The alternate interpretation of the Quantum theory utilizing indefinite metric

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In this paper, we propose an alternate interpretation of the quantum theory using objective physical reality that does not depend on the conventional probability interpretation.

As typical physical phenomena for the probability interpretation, we consider the single-photon interference, single-electron interference, and EPR correlation experiments using photon polarization. For the calculation using the alternate interpretation, the minus sign derived from the covariant quantization of Maxwell's equations, which is associated with the scalar potential of time axis component of four-vector, is taken as it is as an inevitable request from the theory. In addition, geometrical phase is incorporated, which can be recognized as a kind of scalar potential. We show that both conventional and alternate interpretation derive the identical calculation results for these single photon, single electron interference, and EPR correlation.

These alternate calculation processes describe that there is a kind of scalar potential in whole space-time and when there is some geometrical arrangement in the space, the scalar potential forms the oscillatory field of the potential according to the arrangement. It reveals the objective physical reality that the single-photon, single-electron interference, and EPR correlation are generated by the movement of the photons and electrons in the oscillatory field with interference.

In addition, we show that the oscillatory field formation of the scalar potential depending on the geometrical arrangement causes energy fluctuation in the space, which enables removal of infinite zero-point energy and causes spontaneous symmetry breaking and Casimir effect. By recognizing the electromagnetic field as an unitary U(1) gauge field and generalizing it to a special unitary SU(2), we also show the uncharged particle, e.g. neutron, interferences are generated by the geometrical arrangement of the SU(2) gauge field or geometrical phase. Furthermore, we discuss the origin of the scalar potential by distinguishing the space where the substance exists and the vacuum.

Finally, by introducing the extended Lorentz gauge, we propose the alternate solution without physical state and subsidiary condition for the contradiction between Lorentz gauge as an operator equation and the commutation relation in the covariant canonical quantization of Maxwell's equations with the conventional Lorentz gauge.

This paper contains the compilation of published author's papers[1,2] in addition to featured discussions such as the physical reality, uncharged particle interference, geometrical phase and alternate proposal for the contradiction between Lorentz gauge as an operator equation and commutation relation[3].

CONTENTS

I. INTRODUCTION

The standard quantum theory has been constructed based on probability interpretation. An arbitrary state of a microscopic physical system such as an atom or elementary particle is represented by a state vector equate with a vector in Hilbert space. An expected value of a physical quantity is obtained from the eigenvalue equation of an operator representing a physical quantity. That is the outline of the calculation technique of the standard quantum theory based on probability interpretation.

According to the idea of equating this physical state with a vector in Hilbert space, the inner product of the vectors is interpreted as representing the probability that the state of the system exists in the space-time. Calculations using this basic concept are in agreement with experimental results. Without this concept, single photon and electron interference are difficult to explain. In addition, an entangled state exhibiting long-range correlation that seem to contradict relativity has been discussed by probability interpretation.

However, as long as follow this concept, it is difficult to solve the paradoxes associate with a wave packet reduction typified by "Schrödinger's cat" and "Einstein, Podolsky and Rosen (EPR)".[4,5]

To interpret quantum theory without these paradoxes, de Broglie and Bohm proposed the so-called "hidden variable" theory.[6,7] However it is considered "hidden variable" has been rejected by violation of Bell's inequality. The rejection of "hidden variables" due to the violation of Bell's inequality is inconsistent with relativity that relies on the locality of physical laws.

Although the improvement has not been completed so far, some researchers have been trying to improve the quantum theory based on probability interpretation to fit relativity.[8–12]

Various discussions and experiments have been conducted associate with the correctness of the basic concept of quantum theory that requires description of physical phenomena beyond relativity and common sense. For examples, quantum mechanical superpositions by some experiments have been reviewed.[13] The atom interference by using Bose-Einstein condensates (BECs) has been reported experimentally and theoretically.[14,15] The coherence length of an electron or electron-electron interference by using Aharonov-Bohm oscillations in an electronic MZI has been discussed theoretically.[16,17] A plasmonic modulator utilizing an interference of coherent electron waves through Aharonov-Bohm effect has been studied by the author.[18] The entangle states have been widely discussed experimentally and theoretically.[19–24] The photon interference by using nested MZIs and vibrate mirrors has been measured and analyzed.[25,26] The double-slit electron diffraction has been experimentally demonstrated.[27]

These reports associated with quantum phenomena have convinced the validity of the basic concept of the probability interpretation, and the reliability of the standard quantum theory based on the probability interpretation has come to be considered unwavering.

However, these reports just confirmed the agreement between the measurement results and the calculation results based on the basic concept of the probability interpretation, and examined the application of the interference derived from the probability interpretation. They have considered no possibility other than the probability interpretation of quantum theory.

In this paper, we propose an alternate interpretation of the quantum theory without probability interpretation and show the identical calculation results are obtained for single photon interference, single electron interference, and EPR correlation utilizing both conventional and alternate interpretation. Then we also show the alternate interpretation describes objective physical reality that photons and electrons are actually moving in space-time instead of probability interpretation.

According to the alternate interpretation, the concept of pure state of which probabilities are fundamental sense is not necessary in nature. Only the concept of mixed states of which probabilities are statistical sense is physically valid as a natural law. Although the probability interpretation of the standard quantum theory using the mixed state is useful for calculations, it is shown that quantum theory will be deterministic physics without probability interpretation as in classical physics.

In addition, we show that the removal of infinite zero-point energy without artificial subtraction, Casimir effect and spontaneous symmetry breaking can be spontaneously obtained. Furthermore, by generalizing the electromagnetic field of unitary gauge field to special unitary gauge field, we also show uncharged particle interference can be explained as the same mechanism, i, e., by the geometrical arrangement of the SU(2) gauge field or geometrical phase which can be recognized as a kind of scalar potential.

The structure of this paper is as follows.

In chapter II, we summarize the covariant quantization of Maxwell's equation that requires an indefinite metric, which is the essence of this paper, and discuss that the indefinite metric obtained by the quantization should take precedence over the probability interpretation.

In chapter III, we indicate the difference in calculation and interpretation for single-photon interference, single-electron interference and EPR correlation between using the conventional probability interpretation and alternate interpretation. Despite the differences, except for the interpretation, the identical results can be obtained as the observable physical phenomena. In addition, we show a convenient format for calculation named "simple calculation method", which simplifies the calculation of the alternate interpretation. From the calculation results by the alternate interpretation, it becomes clear that the scalar potential is a physical reality with a minus sign of indefinite metric.

In chapter IV, as applications of the alternate interpretation, we show that the removal of infinite zero-point energy, Casimir effect, spontaneous symmetry breaking can be spontaneously obtained utilizing the generalization of the geometrical arrangement in the space discussed in chapter III. We also refer to the general approach for single particle interference include uncharged particle.

In chapter V, the origin of the indefinite metric potential and Maxwell's equations in vacuum, which is the core of the alternate interpretation examined in this paper, is discussed.

In chapter VI, we discuss the contradiction between Lorentz condition as an operator and commutation relation in covariant canonical quantization of Maxwell's equations. Then we propose the alternate method introducing the extended Lorentz gauge that can avoid the contradiction without physical state and subsidiary condition.

In chapter VII, we summarize the alternate interpretation.

II. COVARIANT QUANTIZATION OF MAXWELL'S EQUATIONS

In order to revise the basic concept of the conventional standard quantum theory based on probability interpretation to the alternate interpretation using objective physical reality, covariant quantization of Maxwell's equations using Lorentz gauge is indispensable instead of using Coulumb gauge. That will be shown together with concrete calculations in the following chapters.

The purpose of the quantization described in this chapter is to clarify the introduction of the minus sign required by the indefinite metric, which is necessary for the discussion in the following chapters.

For that purpose, quantization is performed using Fourier transform without going into the details of canonical quantization, namely, the four-vector satisfying the covariant form of Maxwell's equations is expressed by the Fourier transform of plane wave expansion and then those Fourier coefficients are replaced with operators along with setting commutation relations.

The canonical covariant quantization of Maxwell's equations in Lorentz gauge requires the discussions other than the purpose of this chapter, i.e., associate with the selection of Lagrangian density and the setting of commutation relations. Therefore the canonical quantization will be dealt with independently and discussed in Chapter VI.

However, there is no difference in calculations and discussions in the following chapters, whether the quantization in this chapter or the canonical quantization is adopted.

A. Quantization using Lorentz gauge

The subject of the discussion is Maxwell's equations below.

$$\begin{aligned} \left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} - \nabla\left(\nabla\cdot\mathbf{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\right) &= -\mu_0\mathbf{i} \\ \left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi + \frac{\partial}{\partial t}\left(\nabla\cdot\mathbf{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\right) &= -\frac{\rho}{\varepsilon_0} \end{aligned} \tag{1}$$

where μ_0 and ε_0 are the permeability and permittivity of a vacuum respectively.

In the Maxwell's equations (1), the electromagnetic potentials ϕ and $\mathbf{A}$ are expressed as following four-vector in Minkowski space.

$$A^\mu = (A^0,\ A^1,\ A^2,\ A^3) = (\phi/c,\ \mathbf{A}) \tag{2}$$

The charge density ρ and space currents $\mathbf{i}$ are also expressed as following four-currents.

$$j^{\mu} = (j^0,\ j^1,\ j^2,\ j^3) = (c\rho,\ \mathbf{i}) \tag{3}$$

Hence, by setting the axises of Minkowski space as $x^0 = ct,\ x^1 = x,\ x^2 = y,\ x^3 = z$, Maxwell's equations in the Lorenz gauge are expressed as following covariant form

$$\Box A^{\mu} = \mu_0 j^{\mu}, \qquad \partial_{\mu} A^{\mu} = 0 \tag{4}$$

In addition, the conservation of charge

$$\mathrm{div}\ \mathbf{i} + \partial\rho/\partial t = 0 \tag{5}$$

is expressed as following covariant form.

$$\partial_{\mu} j^{\mu} = 0 \tag{6}$$

where,

$$\partial_{\mu} = (1/c\partial t,\ 1/\partial x,\ 1/\partial y,\ 1/\partial z) = (1/\partial x^0,\ 1/\partial x^1,\ 1/\partial x^2,\ 1/\partial x^3) \tag{7}$$

and $\Box$ is the d'Alembertian :$\Box \equiv \partial_{\mu}\partial^{\mu} \equiv \partial^2/c^2\partial t^2 - \Delta$.

The covariant and contravariant vectors can be transformed to each other using the simplest Minkowski metric tensor $\mathsf{g}_{\mu\nu}$ as follows.

$$\mathsf{g}_{\mu\nu} = \mathsf{g}^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{8}$$

$$A_{\mu} = \mathsf{g}_{\mu\nu} A^{\nu}\ ,\ A^{\mu} = \mathsf{g}^{\mu\nu} A_{\nu} \tag{9}$$

The following quadratic form of a four-vector is invariant under a Lorentz transformation.

$$(x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 \tag{10}$$

The wave front equation can be expressed by applying a minus sign to the above quadratic form and can be described by using metric tensor as follows.

$$-\mathsf{g}_{\mu\nu} x^{\mu} x^{\nu} = -x^{\mu} x_{\mu} = x^2 + y^2 + z^2 - c^2 t^2 = 0 \tag{11}$$

This quadratic form, including the minus sign, is also introduced into the inner product of arbitrarily vector and commutation relations in Minkowski space.

Maxwell's equations that satisfy the four-vectors with no four-current in free space can be expressed as the following Fourier transform by plane wave expansion.[28]

$$A_\mu(x) = \int d\tilde{k} \sum_{\lambda=0}^{3} [a^{(\lambda)}(k)\epsilon_\mu^{(\lambda)}(k)e^{-ik\cdot x} + a^{(\lambda)\dagger}(k)\epsilon_\mu^{(\lambda)*}(k)e^{ik\cdot x}] \tag{12}$$

$$\tilde{k} = \frac{d^3k}{2k_0(2\pi)^3} \quad k_0 = |\mathbf{k}| \tag{13}$$

where the unit vector of time-axis direction n and polarization vectors $\epsilon_\mu^{(\lambda)}(k)$ are introduced as $n^2 = 1,\ n^0 > 0$ and $\epsilon^{(0)} = n$, $\epsilon^{(1)}$ and $\epsilon^{(2)}$ are in the plane orthogonal to k and n

$$\epsilon^{(\lambda)}(k) \cdot \epsilon^{(\lambda')}(k) = -\delta_{\lambda,\lambda'} \quad \lambda\ ,\ \lambda' = 1,\ 2 \tag{14}$$

$\epsilon^{(3)}$ is in the plane $(k,\ n)$ orthogonal to n and normalized

$$\epsilon^{(3)}(k) \cdot n = 0\ ,\ \epsilon^{(3)}(k) \cdot \epsilon^{(3)}(k) = -1 \tag{15}$$

Hence $\epsilon^{(0)}$, $\epsilon^{(1)}$ (and $\epsilon^{(2)}$) and $\epsilon^{(3)}$ can be recognized as a polarization vector of scalar wave, transversal waves and a longitudinal wave respectively. Here we take the following the easiest forms as these vectors.

$$\epsilon^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \epsilon^{(1)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \epsilon^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \epsilon^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{16}$$

The quantization is performed by replacing the Fourier coefficients of the four-vector with operators $\hat{A}_\mu \equiv \sum_{\lambda=0}^{3} \hat{a}^{(\lambda)}(k)\epsilon_\mu^{(\lambda)}(k)$ and setting the commutation relations as follows.

$$[\hat{A}_\mu(k),\ \hat{A}_\nu^\dagger(k')] = -\mathsf{g}_{\mu\nu}\delta(k - k') \tag{17}$$

The time-axis component has the opposite sign of the space axes. Because $\langle 0|\hat{A}_0(k)\hat{A}_0^\dagger(k')|0\rangle = -\delta(k - k')$,

$$\langle 1|1\rangle = -\langle 0|0\rangle \int d\tilde{k}|f(k)|^2 \tag{18}$$

where $|1\rangle = \int d\tilde{k} f(k)\hat{A}_0^\dagger(k)|0\rangle$. Therefore, the time-axis component is the source of indefinite metric.

Here, when we identify $|1\rangle$ as the single photon state by the probability interpretation, the probability that there is one photon in space is negative. Therefore the probability interpretation is inapplicable for the covariant quantization.

Mathematically, the negative sign of the inner product of vectors is contrary to the definition of the inner product of vectors in Hilbert space.

In the first place, equating of the physical state vectors $|x\rangle$ and those inner product $\langle x|x\rangle$ with the vectors and those inner product in Hilbert space such as $|x\rangle \geq 0 \quad \langle x|x\rangle = 0 \Leftrightarrow |x\rangle = 0$ was not derived from theory, but estimated and artificially introduced.

The reliability of the identification has been established as a result of accumulating agreement between the calculation results obtained from the probability interpretation and various experimental results.

In addition to the reliability, the mathematical procedures which can reconcile the probability interpretation with the indefinite metric has been developed for example so-called Gupta-Bleuler formalism[29,30] and Nakanishi-Lautrup formalism[31] as will be mentioned in chapter VI. However, such a procedures are not necessarily required theoretically, and can be understood as mathematical techniques to be consistent with the probability interpretation by introducing artificial manipulation such as physical states and subsidiary conditions.

On the other hand, the minus sign is necessarily introduced from Maxwell's equations and the theory of relativity. In the first place, the physical space of the natural world is expressed as Minkowski space even if gravity is ignored or Riemann space if gravity is included, and the metric is not limited to a positive definite value.

In this paper, we accept the introduction of the minus sign as an inevitable request from the theory, and propose an alternate interprettion of the quantum theory without probability interpretation. In the following chapters we show the calculation results of single-photon interference, single-electron interference and EPR correlation using probability interpretation can be faithfully reproduced by introducing the minus sign and a kind of scalar potential (geometrical phase). Then we also show these phenomena will be clear the image of objective physical reality.

If Coulomb gauge is adopted, the scalar potential of the time axis component, which is the source of the indefinite metric, will be ignored. Therefore the discussion in the next chapter becomes difficult. In addition, the explicit covariance of Maxwell's equations is also lost, hence Lorentz gauge should be adopted to construct the basic concept of physical law independent of the coordinate system.

III. SINGLE PHOTON, SINGLE ELECTRON INTERFERENCE AND EPR CORRELATION

In this chapter, we show the calculation methods for the single photon interference, single electron interference, EPR correlation by using both probability and alternate interpretation. We emphasize that the calculation of single-photon and single-electron interference by using probability interpretation imposes the description that a single particle that cannot be further divided must be considered as if it were divided into separate paths on us.

On the other hand, the calculation of single-photon and single-electron interference by using the alternate interpretation reveals that the scalar potential, which is the source of the indefinite metric, forms an oscillatory field due to the geometrical arrangement of the experimental setups. Then we can obtain the image of objective physical reality which describes the inseparable single photon or electron moves in the oscillatory field while interfering with each other.

For EPR correlation, the calculation utilizing polarized photons will be discussed. On the calculation of EPR correlation by using probability interpretation, two photons having polarizations orthogonal to each other must be considered as a correlated photon pair state (Entanglement), which simultaneously splits into different paths and only probabilistically exists in whole space-time. This kind of explanation requires not only the probability interpretation but also the "denial of locality" which contradicts the relativity such that the polarization of the photon of the other path is determined at the very moment when the polarization is found by the polarization measurement of the photon of the other path.

On the other hand, according to the alternate interpretation, the correlated photon pair has the designated polarization that is determined when it generates, and when the polarization direction is measured, those photons are interfered with the scalar potential that forms an oscillatory field due to the geometrical arrangement of the experimental setup. We clarify that there seems to be a non-local long-range correlation beyond the causality due to the interference with the potential.

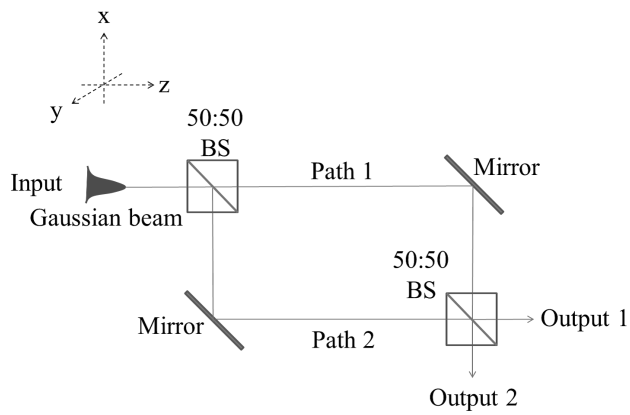

FIG. 1. Schematic view of Mach-Zehnder Interferometer (MZI). BS:Beam Splitter.

A. Single photon interference

1. Calculation using probability interpretation

Figure 1 shows a schematic view of the Mach-Zehnder Interferometer (MZI) and coordinate system.[1]

On the probability interpretation, the calculation of the single photon interference by using Maxwell's equations (1) in free space ($\mathbf{i} = 0$, $\rho = 0$) in Coulomb gauge eliminates the scalar potential ϕ and only uses the quantized vector potentials $\hat{a}$ and $\hat{a}^\dagger$ as the photon annihilation and creation operator respectively.

According to the reference[32], the following electric field operator of which square $\hat{E}^\dagger \hat{E}$ is used as an electric field intensity operator proportional to the photon number, and number state $|n\rangle$ are introduced to calculate the single photon interference of the MZI.

$$\hat{E} = \frac{1}{\sqrt{2}}\hat{a}_1 \exp{(i\theta)} + \frac{1}{\sqrt{2}}\hat{a}_2 \tag{19}$$

where $\hat{a}_{1\mathrm{or}2}$is the photon annihilation operator corresponding to an optical mode passing through path 1 or 2, respectively, θ is a phase difference corresponding to the difference in length between the two paths.

$\hat{a}_{1\mathrm{or}2}$ and $\hat{a}^\dagger_{1\mathrm{or}2}$ are defined along with the expected photon number from the photon creation and annihilation operators $\hat{a}$ and $\hat{a}^\dagger$ in free space (MZI input) before the photons are split into two paths as follows.

$$\langle n|\hat{a}_1^\dagger \hat{a}_1|n\rangle = \langle n|\hat{a}_2^\dagger \hat{a}_2|n\rangle = \langle n|\hat{a}_1^\dagger \hat{a}_2|n\rangle = \frac{1}{2}n \tag{20}$$

$$\hat{a} = \frac{\hat{a}_1 + \hat{a}_2}{\sqrt{2}} \quad , \quad \hat{a}^\dagger = \frac{\hat{a}_1^\dagger + \hat{a}_2^\dagger}{\sqrt{2}} \tag{21}$$

The photon number at the MZI output is calculated as follows by using the squared operator and the number state.[32]

$$\langle \hat{I} \rangle \propto \langle n|\hat{E}^\dagger \hat{E}|n\rangle = \frac{1}{2}\langle n|\hat{a}_1^\dagger \hat{a}_1|n\rangle + \frac{1}{2}\langle n|\hat{a}_2^\dagger \hat{a}_2|n\rangle + \cos\theta \langle n|\hat{a}_1^\dagger \hat{a}_2|n\rangle \tag{22}$$

where $\langle \hat{I} \rangle$ is the expected field intensity proportional to the photon number.

