On a nonlinear differential equation of Lienard type

M.D. Monsia*

Department of Physics, University of Abomey-Calavi, Abomey-Calavi, 01.B P.526, Cotonou, BENIN

Abstract

We propose in this paper a nonlinear differential equation of Lienard type which contains the Ermakov-Pinney equation as special case. A major result is that the exact and explicit general periodic solution may be easily calculated.

Keywords: Lienard equation, Ermakov-Pinney equation, general periodic solution, trigonometric function

Theory

Let us consider [1]

$$\ddot{x}+\frac{1}{2}(\alpha-q)a\,x^{\alpha-q-1}+\frac{qb}{2}x^{-q-1}=0 \quad (1)$$

obtained from the first order differential equation [1]

$$\dot{x}^2x^q+a\,x^\alpha=b \quad (2)$$

where the dot over a symbol means differentiation with respect to time t.

Substituting $\alpha=q+2$, and $b=\frac{a(q+2)}{4}$, into (1), yields as equation

$$\ddot{x}+a\,x+\frac{aq(q+2)}{8}x^{-q-1}=0 \quad (3)$$

where a, b, α and q are arbitrary parameters.

The equation (3) is the proposed nonlinear differential equation. For $q=2$, the equation (3) gives the well-known Ermakov-Pinney equation

$$\ddot{x}+a\,x+\frac{a}{x^3}=0 \quad (4)$$

* Corresponding author : E-mail: monsiadelphin@yahoo.fr

The use of (2) leads to

$$\int \frac{x^{\frac{q}{2}}dx}{\sqrt{1-\frac{4}{q+2}x^{q+2}}} = \sqrt{b}\left(t+K\right) \qquad (5)$$

where K is an arbitrary constant, from which one may secure the exact and explicit general solution of (3) as

$$x(t) = \left[\frac{\sqrt{q+2}}{2}\sin\left(\pm\frac{q+2}{2}\sqrt{a}(t+K)\right)\right]^{\frac{2}{q+2}} \qquad (6)$$

where $q > -2$.

Reference

[1] M. D. Monsia, Analysis of a purely nonlinear generalized isotonic oscillator equation, Math.Phys.,viXra.org/2010.0195v1.pdf (2020).