a property of energy, from the logical point of view ! Without energy no matter could have formed, and without energy & matter no space would exist !  $\rightarrow$  Energy must store the structural information ! Together with Point 7.) this indicates that the curvature of spacetime at the molecular level (crystals), at the atomic level and on the macroscopic level is caused by energy and defined by the "Energy Structure Constant" Phi & base unit 1 which may represent a base energy/wave element  $\rightarrow$  This idea will help to unify General Relativity & Quantum Physics ! If the gravitational singularity (maximum) in M87 really has a dodecahedral structure, then there is strong indication that gravitation, like matter, is defined by the same constant duo : Phi and 1 in Quantum Mechanics and at the cosmic level !

7.) The structure of the M87 black hole ( $\rightarrow$  EHT2017) indicates a dodecahedral structure. Therefore the distribution of matter in gravitational singularities (maxima) seems to be defined essentially by constant Phi & base unit 1! The largescale distribution of matter in the universe seems to be predominantly based on an order-5 dodecahedral honeycomb or "Poincare-Dodecahedral-Space" (see : "EHT2017 may provide evidence for a Poincare Dodecahedral Space Universe")

8.) The natural numbers can be assigned to a defined infinite set of Fibonacci -Number Sequences.

9.) This infinite set of Fibonacci-Number Sequences, and the numbers contained in these sequences, are connected to each other by a complex precisely defined spatial network based on constant Phi.

For the progressing Fibonacci-Sequences towards infinity the connections between the numbers approach constant Phi. → see my study : "Creation of an infinite Fibonacci Number Sequence Table"

10.) Constant Phi (golden mean = 1.618..) must be a fundamental constant of the final equation(s) of the universal mathematical and physical theory. ( $\rightarrow$  It may be the only irrational constant that appears in the(se) equation(s) )

11.) The number-5-oscillation ( $\rightarrow$  the numbers divisible by 5) in the two number sequences 6n+5 (Sequence 1) and 6n+1 (Sequence 2), with n=(0,1,2,3,...), defines the distribution of the prime numbers and non-prime-numbers. The number-5-oscillation defines the starting point and the wave length of defined non-prime-number-oscillations in these Sequences 1+2 (SQ1 & SQ2). (Note : the combination of the two sequences SQ1 & SQ2 is considered here )  $\rightarrow$  weblink to my study : https://arxiv.org/abs/0801.4049 ( or alternatively here : http://vixra.org/abs/1907.0355 )  $\rightarrow$  For a quick overview please see the Chapter 8.5 of this study ("EHT2017 may provide evidence...")

12.) The importance of the number-5-oscillation for the distribution of primes and non-primes is a further indication for the conjecture that the largescale structure of the universe seems to be predominantly (mainly) based on an order-5 Poincare-Dodecahedral-Space structure.  $\rightarrow$  The space structure of the universe seems to be based essentially on the **5.Platonic Solid:** the Dodecahedron ( $\rightarrow$  consisting of 12 regular pentagonal faces, three faces meeting at each vertex)

Time will show if my Conjectures are correct !

## References :

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PHASE SPACES IN SPECIAL RELATIVITY : TOWARDS ELIMINATING GRAVITATIONAL SINGULARITIES

by PETER DANENHOWER  $\rightarrow$  see weblink: https://arxiv.org/pdf/0706.2043.pdf

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About the logic of the prime number distribution - by Harry K. Hahn : https://arxiv.org/abs/0801.4049

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#### <u>Appendix 1</u> : → Here the calculations from Chapter 5

With the algebraic term of constant u we can calculate all square roots of all natural numbers expressed only by constant  $\phi$  and 1:

$$\sqrt{\phi^2 - 2} = \frac{\sqrt{2\sqrt{5} - 2}}{2} ; \text{ we equate the two algebraic terms which represent the same constant !}$$
  

$$\Rightarrow 4\phi^2 - 8 = 2\sqrt{5} - 2 ; \text{ we square both sides and transform}$$
  

$$\phi^2 = \frac{\sqrt{5} + 3}{2} ; (1) \text{ we solve for } \phi^2$$
  

$$\sqrt{5} = 2\phi^2 - 3 ; (2) \text{ we solve for } \sqrt{5}$$

Now we go back to the square root spiral and use the following right triangle :

 $(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2$ ; application of the Pythagorean theorem  $6 = (2\phi^2 - 3)^2 + 1$ ; we replace  $\sqrt{5}$  by equation (2) and transform

$$\Rightarrow 3 = \frac{\phi^4 + 1}{\phi^2} \quad (3) \quad \Rightarrow \quad \sqrt{3} = \sqrt{\frac{\phi^4 + 1}{\phi^2}} \quad (4) \quad ; \text{ square root 3 expressed by } \phi \text{ and } 1$$

Now we use the following right triangle :

$$(\sqrt{3})^2 = (\sqrt{2})^2 + 1^2$$
; application of the Pythagorean theorem & inserting equation (3)

$$\Rightarrow 2 = \frac{\phi^4 + 1}{\phi^2} - 1 \Rightarrow 2 = \frac{\phi^4 - \phi^2 + 1}{\phi^2} \quad (5) \text{ and } \sqrt{2} = \sqrt{\frac{\phi^4 - \phi^2 + 1}{\phi^2}} \quad (6)$$

; square root 2 expressed by  $\phi$  and 1

Now we insert equation (3) in equation (2) :

$$\rightarrow \qquad \sqrt{5} = 2\varphi^2 - \frac{\varphi^4 + 1}{\varphi^2} \rightarrow \sqrt{5} = \frac{\varphi^4 - 1}{\varphi^2} \quad ; \quad (7) \quad ; \qquad \Rightarrow \quad 5 = \left(\varphi^2 - \frac{1}{\varphi^2}\right)^2$$

square root 5 expressed by  $oldsymbol{\phi}$  and 1

Now we use the following right triangle :