Substituting 1 ($n = 1$) for the photon number as a single photon, the above expected value is calculated to be as follows.

$$\langle \hat{I} \rangle \propto \frac{1}{4} + \frac{1}{4} + \frac{1}{2}\cos\theta = \frac{1}{2} + \frac{1}{2}\cos\theta \tag{23}$$

The above calculations can be interpreted that the photon incident from the MZI is passing through both path 1 and 2 simultaneously with probability 1/2 along with the phase difference corresponding to the optical length between the two paths, as is clear from the form of the electric field operator introduced in (19), the expected photon number (20) on each path, the division of the photon creation and annihilation operator (21).

This interpretation is valid as statistical physics that when the intensity of light incident on the MZI is high and the photon number n is large, approximately $n/2$ photons are on one side and $n/2$ photons are on the other side.

This kind of statistical state has been introduced as a mixed state which is multiplied by a density matrix proportional to the probability including the pure state having a fundamental probability. The probability interpretation using the mixed state is considered statistically valid.

However, even if the intensity of light incident on the MZI is very low and it is considered to be a single photon that cannot be split any more, the single photon is considered as a pure state with fundamental probability and passes through both path 1 and 2 simultaneously with probability 1/2.

2. *Calculation using alternate interpretation*

In order to revise the probability interpretation to alternate interpretation with objective physical reality, we examine again the electromagnetic field of the incident photon beam on the MZI in Figure 1.

First, assume that a light beam having an angular frequency ω and a propagation constant β polarized in the x-axis direction propagates in the z-axis direction, and the electric field of the light beam forms, for example, a Gaussian distribution of which cross-sectional shape is well localized in free space. Then, the electric field of the input light beam can be expressed as follows.

$$\mathbf{E} = \mathbf{e}_x \cdot C_E \cdot \exp\left(-\frac{x^2+y^2}{w_0^2}\right) \cdot \cos(\omega t - \beta z) \tag{24}$$

where, $\mathbf{e}_x$ is unit vector parallel to the x-axis. C_E is an arbitrary constant which is proportional to the magnitude of the electric field. w_0 is the radius of the optical beam. $\mathbf{E}$ and $\mathbf{B}$ are expressed by vector and scalar potentials as follows.

$$\begin{aligned} \mathbf{E} &= -\frac{\partial}{\partial t}\mathbf{A} - \nabla\phi \\ \mathbf{B} &= \nabla \times \mathbf{A} \end{aligned} \tag{25}$$

By introducing a vector function $\mathbf{C}$, $\mathbf{A}$ is calculated from (24) and (25) as follows.

$$\begin{aligned} \mathbf{A} &= -\frac{1}{\omega}\mathbf{e}_x \cdot C_E \cdot \exp\left(-\frac{x^2+y^2}{w_0^2}\right) \cdot \sin(\omega t - \beta z) + \mathbf{C} \\ \frac{\partial}{\partial t}\mathbf{C} &= -\nabla\phi \end{aligned} \tag{26}$$

To localize $\mathbf{B}$ to space, we can take $\mathbf{C}$ as an irrotational vector function $\nabla \times \mathbf{C} = 0$. For example, $\mathbf{C}$ and ϕ can be expressed by introducing an arbitrary scalar function λ which satisfies $\mathbf{C} = \nabla\lambda$ and $\nabla\left(\frac{\partial}{\partial t}\lambda + \phi\right) = 0$.

Then $\mathbf{B}$ is expressed as follows

$$\begin{aligned} \mathbf{B} &= \nabla \times \mathbf{A} \\ &= \frac{\beta}{\omega}\mathbf{e}_y \cdot C_E \cdot \exp\left(-\frac{x^2+y^2}{w_0^2}\right) \cdot \cos(\omega t - \beta z) \\ &\quad -\frac{2y}{\omega \cdot w_0^2}\mathbf{e}_z \cdot C_E \cdot \exp\left(-\frac{x^2+y^2}{w_0^2}\right) \cdot \sin(\omega t - \beta z) \end{aligned} \tag{27}$$

Therefore, $\mathbf{E}$ and $\mathbf{B}$ are localized in the free space in the input. In contrast, the vector and scalar potentials are not necessarily localized. The above localized form (24) is one example, other forms which satisfy the Maxwell's equations (1) in free space $\mathbf{i} = 0$ and $\rho = 0$ can be adopted.

Note that, the free space propagation will expand the radius of the Gaussian beam. However, that of the propagated beam $w(z)$ will be approximately 10.5mm when the beam of which launched radius $w_0 = 10\mathrm{mm}$ propagates $z = 100\mathrm{m}$ in free space. This value can be calculated using $w(z) = w_0\sqrt{1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2}$ in the case of wavelength $\lambda = 1\mu\mathrm{m}$. Hence the spatially expansion of the beam will be negligible small in the case of the paths of the MZI are less than several tens meters.

As described above, even if the photon is localized, the potentials are not always localized. In particular, the scalar potential can exist in whole space-time. The alternate interpretation is shown below by utilizing the existence of the scalar potential as the objective physical reality.

First, note that vector potentials and scalar potential are mixed by Lorentz transformation. Therefore from the Lorentz invariance, vector potentials and scalar potential should be equally treated as $\langle 1|\hat{A}_0^\dagger \hat{A}_0|1\rangle = \langle 1|\hat{A}_1^\dagger \hat{A}_1|1\rangle = \langle 1|\hat{A}_2^\dagger \hat{A}_2|1\rangle = \langle 1|\hat{A}_3^\dagger \hat{A}_3|1\rangle$.

For simplicity of calculation, the incident beam in the figure 1 is assumed to be perfectly polarized on the x-axis, and the longitudinal wave that is considered to be unphysical presence is ignored, i.e., $A_2 = 0,\ A_3 = 0$. However, as mentioned above, the scalar potential can exist in the whole space-time. Therefore, the four-vector at the input of the MZI is expressed as follows.

$$A_\mu = (A_0,\ A_1,\ 0,\ 0) \tag{28}$$

It is reasonable for physically phenomenon that the scalar potential is split when there are two paths of the MZI. Here, we examine that a single photon represented by a transverse wave polarized in the x-axis passes through path 1 and the scalar potential A_0 is divided into both paths 1 and 2 with a phase difference θ between the two paths. In this case, the four-vector ($\equiv A_{\mu:(\mathrm{path1})}$) along path 1 and four-vector ($\equiv A_{\mu:(\mathrm{path2})}$) along path 2 of MZI can be expressed as follows.

$$\begin{aligned} A_{\mu:(\mathrm{path1})} &= (\frac{1}{2}e^{i\theta/2}A_0,\ A_1,\ 0,\ 0) \\ A_{\mu:(\mathrm{path2})} &= (\frac{1}{2}e^{-i\theta/2}A_0,\ 0,\ 0,\ 0) \end{aligned} \tag{29}$$

By replacing Fourier coefficients of the four-vector with operators $\hat{A}_\mu \equiv \sum_{\lambda=0}^{3} \hat{a}^{(\lambda)}(k)\epsilon_\mu^{(\lambda)}(k)$ and setting the commutation relations, the single photon interference can be calculated using the potentials as an operator. According to the quantization, photon annihilation operators corresponding to optical modes passing through MZI paths 1 and 2 are defined as $\hat{A}_{\mu:(\mathrm{path1})}$ and $\hat{A}_{\mu:(\mathrm{path2})}$ respectively. Here we must introduce the indefinite metric into the product of the operator as follows.

$$\hat{A}^\dagger \hat{A} = -\mathsf{g}_{\mu\nu}\hat{A}^{\mu\dagger}\hat{A}^{\nu} = -\mathsf{g}^{\mu\nu}\hat{A}^\dagger_\mu \hat{A}_\nu \tag{30}$$

Hence the photon number operator at the output of MZI can be obtained as following expression by using the photon annihilation operator at the point $\hat{A}_{\mu:(\mathrm{path1})} + \hat{A}_{\mu:(\mathrm{path2})}$.

$$-\mathsf{g}^{\mu\nu}\{\hat{A}_{\mu:(\mathrm{path1})} + \hat{A}_{\mu:(\mathrm{path2})}\}^\dagger\{\hat{A}_{\nu:(\mathrm{path1})} + \hat{A}_{\nu:(\mathrm{path2})}\} = -\frac{1}{2}\hat{A}^\dagger_0\hat{A}_0 + \hat{A}^\dagger_1\hat{A}_1 - \frac{1}{2}\hat{A}^\dagger_0\hat{A}_0\cos\theta \tag{31}$$

where, the following relations are used.

$$\begin{aligned}
-\mathsf{g}^{\mu\nu}\hat{A}^\dagger_{\mu:(\mathrm{path1})}\hat{A}_{\nu:(\mathrm{path1})} &= -\tfrac{1}{4}\hat{A}^\dagger_0\hat{A}_0 + \hat{A}^\dagger_1\hat{A}_1 \\
-\mathsf{g}^{\mu\nu}\hat{A}^\dagger_{\mu:(\mathrm{path1})}\hat{A}_{\nu:(\mathrm{path2})} &= -\tfrac{1}{4}e^{-i\theta}\hat{A}^\dagger_0\hat{A}_0 \\
-\mathsf{g}^{\mu\nu}\hat{A}^\dagger_{\mu:(\mathrm{path2})}\hat{A}_{\nu:(\mathrm{path1})} &= -\tfrac{1}{4}e^{i\theta}\hat{A}^\dagger_0\hat{A}_0 \\
-\mathsf{g}^{\mu\nu}\hat{A}^\dagger_{\mu:(\mathrm{path2})}\hat{A}_{\nu:(\mathrm{path2})} &= -\tfrac{1}{4}\hat{A}^\dagger_0\hat{A}_0
\end{aligned}$$

From (31) and $|1\rangle$ with the Lorentz invariance, $\langle\hat{I}\rangle \propto \frac{1}{2} - \frac{1}{2}\cos\theta$ can be calculated. By selecting the proper reference point of the phase difference, (23) is reproduced.

$$\langle\hat{I}\rangle \propto \frac{1}{2} + \frac{1}{2}\cos\theta \tag{32}$$

Here, in order to clarify the motion of the scalar potential and the single photon as an objective physical reality, we further study the setup of the single photon MZI experiment. Since the electromagnetic field has time reversal invariance, there is no particular reason to distinguish the input and output of the MZI. Therefore, the photon annihilation operator at the confluence of the input of the MZI should be represented as the output of the MZI. Then

$$\hat{A}_\mu = \hat{A}_{\mu:(\mathrm{path1})} + \hat{A}_{\mu:(\mathrm{path2})} = \left(\cos\frac{\theta}{2}\hat{A}_0,\ \hat{A}_1,\ 0,\ 0\right) \tag{33}$$

It is physically reasonable to assume that there is clearly single photon at the input of the MZI. If (28) is used to calculate the photon number with the single photon state at the input of the MZI, the result is calculated to be $\langle 1|(-\hat{A}^\dagger_0\hat{A}_0 + \hat{A}^\dagger_1\hat{A}_1)|1\rangle = 0$. Therefore if we erase the scalar potential as $\hat{A}_0 = 0$, we can not obtain the interference. On the other hand, the photon number is 1 when $\theta = \pm N\pi$ (N : odd number) in (33). Therefore, we should recognize the scalar potential at the input of the MZI is nonzero (not empty, i.e., $\hat{A}_0 \neq 0$) but rather cancels each other out with opposite phase waves, i.e., $\cos(\theta/2) = 0$. From this study, it is possible to obtain a picture that the scalar potential generates an oscillatory field like $f(\theta)\cdot\hat{A}_0$ when there is a division (geometrical arrangement) in the space. Where $f(\theta)$

is an oscillatory function of θ. The formation of the oscillatory field can be recognized as a "hidden variable" associate with the EPR correlation, which will be dealt with in the next section. It is possible to interpret that a substantial photon moves in the oscillatory field, which interferes with each other. Therefore, the expected value of the field intensity at any spatial position is calculated as $\langle \hat{I} \rangle \propto \frac{1}{2} + \frac{1}{2}\cos\theta$ using (33). From the discussion, no matter where the substantial photon moves in the space, no photon can be observed at the position where $\theta = \pm N\pi$ (N : odd number) in the space.

In this way, if we accept the indefinite metric required from the covariant quantization without probability interpretation, a clear image of objective physical reality is obtained. That is to say, there is a scalar potential on both paths of the MZI, which forms an oscillatory field caused by the path division of the MZI, and a single photon incident from one path passes through while interfering with the oscillatory field. The formation of the oscillatory field caused by the path division examined here corresponds to the fact that the phase term depending on the potentials introduced on the electron wave function cannot be eliminated in the spatially multiple-connected region in Aharonov-Bohm effect.[33,34]

We can replace the above calculation and picture by an analogy of electronic circuits that the scalar potential corresponds to the bias current (voltage) and the vector potential, which represents a single photon, corresponds to a signal current (voltage) added to the bias current (voltage). However, in this case, the bias current (voltage) is not direct current (DC) (or voltage), but an alternating bias current (AC) (or voltage) that interferes with the signal current (voltage), which causes output fluctuations. In summary, an observed signal fluctuates when the signal is added on a fluctuating reference. In addition, if we use the analogy of homodyne detection wireless communication or optical communication, the scalar potential corresponds to a continuously oscillating local oscillator placed at the receiving end and a single photon corresponds to a signal. This corresponds to extracting the signal information by the interference between the continuous wave of the local oscillator and the signal at the receiving end, and the image shows that each point in whole space-time has a local oscillator.

In the above calculations, the scalar potential that requires the indefinite metric was treated as an operator as a physical reality. Following the standard quantum theory, we call the form Heisenberg picture in which the indefinite metric of the scalar potential is imposed on an operator. Then, we can call the form Schrödinger picture in which the indefinite metric of the scalar potential is imposed on a state vector.

In Schrödinger picture, the expected field intensity can be calculated using the state along path 1 and path 2, and the photon annihilation operator at the output in Schrödinger picture $\hat{A}_S$, which is proportional to the electric field operator $\hat{E} \propto \hat{A}_S$ at the output 1 (or 2). For a more detailed definition is as follows. The operators $\hat{A}$, $\hat{A}_S$ and state $|1\rangle$, $|1\rangle_S$ can be translated by using the Hamiltonian $\hat{\mathcal{H}}$ as $\hat{A} = e^{i\hat{\mathcal{H}}t/\hbar}\hat{A}_S e^{-i\hat{\mathcal{H}}t/\hbar}$ and $|1\rangle_S = e^{-i\hat{\mathcal{H}}t/\hbar}|1\rangle$ respectively.

Here the state along path 1 can be expressed as $\frac{1}{2}e^{i\theta/2}|1\rangle_{S0} + |1\rangle_{S1}$ according to (29), where $|1\rangle_{S0}$ expresses the state which there is only scalar potential A_0 along path 1, and $|1\rangle_{S1}$ expresses the state which there is x-polarized single photon along path 1 respectively.

In the same way, the state along path 2 can be expressed as $\frac{1}{2}e^{-i\theta/2}|1\rangle_{S0} + |0\rangle_S$ according to (29), where $|0\rangle_S$ expresses the state which there is no photon along path 2. $|0\rangle_S$ can be identified as "ideal vacuum" as will be discussed in section IV A. However $|0\rangle_S \equiv 0$ which means ${}_S\langle 0|0\rangle_S = 0 \neq 1$. We can neglect the "ideal vacuum" in the calculation.

Then the state of the output can be calculated as following the sum of the both states.

$$\cos\frac{\theta}{2}|1\rangle_{S0} + |1\rangle_{S1} \tag{34}$$

Therefore the expected value of the field intensity using the probability interpretation is also reproduced by using Schrödinger picture as follows.

$$\begin{aligned} \langle\hat{I}\rangle \;&\propto\; \left(\cos\frac{\theta}{2}{}_{S0}\langle 1| +_{S1}\langle 1|\right)\hat{A}_S^\dagger\hat{A}_S\left(\cos\frac{\theta}{2}|1\rangle_{S0} + |1\rangle_{S1}\right) \\ &= -\cos^2\frac{\theta}{2} + 1 = \frac{1}{2} - \frac{1}{2}\cos\theta = \frac{1}{2} + \frac{1}{2}\cos\theta' \end{aligned} \tag{35}$$

where $\theta' = \theta \pm \pi$(the proper reference point of the phase difference), and according to (30), $-_{S0}\langle 1|\hat{A}_S^\dagger\hat{A}_S|1\rangle_{S0} =_{S1} \langle 1|\hat{A}_S^\dagger\hat{A}_S|1\rangle_{S1} = 1,_{S0}\langle 1|1\rangle_{S1} =_{S1}\langle 1|1\rangle_{S0} = 0$ are used.

In the above discussion, we have expressed the operators and states along path 1 and 2. However, an objective physical reality is that the oscillatory field of the scalar potential is formed due to the geometrical arrangement that divides the space into two, and a single photon passes through (or exists) in it. That is, in Heisenberg picture, the x-polarized single photon $\hat{A}_1$ passes through (or exists) in the oscillatory field of the scalar potential $\cos\frac{\theta}{2}\hat{A}_0$. In Schrödinger picture, the x-polarized single photon $|1\rangle_{S1}$ passes through (or exists) in the oscillatory field of the scalar potential $\cos\frac{\theta}{2}|1\rangle_{S1}$ formed on the vacuum $|0\rangle_{S0}$.

It is convenient to calculate the single photon interference in Heisenberg picture by introducing the following operator $\hat{A}'_0$ instead of $\hat{A}_0$ using the above operator $\hat{A}_1$.

$$
\begin{aligned}
\hat{A}'_0 &= \frac{1}{2}\gamma e^{i\theta/2}\hat{A}_1 - \frac{1}{2}\gamma e^{-i\theta/2}\hat{A}_1 \\
\hat{A}'^{\dagger}_0 &= \frac{1}{2}\gamma e^{-i\theta/2}\hat{A}^{\dagger}_1 - \frac{1}{2}\gamma e^{i\theta/2}\hat{A}^{\dagger}_1
\end{aligned} \tag{36}
$$

where $\gamma^2 = -1$ (i.e., γ corresponds to the square root of the determinant of Minkowski metric tensor $\sqrt{|\mathsf{g}_{\mu\nu}|} \equiv \sqrt{\mathsf{g}} \equiv \sqrt{-1} = \gamma$)

By using the operator, the expected field intensity $\langle \hat{I} \rangle \propto \langle 1|(\hat{A}'_0 + \hat{A}_1)^{\dagger}(\hat{A}'_0 + \hat{A}_1)|1\rangle$ can be calculated as follows.

$$
\begin{aligned}
\hat{A}'^{\dagger}_0\hat{A}'_0 &= -\frac{1}{4}\hat{A}^{\dagger}_1\hat{A}_1 - \frac{1}{4}\hat{A}^{\dagger}_1\hat{A}_1 + \frac{1}{4}e^{i\theta}\hat{A}^{\dagger}_1\hat{A}_1 + \frac{1}{4}e^{-i\theta}\hat{A}^{\dagger}_1\hat{A}_1 \\
&= -\frac{1}{2}\hat{A}^{\dagger}_1\hat{A}_1 + \frac{1}{2}\hat{A}^{\dagger}_1\hat{A}_1\cos\theta \\
\hat{A}^{\dagger}_1\hat{A}'_0 &= \frac{1}{2}\gamma e^{i\theta/2}\hat{A}^{\dagger}_1\hat{A}_1 - \frac{1}{2}\gamma e^{-i\theta/2}\hat{A}^{\dagger}_1\hat{A}_1 \\
\hat{A}'^{\dagger}_0\hat{A}_1 &= \frac{1}{2}\gamma e^{-i\theta/2}\hat{A}^{\dagger}_1\hat{A}_1 - \frac{1}{2}\gamma e^{i\theta/2}\hat{A}^{\dagger}_1\hat{A}_1
\end{aligned} \tag{37}
$$

Finally, we can obtain the following result.

$$
\begin{aligned}
\langle 1|\hat{A}^{\dagger}_1\hat{A}_1|1\rangle &= 1 \\
\langle 1|\hat{A}'^{\dagger}_0\hat{A}'_0|1\rangle &= -\frac{1}{2} + \frac{1}{2}\cos\theta \\
\langle 1|\hat{A}^{\dagger}_1\hat{A}'_0|1\rangle &= \frac{1}{2}\gamma e^{i\theta/2} - \frac{1}{2}\gamma e^{-i\theta/2} \\
\langle 1|\hat{A}'^{\dagger}_0\hat{A}_1|1\rangle &= \frac{1}{2}\gamma e^{-i\theta/2} - \frac{1}{2}\gamma e^{i\theta/2}
\end{aligned}
$$

$$
\langle 1|\hat{A}^{\dagger}_1\hat{A}_1|1\rangle + \langle 1|\hat{A}'^{\dagger}_0\hat{A}'_0|1\rangle + \langle 1|\hat{A}^{\dagger}_1\hat{A}'_0|1\rangle + \langle 1|\hat{A}'^{\dagger}_0\hat{A}_1|1\rangle = \frac{1}{2} + \frac{1}{2}\cos\theta \tag{38}
$$

This form is equivalent to using the following four-vector instead of (29).

$$
\begin{aligned}
\hat{A}_{\mu:(\mathrm{path1})} &\equiv (0,\ \hat{A}_1,\ 0,\ 0) \\
\hat{A}_{\mu:(\mathrm{path2})} &\equiv (\frac{1}{2}ie^{i\theta/2}\hat{A}_0 - \frac{1}{2}ie^{-i\theta/2}\hat{A}_0,\ 0,\ 0,\ 0)
\end{aligned} \tag{39}
$$

We call this expression "simple calculation method" in this paper. When this simple calculation method is interpreted by physical reality, the single photon passes through only path 1 and the oscillatory field caused by scalar potential exists only on path 2. Although the picture of oscillatory field formation by the scalar potential remains, we lost the natural picture discussed based on the Gaussian light beam that the scalar potential exists in whole space-time. Furthermore, as will be described later, it obscures the generalized picture of the oscillatory field formation by arbitrary number paths. Therefore, we should understand that it does not faithfully describe the objective physical reality, though this simple calculation method is a convenient format for reproducing the calculation results.

