$\square$

## Fermat's Last Theorem as a consequence of the little one

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## Abstract

In one of Fermat's equivalent equalities, the 3rd digit in the sum of powers $a^{\wedge} n+b^{\wedge} n-c^{\wedge} n$ is not zero and there is a single-valued function of only the last digits $a^{\prime}, b^{\prime}, c^{\prime}$; therefore it cannot be zeroed out with the 2nd and 3rd digits in the sum of bases a+b-c.
Apart from the simplest foundations of the theory of a prime number and the consequences of the little theorem, this is, strictly speaking, the proof of the FLT in the first case.
See the proof of the second case here: https://vixra.org/pdf/1908.0072v1.pdf.
In memory of wife, mother and grandmother

## Fermat's Theorem:

Equality (for prime degree $n>2$; все числа даны в базе $n$ )
$\left.1^{*}\right) a^{n}+b^{n}-c^{n}=0$ in positive integers $a, b, c$ does not exist.

The notation and lemmas /Pour les preuves des lemmes, voir l'annexe in https://vixra.org/pdf/1908.0072v1.pdf и https://vixra.org/pdf/1707.0410v1.pdf )
a', a", a'" - 1st, 2nd, 3rd digit from the end in the number a;
$\mathrm{a}_{[2]}, \mathrm{a}_{[3]}, \mathrm{a}_{[4]}-$ two-, three-, four-digit ending of the number a ;
nn - n*n.
$S(g), S\left(g^{n}\right), S\left(g^{n n}\right)$, sum of $g, g^{n}, g^{n n}, g=1,2, \ldots n-1, G=(1,2, \ldots n-1)$, where L1a. $S\left(g^{1}\right)_{[2]}=0 v$ with the second digit $v=(n-1) / 2$ (see sum of arithmetic progression); $S\left(g^{n}\right)_{[3]}=00 \mathrm{v}$; $\mathrm{S}\left(\mathrm{g}^{\mathrm{nn}}\right)_{[4]}=000 \mathrm{v}$; etc. (When calculating the sums, the terms are pre-summed in pairs equally spaced from the ends of the series.)

If digit a' is not 0 , then
L1. $\left(a^{n-1}\right)^{\prime}=1$ (Fermat's little theorem); $\left(a^{n-1}\right)^{n}{ }_{[2]}=01 ;\left(a^{n-1}\right)^{n n}{ }_{[3]}=001$.
L1c. $\left(a^{\prime n}-a^{\prime}\right)_{[11]}=0 ;\left(a^{\prime n n}-a^{\prime n}\right)_{[2]}=0 ;\left(a^{\prime n n}-a^{\prime n n}\right)_{[3]}=0$.
L2a (key!). There is such a digit $d$ that the second digit ( $\mathrm{d}^{\mathrm{n}}$ )" in the number $\mathrm{d}^{\mathrm{n}}$ is not zero. (Indeed, if second digits in all $\mathrm{d}^{\mathrm{n}}$ are equal to zero, then the second digit of the sum of the number series $d^{n}$, where $d=1,2, \ldots n-1$, is not zero and is equal to ( $\mathrm{n}-1$ )/2, which is incorrect. See L1a.)
L2b. Similarly: there is a digit d such that digit ( $\left.\mathrm{d}^{\mathrm{nn}}\right)^{\prime \prime}$ is not zero.

L2c. There is a digit d such that the digit [ $\left.d^{n n}\left(a^{n n}+b^{n n}-c^{n n}\right)\right]{ }^{\prime \prime}$, where $(a+b-c)^{\prime}=0 n$ $(\mathrm{abc})^{\prime}=/=0$, is not zero. (The proof is the same as in the case of L2a.)

L3. For $k>1$, the $k$-th digit in the number $a^{n}$ does not depend on the $k$-th digit of the base a. (Corollary from Newton's binomial in prime base.)
L3a. Consequence. If $a^{\prime}$ is not equal to 0 , then digits $a^{n}{ }_{[2]}$ and $a^{n n}{ }_{[3]}$ are functions of only a' and do not depend on the digits of higher ranks.
$2 a^{*}$ ) In Fermat's equality $1^{*}$ two-digit endings of numbers $a, b, c$, not multiples of $n$, there are two-digit endings of degrees $a^{\prime n}, b^{\prime n}, c^{\prime n}$.
$2 b^{*}$ ) Therefore, the number a (like $b$ and $c$ ) can be represented as $a=a^{\prime n}+A n^{2}$, where $\mathrm{A}=\left(\mathrm{a}-\mathrm{a}_{[2]}\right) / \mathrm{n}^{2}$, and the number $\mathrm{a}^{\mathrm{n}}$ (and $\mathrm{b}^{\mathrm{n}}$ and $\mathrm{c}^{n}$ ) can be represented as
$\left.3^{*}\right) a^{n}=\left(a^{\prime n}+a^{\circ} n^{2}\right)^{n}=a^{\prime n n}+A n^{3}$ (similarly for $b^{n}$ and $\left.c^{n}\right)$, with the value $\left(a^{\prime n n}+b^{\prime n n}-c^{\prime n n}\right)_{[3]}=0$ in the original equality $1^{*}$.

And now the equality $1^{*}$ can be written by four-digit endings in the form:
$\left.4^{*}\right)\left(a^{\prime n n}+b^{\prime n n}-c^{\prime n n}\right)_{[4]}+(a+b-c)^{\prime} n^{3}+F n^{4}=0$.

## Proof of the last theorem in the first case $-(a b c)^{\prime}=/=0$

According to L2c, in at least one of the $\mathrm{n}-1$ equivalent equalities obtained from equality $1^{*}$ by multiplying it by the numbers $\mathrm{g}^{\mathrm{nnn}}$, where $\mathrm{g}=1,2, \ldots \mathrm{n}-1$, the third digit in the number $\left(a^{n}+b^{n}-c^{n}\right)$ Is NOT equal to zero, since two-digit endings of the base $a$, $b, c$ are two-digit endings of degrees $a^{\prime n}, b^{\prime n}, c^{\prime n}$ (see $2 a^{*}$ ), and three-digit endings of degrees $a^{n}, b^{n}, c^{n}$ are three-digit endings of degrees $a^{\prime n n}, b^{\prime n n}, c^{\prime n n}$, which are single-valued functions of only the last digits $\mathrm{a}^{\prime}, \mathrm{b}$ ', $\mathrm{c}^{\prime}$ and, therefore, by changing the values of the second and third digits of the bases $a, b, c$, the value of the third digit cannot be changed!

Thus, in one of the equivalent equalities $1 a^{*}$, the third digit of the number $a^{n}+b^{n}-c^{n}$ is not equal to zero, which proves the truth of the first case of FLT.
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