Full Relativistic Compton Edge

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Abstract

The original Compton wavelength published by Arthur Compton [1] in 1923 was not full relativistic, but this has been extended to a full relativistic Compton wavelength by Haug [2]. This means also the standard Compton edge not is full relativistic, here we extend the Compton edge to be full relativistic.

Key Words: Compton wavelength, Compton edge, Photon energy.

1 Introduction: Standard Compton Edge

First we will quickly go through the non-relativistic Compton edge, this is all well known, but is a good refreshment before we extend it to a full relativistic Compton edge. In 1923 Arthur Compton published a paper on what today is known as Compton scattering. It is basically to shoot a photon at an electron. One are measuring the wavelength of the photon before it hits the electron and after it has hit the electron, as well as the angle between the incoming and outgoing photon. Arthur Compton came to the following formula

$$\lambda_2 - \lambda_1 = \frac{\hbar}{m_e c}(1 - \cos\theta) \tag{1}$$

where λ_1 is the wavelength of the photon before it hits the electron, and λ_2 is the wavelength of the photon after it has hit the electron, and θ is the angle between the incoming and outgoing photon. Further m_e is the electron mass in kg and $\hbar$ is the reduced Planck constant, and c is the speed of light in vacuum. Re-written in terms of the incoming and outgoing photon energy we have

$$\begin{aligned} \frac{\hbar c}{E_2} - \frac{\hbar c}{E_1} &= \frac{\hbar}{m_e c}(1 - \cos\theta) \\ \frac{1}{E_2} - \frac{1}{E_1} &= \frac{1}{m_e c^2}(1 - \cos\theta) \end{aligned} \tag{2}$$

Next, we solve the equation above with respect to E_2 and get

$$E_2 = \frac{E_1}{1 + \frac{E_1(1-\cos\theta)}{m_e c^2}} \tag{3}$$

and solved with respect to E_1 we get

$$E_1 = \frac{E_2}{1 - \frac{E_2(1-\cos\theta)}{m_e c^2}} \tag{4}$$

The energy transferred to the electron is $E_T = E_1 - E_2$. The maximum energy transferred is when the angle θ is 180 degrees, and since $\cos 180 = -1$ we get

$$E_T = E_1 - \frac{E_1}{1 + \frac{2E_1}{m_e c^2}} \tag{5}$$

This is the standard well-known Compton edge, see also [3].

2 Relativistic Compton Edge

The Compton scattering formula is not full relativistic. It naturally assumes that the speed of the photons moves at the speed of light so in this respect it is relativistic, but it assumes the electron is at rest at the moment the photon hits the electron. This is necessarily not the case, as the electron could be moving also before and when it is hit at a velocity v. When we adjust for also that the electron is moving before and at the moment it is hit we get

$$\lambda_2 - \lambda_1 = \frac{\hbar}{m_e \gamma c}(1 - \cos\theta) \tag{6}$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, is the standard Lorentz factor. For a derivation of this see [2]. This leads to an outgoing photon energy of

$$E_2 = \frac{E_1}{1 + \frac{E_1(1-\cos\theta)}{m_e c^2 \gamma}} \tag{7}$$

and solved with respect to E_1 we get

$$E_1 = \frac{E_2}{1 - \frac{E_2(1-\cos\theta)}{m_e c^2 \gamma}} \tag{8}$$

and we end up with a relativistic Compton edge of

$$E_T = E_1 - \frac{E_1}{1 + \frac{2E_1}{m_e c^2 \gamma}} \tag{9}$$

That this is different than the standard well known Compton edge could potentially have implications for interpretation of Compton scattering experiments already done and done in the future, as well as on related topics such as spectrography, and even in controversial topics such as claims about cold fusion where discussions about the Compton edge has been a topic.

This could also be interesting to see in relation to a new maximum velocity derived for elementary particles, that replaces $v < c$ with $v \leq c\sqrt{1 - \frac{l_p^2}{\lambda^2}} < c$, see [4–8]. This would lead to a minimum limit on the maximum Compton edge, that would be equal to

$$\begin{aligned}
E_T &\geq E_1 - \frac{E_1}{1 + \frac{2E_1}{m_p c^2}} \\
E_T &\geq \frac{hc}{\lambda_1} - \frac{\frac{hc}{\lambda_1}}{1 + \frac{2\frac{hc}{\lambda_1}}{\frac{h}{l_p 2\pi}\frac{1}{c}c^2}} \\
E_T &\geq \frac{1}{\lambda_1} - \frac{\frac{1}{\lambda_1}}{1 + \frac{2\frac{1}{\lambda_1}}{\frac{1}{l_p 2\pi}}} \\
E_T &\geq \frac{1}{\lambda_1} - \frac{\frac{1}{\lambda_1}}{1 + \frac{4\pi l_p}{\lambda_1}}
\end{aligned} \tag{10}$$

As we normally have a photon wavelength much larger than the Planck length, $\lambda_1 >> l_p$, this means the minimum limit on the maximum Compton edge is close to zero, but not zero.

References

[1] A. H. Compton. A quantum theory of the scattering of x-rays by light elements. *Physical Review*, 21(5):483, 1923. URL https://journals.aps.org/pr/abstract/10.1103/PhysRev.21.483.

[2] E. G. Haug. Derivation of a relativistic Compton wavelength. *https://vixra.org/abs/2002.0523*, . URL https://vixra.org/abs/2002.0523.

[3] G. Knoll. *Radiation Detection and Measurement 4th Edition*. Wales, University of Wales Press; 4 edition, 2000.

[4] E. G. Haug. The gravitational constant and the Planck units. A simplification of the quantum realm. *Physics Essays*, 29(4):558, 2016. URL https://doi.org/10.4006/0836-1398-29.4.558.

[5] E. G. Haug. Does special relativity lead to a trans-Planckian crisis? *Applied Physics Research*, 12(1), 2020.

[6] E. G. Haug. The ultimate limits of the relativistic rocket equation. The Planck photon rocket. *Acta Astronautica*, 136, 2017.

[7] E. G. Haug. Collision space-time: Unified quantum gravity. *Physics Essays*, 33(1):46, 2020. URL https://doi.org/10.4006/0836-1398-33.1.46.

[8] E. G. Haug. Re-thinking the foundation of physics, its relation to quantum gravity and quantum probabilities: Unification of gravity and quantum mechanics. *https://vixra.org/abs/2012.0089*, . URL https://vixra.org/abs/2012.0089.