The Schrödinger picture of the simple calculation method can be expressed as follows according to (36) and (39).

The state along path 1 is $|1\rangle_{S1}$, the state along path 2 is $\left(\frac{1}{2}\gamma e^{i\theta/2} - \frac{1}{2}\gamma e^{-i\theta/2}\right)|1\rangle_{S0}$. Here we define

$$|\zeta\rangle \equiv \left(\frac{1}{2}\gamma e^{i\theta/2} - \frac{1}{2}\gamma e^{-i\theta/2}\right)|1\rangle_{S0} \tag{40}$$

Then the expected value of the field intensity using the probability interpretation is also reproduced by using Schrödinger picture of the simple calculation method.

$$\begin{aligned}\langle\hat{I}\rangle &\propto \left({}_{S1}\langle 1| + \langle\zeta|\right)\hat{A}_S^\dagger\hat{A}_S\left(|1\rangle_{S1} + |\zeta\rangle\right)\\ &= 1 + \langle\zeta|\hat{A}_S^\dagger\hat{A}_S|\zeta\rangle = 1 - \sin^2\frac{\theta}{2} = \cos^2\frac{\theta}{2} = \frac{1}{2} + \frac{1}{2}\cos\theta \end{aligned}\tag{41}$$

Because $\langle\zeta|\zeta\rangle = -\frac{1}{2} + \frac{1}{2}\cos\theta$ and $\langle\zeta|\zeta\rangle \leq 0$ when $\theta \neq \pm N\pi$(N: even number), $|\zeta\rangle$ is an indefinite metric vector. Therefore we can understand that the above calculation represents Schrödinger picture in which the indefinite metric of the scalar potential is imposed on a state vector.

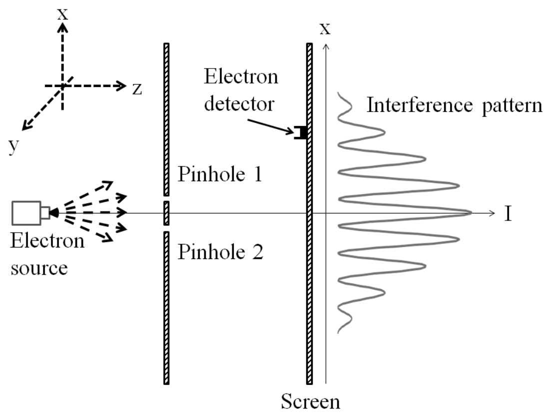

FIG. 2. Schematic view of a typical setup for the single electron interference experiment. : The electron emitted from the electron source passes through the two pinholes and is detected by the electron detector on the screen, and the detection frequency is recorded as an interference pattern on the screen.

B. Single electron interference

1. Calculation using probability interpretation

Figure 2 shows the schematic view of a typical setup for the single electron interference experiment.[1,35] This experimental setup is also the equivalent setup for single photon interference which has the divided path.

In the quantum mechanical description using probability interpretation, the single-electron interference in the figure 2 is calculated using the following probability amplitude

$$\phi_1 = \langle x|1\rangle\langle 1|s\rangle \ , \ \phi_2 = \langle x|2\rangle\langle 2|s\rangle \tag{42}$$

and probability (density) of finding the electron on the screen.[35]

$$P_{12} = |\phi_1 + \phi_2|^2 \tag{43}$$

Where $\phi_1 = \langle x|1\rangle\langle 1|s\rangle$ and $\phi_2 = \langle x|2\rangle\langle 2|s\rangle$ are composed of probability amplitudes as follows.

$\langle 1_{\text{or}}2|s\rangle \equiv \langle$ electron arrives at pinhole 1 or 2 | electron leaves s (electron source) $\rangle$

$\langle x|1_{\text{or}}2\rangle \equiv \langle$ electron arrives at screen x | electron leaves pinhole 1 or 2 $\rangle$

When either pinhole 1 or 2 is closed, the each and total probabilities are calculated to be $P_1 = |\phi_1^2|$, $P_2 = |\phi_2^2|$ and $P = P_1 + P_2 \neq P_{12}$. In this case, the interference when both pinholes open at the same time will not be reproduced. Therefore the probability interpretation that a single electron passes through both pinholes simultaneously has been introduced, though it cannot be further divided.

This single electron interference also gives the same interference of the single photon interference described in section III A.

In probability interpretation, the charge operator is defined instead of the photon number operator defined in the calculation of the single photon interference, and the states of the single electron passing through path 1 and path 2 are introduced. The single electron interference can be calculated by using the charge operator and the above two states.

Specifically, we replace the n-photon number state with the electron-number state $|n\rangle$ where n electrons are present, and the photon number operator $\mathbf{n} = \hat{a}^\dagger \hat{a}$ composed of the operator $\hat{a}$ of the expression (21) is replaced by defining the charge operator $\mathbf{Q} \equiv \int d^3x j_0(x)$.

Where $j_0(x)$ is the 0-th component of four-current $j_\mu = (q, \mathbf{i})$, $\partial^\mu j_\mu = \frac{\partial q}{\partial t} + \nabla \cdot \mathbf{i} = 0$ Because the charge operator satisfies $\mathbf{Q}|n\rangle = nq|n\rangle$, n electronic states are eigenstates of $\mathbf{Q}$.[28,36]

Here the state $|1\rangle_{1\&2}$ that a single electron simultaneously passes through both path 1 and 2 with probability 1/2 can be expressed as follows.

$$|1\rangle_{1\&2} = \frac{1}{\sqrt{2}}|1\rangle \exp i\theta + \frac{1}{\sqrt{2}}|1\rangle \tag{44}$$

where, θ is the phase corresponding to the difference of the path length. Then expected charge intensity $\langle \hat{I} \rangle$ can be calculated as follows.

$$\langle \hat{I} \rangle \propto \langle 1|_{1\&2} \mathbf{Q} |1\rangle_{1\&2} = q\,(1 + \cos\theta) \propto q\left(\frac{1}{2} + \frac{1}{2}\cos\theta\right) \tag{45}$$

This is the same expression as the single photon interference in the previous section.

2. Calculation using alternate interpretation

In order to revise the probability interpretation to alternate interpretation with objective physical reality, we start by reexamining the experimental setup in figure 2 with classical electromagnetism.

In figure 2, an isolated charge q moving with an arbitrary velocity $\mathbf{v}$ in free space generates the scalar and vector potentials called Liénald and Wiechert potentials as follows.[37]

$$\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r\left(1-\frac{v_r}{c}\right)} \ \ , \ \mathbf{A} = \frac{\mu_0}{4\pi}\frac{q\mathbf{v}}{r\left(1-\frac{v_r}{c}\right)} \tag{46}$$

where ε_0, μ_0 and c are the permittivity, permeability and the speed of light respectively. v_r is the component of charge velocity in the direction of the radius vector $\mathbf{r}$ drawn from charge to observer. $r = |\mathbf{r}|$ is the distance in three-dimensional configuration space from the charge to the observer.

Therefore the scalar and vector potentials definitely passes through (or exist) not only the pinhole the electron passes through but also the opposite pinhole.

Thus, similar to the single photon interference described in the previous section, we can obtain the picture that the electron as an objective physical reality propagates through the pinhole on one side and the potentials exist on both side. The potentials are composed of both existing potentials in whole space-time and generated potentials by the movement of the charge. Hence, the electron wave functions should be expressed as follows in such a case.

$$\begin{aligned} \psi_1' &= \psi_1 \cdot \exp\left[i\frac{q}{\hbar}\int_{s\to\text{Pinhole1}\to\text{screen}}(\phi dt - \mathbf{A}\cdot d\mathbf{x})\right] \\ \psi_2' &= \psi_2 \cdot \exp\left[i\frac{q}{\hbar}\int_{s\to\text{Pinhole2}\to\text{screen}}(\phi dt - \mathbf{A}\cdot d\mathbf{x})\right] \end{aligned} \tag{47}$$

where ψ_1' and ψ_2' are the electron wave functions on the screen passing through pinhole 1 and 2 with the potentials respectively. ψ_1 and ψ_2 are the electron wave functions heading to pinhole 1 and 2 at the electron source without the effects of the potentials. Note that because a single electron shows no effects of a "self-generated field", the potentials should not include the "self-generated potentials". For example, when the two electron propagate from the electron source to the detector via pinhole 1 and 2 respectively, the electron propagates via pinhole 1 is affected the potentials generated by electron propagates via pinhole 2 and vice versa.

The following expression is the probability of finding an electron on the screen by using probability interpretation.

$$P_{12} \propto |\psi'|^2 = |\psi_1' + \psi_2'|^2 = |\psi_1|^2 + |\psi_2|^2 - 2\mathrm{Re}\left(\exp\left[i\frac{q}{\hbar}\oint_{s\to 1\to\text{screen}\to 2\to s}(\phi dt - \mathbf{A}\cdot d\mathbf{x})\right]\psi_1^*\psi_2\right) \tag{48}$$

where 1 and 2 of the integration path denote pinhole 1 and 2 respectively.

Although the probability interpretation insists the single electron passes through pinholes 1 and 2 simultaneously, the expression (48) is derived form the picture that the single electron passes through a pinhole on one side assuming that there are potentials on both sides as mentioned above. Hence this is considered to be the expected value of the charge intensity observed on the screen.

However, (48) uses the wave function. We need to shift from the wave function to the state vector to clarify the picture. Note that (48) is a formula equivalent to Aharonov-Bohm effect.[33] The phase term can be eliminated in a single connected region where the spatial region can contract to a single point but cannot be eliminated in multiple-connected region where the spatial region can not contract to a single point.

In the following description, we will show that the phase term can be introduced as a geometric phase, so-called Berry phase, even when there is no existing or generating potentials as described above.

Here we first consider in the case of single electron passes through pinhole 1 or 2, and arrive at the detector.

In this case, we consider the scattering problem of Schrödinger equation for the single electron interference.

The setup of the calculation is that an electron passes through only pinhole 1 (or 2; we use 1 below.) and is affected by the scalar potential of both pinholes as described such existence in previous section. First, when there are no pinholes and no scalar potential, the time dependent solution $|\psi(t)\rangle$ of the electron state obeys following Schrödinger equation.

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = H_0|\psi(t)\rangle \tag{49}$$

where H_0 is Hamiltonian without perturbation (scalar potential). Then we can obtain

$$|\psi(t)\rangle = c_1 \exp(-\frac{iEt}{\hbar})|\psi_1\rangle \tag{50}$$

where c_1, E and $|\psi_1\rangle$ are an arbitrary constant, eigenvalue (energy) of H_0 and the one electron eigenstate heading to pinhole 1 respectively. Next, when the scalar potential exists, the total Hamiltonian is described as $H = H_0 + q\phi$ and we can assume that the solution is an extension of an arbitrary constant c_1 to a time function.

$$|\psi(t)\rangle = c_1(t) \exp(-\frac{iEt}{\hbar})|\psi_1\rangle \tag{51}$$

Note that even if ϕ does not depend on time t or $\phi = 0$, the system, in this case, the splitting and recombination of a beam may be regarded as going backwards in time along one beam and returning along the other beam to its original state at the same time: cyclic evolution.[38] In this subsection, we assume the $|\psi(t)\rangle$ can be expressed as in (51), whereas $c_1(t)$ is justified by the references[38–40], which is discussed in subsection IV E.

By applying the solution (51) to Schrödinger equation of total Hamiltonian, we can obtain the following expression for $c_1(t)$.

$$i\hbar \exp(-\frac{iEt}{\hbar})\frac{\partial c_1(t)}{\partial t}|\psi_1\rangle = \exp(-\frac{iEt}{\hbar})c_1(t)q\phi|\psi_1\rangle \tag{52}$$

By applying $\langle\psi_1|$ to (52) from the left side, the partial differential equation for $c_1(t)$ is obtained as follows.

$$i\hbar\frac{\partial c_1(t)}{\partial t} = c_1(t)q\phi \tag{53}$$

Therefore $c_1(t)$ is obtained as $c_1(t) = c_0 \exp(-i\frac{q}{\hbar}\int \phi dt)$, where $|c_0|^2 = 1$ normalized constant.

Since ϕ is on multiple-connected region when there are pinholes, $c_1(t)$ needs to be added along s $\rightarrow$ Pinhole1 $\rightarrow$ screen and s $\rightarrow$ Pinhole2 $\rightarrow$ screen as follows.

$$c_1(t) = \frac{1}{2}\exp(-i\frac{q}{\hbar}\int_1 \phi dt) + \frac{1}{2}\exp(-i\frac{q}{\hbar}\int_2 \phi dt) \tag{54}$$

where the subscript 1 or 2 of the integral means $s\rightarrow$ Pinhole1 or Pinhole2 $\rightarrow$ screen as same in (47). In addition we assume $c_1(0) = 1$ and the coefficients 1/2 because ϕ is symmetric with respect to Pinhole1 and Pinhole2. Hence the following solution is obtained.

$$|\psi(t)\rangle = \frac{1}{2}\exp(-i\frac{q}{\hbar}\int_1 \phi dt)\exp(-\frac{iEt}{\hbar})|\psi_1\rangle + \frac{1}{2}\exp(-i\frac{q}{\hbar}\int_2 \phi dt)\exp(-\frac{iEt}{\hbar})|\psi_1\rangle \tag{55}$$

Note that this expression corresponds to that the direct product of the oscillatory field of the scalar potential caused by the geometrical arrangement and one electron state as follows.

$$(\frac{1}{2}e^{i\theta/2}\mathbf{A}_0|1\rangle + \frac{1}{2}e^{-i\theta/2}\mathbf{A}_0|1\rangle) \otimes |\psi_1\rangle = \frac{1}{2}e^{i\theta/2}\mathbf{A}_0|1\rangle \otimes |\psi_1\rangle + \frac{1}{2}e^{-i\theta/2}\mathbf{A}_0|1\rangle \otimes |\psi_1\rangle \tag{56}$$

Therefore the expected charge intensity of the single electron interference can be calculated using the above solution as follows.

$$\begin{aligned} \langle \hat{I} \rangle &\propto \langle\psi(t)|\mathbf{Q}|\psi(t)\rangle \\ &= \frac{1}{4}q\langle\psi_1|\psi_1\rangle + \frac{1}{4}q\langle\psi_1|\psi_1\rangle + \frac{1}{4}q\exp(i\frac{q}{\hbar}\oint \phi dt) + \frac{1}{4}q\exp(-i\frac{q}{\hbar}\oint \phi dt) \\ &= \frac{1}{2}q + \frac{1}{2}q\cos(\frac{q}{\hbar}\oint \phi dt) \equiv \frac{1}{2}q + \frac{1}{2}q\cos\theta \end{aligned} \tag{57}$$

where the trajectory of the line integral $\oint$ is as same as (48). This result stands for the single electron interference is not caused by extrinsically generated potentials, but by the oscillatory field of the intrinsically existing scalar potential formed according to the geometrical arrangement of the space.

The relativistic generalization of (54) is given as follows.[33]

$$c_1(t) \;=\; \frac{1}{2}\exp(-i\frac{q}{\hbar}\int_1(\phi dt - \mathbf{A}\cdot d\mathbf{x}) + \frac{1}{2}\exp(-i\frac{q}{\hbar}\int_2(\phi dt - \mathbf{A}\cdot d\mathbf{x}) \tag{58}$$

(Note: This correspond to use $H = H_0 + q\phi - q\mathbf{A}$, however such Hamiltonian is inconsistent with second-order partial differential Schrödinger equation utilizing covariant derivative as described later in section IV E. If we consider first-order partial differential Dirac equation, similar Hamiltonian $H = H_0 + \gamma^0 q\phi - \gamma^i qA_i$ $(i = 1, 2, 3)$ is obtained, where γ^μ are Dirac's gamma matrices and the wave function has 4 components.)

Hence the expected charge intensity of the single electron interference using the relativistic generalization is expressed as follows.

$$\begin{aligned}\langle\hat{I}\rangle \;&\propto\; \frac{1}{2}q + \frac{1}{2}q\cos(\frac{q}{\hbar}\oint\phi_{\text{int}}dt + \frac{q}{\hbar}\oint(\phi_{\text{ext}}dt - \mathbf{A}\cdot d\mathbf{x})) \\ &\equiv\; \frac{1}{2}q + \frac{1}{2}q\cos(\theta + \frac{q}{\hbar}\oint(\phi_{\text{ext}}dt - \mathbf{A}\cdot d\mathbf{x}))\end{aligned} \tag{59}$$

where we split the scalar potential ϕ into intrinsically existing term ϕ_{int} and extrinsically generated term ϕ_{ext} such as Faraday cage in Ref[33] by considering Aharonov-Bohm effect, i.e., first term θ in *cos* generates single electron interference and second term generates Aharonov-Bohm effect which shifts the interference pattern. Note that we can also split the vector potential $\mathbf{A}$ into $\mathbf{A} = \mathbf{A}_{\text{int}} + \mathbf{A}_{\text{ext}}$.

In (52), we considered the existing scalar potential ϕ. If there is no electromagnetic potential in space, no phase term is generated.

However, even if we try to calculate the "self-generating" electromagnetic potentials in (46), the potentials can not be defined in terms of the position of electrons on the path because $r = 0$. This is one of the major problems from classical physics that is brought into the quantum fields theory, which gives infinite self-energy, and there is still no fundamental theoretical solution.

Therefore, in this paper, we follow the conventional treatment that a single electron is not affected by the "self-generating" potentials. Instead, we will show that the introduction of the geometric phase noted by Berry[41] yields a phase term that is formally identical to the case where there are existing potentials, as follows.

The geometric phase noted by Berry as follows, which was introduced by slowly transport or adiabatic evolution, was generalized for any cyclic evolution in the projective Hilbert space by Y. Aharonov and J. Anandan.[38] and all quantum evolutions, not merely cyclic evolutions.[39,40]

Note that the following discussion for single electron interference is based on the original concept by Berry, but it should be discussed based on the reference[38–40] which generalizes the adiabatic theorem (we call this Berry-Aharonov-Anandan phase, BAA phase). The calculation by the BAA phase is discussed in the calculation of neutron interference later in subsection IV E. The single electron interference should also be calculated by the same BAA phase as in subsection IV E though the following result is unchanged.

