On a nonlinear oscillator equation of Lienard type

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Abstract

We propose in this paper a nonlinear oscillator equation of Lienard type which contains the Ermakov-Pinney equation as special case. A major result is that the exact and explicit general periodic solution may be easily calculated.

Keywords: Lienard equation, Ermakov-Pinney equation, general periodic solution, trigonometric function

Theory

Let us consider [1]

$$\ddot{x} + \frac{1}{2}(\alpha - q)a\,x^{\alpha-q-1} + \frac{qb}{2}x^{-q-1} = 0 \tag{1}$$

obtained from the first order differential equation [1]

$$\dot{x}^2 x^q + a\,x^\alpha = b \tag{2}$$

where the dot over a symbol means differentiation with respect to time t.

Substituting $\alpha = q+2$, and $b = \frac{a(q+2)}{4}$, into (1), yields as equation

$$\ddot{x} + a\,x + \frac{aq(q+2)}{8}x^{-q-1} = 0 \tag{3}$$

where a, b, α and q are arbitrary parameters.

The equation (3) is the proposed nonlinear oscillator where $q > -2$. For $q = 2$, the equation (3) gives the well-known Ermakov-Pinney equation

$$\ddot{x} + a\,x + \frac{a}{x^3} = 0 \tag{4}$$

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The use of (2) leads to

$$\int \frac{x^{\frac{q}{2}}dx}{\sqrt{1-\frac{4}{q+2}x^{q+2}}} = \sqrt{b}\left(t+K\right) \tag{5}$$

where K is an arbitrary constant, from which one may secure the exact and explicit general solution of (3) as

$$x(t) = \left[\frac{\sqrt{q+2}}{2}\sin\left(\frac{q+2}{2}\sqrt{a}(t+K)\right)\right]^{\frac{2}{q+2}} \tag{6}$$

Reference

[1] M. D. Monsia, Analysis of a purely nonlinear generalized isotonic oscillator equation, Math.Phys.,viXra.org/2010.0195v1.pdf (2020).