On properties of Solutions of a generalized quadratic Lienard equation

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Abstract

We present a quadratic Lienard equation which contains several nonlinear equations as special cases. The equation is exactly and explicitly integrable despite of the presence of quadratic term. The general solution is expressed as a power law of a sine function of time. This allows to compare the properties of solutions with those obtained by phase plane analysis.

Theory

We consider a quadratic Lienard type equation

$$\ddot{u}+\frac{\dot{u}^2}{u}\left(\frac{p}{3}-1\right)+\frac{3a^2}{p}u-\frac{c}{p}u^{-\left(\frac{p}{3}-1\right)}=0 \qquad (1)$$

where $a, c,$ and p are arbitrary parameters ($p \neq 0$).

For $p=3,$ (1) reduces to

$$\ddot{u}+a^2u=c \qquad (2)$$

So equation (1) is a generalization of the well known linear harmonic oscillator equation. This shows the physical importance of equation (1). Using the generalized Sundman transformation introduced recently by Adjaï et coworkers [1], the equation may be solved explicitly in terms of sine function.

Reference

[1] D. K. K. Adjaï, L. H. Koudahoun, J. Akande, Y. J. F. Kpomahou, M. D. Monsia, Solutions of the Duffing and Painlevé-Gambier equations by generalized Sundman transformation, Journal of Mathematics and Statistics 14 (1) (2018) 241-252, doi:10.3844jmssp.2018.241.252.

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