The introduction of the geometric phase $\gamma(C)$, where C is closed loop as described below, is formally the same as replacing $c_1(t)$ in (51) with $\exp[i\gamma(t)]$ which means we consider the Hamiltonian $H(\mathbf{r}(t))$ with varying parameters $\mathbf{r}(t) = (x, y, z)$ instead of an unperturbed Hamiltonian H_0 in (49) and the parameter dependent eigenstate $|\psi_1(\mathbf{r}(t))\rangle$ instead of time dependent state $|\psi_1(t)\rangle$ in addition to the parameter dependent eigenvalue (energy) $E(\mathbf{r}(t))$ instead of stationary eigenvalue E. In these replacements, the specific solution of the Schrödinger equation is obtained as following form.

$$\begin{aligned} |\psi(t)\rangle &= \exp\left[i\gamma(t) - \frac{i}{\hbar}\int_0^t E(\mathbf{r}(s))ds\right]|\psi_1(\mathbf{r}(t))\rangle \\ &\equiv \exp\left[i\gamma(t) + i\gamma_d(t)\right]|\psi_1(\mathbf{r}(t))\rangle \end{aligned} \tag{60}$$

Substituting (60) into Schrödinger equation (49) with (50) and the above replacement yields

$$\frac{\partial\gamma(t)}{\partial t}|\psi_1(\mathbf{r}(t))\rangle = i\frac{\partial}{\partial t}|\psi_1(\mathbf{r}(t))\rangle \tag{61}$$

Multiplying $\langle\psi_1(\mathbf{r}(t))|$ (multiplying the other eigenstates is zero due to adiabatic assumption) by the left-hand side leads to

$$\begin{aligned} \gamma(t) &= i\int_0^t \langle\psi_1(\mathbf{r}(s))|\frac{\partial}{\partial s}|\psi_1(\mathbf{r}(s))\rangle ds \\ &= i\int_{\mathbf{r}(0)}^{\mathbf{r}(t)} \langle\psi_1(\mathbf{r})|\nabla_{\mathbf{r}}|\psi_1(\mathbf{r})\rangle \cdot d\mathbf{r} \end{aligned} \tag{62}$$

where $\partial/\partial t = \sum_i dx_i/dt \cdot \partial/\partial x_i$ is used. When $\mathbf{r}(t)$ travels in a closed loop C on parameter space, i.e., $\mathbf{r}(0) = \mathbf{r}(T)$, the total phase change of $|\psi\rangle$ is given by

$$|\psi(T)\rangle = \exp\left[i\gamma(C) - \frac{i}{\hbar}\int_0^T E(\mathbf{r}(s))ds\right]|\psi_1(\mathbf{r}(0))\rangle \tag{63}$$

Note that the second term of the phase $\equiv \gamma_d(T)$ is called dynamical phase in comparison to the geometric phase.

Because the two paths along $C_1 \equiv$ s $\rightarrow$ Pinhole1 $\rightarrow$ screen and $C_2 \equiv$ s $\rightarrow$ Pinhole2 $\rightarrow$ screen which forms a closed loop $C \equiv C_1 - C_2$ are on multiple-connected region in this setup, the solution (60) needs to be added up the phase changes $c_1(t) \equiv \exp(i\gamma(t))$ along C_1 and C_2 as same as (54) by taking the $\mathbf{r}(t)$ as an external parameter in the real space itself.

$$\begin{aligned}|\psi(t)\rangle &= \frac{1}{2}\exp\left[i\gamma(C_1) + i\gamma_d(C_1)\right]|\psi_1(\mathbf{r}(t))\rangle \\ &\quad + \frac{1}{2}\exp\left[i\gamma(C_2) + i\gamma_d(C_2)\right]|\psi_1(\mathbf{r}(t))\rangle \end{aligned} \tag{64}$$

Note that because $|\psi_1(\mathbf{r}(t))\rangle$ is the one electron eigenstate heading to pinhole 1, the real single electron travels along C_1 but the phase difference due to the geometrical arrangement of the space is introduced. This is the identical form as (56) by equating the phases of the scalar potential with the geometrical phases.

By using (64), the expected charge intensity of the single electron interference can be calculated as same manner as (57).

$$\begin{aligned}\langle\hat{I}\rangle &\propto \langle\psi(t)|\mathbf{Q}|\psi(t)\rangle \\ &= \frac{1}{4}q + \frac{1}{4}q + \frac{1}{4}q\exp\left[i\gamma(C) + i\gamma_d(C)\right] + \frac{1}{4}q\exp\left[-i\gamma(C) - i\gamma_d(C)\right] \\ &= \frac{1}{2}q + \frac{1}{2}q\cos(\gamma(C) + \gamma_d(C))\end{aligned} \tag{65}$$

where the normalization of the eigenstate $\langle\psi_1(\mathbf{r}(t))|\psi_1(\mathbf{r}(t))\rangle = 1$ is used.

Note that because the following discussion will deviate from the adiabatic assumption, it should be discussed based on the reference[38–40] described later in subsection IV E.

When we asuume $\gamma_d(C) = -\frac{1}{\hbar}\{\int_0^{t_1} E(\mathbf{r}(s))ds - \int_0^{t_2} E(\mathbf{r}(s))ds\} = -\frac{E}{\hbar}\Delta t$, $\gamma(C)$ can be calculated as follows, where $\Delta t = t_1 - t_2$ is the difference in travel time of the electron motion along C_1 and C_2.

The time evolution eigenstate $|\psi_1(\mathbf{r}(t))\rangle$ of the free (without potentials) electron wave propagating in space with the angular frequency ω and energy E can be expressed as follows.

$$|\psi_1(\mathbf{r}(t))\rangle = \exp(-i\omega t)\exp(-i\frac{E}{\hbar}t)|\psi_{1\mathrm{res}}(\mathbf{r}(0))\rangle \tag{66}$$

The direct substitution of (66) into (62) leads to

$$\begin{aligned}\gamma(t) &= \int_0^t \omega + \frac{E}{\hbar}\langle\psi_{1\mathrm{res}}(\mathbf{r}(0))|\psi_{1\mathrm{res}}(\mathbf{r}(0))\rangle ds + i\int_0^t \langle\psi_{1\mathrm{res}}(\mathbf{r}(0))|\frac{\partial}{\partial s}|\psi_{1\mathrm{res}}(\mathbf{r}(0))\rangle ds \\ &= \int_0^t (\omega + \frac{E}{\hbar})ds \end{aligned} \tag{67}$$

Therefore

$$\gamma(C) + \gamma_d(C) = \gamma(C_1) - \gamma(C_2) - \frac{E}{\hbar}\Delta t = (\omega + \frac{E}{\hbar})\Delta t - \frac{E}{\hbar}\Delta t = \omega\Delta t \tag{68}$$

Hence the expected charge intensity of the single electron interference (65) is as follows.

$$\langle\hat{I}\rangle \propto \frac{1}{2}q + \frac{1}{2}q\cos(\omega\Delta t) \tag{69}$$

This expression is identical to the following traditional calculation for the single electron interference.[35]

$$P_{12} = |\phi_1 + \phi_2|^2 = |\phi_1|^2 + |\phi_2|^2 + 2|\phi_1||\phi_2|\cos\delta \tag{70}$$

where P_{12}, ϕ_1 and ϕ_2 are given in (42) and (43), δ is the phase difference between ϕ_1 and ϕ_2.

In the above discussion does not include the varying potentials utilizing some devices such as an infinite length solenoid coil or annular solenoid coil that penetrates the integral loop. In such case, external potentials $\mathbf{A}_{\mathrm{ext}}$ is generated and varies according to the flux generated by the device, which shifts the fringe pattern of the expected charge intensity. This kind of fringe shift originate in Aharonov-Bohm effect has been experimentally verified by A. Tonomura.[42]

As described in the above (though the description should be replaced by BAA phase discussed in later subsection IV E), we can obtain the picture of the objective physical reality that the space is divided by two pinholes which causes an oscillatory field (phase distribution) similar to the scalar potential corresponds to the indefinite metric, and a single electron passes through one pinhole placed in the oscillatory field (or geometrical phase distribution) and arrives at the detector placed in the same oscillatory field.

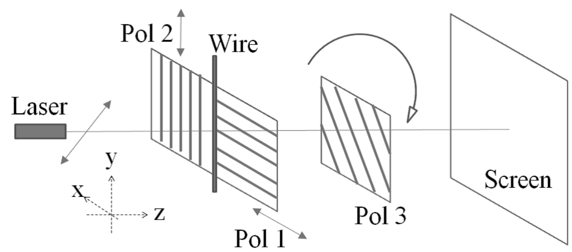

FIG. 3. Typical setup for the Quantum Eraser. Pol1 and Pol2 are fixed linear polarizers with polarizing axes perpendicular (x and y). Pol3 is a revolvable linear polarizer.[2,43]

C. EPR correlation

1. Calculation using probability interpretation

For the calculation of EPR correlation using photon polarization by probability interpretation, it is helpful to first examine an experimental setup using polarizers called Quantum Eraser shown in figure 3.[2,43]

The phenomenon of the Quantum Eraser is outlined below.

In case of no polarizers, we can observed an interference pattern composed of dark and bright fringes on the screen because light passing on the left of the wire is combining, or "interfering," with light passing on the right-hand side. This means " we have no information about which path each photon went".

In case of polarizers 1 and 2, which are called "which-path markers", are positioned right behind the wire as shown in Figure 3, the launched light polarized in 45° direction from the laser is polarized in perpendicular (x-polarized and y-polarized) by these polarizers. This means "which-path makers" have made available the information about which path each photon went. Therefore we can observe no interference pattern on the screen .

When polarizer 3 with the polarization angle +45° or -45° is subsequently inserted in front of the screen in addition to the "which-path makers", the interference pattern reappears due to the invalidation of the information of "which-path makers" by polarizer 3.

The mathematical description of the erasure and reappearance of the interference pattern is as follows. The x- and y-polarized photon passing through polarizer 1 and 2 can be expressed by following quantum-superposition state.

$$|x\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \tag{71}$$

and

$$|y\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle \tag{72}$$

where "+" and "-" stand for polarizations +45° and -45° with respect to x.

As seen in the left-hand side of equations. (71) and (72), the photons passing through polarizers 1 and 2 are polarized at right angles to each other. This prevent the interference pattern. In other words, "which-path makers" have enabled information about which path each photon went. The right side of equations (71) and (72) have the same polarization states, but the interference patterns created by the polarizer states of +45° and -45° are inverse fringes and annihilate each other. Therefore, there are no interference patterns in the sum total of the images.

The subsequent insertion of the polarizer 3 with the polarization angle +45° or -45° can allow only $|+\rangle$ or $|-\rangle$ to pass through polarizer 3. Therefore, either $|+\rangle$ or $|-\rangle$ of both equations (71) and (72) can create the interference pattern again, which means that we can not identify which-path the photons had passed through, i.e., polarizer 3 has disabled the information of "which-path makers".

As is clear from (71) and (72), enable/disable information about which side of the wire each photon passed through is interpreted by the states that contains +45° and -45° polarized single photon with a probability of 1/2 even if the laser light intensity becomes extremely low and it becomes an indivisible single photon.

In order to calculate the single photon interference by probability interpretation, we replace the side of the polarizer 1 with path 1 and the side of polarizer 2 with path 2 of MZI in the calculation of single photon interference in the previous section. In addition, the ratio of photon passage by polarizers 1, 2 and 3 are replaced with those rotation angles. We examine both x-polarized and y-polarized photon emitted from the laser in the calculation. These calculations give the same results as the EPR correlation described later.

In the calculation, all angles of the polarizers are based on the x axis. The x-polarized light beam emitted from the laser should not be able to pass through polarizer 2, which allows only y-polarization to pass. However, probability interpretation asserts that x-polarized light is a superposition of polarization states different from x-polarized light like (71). Therefore, the calculation is possible by replacing the probability that a photon can pass with a phase that is proportional to the difference between the polarization angle of the photon and the angle of the polarizer.

The probability that an x-polarized photon can pass through the side of the polarizer 1 is described as the angle of the polarizer 1. In this case, since it is originally polarized on the x axis, the phase can be selected as 0. The photon polarized in the x-axis after passing through the polarizer 1 is described by introducing the rotation angle $|\phi|$ of the polarizer 3 corresponding to the probability that the photon can pass through the polarizer 3 into the phase.

On the other hand, the x-polarized photon passing through the polarizer 2 is described as a rotation angle $\pi/2$ corresponding to the probability that the photon can pass through the polarizer 2.

The rotation angle $\frac{\pi}{2}-|\phi|$ by the polarizer 3 is further introduced into the photon passing through the polarizer 2. In addition, the phase difference θ between the polarizer 1 and 2 is introduced on the x-polarized photon. Now we can define the following electric field operator.

$$\begin{aligned}\hat{E} &= \frac{1}{\sqrt{2}}\hat{a}_1 \exp\left(i|\phi|+\theta\right) + \frac{1}{\sqrt{2}}\hat{a}_2 \exp\left(i\frac{\pi}{2}\right)\exp\left\{i\left(\frac{\pi}{2}-|\phi|\right)\right\} \\ &= \frac{1}{\sqrt{2}}\hat{a}_1 \exp\left(i|\phi|+\theta\right) + \frac{1}{\sqrt{2}}\hat{a}_2 \exp\left\{i\left(\pi-|\phi|\right)\right\} \end{aligned} \tag{73}$$

The interference on the screen is calculated as same as in the previous section.

$$\langle\hat{I}\rangle \propto \langle n|\hat{E}^\dagger\hat{E}|n\rangle = \frac{1}{2}\langle n|\hat{a}_1^\dagger\hat{a}_1|n\rangle + \frac{1}{2}\langle n|\hat{a}_2^\dagger\hat{a}_2|n\rangle - \cos\left(\theta+2|\phi|\right)\langle n|\hat{a}_1^\dagger\hat{a}_2|n\rangle \tag{74}$$

$\langle\hat{I}\rangle$ is the expected field intensity, which is proportional to the number of photons, as in the previous section. In the probability interpretation, the above expected value is calculated as follows, including the case of a single photon of which photon number is 1 ($n=1$).

$$\langle\hat{I}\rangle \propto \frac{1}{2} - \frac{1}{2}\cos\left(2|\phi|+\theta\right) \tag{75}$$

In the case of the y-polarized light beam emitted from the laser, we transpose the phase θ to the y-polarized side and introduce the phases of the polarizers in the same way as for the x-polarized light. Then the electric filed operator can be calculated as follows,

$$\hat{E} = \frac{1}{\sqrt{2}}\hat{a}_1 \exp\left\{i\left(\frac{\pi}{2}+|\phi|\right)\right\} + \frac{1}{\sqrt{2}}\hat{a}_2 \exp\left\{i\left(\frac{\pi}{2}-|\phi|+\theta\right)\right\} \tag{76}$$

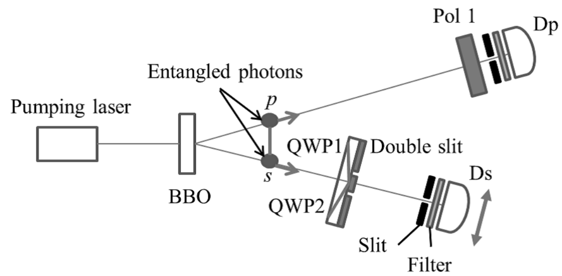

FIG. 4. Typical setup for the Delayed Choice Quantum Eraser. QWP1 and QWP2 are quarter-wave plates aligned in front of the double slit with fast axes perpendicular. Pol1 is a linear polarizer. BBO ($\beta-\mathrm{BaB_2O_4}$) crystal generates entangled photons by spontaneous parametric down-conversion[22]..

Finally, we can obtain as follows.

$$\langle \hat{I} \rangle \propto \frac{1}{2} + \frac{1}{2}\cos\left(2|\phi| - \theta\right) \tag{77}$$

Here, if the laser beam contains one or more x-polarized and y-polarized photons each with a probability of 1/2, the intensity of the interference is given by the sum of (75) and (77) as follows.

$$\langle \hat{I} \rangle \propto 1 - \frac{1}{2}\cos\left(2|\phi| + \theta\right) + \frac{1}{2}\cos\left(2|\phi| - \theta\right) \tag{78}$$

In the expression, when $\phi = \pm\pi,\ \pm\frac{1}{2}\pi$, it becomes $\langle I \rangle \propto 1$ and the interference fringes are erased, $\phi = \pm\frac{1}{4}\pi$ becomes $\langle I \rangle \propto 1 \pm \sin\theta$ and the interference fringe is reproduced.

Even if the laser intensity is very weak and consists of an x-polarized single photon and a y-polarized single photon, (78) is satisfied.

The delayed-choice Quantum Eraser for examining the EPR correlation below corresponds to the case that the emitted photon from the laser is an x-polarized single photon and a y-polarized single photon of this Quantum Eraser. The intensity of the photon interference obtained in the experiment is the same as (75) and (77).

Figure 4 shows the typical setup for the Delayed Choice Quantum Eraser for examining EPR correlation. This experimental setup is similar setup where the polarizer 1 plays the role of the polarizer 3 in figure 3 of Quantum Eraser. In this setup, the BBO is excited by the light emitted from the laser to create p and s photons of which polarizations are orthogonal to each other. These photons propagate in different paths, and p is scanned by the polarizer 1 and measured by the detector Dp. The photon s is measured by Ds through the quarter-wave plates 1 and 2 of which fast axes are orthogonal to each other and the double slit on the back surface thereof, and further through the slit and the filter.

In this experiment, the setup is arranged according to the following order, and the result in each arrangements is obtained.

1. When polarizer 1, QWP1 and QWP2 are removed, the coincidence counts of Ds versus Dp shows an intensity fluctuation of the photon which indicates the interference by the double slit.

2. When QWP1 and QWP2 are installed, the intensity fluctuation of the coincidence counts is erased.

3. The polarizer 1 is additionally installed in the direction that matches the fast axis of QWP1. Then, the intensity fluctuation of the coincidence counts reappears as the interference.

4. Subsequently, the angle of the polarizer 1 is rotated by 90 degrees. Then, the intensity fluctuation of the coincidence counts shows the interference of which the bright and dark reversed.

From this procedure, the polarizer 1 corresponds to the polarizer 3 of Quantum Eraser described above and looks like erases or reappears the information about which path (QWP1 or QWP2) each photon went. That is a phenomenon similar to the above-mentioned Quantum Eraser. Moreover, although the polarization direction setting of the photon p and the observation of the interference by the photon s are performed at spatially separated positions, the polarization of these photons is completely correlated with one side being x-polarized and the other side being y-polarized. Here the interference in the opposite phase obtained in steps 3 and 4 are given by (75) and (77) calculated by the Quantum Eraser described above.

When the distance from BBO to Dp is arranged longer than the distance from BBO to the double slit, such arrangement becomes a delayed choice because the choice of the polarization direction of the photon p by the polarizer 1 is performed after the observation of the interference by the photon s. In this arrangement, the presence or absence of the interference can also be observed by the same procedure as above. This seems that the erasing or reproduction of the information of “which-path marker” by the polarizer 1 was determined after the observation. Note that the experiment did not allow for the observer to choose the polarization angle in the time period after photon s was detected and before detection of p, as discussed in the reference[22].

When s and p are single photons, the probability interpretation asserts that the photon s and p do not have a state of x polarization (or y polarization) that is fixed from the beginning such as $|x\rangle_{\mathrm{s}}$ or $|y\rangle_{\mathrm{s}}$ and $|x\rangle_{\mathrm{p}}$ or $|y\rangle_{\mathrm{p}}$. Instead they are in the superposition states of those polarizations as follows.

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle_{\mathrm{s}}|y\rangle_{\mathrm{p}} + |y\rangle_{\mathrm{s}}|x\rangle_{\mathrm{p}}\right) \tag{79}$$

Such perfect correlated state existing in a spatially separated region is called correlated photon pair state (Entanglement). According to the probability interpretation for the entanglement, both photon s and p propagate each path as superposition of x-polarized and y-polarized state (79).

This postulates the existence of long-range correlation beyond the causality that contradicts the locality of physical laws, i.e., theory of relativity, because even if the photons are far apart from each other, when the state of one photon (s or p) is observed and determined to be $|x\rangle$ then that of the other photon (p or s) suddenly changes from the quantum-superposition state into $|y\rangle$. Historically, EPR paper[5] written by "Einstein, Podolsky and Rosen (EPR)" first pointed out the imperfections of the standard quantum theory by taking up such non-local correlation states. At present, the correlated photon pair state (Entanglement) is also called the EPR state.

2. Calculation using alternate interpretation

Here we revise the incomprehensible probability interpretation such as correlated photon pair state to alternate interpretation using objective physical reality.

The results of the interference (75) and (77) are obtained in both setups in figure 3 and figure 4 as mentioned above. These results has been thought to be peculiar to quantum theory, which cannot be obtained by classical calculation assuming that a single photon has a fixed polarization. The violation of Bell's inequality supports the calculation results of the quantum theory.[44,45]

However, the obtaining the identical calculation results both probability and alternate interpretation using objective physical reality with locality means that Bell's inequality violates in both cases.

In the following, the results of the interference (75) and (77) are reproduced with a picture of the objective physical reality by replacing the side of the polarizer 1 as the photon path s and the side of the polarizer 2 as the photon path p in the setup of figure 4 respectively.

As a picture of objective physical reality, we assume the laser creates either fixed x- or y-polarized single photon, or two photons with fixed x- and y-polarized single photon. Since there is a scalar potential in whole space-time as described in the calculation of single photon interference, the four-vector in the space from the laser output to immediately before passing through the polarizer 1 and 2 is expressed as follows.

$$A_\mu = (A_0,\ A_1,\ A_2,\ 0) \tag{80}$$

Where, in order to simplify the calculation, the non-physical longitudinal wave was ignored as $A_3 = 0$.