 $(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2$ ; application of the Pythagorean theorem & inserting equation (7)

$$\Rightarrow \quad 6 = \left(\frac{\phi^4 - 1}{\phi^2}\right)^2 + 1 \quad \Rightarrow \quad 6 = \frac{\phi^8 - \phi^4 + 1}{\phi^4} \quad (8) \text{ and } \sqrt{6} = \sqrt{\frac{\phi^8 - \phi^4 + 1}{\phi^4}} \quad (9)$$

We can now continue and use the following right triangles of the square root spiral :

 $(\sqrt{7})^2 = (\sqrt{6})^2 + 1^2$ ; application of the Pythagorean theorem & inserting equation (8)

$$\Rightarrow \quad 7 = \frac{\varphi^8 + 1}{\varphi^4} \quad (10) \qquad \Rightarrow \quad \sqrt{7} = \sqrt{\frac{\varphi^8 + 1}{\varphi^4}} \quad (11) \qquad \Rightarrow \quad 7 = \varphi^4 + \frac{1}{\varphi^4}$$

In the same way we can now calculate all square roots of all natural numbers with the next right triangles :

$$\Rightarrow \qquad 8 = \frac{\phi^8 + \phi^4 + 1}{\phi^4} \quad (12) \text{ and } \sqrt{8} = \sqrt{\frac{\phi^8 + \phi^4 + 1}{\phi^4}} \quad (13)$$

$$\Rightarrow \quad 10 = \frac{\phi^8 + 3\phi^4 + 1}{\phi^4} \quad (14) \text{ and } \sqrt{10} = \sqrt{\frac{\phi^8 + 3\phi^4 + 1}{\phi^4}} \quad (15)$$

$$\Rightarrow \qquad 11 = \frac{\phi^8 + 4\phi^4 + 1}{\phi^4} \quad (16) \text{ and } \sqrt{11} = \sqrt{\frac{\phi^8 + 4\phi^4 + 1}{\phi^4}} \quad (17)$$

$$\Rightarrow 12 = \frac{\phi^8 + 5\phi^4 + 1}{\phi^4} (18) \text{ and } \sqrt{12} = \sqrt{\frac{\phi^8 + 5\phi^4 + 1}{\phi^4}} (19)$$

From the above shown formulas (equations 3 to 19) we can read a general rule for all natural numbers > 10 : Note :  $\rightarrow$  The expression (3+n) in the rule can be replaced by products or sums of the equations (3) to (13)

$$\Rightarrow (10+n) = \frac{\phi^{8} + (3+n)\phi^{4} + 1}{\phi^{4}} (20) \text{ and } \sqrt{(10+n)} = \sqrt{\frac{\phi^{8} + (3+n)\phi^{4} + 1}{\phi^{4}}} (30)$$

With these formulas we can express all natural numbers and their square roots only with  $\varphi$  and 1! This is a very interesting discovery, because it allows to describe probably most (if not all) geometrical objects only with  $\varphi$  and 1!

If we transfom the equations (3) to (19) into the standard-form for polynomials then we get the following equations :

**0** = 
$$\phi^4 - 3\phi^2 + 1$$
 (40) or **0** =  $\phi^8 - 7\phi^4 + 1$  (50)

# Abstract :

#### Appendix 2: Infinite Fibonacci Number Sequence Table : Sequences No. 1 to 33 shown (F1 – F33): -> Weblink to the explanatory Study - by Dipl. Ing. (FH) Harry K. Hahn ( $\rightarrow$ Note : Fibonacci Number Sequences are defined by Constant Phi )

Assistance: A Fibonaci-Sequences-Table was developed, which contains infinite Fibonaci-Sequences. This was achieved with the help of research results from an extensive botanical study. This study coarnined the phyllotactic patterns (Fibonaci-Sequences) which appear in the tree-species "Pinus mugo" at different altitudes (from \$50m up to 2500m ) with the increase of altitude above around 2000m the phyllotactic patterns (baneco siderably, the number of patterns (fibonaci-Sequences) grows from 3 to 12, and the relative frequency of the main Fibonaci-Sequences are which Fibonaci-Sequences in the plant dearity is linked to environmental (physical) factors changing with altitude. Especially changes in temperature. / radiation- conditions seem to be the main cause which fibonaci-Sequences in which fibonaci-Sequences. This was achieved that the help of research vector of the radia fibonaci. Sequences have be the main cause which fibonaci-Sequences in which fibonaci-Sequences in which fibonaci-Sequences is the plant dearity is linked to environmental (physical) factors changing with altitude. Especially changes in temperature. / radiation- conditions seem to be the main cause which defines which and constraints in its obvis interestings spatial dependencies between numbers of different Fibonaci-Sequences. The developed (to constant H) is in botany Phyllotactic setures in which frequency. The developed is obvis interestings spatial dependencies between numbers of different Fibonaci-Sequences. The spatial seture is in botany frequencies describes the arrangement of leaves on spiral paths on a plant's stem. Phyllotactic spirals and the spiral seture is in botany Phyllotactic setures in numbers of different Fibonaci Sequences. The instance of the spiral seture is in a stem. Phyllotactic spirals and the area constended to each other, by the solden arcs is in botany Phyllotactic setures in anter. But the true cause of Phyllotactic spirals and the arcs spiral seture is in the true seture of the spiral seture is in the true cau