First, we calculate in the case of there exists an x-polarized single photon and scalar potential. This case corresponds to the calculated situation for (75). When the x-polarized single photon and the scalar potential pass through both polarizers, the x-polarized photon can only pass through polarizer 1 but not polarizer 2. When the scalar potential is divided on each side, a phase difference is introduced associated with the division. Therefore the following expressions are appropriate for the four-vectors after passing through each polarizer.

$$A_{(\text{x pol 1})\ \mu} = \left(\frac{1}{2}e^{i\theta_x/2}A_{(x)0},\ A_{(x)1},\ 0,\ 0\right)\ ,\ A_{(\text{x pol 2})\ \mu} = \left(\frac{1}{2}e^{-i(\theta_x/2+\pi/2)}A_{(x)0},\ 0,\ 0,\ 0\right) \tag{81}$$

Here, a phase shift $\pi/2$ based on x-polarized photon is introduced into the scalar potential passing through the polarizer 2. Where, the phase difference due to the division of the scalar potential is divided into both sides for the symmetrical description as θ_x.

When the x-polarized photon polarized by the polarizer 1 that represent the physical reality of real x-polarized photon among this four-vector is trying to pass through the polarizer 3, it can not pass through the polarizer 3 of which rotation angle is not x axis. However, because the scalar potential passes through with obtaining $|\phi|$ phase shift, which corresponds to the rotation angle of the polarizer 3, the phase term becomes $i\left(|\phi| + \theta_x/2\right)$.

On the other hand, the scalar potential that has passed through the polarizer 2 is polarized in y-direction. Therefore in the case of the rotation angle of the polarizer 3 is measured from the x-axis (x=0) , $\pi/2$ is added to the rotation angle of the polarizer 3, then the phase becomes $-i\left(|\phi| + \theta_x/2 + \pi\right)$.

Hence the four-vector at the polarizer 3 is expressed as follows.

$$\begin{aligned} A_{(\mathrm{x\ pol\ 1,\ 2\to3})\ \mu} &\equiv A_{(\mathrm{x\ pol\ 1\to3})\ \mu} + A_{(\mathrm{x\ pol\ 2\to3})\ \mu} \\ &= \left(\frac{1}{2}e^{i(|\phi|+\theta_x/2)}A_{(x)0} + \frac{1}{2}e^{-i(|\phi|+\theta_x/2+\pi)}A_{(x)0},\ A_{(x)1},\ 0,\ 0\right) \end{aligned} \tag{82}$$

Here from Lorentz invariance, $\langle 1|A_0^\dagger A_0|1\rangle = \langle 1|A_1^\dagger A_1|1\rangle = \langle 1|A_2^\dagger A_2|1\rangle = 1$ is required as in single photon interference. Then the following interference can be obtained.

$$\langle I_s\rangle \propto \langle 1|A^\dagger_{(\mathrm{x\ pol\ 1,\ 2\to3})}A_{(\mathrm{x\ pol\ 1,\ 2\to3})}|1\rangle = \frac{1}{2} + \frac{1}{2}\cos(2|\phi| + \theta_x) \tag{83}$$

Note that $A^\dagger A \equiv -\mathsf{g}_{\mu\nu}A^{\mu\dagger}A^\nu = -A_0^\dagger A_0 + A_1^\dagger A_1 + A_2^\dagger A_2 + A_3^\dagger A_3$.

This can be interpreted that the interference between the oscillatory field caused by the scalar potential and x-polarized photon determines the photon intensity that can pass through the polarizer 3.

Similarly, in the case of there exists y-polarized single photon and scalar potential, the phase difference of the divided scalar potential can be θ_y as follows.

$$A_{(\mathrm{y\ pol\ 1})\ \mu} = \left(\frac{1}{2}e^{i(\theta_y/2+\pi/2)}A_{(y)0},\ 0,\ 0,\ 0\right)\ ,\ A_{(\mathrm{y\ pol\ 2})\ \mu} = \left(\frac{1}{2}e^{-i\theta_y/2}A_{(y)0},\ 0,\ A_{(y)2},\ 0\right) \tag{84}$$

$$\begin{aligned} A_{(\mathrm{y\ pol\ 1,\ 2\to3})\ \mu} &\equiv A_{(\mathrm{y\ pol\ 1\to3})\ \mu} + A_{(\mathrm{y\ pol\ 2\to3})\ \mu} \\ &= \left(\frac{1}{2}e^{i(|\phi|+\theta_y/2+\pi/2)}A_{(y)0} + \frac{1}{2}e^{-i(|\phi|+\theta_y/2+\pi/2)}A_{(y)0}, 0,\quad A_{(y)2},\ 0\right) \end{aligned} \tag{85}$$

Then the following interference can be obtained.

$$\langle I_p\rangle \propto \langle 1|A^\dagger_{(\mathrm{y\ pol\ 1,\ 2\to3})}A_{(\mathrm{y\ pol\ 1,\ 2\to3})}|1\rangle = \frac{1}{2} + \frac{1}{2}\cos(2|\phi| + \theta_y) \tag{86}$$

Choosing the phase difference due to the division of the scalar potential as $\theta \equiv \theta_x - \pi = -\theta_y$, the identical interferences (75) and (77) are calculated as follows..

$$\langle I_s\rangle \propto \frac{1}{2} - \frac{1}{2}\cos(2|\phi| + \theta)\ ,\qquad \langle I_p\rangle \propto \frac{1}{2} + \frac{1}{2}\cos(2|\phi| - \theta) \tag{87}$$

Furthermore, when there are x-polarized single photon, y-polarized single photon and scalar potential, the four-vector at polarizer 3 is the sum of (82) and (85), we can reproduce (78) by simple calculation as follows.

$$\langle 1|A^{\dagger}_{(\mathrm{x,\ y\ pol\ 1,\ 2\to 3})}A_{(\mathrm{x,\ y\ pol\ 1,\ 2\to 3})}|1\rangle = 1-\frac{1}{2}\cos(2|\phi|+\theta)+\frac{1}{2}\cos(2|\phi|-\theta) \tag{88}$$

Now, we have shown that the identical results (75), (77) and (78) can be obtained by using the indefinite metric as it is. In addition, we show that the identical results can be obtained by the simple calculation method.

In the simple calculation method, we consider the state that only the scalar potential exists without real photon and forms the oscillatory field. This state is represented by $|\zeta\rangle$ as shown in the single photon calculation. Therefore, in this experimental setup, the state of x-polarized single photon in the space should be replaced by $|x\rangle + |\zeta\rangle$ instead of $|x\rangle$. However, when the scalar potential exists in a space without division, the phase difference due to the path difference is $\theta = 0$, and the oscillatory field due to the interference of the scalar potential is not formed, so $\langle\zeta|\zeta\rangle = 0$.

Here we note the above and choose the states of the x-polarized single photon and the y-polarized single photon as follows.

$$\begin{aligned}
|x\rangle + |\zeta_{\phi,x}\rangle &= |x\rangle + \frac{1}{2}\gamma e^{i|\phi|}e^{-i\theta/2}|x\rangle - \frac{1}{2}\gamma e^{-i|\phi|}e^{i\theta/2}|x\rangle \\
|y\rangle + |\zeta_{\phi+\frac{1}{2}\pi,y}\rangle &= |y\rangle + \frac{1}{2}\gamma e^{i(|\phi|+\frac{1}{2}\pi)}e^{i\theta/2}|y\rangle - \frac{1}{2}\gamma e^{-i(|\phi|+\frac{1}{2}\pi)}e^{-i\theta/2}|y\rangle
\end{aligned} \tag{89}$$

By using the states we can calculate the interference in the presence of the x-polarized single photon as follows.

$$\begin{aligned}
\langle I\rangle &\propto (\langle x| + \langle\zeta_{\phi,x}|)(|x\rangle + |\zeta_{\phi,x}\rangle) = \langle x|x\rangle - \frac{1}{2}\langle x|x\rangle - \frac{1}{2}\langle x|x\rangle\cos(2|\phi|+\theta) \\
&= \frac{1}{2} - \frac{1}{2}\cos(2|\phi|+\theta)
\end{aligned} \tag{90}$$

This is identical with (75). Similarly, we can calculate the identical interference (77) in the presence of the y-polarized single photon.

Finally, we can calculate the interference in the presence of both x-polarized and y-polarized photons as follows,

$$\langle I\rangle \propto \left(\langle x| + \langle\zeta_{\phi,x}| + \langle y| + \langle\zeta_{\phi+\frac{1}{2}\pi,y}|\right)\left(|x\rangle + |\zeta_{\phi,x}\rangle + |y\rangle + |\zeta_{\phi+\frac{1}{2}\pi,y}\rangle\right) \tag{91}$$

where $\langle x|y\rangle = \langle y|x\rangle = 0$, then

$$\langle I\rangle \propto \left(\langle x| + \langle\zeta_{\phi,x}|\right)\left(|x\rangle + |\zeta_{\phi,x}\rangle\right) + \left(\langle y| + \langle\zeta_{\phi+\frac{1}{2}\pi,y}|\right)\left(|y\rangle + |\zeta_{\phi+\frac{1}{2}\pi,y}\rangle\right) \tag{92}$$

This is the sum of the result (75) when only x-polarized photon exists and the result (77) when only y-polarized photon exists. Therefore (78) is reproduced.

$$\langle I\rangle \propto 1 - \frac{1}{2}\cos\left(2|\phi| + \theta\right) + \frac{1}{2}\cos\left(2|\phi| - \theta\right) \tag{93}$$

This simple calculation method is the Schrödinger picture in which the calculation of an indefinite metric is imposed on the state vector. As in the case of single-photon interference, we can calculate the interference using Heisenberg picture in which the calculation of an indefinite metric is imposed on the operator. In Heisenberg picture, the photon number operator is replaced with $\mathbf{n} = (\hat{A}_1^\dagger + \hat{A}_p^\dagger)(\hat{A}_1 + \hat{A}_p)$. Where, $\hat{A}_1$ and $\hat{A}_p$ (p : polarization $=$ $x,\ y, \cdots, etc.$) are the photon annihilation operator that represents a real photon and a photon annihilation operator that represents a scalar potential polarized in the p direction. These operators can be defined as follows.

$$\hat{A}_x = \frac{1}{2}\gamma e^{i|\phi|}e^{-i\theta/2}\hat{A}_1 - \frac{1}{2}\gamma e^{-i|\phi|}e^{i\theta/2}\hat{A}_1\ ,\ \hat{A}_x^\dagger = \frac{1}{2}\gamma e^{-i|\phi|}e^{i\theta/2}\hat{A}_1^\dagger - \frac{1}{2}\gamma e^{i|\phi|}e^{-i\theta/2}\hat{A}_1^\dagger \tag{94}$$

We can reproduce (90) by using the above operators of Heisenberg picture.

$$\langle I\rangle = \langle n|(\hat{A}_1^\dagger + \hat{A}_x^\dagger)(\hat{A}_1 + \hat{A}_x)|n\rangle = \langle n|\mathbf{n}_1|n\rangle + \langle n|\hat{A}_x^\dagger\hat{A}_x|n\rangle \propto \frac{1}{2} - \frac{1}{2}\cos\left(2|\phi| + \theta\right) \tag{95}$$

Note that the Heisenberg picture imposes the indefinite metric calculation on the operator, so we should replace the ordinary photon annihilation operator for x-polarization $\hat{A}_1$ with $\hat{A}_1 + \hat{A}_x$, where $\hat{A}_x$ is the photon annihilation operator derived from the scalar potential $\hat{A}_x$. Therefore, the photon number operator in the presence of x-polarized and y-polarized photons is $(\hat{A}_1 + \hat{A}_x) + (\hat{A}_2 + \hat{A}_y)$. We can reproduce (93) as follows by choosing the appropriate phase of these replacements.

$$\begin{aligned}\langle I\rangle &= \langle n|(\hat{A}_1^\dagger + \hat{A}_x^\dagger + \hat{A}_2^\dagger + \hat{A}_y^\dagger)(\hat{A}_1 + \hat{A}_x + \hat{A}_2 + \hat{A}_y)|n\rangle \\ &= \langle n|\mathbf{n}_1|n\rangle + \langle n|\hat{A}_x^\dagger\hat{A}_x|n\rangle + \langle n|\mathbf{n}_2|n\rangle + \langle n|\hat{A}_y^\dagger\hat{A}_y|n\rangle \\ &\propto 1 - \frac{1}{2}\cos\left(2|\phi| + \theta\right) + \frac{1}{2}\cos\left(2|\phi| - \theta\right)\end{aligned} \tag{96}$$

Where, we assume that there are the same number of x-polarized and y-polarized photons. $\langle n|\mathbf{n}_1|n\rangle \equiv \langle n|\hat{A}_1^\dagger \hat{A}_1|n\rangle = \langle n|\mathbf{n}_2|n\rangle \equiv \langle n|\hat{A}_2^\dagger \hat{A}_2|n\rangle = n$

Under the condition $|n\rangle \equiv |n\rangle_x + |n\rangle_y$, we can calculate equivalent relations to those in conventional quantum theory such as $\hat{A}_1|n\rangle = \hat{A}_1|n\rangle_x + \hat{A}_1|n\rangle_y = \sqrt{n}|n-1\rangle_x$, $\hat{A}_2|n\rangle = \hat{A}_2|n\rangle_x + \hat{A}_2|n\rangle_y = \sqrt{n}|n-1\rangle_y$ and $\langle n|\hat{A}_1^\dagger \hat{A}_2|n\rangle = \langle n|\hat{A}_2^\dagger \hat{A}_1|n\rangle = 0$. Where $|n\rangle_x, |n\rangle_y$ is the n photon number state of the x- and y-polarized photon respectively.

Using the alternate interpretation, it is also possible that the polarization direction of the photon pair created in BBO in figure 4 is controlled by the scalar potential as describe below.

The scalar potential exists in whole space-time even if there is no photons. Therefore when the angle of the polarizer 1 is set to $|\phi|$, the scalar potential is affected by the setting. This state can be expressed as the sum of $|0\rangle$ where no photon exists and $|\zeta_{|\phi|}\rangle$ where the scalar potential is oriented by the rotation of the polarizer 1. This state can be expressed as follows by a simple calculation method.

$$|0\rangle + |\zeta_{|\phi|}\rangle = |0\rangle + \frac{1}{2}\gamma e^{i(|\phi|-|\psi|)} e^{i\theta/2}|0\rangle - \frac{1}{2}\gamma e^{-i(|\phi|-|\psi|)} e^{-i\theta/2}|0\rangle \tag{97}$$

Where, ψ is the polarization angle of the photon s that would be created by the BBO and propagates to the direction of the polarizer 1. The propagation speed of the scalar potential is equal to the speed of light because it is an electromagnetic potential. Therefore, the state that BBO creates photon $|\psi\rangle$ with polarization angle $|\psi|$ is calculated to be the product of state which represents the scalar potential arrived at BBO and photon creation operator $\hat{A_{|\psi|}}^\dagger$ of which polarization angle is $|\psi|$ as follows.

$$\hat{A_{|\psi|}}^\dagger|0\rangle + \hat{A_{|\psi|}}^\dagger|\zeta_{|\phi|}\rangle = |\psi\rangle + \frac{1}{2}\gamma e^{i(|\phi|-|\psi|)} e^{i\theta/2}|\psi\rangle - \frac{1}{2}\gamma e^{-i(|\phi|-|\psi|)} e^{-i\theta/2}|\psi\rangle \tag{98}$$

Note that the scalar potential propagates along a single path to BBO in this setup, hence the phase is $\theta = 0$. Therefore

$$\langle I\rangle \propto \frac{1}{2} + \frac{1}{2}\cos\left(2|\phi| - 2|\psi|\right) \tag{99}$$

Because a single photon is created, $\langle I\rangle = 1$ is required and $\psi = \phi$ is obtained. That is to say, photons are created at the setting angle of the polarizer 1.

Where, even if the photon pair is created with random polarization from BBO regardless of the setting angle of the polarizer 1, the polarizations of the photon pair are orthogonal to each other. Hence, (87) is observed due to the interference between the classical local perfect correlation determined when the photon pair is created and the oscillatory field of the scalar potential.

Here, the alternate interpretation of the EPR correlation can be explained using the objective physical reality, which the real photons move in the oscillatory field of the scalar potential that causes interference. Then, at the polarizer, it is possible to interpret that the photon of the polarization different from that before the interference is created by the interference between the real photon and the scalar potential, and the photon before the interference is annihilated. The alternate interpretation is consistent with the quantum field theory that particles are created and annihilated by the excitation of vacuum. In addition, it has been said that the conventional quantum theory based on probability interpretation was proved to be correct due to the violation of Bell's inequality by various experimental setups of EPR correlation devised so as not to have a loophole.[46] However even if we use the alternate interpretation based on objective physical reality, we can obtain the identical results as conventional quantum theory, therefore the violation of Bell's inequality does not show the denial of locality of physical laws and physical reality, nor does it support the validity of conventional quantum theory based on probability interpretation.

Note that, in the above calculation of the alternate interpretation for EPR correlation, the two phases are introduced, one is from the space division θ and the other is from the rotation angle of the polarizer ϕ. The former is obviously a geometrical phase on real space such as Aharonov-Bohm effect[33] and the later can be identified as so-called Pancharatnam's phase[47] which is a geometrical phase on Poincaré sphere. Therefore we can understand that EPR correlation is caused by two types of the geometrical phases. The detailed study of the relation between various physical phenomena and the geometrical phases must be needed but it is the future work.

IV. APPLICATION

We revealed that by examining photons, electrons, and scalar potentials as objective physical reality without using probability interpretation, the calculation results of single-photon interference, single-electron interference, and EPR correlation agree with those obtained by probability interpretation in the preceding chapters. Here, we will discuss some phenomena peculiar to quantum theory that can be explained by alternate interpretation using the objective physical reality.

For the discussion, it is first necessary to generalize the formation of the oscillatory field by the scalar potential discussed in the preceding chapters. The discussions in the preceding chapters are all setups where there is two paths that divides the space into two. We will generalize the above setups and examine what kind of oscillatory field is formed when the whole space-time is divided into three or four to any number (arbitrary geometrical arrangement), and what kinds of physical phenomena are caused by that oscillatory field. As a result of the examination, it is shown that the fluctuation of zero-point energy of the electromagnetic field, Casimir effect, and the spontaneous symmetry breaking are naturally derived with the image of objective physical reality.

We also present a general approach for single particle interference associated with the generalization.

A. Generalization of the geometrical arrangement

The division into two paths dealt with in the preceding chapters divides the scalar potential into two, and the divided coefficient was set to 1/2 and the phase difference was set to θ. In order to extend that into an arbitrary number of divided paths, we introduce divided coefficients r_j and phases θ_j of the scalar potential. Then, the scalar potential can be expressed as follows, where M is the number of divided paths.

$$\sum_{j=1}^{M} r_j e^{i\theta_j} A_0 \tag{100}$$

where $\sum_{j=1}^{M} r_j = 1$. The case of the preceding chapters correspond to $M = 2$, $r_1 = r_2 = 1/2$ and $\theta_1 = -\theta_2 = \theta/2$ etc. Therefore, the expected value of the number of photons can be calculated by the following photon annihilation operator when there is an x-polarize single photon in arbitrary number of divided paths.

$$\hat{A}_\mu = \left(\sum_{j=1}^{M} r_j e^{i\theta_j} \hat{A}_0,\ \hat{A}_1,\ 0,\ 0 \right) \tag{101}$$

Then

$$\begin{aligned} \langle \hat{I} \rangle &\propto \langle 1 | - \mathrm{g}^{\mu\nu} \hat{A}_\mu^\dagger \hat{A}_\nu | 1 \rangle \\ &= - \left\{ \sum_{j=1,\ k=1}^{M} r_j r_k e^{i(\theta_j - \theta_k)} \right\} \langle 1 | \hat{A}_0^\dagger \hat{A}_0 | 1 \rangle + \langle 1 | \hat{A}_1^\dagger \hat{A}_1 | 1 \rangle \\ &= - \left\{ (r_1^2 + r_2^2 + \cdot + r_M^2) + \sum_{j \neq k}^{M} r_j r_k e^{i(\theta_j - \theta_k)} \right\} + 1 \end{aligned} \tag{102}$$

Because $0 \leqq r_j \leqq 1$, then $0 \leqq \langle 1 | - \mathrm{g}^{\mu\nu} \hat{A}_\mu^\dagger \hat{A}_\nu | 1 \rangle \leqq 1$. When $M \to \infty$ and the phases are completely random, $\sum_{j=1}^{\infty} r_j e^{i\theta_j}$ converges with 0. The division of the path is caused by the existence of matter in space-time, and $M \to \infty$ can be regarded as the final physical space where there are many innumerable matter with maximum entropy. We call this “final vacuum” in this paper.

On the other hand, $M = 1$ in (100) can be regarded as a space-time without any geometrical arrangement in space. We call this “initial vacuum” in this paper.

In addition, we call the vacuum with $1 < M < \infty$ “real vacuum” and call the vacuum with no potentials such like an empty container “ideal vacuum”.

Expressing the generalization by a simple calculation method, the following operators and states can be applied to arbitrary geometrical arrangement instead of (36) and (40).

$$\begin{aligned}\hat{A}'_0 &= \gamma\sum_{j=1}^{\infty} r_j e^{i\theta_j}\hat{A}_1 - \gamma\sum_{j=1}^{\infty} r_j e^{-i\theta_j}\hat{A}_1 \\ \hat{A}'^{\dagger}_0 &= \gamma\sum_{j=1}^{\infty} r_j e^{-i\theta_j}\hat{A}^{\dagger}_1 - \gamma\sum_{j=1}^{\infty} r_j e^{i\theta_j}\hat{A}^{\dagger}_1\end{aligned} \tag{103}$$

$$|\zeta\rangle \equiv \left(\gamma\sum_{j=1}^{\infty} r_j e^{i\theta_j} - \gamma\sum_{j=1}^{\infty} r_j e^{-i\theta_j}\right)|1\rangle_S \tag{104}$$

From the generalization of the divided path, it can be seen that when some geometrical arrangement is brought into the "initial vacuum" and the path is divided, the expected value fluctuates depending on the oscillatory field.

B. Zero point energy of the electromagnetic fields

The photon creation and annihilation operators obtained from conventional quantization procedure for quantum optics in Coulomb gauge have relationships with harmonic oscillator as follows. (We examine only x-polarized photon for simplicity.)

$$\begin{aligned} \hat{A}_1 &= \frac{1}{\sqrt{2\hbar\omega}}\left(\omega\hat{q} + i\hat{p}\right) \\ \hat{A}_1^\dagger &= \frac{1}{\sqrt{2\hbar\omega}}\left(\omega\hat{q} - i\hat{p}\right) \end{aligned} \tag{105}$$

where $\hat{q}$ and $\hat{p}$ are position and momentum operators obeying the commutation relation $[\hat{q}, \hat{p}] = i\hbar$. Hamiltonian of harmonic oscillator is expressed as follows.

$$\hat{\mathcal{H}} = \frac{1}{2}\left(\hat{p}^2 + \omega^2\hat{q}^2\right) \tag{106}$$

Then following relations are obtained.

$$\begin{aligned} \hat{A}_1^\dagger\hat{A}_1 &= \frac{1}{2\hbar\omega}\left(\hat{p}^2 + \omega^2\hat{q}^2 + i\omega\hat{q}\hat{p} - i\omega\hat{p}\hat{q}\right) \\ &= \frac{1}{\hbar\omega}\left(\hat{\mathcal{H}} - \frac{1}{2}\hbar\omega\right) \\ \hat{A}_1\hat{A}_1^\dagger &= \frac{1}{\hbar\omega}\left(\hat{\mathcal{H}} + \frac{1}{2}\hbar\omega\right) \end{aligned} \tag{107}$$

From (107) and $\langle 0|\hat{A}_1^\dagger\hat{A}_1|0\rangle = 0$, zero-point energy conventionally has been recognized as $\langle 0|\hat{\mathcal{H}}|0\rangle = \frac{1}{2}\hbar\omega$, i.e.,

$$\langle 0|\hat{A}_1^\dagger\hat{A}_1|0\rangle = \frac{1}{\hbar\omega}\langle 0|\left(\hat{\mathcal{H}} - \frac{1}{2}\hbar\omega\right)|0\rangle = \frac{1}{\hbar\omega}\left(\langle 0|\hat{\mathcal{H}}|0\rangle - \frac{1}{2}\hbar\omega\right) = 0 \tag{108}$$

This conventional fixed zero-point energy originates from the definition of the photon creation and annihilation operators in (105) without the scalar potentials. However we have obtained the idea that there exists the potentials in whole space-time. Then we should replace (105) with followings by using the operators in (36).

$$\begin{aligned} \hat{A}_0' + \hat{A}_1 &= \frac{1}{\sqrt{2\hbar\omega}}\left(\omega\hat{q} + i\hat{p}\right) \\ \hat{A}_0'^\dagger + \hat{A}_1^\dagger &= \frac{1}{\sqrt{2\hbar\omega}}\left(\omega\hat{q} - i\hat{p}\right) \end{aligned} \tag{109}$$

Therefore Hamiltonian will be the same expression of the interference as follows.

$$\hat{\mathcal{H}} = \hbar\omega\left(-g^{\mu\nu}\hat{A}_\mu^\dagger\hat{A}_\nu\right) + \frac{1}{2}\hbar\omega \tag{110}$$

Hence the energy of single photon state also fluctuates.

$$\langle 1|\hat{\mathcal{H}}|1\rangle = -\frac{1}{2}\hbar\omega\langle 1|\hat{A}_0^\dagger\hat{A}_0|1\rangle\cos\theta + \frac{1}{2}\hbar\omega\langle 1|\hat{A}_1^\dagger\hat{A}_1|1\rangle + \frac{1}{2}\hbar\omega \tag{111}$$

The single photon can be observed at some point with $\theta = \pm N\pi$, $(N:\text{oddnumber})$. By using Lorentz invariance $\langle 1|\hat{A}_0^\dagger\hat{A}_0|1\rangle = \langle 1|\hat{A}_1^\dagger\hat{A}_1|1\rangle$

$$\begin{aligned}\langle 1|\hat{\mathcal{H}}|1\rangle &= \frac{1}{2}\hbar\omega\langle 1|\hat{A}_0^\dagger\hat{A}_0|1\rangle + \frac{1}{2}\hbar\omega\langle 1|\hat{A}_1^\dagger\hat{A}_1|1\rangle + \frac{1}{2}\hbar\omega \\ &= \langle 1|\hat{A}_1^\dagger\hat{A}_1|1\rangle\hbar\omega + \frac{1}{2}\hbar\omega = \hbar\omega \end{aligned}\tag{112}$$

Therefore, $\langle 1|\hat{A}_1^\dagger\hat{A}_1|1\rangle = \frac{1}{2}$. This leads to the replacement of the expected photon number as follows.

$$\langle 0|\hat{A}_1^\dagger\hat{A}_1|0\rangle = -\frac{1}{2}, \quad \langle 1|\hat{A}_1^\dagger\hat{A}_1|1\rangle = \frac{1}{2}, \quad \langle 2|\hat{A}_1^\dagger\hat{A}_1|2\rangle = \frac{3}{2}, \quad \cdots \tag{113}$$

Generally, $\langle 0|\hat{A}_1^\dagger\hat{A}_1|0\rangle$ is considered to be 0. However, we should accept $\langle 0|\hat{A}_1^\dagger\hat{A}_1|0\rangle = -\frac{1}{2}$ which requires indefinite metric. We will revise this calculation later in this section. Even with the revision, the conclusion that the zero-point energy fluctuates remains unchanged.

Then absolute value of the single photon interference moves depending on the selection of $\langle 0|\hat{A}_1^\dagger\hat{A}_1|0\rangle$. However $\langle\hat{I}\rangle \propto \frac{1}{2} \pm \frac{1}{2}\cos\theta$ remains unchanged.

By using the expectation value, zero-point energy is calculated to be as follows.

$$\begin{aligned}\langle 0|\hat{\mathcal{H}}|0\rangle &= -\hbar\omega\left\{(r_1^2 + r_2^2 + \cdot + r_M^2) + \sum_{j\neq k}^{M} r_j r_k e^{i(\theta_j-\theta_k)}\right\}\langle 0|\hat{A}_1^\dagger\hat{A}_1|0\rangle \\ &\quad +\hbar\omega\langle 0|\hat{A}_1^\dagger\hat{A}_1|0\rangle + \frac{1}{2}\hbar\omega \\ &= \frac{1}{2}\hbar\omega\left\{(r_1^2 + r_2^2 + \cdot + r_M^2) + \sum_{j\neq k}^{M} r_j r_k e^{i(\theta_j-\theta_k)}\right\}\end{aligned}\tag{114}$$

This shows that the zero-point energy fluctuates such that $0 \leq \langle 0|\hat{\mathcal{H}}|0\rangle \leq \frac{1}{2}\hbar\omega$ depending on the path division. This also explains the spontaneous symmetry breaking. From this, infinite zero-point energy due to infinite degrees of freedom is removed when the phase is completely random with $M \to \infty$ in (114).

In the above discussion, we replaced the operator (105) with (109). However, if the operator $\hat{A}_0$ is introduced, we should replace the commutation relation of the position and momentum operators $[\hat{q}, \hat{p}] = i\hbar$ with $[\hat{q_\mu}, \hat{p_\nu}] = -g_{\mu\nu}i\hbar$ and examine the time axis component as follows.

$$
\begin{aligned}
\hat{A}_0^\dagger \hat{A}_0 &= \frac{1}{2\hbar\omega}\left(\hat{p_0}^2 + \omega^2 \hat{q_0}^2 + i\omega \hat{q_0}\hat{p_0} - i\omega \hat{p_0}\hat{q_0}\right) \\
&= \frac{1}{\hbar\omega}\left(\hat{\mathcal{H}}_0 + \frac{1}{2} i\omega [\hat{q_0}, \hat{p_0}]\right) = \frac{1}{\hbar\omega}\left(\hat{\mathcal{H}}_0 + \frac{1}{2}\hbar\omega\right) \\
\hat{A}_0 \hat{A}_0^\dagger &= \frac{1}{\hbar\omega}\left(\hat{\mathcal{H}}_0 - \frac{1}{2}\hbar\omega\right)
\end{aligned} \tag{115}
$$

where $\hat{\mathcal{H}}_0$ is the Hamiltonian corresponds to the energy of time axis. Hence, the total Hamiltonian $\hat{\mathcal{H}}$ can be calculated as follows using $\hat{\mathcal{H}}_1$ (Hamiltonian of the spatial component of (107))

$$
\hat{\mathcal{H}} \equiv \hat{\mathcal{H}}_1 - \hat{\mathcal{H}}_0 = \hbar\omega\left(-g^{\mu\nu}\hat{A}_\mu^\dagger \hat{A}_\nu\right) + \frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega \tag{116}
$$

Here we note that similar to the discussion the above, the last term obtained from the commutation relation between the position and momentum operator of the time axis component should include a mathematical expression that incorporates the phase by the path division, similar to $\{\quad\}$ in (102). We set the mathematical expression to f, then the zero-point energy can be expressed as follows.

$$
\begin{aligned}
\langle 0|\hat{\mathcal{H}}|0\rangle &= \hbar\omega\langle 0|\left(-g^{\mu\nu}\hat{A}_\mu^\dagger \hat{A}_\nu\right)|0\rangle + \frac{1}{2}\hbar\omega\langle 0|0\rangle + \frac{1}{2}\hbar\omega\langle 0|f|0\rangle \\
&= \frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega f
\end{aligned} \tag{117}
$$

In order to determine f, we calculate the energy of 1-photon state with the help of (102). Finally, the following zero-point energy is obtained.

$$
\begin{aligned}
\langle 1|\hat{\mathcal{H}}|1\rangle &= \hbar\omega\langle 1|\left(-g^{\mu\nu}\hat{A}_\mu^\dagger \hat{A}_\nu\right)|1\rangle + \frac{1}{2}\hbar\omega\langle 1|1\rangle + \frac{1}{2}\hbar\omega\langle 1|f|1\rangle \\
&= \hbar\omega - \hbar\omega\left\{(r_1^2 + r_2^2 + \cdot + r_M^2) + \sum_{j\neq k}^{M} r_j r_k e^{i(\theta_j - \theta_k)}\right\} + \frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega f
\end{aligned} \tag{118}
$$

This is always single photon energy $\hbar\omega$ in whole space-time. Therefore, in the case of "initial vacuum" with $M = 1$ split path, $\{\quad\}$ will be 1, i.e., $f = 1$. In the case of "final vacuum" with $M \to \infty$ split paths, $\{\quad\}$ will be 0, i.e., the value will be $f = -1$. Otherwise, i.e., "real vacuum", the value fluctuates such as $-1 < f < 1$ depending on $\{\quad\}$. Therefore, the zero-point energy fluctuates as follows depending on the geometrical arrangement existing in the space.

$$
0 \leq \langle 0|\hat{\mathcal{H}}|0\rangle = \frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega f \leq \hbar\omega \tag{119}
$$

C. Casimir effect

The zero-point energy has been measured through Casimir effect.[48–52] The following circumstance can be identified as a typical setup for the measurement of Casimir effect. From the discussion in the zero-point energy, if a certain space that is not "ideal vacuum" but "initial vacuum" is prepared and a certain geometrical arrangement, e.g. two parallel plates, is placed in the space, then the zero-point energy of the space and geometrical arrangement are calculated to be $\frac{1}{2}\hbar\omega$ and $0 \leq \langle 0|\hat{\mathcal{H}}|0\rangle \leq \frac{1}{2}\hbar\omega$ using (114) respectively or $\hbar\omega$ and $0 \leq \langle 0|\hat{\mathcal{H}}|0\rangle \leq \hbar\omega$ using (119) respectively. Because the energy of the geometrical arrangement is not exceed that of the space, the geometrical arrangement is subjected to a compressive stress from the space in both calculations.

This kind of attractive force derived from the energy difference between the geometrical arrangements is identical with the basic concept of Van der Waals force which will be the origin of Casimir effect.[53]

D. Spontaneous symmetry breaking

Conventional approach of the spontaneous symmetry breaking, which explores the possibility of $\mathbf{Q}|0\rangle_q \neq 0$ or $|0\rangle_q$ is not an eigenstate of $\mathbf{Q}$, has been discussed using Goldstone boson or Higgs boson.[28,54]. Where $|0\rangle_q$ is vacuum state.

However, as mentioned in the previous chapters the phase difference is generated by the geometrical arrangement of the space which corresponds to oscillatory field of the scalar potential. According to (65), when the phase difference $\gamma(C) = n\pi$ ($n =$ oddnumber), the single electron state $|\psi(t)\rangle$ of (64) can be identified as the initial vacuum $|\phi_0\rangle$ because $|\phi_0\rangle\langle\phi_0|\mathbf{Q}|\phi_0\rangle = 0$ instead of $|0\rangle_q$ which can be identified as "ideal vacuum". If $|\phi_0\rangle$ is an eigenstate of $\mathbf{Q}$, i.e., $\mathbf{Q}|\phi_0\rangle = \alpha|\phi_0\rangle$, then $\langle\phi_0|\mathbf{Q}|\phi_0\rangle = \alpha\langle\phi_0|\phi_0\rangle = 0$, where α is an eigenvalue. Therefore the "initial vacuum" $|\phi_0\rangle$ is an eigenstate of $\mathbf{Q}$.

When the phase difference θ of the scalar potentials changes, the vacuum state $|\phi_0\rangle$ becomes "real vacuum" of which eigenvalue fluctuates between q and 0. The eigenvalue fluctuation means the vacuum state is not an eigenstate of $\mathbf{Q}$ but the function of the phase difference θ and geometrical arrangement typified by M in the previous chapters. This expresses the spontaneous symmetry breaking.

There is no fluctuation in the space with completely random phases as shown in section IV A. That is to say, there is symmetry. We can identify such space as "final vacuum". However the fluctuation gains entrance into the "final vacuum" when the geometrical arrangement in the space changes, and eventually the symmetry is broken towards "initial vacuum" where has no geometrical arrangement.

This idea seems little uncomfortable but reasonable when the origin is vice versa, i.e., "initial vacuum" instead of "final vacuum", i.e., we should consider the real physical space has no symmetry but the space gains symmetry when $M \to \infty$. This vice versa idea is similar to the fact that the magnetic poles of a ferromagnetic are aligned in one direction at extremely low temperatures which corresponds to "initial vacuum" with no symmetry, but are dispersed in some directions as the temperature rises which corresponds to "real vacuum", and finally dispersed in completely random directions above Curie temperature and the magnetism disappears which corresponds to "final vacuum" with symmetry.

The above discussion that the real vacuum is filled with potentials of which state exists under (or above) the original ground state is identical with the spontaneous symmetry breaking using the analogy of superconductivity when we replace $\mathbf{Q}$ or $\hat{\mathcal{H}}$ with energy level reported by Y. Nambu and G. Jona-Lasinio.[55,56] When the geometrical arrangement and phase is fixed, the one vacuum is selected and the selection breaks the symmetry of vacuum.

In addition, as described in section IV B, the fluctuation of the zero-point energy of electromagnetic fields $0 \leq \langle 0|\hat{\mathcal{H}}|0\rangle \leq \frac{1}{2}\hbar\omega$ (or $\hbar\omega$) also shows that the vacuum is not the eigenstate of the photon number. That is the spontaneous symmetry breaking. Eventually, the spontaneous symmetry breaking is caused by the geometrical arrangement of space-time and phase, which produces the oscillatory field of the scalar potential (time-axis component of a vector field). This argument derived from the oscillatory field might seem like peculiar but it is consistent with the conventional idea of spontaneous symmetry breaking that the vacuum is degenerate.

E. General approach for single particle interference

By generalizing (35), single particle interference can be described as follow.

$$\begin{aligned} \langle I \rangle &= (\langle \phi| + \langle \zeta|)\, \mathbf{F}\, (|\phi\rangle + |\zeta\rangle) \\ &= f + \langle \zeta|\mathbf{F}|\zeta\rangle + f\langle \phi|\zeta\rangle + f\langle \zeta|\phi\rangle \end{aligned} \tag{120}$$

When $\langle \zeta|\mathbf{F}|\zeta\rangle + f\langle \phi|\zeta\rangle + f\langle \zeta|\phi\rangle = -\frac{1}{2}f + \frac{1}{2}f\cos\theta$, single particle interferences of $\mathbf{F}$ by 2-path geometrical arrangement, i.e., $\langle I \rangle = f\left\{\frac{1}{2} + \frac{1}{2}\cos\theta\right\}$ is generated. Where $\mathbf{F}$ is an arbitrary observable operator of the particle, $|\phi\rangle$ is an eigenstate of $\mathbf{F}$, f is the eigenvalue of $\mathbf{F}$ under state $|\phi\rangle$ and $|\zeta\rangle$ is an indefinite metric vector expressing the oscillatory field of the scalar potential.

When $\mathbf{F}$ is number operator of the particle $\mathbf{n}$, and $|\phi\rangle$ is the single particle state of (120), the expected number of the particle fluctuates as follows.

$$\begin{aligned} (\langle 1| + \langle \zeta|)\, \mathbf{n}\, (|1\rangle + |\zeta\rangle) &= 1 + \langle \zeta|\mathbf{n}|\zeta\rangle + \langle 1|\zeta\rangle + \langle \zeta|1\rangle \\ &= \frac{1}{2} + \frac{1}{2}\cos\theta \end{aligned} \tag{121}$$

In the case of arbitrary geometrical arrangement, the expected number of the particle can be calculated as follows using similar expression (103), (104) and (102).

$$\langle \hat{I} \rangle \propto -\left\{ (r_1^2 + r_2^2 + \cdot + r_M^2) + \sum_{j \neq k}^{M} r_j r_k e^{i(\theta_j - \theta_k)} \right\} + 1 \tag{122}$$

The above generalization is based on the electromagnetic field, which is a commutative (Abelian) gauge field, i.e., U(1) gauge field : unitary group of dimension 1, in the preceding chapters. The above discussion can be generalized to the non-commutative (non-Abelian) gauge field as follows.

The existence of the electromagnetic field can be derived, so called gauge transformation of the second kind, as a gauge field by gauge invariance of the phase transformation of the electron wave function ψ with conservation of charge Q as follows.

$$\psi' = U\psi \tag{123}$$

where $U = e^{i\chi Q}$ is a unitary transformation, χ is an arbitrary function of space-time point $(x) \equiv (\mathbf{x}, t)$.

Note that the conservation of charge Q means as follows. We can express the wave function Ψ in the initial state composed of elementary particles $a, b, \cdots$ with charges $Q_a, Q_b, \cdots$ as the product of the wave functions of each elementary particle $\psi_a, \psi_b, \cdots$. Even if the initial state changes to the final state composed of elementary particles $a', b', \cdots$, the law of conservation of charge holds, i.e., $Q_a + Q_b + \cdots = Q_{a'} + Q_{b'} + \cdots$.

When the phase factors $e^{i\chi Q_a}, e^{i\chi Q_b}, \cdots$ are multiplied to the wave functions, the phase factor $e^{i\chi(Q_a+Q_b+\cdots)} = e^{i\chi Q}$ is multiplied to the initial wave function, where Q is the total charge of the initial state. From the law of conservation of charge, the same phase factor $e^{i\chi(Q_a+Q_b+\cdots)} = e^{i\chi(Q_{a'}+Q_{b'}+\cdots)} = e^{i\chi Q}$ is applied to the final state.

However, the derivative operation by ∂_μ to the above transformation $U\psi$ is calculated to be as follows.

$$\partial_\mu \psi' \;=\; e^{i\chi Q}\left\{\partial_\mu + iQ\partial_\mu \chi\right\}\psi \tag{124}$$

While the kinetic term which includes derivative operators such as momentum changes, the gauge transformation of the second kind should be the phase transformation that does not change in physical quantities because of the local causality.

Therefore the derivative operator ∂_μ is replaced with covariant derivative $D_\mu = \partial_\mu - iQA_\mu$ to recover the invariance of the physical quantities, where A_μ is a four-vector. By using this replacement with the following gauge transformations,

$$\begin{aligned} \psi' &= e^{i\chi Q}\psi \\ A'_\mu &= A_\mu + \partial_\mu \chi \end{aligned} \tag{125}$$

we can calculate as follows.

$$\begin{aligned} (D_\mu\psi)' &= \left\{\partial_\mu - iQA'_\mu\right\} e^{i\chi Q}\psi \\ &= e^{i\chi Q}\left\{(\partial_\mu + iQ\partial_\mu\chi) - i(QA_\mu + Q\partial_\mu\chi\right\}\psi \\ &= e^{i\chi Q}\left\{\partial_\mu - iQA_\mu\right\}\psi = e^{i\chi Q} D_\mu\psi \end{aligned} \tag{126}$$

Hence, the invariance of the physical quantities, e.g., momentum $p_\mu = (E/c, \mathbf{p}) = i\hbar\partial_\mu$, is obtained.

$$< \psi', P'_\mu \psi' >=< e^{i\lambda Q}\psi, e^{i\lambda Q} P_\mu \psi >=< \psi, P_\mu \psi > \tag{127}$$

where $P_\mu = i\hbar D_\mu = i\hbar(\partial_\mu - iQA_\mu)$.

The gauge field A_μ, as described above, is introduced because of the existence of the gauge transformation associate with the conservation of charge Q and the local causality. In quantum electrodynamics, the gauge field A_μ is equated with the electromagnetic potential A_μ. Because $U = e^{i\chi Q}$ is a unitary transformation of dimension 1, the gauge field A_μ is called U(1) gauge field.

Extending the above procedure to a multicomponent wave function such as nucleon with isotopic spin conservation, the non-commutative (non-Abelian) gauge field B_μ is introduced.

For example, the wave function of nucleon ψ can be described by two-component composed of proton p and neutron n which are spin 1/2 fermions with z-axis isotopic spin $I_3 = 1/2$ and $I_3 = -1/2$ respectively. The subscript 3 of the isotopic spin $\boldsymbol{I}$ is used because it is not the rotation of real three-dimensional space x, y, z, but the rotation of the three-dimensional isotopic space. The invariance under local isotopic spin rotations leads to formulating a principle of isotopic gauge invariance and the existence of a vector field B_μ ($= 2\boldsymbol{b}_\mu \cdot \boldsymbol{I}$) which has the same relation to the isotopic spin that the electromagnetic field has to the electric charge as discussed above. The specific expression is as follows.

$$\psi' = S\psi \tag{128}$$

where $S = e^{i\boldsymbol{\theta}\cdot\boldsymbol{I}}$ is a 2×2 special unitary (SU(2)) matrix, $\boldsymbol{\theta}$ is the vector that specifies the axis and angle of the rotation in isotopic space. The isotopic spin $\boldsymbol{I}$ is described as $\boldsymbol{I} = \frac{\boldsymbol{\tau}}{2}$ using following Pauli matrix $\boldsymbol{\tau}$.

$$\tau_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \tau_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{129}$$

The subscripts 1, 2, 3 are used accordance with those of $\boldsymbol{I}$.

When $\boldsymbol{\theta}$ is extended as an arbitrary function of space-time point $(x) \equiv (\mathbf{x}, t)$, the derivative operator ∂_μ is replaced with the following covariant derivative D_μ by introducing the vector fields B_μ not to change in physical quantities, as same in the case of U(1) gauge transformation.[57]

$$D_\mu = (\partial_\mu - i\epsilon B_\mu)\psi \tag{130}$$

with the following gauge transformation.

$$B'_\mu = SB_\mu S^{-1} + \frac{i}{\epsilon} S\partial_\mu S^{-1} \tag{131}$$

where ϵ is an arbitrary constant called coupling constant and $B_\mu = 2\boldsymbol{b}_\mu \cdot \boldsymbol{I}$ respectively. (Bold-face letters denote three-component vectors in isotopic space.)

Because $S = e^{i\boldsymbol{\theta}\cdot\boldsymbol{I}}$ is a special unitary transformation of dimension 2, the vector field B_μ ($\boldsymbol{b}_\mu$) is called SU(2) gauge field. The generalized Aharonov-Bohm effect by the SU(2) gauge field had been discussed by Wu and Yang.[34].

Therefore the vacuum is filled with the gauge fields which interact with the particle (e.g., nucleon) as same as in the discussion of subsection III B.

We can obtain the similar discussion and calculation of the uncharged particle such as neutron interference[58] originating from B_μ field by replacing the four-vector potential A_μ in subsection III B with B_μ.

Even if there are no B_μ in space, the geometrical phase $\gamma(C)$ is introduced by the similar calculations as in (60) to (68) where $|\psi(t)\rangle$ and $\gamma(t)$ are replaced with multicomponent state and unitary matrix respectively. Generalizing the geometrical phase of the non-Abelian gauge fields instead of Abelian gauge fields has been discussed by Frank Wilczek and A. Zee[59].

Note that The neutron couples to magnetic fields via its permanent magnetic dipole moment which can cause the Larmor precession. Therefore the neutral particle possessing magnetic dipole moment encircling a line charge shows phase shift similar to Aharonov-Bohm effect, which has been theoretically shown by Aharonov and Casher[60]. In addition, the neutrons interferences by the geometrical phase (Berry's phase) had been experimentally reported in[61,62] and introduced in[58]. In these experiments, the Berry's phase have been treated as an additional phase that adds to the dynamic phase caused by the external magnet field interacting with the magnetic moment of the neutral particle.

Here we take the neutron interference caused by precession in a magnetic field B, which was devised to experimentally demonstrate spinor rotation in references[63,64], to calculate the single neutron interference. These experimental results and standard mathematical treatments are summarized in[65].

The experimental setup is similar to the single-electron interference shown in Figure 2, with the electron source replaced by a thermal neutron source, a DC magnetic field B inserted in the path from the neutron source to the screen via pinhole 2 and the two detectors placed on the extension of path 1 and path 2 of the $x = 0$ instead of a moving detector on the screen. In addition, the pinholes are replaced with a single silicon perfect crystal which act as a mirror. For the sake of explanation, the term “pinhole” will be used in place of this mirror in the following.

The conventional interpretation in this setup, the state ket vector going via pinhole 2 suffers a phase change $\exp^{\mp i\omega T/2}$, where T is the time spent in the $B \neq 0$ region and ω is the spin-precession frequency

$$\omega = \frac{g_n e B}{m_p c}, \ (g_n \simeq -1.91) \tag{132}$$

for the neutron with a magnetic moment of $g_n e\hbar/2m_p c$, where m_p is the neutron mass and $g_n \simeq -1.91$ is the neutron magnetic moment in units of $-e\hbar/2m_p c$.

When path from the neutron source to the screen via pinhole 1 and via pinhole 2 meet again in the interference region (screen) of Figure 2, the amplitude of the neutron arriving via pinhole 2 is

$$c_2 = c_2(B=0)\exp^{\mp i\omega T/2} \tag{133}$$

while the amplitude of the neutron arriving via pinhole 1 is c_1, independent of B.

where $c_{1,2}$ are the coefficients of the time evolution of the state ket from a general ket $|\alpha\rangle$ for this setup as follows. Note that since the experiment was done with mono-energetic unpolarized neutrons, each interference is due to one of the spins, but the spin-up and spin-down neutron contributions must be considered together.

$$|\alpha;t\rangle = c_{1,2}|+\rangle\langle+|\alpha\rangle, \ \text{or } c_{1,2}|-\rangle\langle-|\alpha\rangle \tag{134}$$

The base kets of the spin $\frac{1}{2}$ systems $|S_z;\pm\rangle$ are denoted for brevity as $|\pm\rangle$ and

$$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad 1 = |+\rangle\langle+| + |-\rangle\langle-| \tag{135}$$

Therefore the intensity observable in the interference region ($\propto \langle\alpha;t|\alpha;t\rangle$) must exhibit a sinusoidal variation

$$\cos\left(\frac{\mp\omega T}{2} + \delta\right) \tag{136}$$

where δ is the phase difference between c_1 and $c_2(B=0)$.

The above result is based on the quantum superposition state as follows.

$$|\alpha;t\rangle = c_1|+\rangle\langle+|\alpha\rangle + c_2|+\rangle\langle+|\alpha\rangle \ \text{ or } \ |\alpha;t\rangle = c_1|-\rangle\langle-|\alpha\rangle + c_2|-\rangle\langle-|\alpha\rangle \tag{137}$$

When we take $c_1 = 1/2$, $c_2 = \pm 1/2 \exp^{\mp i\omega T/2+\delta}$ on the extension of path 1 (take + of $\pm$) and path 2 (take - of $\pm$) respectively, the counting rates at detectors are expected to be as follows considering the spin-up and spin-down neutron contributions.

$$\begin{aligned} I_1 = I_1(+) + I_1(-) &= \left[\frac{1}{2} - \frac{1}{2}\cos\left(\frac{\omega T}{2} + \delta\right)\right] + \left[\frac{1}{2} - \frac{1}{2}\cos\left(-\frac{\omega T}{2} + \delta\right)\right] \\ &= 1 - \cos\delta\cos\left(\frac{\omega T}{2}\right) \end{aligned} \tag{138}$$

and

$$\begin{aligned} I_2 = I_2(+) + I_2(-) &= \left[\frac{1}{2} + \frac{1}{2}\cos\left(\frac{\omega T}{2} + \delta\right)\right] + \left[\frac{1}{2} + \frac{1}{2}\cos\left(-\frac{\omega T}{2} + \delta\right)\right] \\ &= 1 + \cos\delta\cos\left(\frac{\omega T}{2}\right) \end{aligned} \tag{139}$$

The instrumental parameters α and γ with an average intensity ratio $\gamma/\alpha = 2.6$, experimentally confirmed in Ref[66], have been adopted in Ref[63] and the following counting rates have been shown.

$$\begin{aligned} I_1 &= \gamma - \alpha\cos\delta\cos\left(\frac{\omega T}{2}\right) \\ I_2 &= \alpha\left[1 + \cos\delta\cos\left(\frac{\omega T}{2}\right)\right] \end{aligned} \tag{140}$$

On the other hand, the alternate interpretation does not need quantum superposition state. Instead the phase term depending on the geometrical arrangement is incorporated. The calculation process for the phase term is the same as (51) through (55), where along the path 2 in the $B \neq 0$ the following potential is added to Hamiltonian instead of $q\phi$.

$$V = -\left(\frac{e}{m_p c}\right)\mathbf{S}\cdot\mathbf{B} = \omega S_z \tag{141}$$

$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{\hbar}{2}\{|+\rangle\langle+| - |-\rangle\langle-|\} \tag{142}$$

where $\mathbf{S}$ and S_z are a spin matrix and z-axis component of the matrix respectively. Hence the time dependent coefficient (54) in this case can be expressed as follows.

$$c_1(t) = c_1\exp(-i\frac{1}{\hbar}\int_1 \omega S_z dt) + c_2\exp(-i\frac{1}{\hbar}\int_2 \omega S_z dt) \tag{143}$$

Because $B = 0$ along path 1, the integral of the first term is 0. Therefore

$$\begin{aligned} c_1(t) &= c_1 + c_2 \exp(-i\frac{1}{\hbar}\int_2 \omega S_z dt) = c_1 + c_2 \exp(-i\frac{\omega T}{2}\tau_3) \\ &= c_1 + c_2 \exp(-i\frac{\omega T}{2}\{|+\rangle\langle+| - |-\rangle\langle-|\}) \\ &= c_1 + c_2 \exp(\mp i\frac{\omega T}{2}) \end{aligned} \tag{144}$$

where we split the Pauli matrix by the sign of the spin-up and spin-down cases in the last row. The expression of the ket vector substituting this coefficient is identical to the quantum superposition state for the above conventional interpretation (137). Therefore, regardless of which path the launched neutron passes, the phase depending on the geometrical arrangement is incorporated into the ket vector which is exactly the same expression as the quantum superposition state in the conventional interpretation.

Hence, the single particle interference, whether charged or not, can be caused by the geometrical arrangement of the space. Note that the potential $q\phi$ from (51) through (55) is replaced by V of (142), which can be regarded as the interference is caused by the scalar potential.

Even if $V = 0$, the interference is also generated by so-called Berry's phase (Berry-Aharonov-Anandan phase), geometrical phase $\gamma(C)$, as described in subsection III B 2 but generalized version[38–40], which depends only on the geometry of the space and not on the charge or magnetic dipole moment of the particle.

The original Berry phase was reformulated and generalized from nonadiabatic cyclic evolutions to all quantum evolutions, not merely cyclic evolutions by using the projective Hilbert space $\mathcal{P}_{\mathcal{H}}$, which is defined as the set of rays of the Hilbert space $\mathcal{H}$.

According to[38], arbitrarily $|\psi\rangle,\ |\varphi\rangle \in S_{\mathcal{H}}$(Hilbert sphere) $\equiv \{|\psi\rangle \in \mathcal{H} \mid \langle\psi|\psi\rangle = 1\}$ which evolves according to the Schrödinger equation $H(t)|\psi(t)\rangle = i\hbar(d/dt)|\psi(t)\rangle$, are regarded as an equivalence relation associated with $e^{i\eta} \in U(1)$, i.e., $|\psi\rangle = e^{i\eta}|\varphi\rangle \to |\psi\rangle \sim |\varphi\rangle$.

Hence the projection map

$$\pi:\ S_{\mathcal{H}} \to \mathcal{P}_{\mathcal{H}}\ ,\ \text{i.e., } \pi(|\psi\rangle) = \{|\tilde{\psi}\rangle:\ |\tilde{\psi}\rangle = c|\psi\rangle \sim |\psi\rangle, c \text{ is a complex number}\} \tag{145}$$

can be defined. Then $|\psi(t)\rangle$ defines a curve $C:\ [0,\tau]$ in $S_{\mathcal{H}}$ with $\tilde{C} \equiv \pi(C)$ being a closed curve in $\mathcal{P}_{\mathcal{H}}$.

When the state $|\psi(t)\rangle \in \mathcal{H}$ is defined as $|\tilde{\psi}(t)\rangle = e^{-if(t)}|\psi(t)\rangle$ where $e^{-if(t)} \in U(1)$, $f(t)$

is calculated by substituting into the Schrödinger equation as follows.

$$\begin{aligned} f(t) &= \int_0^t \langle\tilde{\psi}(s)|i\frac{d}{ds}|\tilde{\psi}(s)\rangle ds - \frac{1}{\hbar}\int_0^t \langle\psi(s)|H|\psi(s)\rangle ds \\ &= \gamma(t) + \gamma_d(t) \end{aligned} \tag{146}$$

where

$$\gamma(t) = \int_0^t \langle\tilde{\psi}(s)|i\frac{d}{ds}|\tilde{\psi}(s)\rangle ds = i\int_{p(0)}^{p(t)} \langle\tilde{\psi}|d|\tilde{\psi}\rangle \tag{147}$$

$$\gamma_d(t) = -\frac{1}{\hbar}\int_0^t \langle\psi(s)|H|\psi(s)\rangle ds \tag{148}$$

This description is an extension of (60) to the projective Hilbert space $\mathcal{P}_{\mathcal{H}}$. This $\gamma(t)$ does not need adiabatic assumption and closed loop C formed by $\mathbf{r}(t)$ on $[0,T]$, instead only needs a curve $p:[0,t]\to\mathcal{P}_{\mathcal{H}}$ with $p(0)=p(T)$ which forms closed curve $\tilde{C}$ in $\mathcal{P}_{\mathcal{H}}$. By utilizing the projective Hilbert space $\mathcal{P}_{\mathcal{H}}$, the single particle interference can be calculated in the same way as discussed after (60). Since the setup has no interaction term with the spins, the following calculation is a neutron interference for one of the spins. The setup of the original neutron interference[63,64,67] consisted of mirrors, analyzer, etc. made of perfect Si crystals instead of pinholes and screen, but in the following, the terms pinholes and screen, etc. are replaced by the corresponding components.

The two paths along $C_1 \equiv \mathrm{s} \to \mathrm{Pinhole1} \to \mathrm{screen}$ and $C_2 \equiv \mathrm{s} \to \mathrm{Pinhole2} \to \mathrm{screen}$ in real space correspond to $\tilde{C}_1$ and $\tilde{C}_2$ in $\mathcal{P}_{\mathcal{H}}$. Therefore the state passed through Pinhole 1 is expressed as follows, as in (64).

$$\begin{aligned} |\psi(t)\rangle &= \frac{1}{2}\exp\left[i\gamma(\tilde{C}_1)+i\gamma_d(C_1)\right]|\tilde{\psi}_1(t)\rangle \\ &\quad +\frac{1}{2}\exp\left[i\gamma(\tilde{C}_2)+i\gamma_d(C_2)\right]|\tilde{\psi}_1(t)\rangle \end{aligned} \tag{149}$$

where $|\tilde{\psi}_1(t)\rangle = \pi(|\psi_1(t)\rangle) \in \mathcal{P}_{\mathcal{H}}$ is the projective state of one neutron state heading to pinhole 1 in $\mathcal{H}$, the real single neutron travels along C_1 in real space.

The initial state $|\psi(0)\rangle \in \mathcal{H}$ at the particle source s and the final state $|\psi(T)\rangle \in \mathcal{H}$ at the particle source s (where this is the final s of s $\to$ Pinhole1 $\to$ screen $\to$ Pinhole2 $\to$ s) can be different, but the difference is only phase factor when the system of single particle interference with only geometrical arrangements like pinholes or slits which has no interaction term. This indicates $|\tilde{\psi}(0)\rangle = \tilde{\psi}(T)\rangle \in \mathcal{P}_{\mathcal{H}}$, i. e., $\tilde{C}$ is closed curve in $\mathcal{P}_{\mathcal{H}}$.

Therefore the intensity of single particle interference, including neutrons, electrons, and any other particles which obey Schrödinger equation, is calculated as follows.

$$
\begin{aligned}
\langle \hat{I} \rangle &\propto \langle \psi(t) | \psi(t) \rangle \\
&= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \exp\left[i\gamma(\tilde{C}) + i\gamma_d(C) \right] + \frac{1}{4} \exp\left[-i\gamma(\tilde{C}) - i\gamma_d(C) \right] \\
&= \frac{1}{2} + \frac{1}{2} \cos(\gamma(\tilde{C}) + \gamma_d(C))
\end{aligned} \tag{150}
$$

where the normalization of the projective state $\langle \tilde{\psi}_1(t) | \tilde{\psi}_1(t) \rangle = 1$ is used.

When the energy of the system is E, $\gamma_d(C) = -\frac{1}{\hbar} \oint_C E dt = -\frac{E}{\hbar}(t_1 - t_2) \equiv -\frac{E}{\hbar} \Delta t$, where $\Delta t = t_1 - t_2$ is the difference in travel time of the neutron motion along C_1 and C_2.

Then $\gamma(\tilde{C})$ can be calculated as same manner in (66). The time evolution projective state $|\tilde{\psi}(t)\rangle \in \mathcal{P}_{\mathcal{H}}$ without interaction having the angular frequency ω and energy E of the neutron wave propagating in space can be chosen as follows.

$$
|\tilde{\psi}_1(t)\rangle = \exp i(-\omega t) \exp i(-\frac{E}{\hbar} t) |\tilde{\psi}_1(0)\rangle \tag{151}
$$

where $\omega + \frac{E}{\hbar} = 2n\pi/T$ (n: integer) due to $|\tilde{\psi}_1(0)\rangle = |\tilde{\psi}_1(T)\rangle$

The direct substitution of (151) into (147) leads to

$$
\gamma(t) = \int_0^t (\omega + \frac{E}{\hbar}) ds \tag{152}
$$

Therefore

$$
\gamma(\tilde{C}) \equiv \gamma(\tilde{C}_1) - \gamma(\tilde{C}_2) \tag{153}
$$

is calculated as $(\omega + \frac{E}{\hbar})\Delta t$, where $\Delta t = t_1 - t_2$ is the difference in travel time of the projective neutron motion along $\tilde{C}_1$ and $\tilde{C}_2$, which is identical to the real neutron motion along C_1 and C_2.

Because $\gamma(\tilde{C}) + \gamma_d(C) = (\omega + \frac{E}{\hbar})\Delta t - \frac{E}{\hbar}\Delta t = \omega \Delta t$, the expected neutron flux intensity of the single neutron interference (150) is as follows.

$$
\langle \hat{I} \rangle \propto \frac{1}{2} + \frac{1}{2} \cos(\omega \Delta t) \tag{154}
$$

This is identical to the intensity modulation of the neutron beam by the phase shifter made of Al-sheet in reference[67].

Although the above calculation differs from the standard probability interpretation using quantum-superposition state (137), it can be expressed in the same form as the quantum-superposition state by introducing a phase difference due to the geometric arrangement of the system with two paths.

We can identify $i\langle\tilde{\psi}|\frac{d}{dt}|\tilde{\psi}\rangle$ in (147) as the scalar potential ϕ, the transformation $|\tilde{\psi}\rangle \rightarrow |\tilde{\psi}'\rangle = \exp(-i\Lambda)|\tilde{\psi}\rangle$ yields $\phi' = \phi + \frac{d}{dt}\Lambda$. Then we again can identify $i\langle\tilde{\psi}|d|\tilde{\psi}\rangle = i\langle\tilde{\psi}|\nabla|\tilde{\psi}\rangle \cdot d\mathbf{x}$ in (147) using ϕ after transformation as the vector potential $-\mathbf{A} \cdot d\mathbf{x}$. Choosing $\Lambda = \omega t$ yields

$$\gamma(C) = \oint \omega dt - \oint \mathbf{A} \cdot d\mathbf{x} \tag{155}$$

This corresponds to replacing ϕ with ω in Aharonov-Bohm phase.

The choice (151) makes the scalar potentials extract, the vector potential vanish and cancels out $\gamma_d(C)$.

In this correspondence suggests that the single neutron interference is caused by the oscillatory field (phase distribution) similar to the scalar potential which corresponds to the indefinite metric formed due to the geometrical arrangement.

As can be seen from the above calculation process, this interference is an effect of the geometrical arrangement of the space that appears for all particles, regardless of the presence or absence of charge or interaction.

The same discussion above can be used to explain several single-particle self-oscillations, including neutrino oscillations[68,69].

V. ORIGIN OF THE INDEFINITE METRIC POTENTIAL

In this paper, we have revised the quantum theory using the oscillatory field of the scalar potential, which is the source of the indefinite metric. Although this scalar potential is a well-known quantity in classical electromagnetism, we examine this scalar potential again and extract the meaning of the alternate interpretation of this paper and future issues.

In quantum optics, we can usually use Coulomb gauge to divide the electric field and current density as follows.[32]

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_{\mathrm{T}} + \mathbf{E}_{\mathrm{L}}, \qquad \nabla \cdot \mathbf{E}_{\mathrm{T}} = 0, \qquad \nabla \times \mathbf{E}_{\mathrm{L}} = 0 \\ \mathbf{i} &= \mathbf{i}_{\mathrm{T}} + \mathbf{i}_{\mathrm{L}}, \qquad \nabla \cdot \mathbf{i}_{\mathrm{T}} = 0, \qquad \nabla \times \mathbf{i}_{\mathrm{L}} = 0 \end{aligned} \tag{156}$$

where the indexes "T" and "L" stand for "Transverse" and "Longitudinal", respectively. "Transverse" components of Maxwell's equations using electromagnetic potentials can be expressed as follows.

$$\begin{aligned} \nabla \times \mathbf{E}_{\mathrm{T}} &= -\frac{\partial \mathbf{B}}{\partial t} \ , \qquad \nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}_{\mathrm{T}}}{\partial t} + \mu_0 \mathbf{i}_{\mathrm{T}} \\ \mathbf{E}_{\mathrm{T}} &= -\frac{\partial \mathbf{A}}{\partial t} \ , \qquad \nabla \cdot \mathbf{B} = 0 \end{aligned} \tag{157}$$

where $\mathbf{B}$ is the magnetic field. In addition, "Longitudinal" components can be expressed as follows.

$$\begin{aligned} \mathbf{E}_{\mathrm{L}} &= -\nabla \phi \ , \qquad \nabla \cdot \mathbf{E}_{\mathrm{L}} = \frac{\rho}{\epsilon_0} \\ \mathbf{i}_{\mathrm{L}} &= \epsilon_0 \nabla \frac{\partial \phi}{\partial t} \quad = \quad -\epsilon_0 \frac{\partial \mathbf{E}_{\mathrm{L}}}{\partial t} \end{aligned} \tag{158}$$

Therefore the transverse and longitudinal components seem associated with the magnetic field variation and with charges as the regular scalar potential, respectively.

However, these associations depend on coordinate systems. "Transverse" and "Longitudinal" components are mixed by Lorentz transformation. Then the associations lost meaning, which is the important assertion of relativity.[70]

This relative association justifies the identification of scalar potentials and vector potentials, i.e., identify the number operators as $\langle 1|A_0^\dagger A_0|1\rangle = \langle 1|A_1^\dagger A_1|1\rangle = \langle 1|A_2^\dagger A_2|1\rangle = 1$ by Lorentz invariance. Hence we would better use Maxwell's equations (4) in Lorentz gauge, because Coulomb gauge removes the explicit covariance of Maxwell's equations.

Maxwell's equations in Lorentz gauge can be express as follows by utilizing the linearity of the equation (4).

$$\begin{aligned} \Box A^{\mu} &= \Box(A^{\mu}_{(\mathrm{mat})} + A^{\mu}_{(\mathrm{vac})}) = \mu_0 j^{\mu} \\ \partial_{\mu} A^{\mu} &= \partial_{\mu}(A^{\mu}_{(\mathrm{mat})} + A^{\mu}_{(\mathrm{vac})}) = 0 \end{aligned} \tag{159}$$

where index "mat" and "vac" mean "matter" associated with four-current and "vacuum", respectively. Because we can naturally assume that there are no four-current in vacuum, $A^{\mu}_{(\mathrm{mat})}$ and $A^{\mu}_{(\mathrm{vac})}$ obey the following Maxwell's equations respectively.

$$\Box A^{\mu}_{(\mathrm{mat})} = \mu_0 j^{\mu} \ , \qquad \partial_{\mu} A^{\mu}_{(\mathrm{mat})} = 0 \tag{160}$$

$$\Box A^{\mu}_{(\mathrm{vac})} = 0 \ , \qquad \partial_{\mu} A^{\mu}_{(\mathrm{vac})} = 0 \tag{161}$$

The substantial photon excited by the four-current will be expressed by equation (160). Note that equation (161) replaced $A^{\mu}_{(\mathrm{vac})}$ with $A^{\mu}_{(\mathrm{mat})}$ can express the motion of the potentials associated with the four-current in the spatial domain where is far from and does not contain the four-current.

In contrast, equation (161) expresses the motion of the potentials that have nothing to do with "matter" in vacuum. This evokes us the concept of an ether such that vacuum is the sea filled with the potentials. The special relativity[70] has negated the static ether, but the above filling potentials propagate at the speed of light rather than static entities. Aharonov-Bohm effect clearly indicates that not only electromagnetic fields but also the potentials can make electron interference.[33,71,72] Similarly, the filling potentials in equation (161) can make interference with substantial photon. This is similar to the concept of the non-integrable phase factor associated with the multiple-connected region of the space, which was also noted in Aharonov-Bohm effect.[34] As the space is divided, a phase term associated with the division is introduced and cause interference.

We generally use A^{μ} in equation (159) to calculate photon related phenomena unconsciously, i.e., without separating them into "matter" and "vacuum". Unfortunately, we will not be able to distinguish between $A^{\mu}_{(\mathrm{mat})}$ and $A^{\mu}_{(\mathrm{vac})}$. This is very similar to distinguishing between sea spray and seawater. In fact, no separation is necessary, as both are ever-changing potentials derived from the same Maxwell's equations (159). Hence the filling potentials in vacuum can expel and incorporate the potentials associated with "matter", which gives us the description that vacuum creates and annihilates substantial photon.

The source of the indefinite metric scalar potential examined in this paper seems to be the time-axis component of the potential (161) in vacuum though it is difficult to distinguish.

On the other hand, Maxwell's equations, which has been formulated as a compilation of classical electromagnetism based on experimental facts of electric and magnetic phenomena from the Faraday era, can be considered as (160).

Here we recognize that (160) is derived from experimental facts, and (161) is derived as a gauge field under the local commutative gauge transformation regardless of experimental facts. Then the recognition emphasize the similarity between the introduction of the gauge field due to the invariance of the local phase transformation on the space-time structure and the phenomenon of vacuum fluctuation due to the geometrical arrangement of the space examined in this paper.

In the analogy of the electronic circuit or the communication using homodyne detection mentioned in the preceding chapter, (161) corresponds to the bias current (voltage) or the continuous wave generated from the local oscillator, and (160) corresponds to the signal wave. If the bias is a stable DC current (voltage), there is no problem in extracting only the AC current (voltage) as a signal. However when the DC current (voltage) fluctuates, the signal as the AC current (voltage) also fluctuates.

It is useful for engineering applications to examine the actual physical phenomenon as the circuit or communication in which the DC current (voltage) fluctuates.

VI. CONTRADICTION OF THE COVARIANT CANONICAL QUANTIZATION

In this chapter, we examine the contradiction between Lorentz gauge and commutation relations in covariant canonical quantization of Maxwell's equations on conventional method, and propose alternate method.

A. Canonical quantization and contradiction of commutation relations

First, an overview of canonical quantization and conventional method are outlined below.

The subject is Maxwell's equations in free space with zero four-current as follows again.

$$\Box A^{\nu} - \partial_{\mu}\partial^{\nu}A^{\mu} \;=\; 0 \tag{162}$$

In order to adopt canonical quantization, the following classical lagrangian density has been introduced.

$$\begin{aligned}\pounds_{class} \;&=\; -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \equiv -\frac{1}{4}(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu})(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}) \\ &=\; -\frac{1}{2}\partial_{\mu}A_{\nu}\partial^{\mu}A^{\nu} + \frac{1}{2}\partial_{\mu}A_{\nu}\partial^{\nu}A^{\mu}\end{aligned} \tag{163}$$

Indeed the following Euler-Lagrange equation gives Maxwell's equations (162).

$$\partial_{\mu}\frac{\partial\pounds_{class}}{\partial(\partial_{\mu}A_{\nu})} - \frac{\partial\pounds_{class}}{\partial A_{\nu}} = 0 \tag{164}$$

By using this lagrangian density, the canonically conjugate variables $\pi^{i},\ i = (1,\ 2,\ 3)$ can be defined as follows.

$$\begin{aligned}\pi^{i} \;&=\; \frac{\partial\pounds_{class}}{\partial\dot{A}_{i}} = -\frac{1}{4}\frac{\partial}{\partial\dot{A}_{i}}(F_{0i}F^{0i}) \\ &=\; -\frac{1}{4}\frac{\partial}{\partial\dot{A}_{i}}((\partial_{0}A_{i} - \partial_{i}A_{0})(\partial^{0}A^{i} - \partial^{i}A^{0}) + (\partial_{i}A_{0} - \partial_{0}A_{i})(\partial^{i}A^{0} - \partial^{0}A^{i})) \\ &=\; \partial^{i}A^{0} - \partial^{0}A^{i}\end{aligned} \tag{165}$$

However the conjugate variable π^{0} can not be defined as follows.

$$\pi^{0} = \frac{\partial\pounds_{class}}{\partial\dot{A}_{0}} = 0 \tag{166}$$

Therefore a number of fixing gauge conditions have been proposed. Well-known gauges are Coulomb gauge $\nabla \cdot \mathbf{A} = 0$ and Lorentz gauge $\partial_\mu A^\mu = 0$. Because Coulomb gauge spoils the explicit covariance due to the separation of A_0 from the four-vector, fixing Lorentz gauge has been examined by using following lagrangian density.

$$\pounds_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial_\rho A^\rho)^2 \tag{167}$$

When we use the lagrangian density (167), Maxwell's equations in Lorentz gauge can be obtained from Euler-Lagrange equation (164).

$$\Box A^\nu \ = \ 0 \tag{168}$$

Here the action integral of (163) is as follows.

$$\begin{aligned} S \ &\equiv \ \int d^4x \pounds_{class} \\ &= \ \int d^4x(-\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu + \frac{1}{2}\partial_\mu A_\nu \partial^\nu A^\mu) \end{aligned} \tag{169}$$

The second term of the above integral is calculated to be $\frac{1}{2}(\partial_\mu A^\mu)^2$ by partial integration. Then the lagrangian density (167) which derives Maxwell's equations (168) can be calculated to be following lagrangian density.

$$\pounds_0' = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu \tag{170}$$

The canonically conjugate variables can be obtained by using this lagrangian density as follows .

$$\pi^\mu = \frac{\partial \pounds_0'}{\partial \dot{A}_\mu} \ = \ -\dot{A}^\mu \tag{171}$$

Here quantization is performed by replacing the fields with operators and set the following equal-time commutation relations.

$$\begin{aligned} [A^\mu(\mathbf{x},t), \pi^\nu(\mathbf{x}',t)] \ &= \ -[A^\mu(\mathbf{x},t), \dot{A}^\nu(\mathbf{x}',t)] \\ &= \ i\mathrm{g}^{\mu\nu}\delta^3(\mathbf{x}-\mathbf{x}') \end{aligned} \tag{172}$$

$$[A^\mu(\mathbf{x},t), A^\nu(\mathbf{x}',t)] = [\pi^\mu(\mathbf{x},t), \pi^\nu(\mathbf{x}',t)] = 0 \tag{173}$$

However (172) and (173) derive the following relations.

$$[\partial_\mu A^\mu(\mathbf{x},t), A^\nu(\mathbf{x}',t)] = i\mathrm{g}^{0\nu}\delta^3(\mathbf{x}-\mathbf{x}') \neq 0 \tag{174}$$

Hence (174) is inconsistent with Lorentz gauge $\partial_\mu A^\mu = 0$ as an operator.

Therefore some other lagrangian densities have been proposed. The following lagrangian with auxiliary scalar field B, which is called Nakanishi-Lautrup formalism, will be the most comprehensive form[31].

$$\pounds_{NL} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + B\partial^\mu A_\mu + \frac{1}{2}\alpha B^2 \tag{175}$$

Where α is an arbitrarily real parameter. The inconsistency between Lorentz gauge $\partial_\mu A^\mu = 0$ and (174) can be avoided by using the lagrangian (175), introduction of physical states $|\text{phys}\rangle$ and a restriction of Lorentz gauge in terms of the physical states defined by a subsidiary condition, i.e., $\langle\text{phys}|\partial_\mu A^\mu|\text{phys}\rangle = 0$.

B. Extended Lorentz gauge

The approach using (175) seems to be an artificially imposed mathematical technique by introducing an unreal physical field B and unphysical man-made mathematical formality called "subsidiary condition". That is to say, there is a tendency to describe by artificial mathematical techniques to be consistent with probability interpretation and seem to deviate from the mathematical description of natural laws. In addition, the approach has been introduced for avoidance of negative norm as premises for "probability interpretation". However, as mentioned in this paper, the indefinite metric can express the physical reality that is inevitably required by theory, which is indispensable to the law of nature. Just because it violates the probability interpretation, it might be fatal fault to remove the indefinite metric by making full use of mathematical techniques according to the artificially introduced hypothesis and conditions.

Hence, we examine the lagrangian density (170) and Maxwell's equations in Lorentz gauge (168) again. It is to be noted that Lorentz gauge is dispensable for deriving (168). Indeed (168) is derived from lagrangian density (167) or (170) independently of Lorentz gauge. Alternatively the following condition is indispensable from (162).

$$\partial_\mu \partial^\nu A^\mu = 0 \tag{176}$$

Hence from Lorentz invariance

$$\partial_\mu A^\mu = \epsilon(\text{scaler}) \tag{177}$$

This condition (we call this "extended Lorentz gauge") also has the following gauge invariance of (162) by introducing an arbitrary scalar function χ.

$$A'^{\mu} = A^{\mu} + \partial^{\mu}\chi \tag{178}$$

Although this replacement has been well known, we can imagine that four-vector A^{μ} move on the bias vector $\partial^{\mu}\chi$ such as AC signal and bias current (voltage) of an electric circuit or communication using homodyne detection.

By choosing $\Box\chi = 0$, (177) can be obtained repeatedly as follows.

$$\partial_{\mu}A'^{\mu} = \partial_{\mu}A^{\mu} + \Box\chi = \partial_{\mu}A^{\mu} = \epsilon \tag{179}$$

Hence

$$A^{\mu} = A^{\mu}_{L} + f^{\mu} \tag{180}$$

where $A^{\mu}_{L}$and $f^{\mu} = f^{\mu}(\mathbf{x}, t)$ are a general solution of Lorentz gauge, i.e., $\partial_{\mu}A^{\mu}_{L} = 0$, and a linear formula as a function of $\mathbf{x}, t$ with $\partial_{\mu}f^{\mu} = \epsilon$ respectively. The most common linear formula f^{μ} is the same as a coordinate transformation in form described as follows.

$$f^{\mu} = \beta(a^{\mu}_{\nu}x^{\nu} + b^{\mu}) \tag{181}$$

where β is a constant for fixing the appropriate dimension. Therefore

$$\epsilon = \beta(a^{0}_{0} + a^{1}_{1} + a^{2}_{2} + a^{3}_{3}) = \mathrm{Tr}(\varepsilon) \tag{182}$$

Where, ε is 4×4 matrix with matrix elements a^{μ}_{ν} multiplied by β. Here we replace ϵ with operator $\hat{\epsilon}$ and substitute for (174)

$$[\partial_{\mu}A^{\mu}(\mathbf{x}, t), A^{\nu}(\mathbf{x}', t)] = [\hat{\epsilon}, A^{\nu}(\mathbf{x}', t)] = i\mathbf{g}^{0\nu}\delta^{3}(\mathbf{x} - \mathbf{x}') \neq 0 \tag{183}$$

C. Discussion

We examine the commutation relation $[\partial_{\mu}A^{\mu}, A] = [\hat{\epsilon}, A] \equiv \hat{\epsilon}A - A\hat{\epsilon} \neq 0$ by using matrix representation of the operator. The matrix representation of the photon creation and annihilation operators, $A^{\dagger}$ and A, and photon number state vectors $|\mathbf{n}\rangle$ are expressed as follows.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & \sqrt{2} & 0 & \cdots \\ 0 & 0 & 0 & \sqrt{3} & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad A^\dagger = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & 0 & \cdots \\ 0 & \sqrt{2} & 0 & 0 & \cdots \\ 0 & 0 & \sqrt{3} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{184}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}, |1\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix}, |2\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix}, |3\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ \vdots \end{bmatrix}, \cdots \tag{185}$$

When $\hat{\epsilon} = \epsilon I$, where I is an identity matrix or operator, the operator $\hat{\epsilon}$ just serves as the constant scalar $[\hat{\epsilon}, A] = \epsilon[I, A] = \epsilon(IA - AI) = 0$, then (183) can not be obtained. From (182), we should adopt the operator $\hat{\varepsilon}$ that satisfies $\mathrm{Tr}(\hat{\varepsilon}) = \Sigma\varepsilon^{ii} = \epsilon$ when we replace ϵ with the operator $\hat{\epsilon}$. Hence, the following matrix representation can be adopted as $\hat{\epsilon}$, where we set all diagonal elements to 0 for simplicity. Even if the diagonal element is not 0, the same result as below can be obtained.

$$\hat{\epsilon} = \begin{bmatrix} \varepsilon^{00} & 0 & 0 & 0 & \cdots \\ 0 & \varepsilon^{11} & 0 & 0 & \cdots \\ 0 & 0 & \varepsilon^{22} & 0 & \cdots \\ 0 & 0 & 0 & \varepsilon^{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ & & & & \end{bmatrix} \tag{186}$$

Here we can calculate as follows.

$$
\begin{aligned}
[\hat{\epsilon}, A] &\equiv \hat{\epsilon}A - A\hat{\epsilon} \\
&= \begin{bmatrix} 0 & \varepsilon^{00} & 0 & 0 & \cdots \\ 0 & 0 & \sqrt{2}\varepsilon^{11} & 0 & \cdots \\ 0 & 0 & 0 & \sqrt{3}\varepsilon^{22} & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} - \begin{bmatrix} 0 & \varepsilon^{11} & 0 & 0 & \cdots \\ 0 & 0 & \sqrt{2}\varepsilon^{22} & 0 & \cdots \\ 0 & 0 & 0 & \sqrt{3}\varepsilon^{33} & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \\
&= \begin{bmatrix} 0 & \varepsilon^{00} - \varepsilon^{11} & 0 & 0 & \cdots \\ 0 & 0 & \sqrt{2}(\varepsilon^{11} - \varepsilon^{22}) & 0 & \cdots \\ 0 & 0 & 0 & \sqrt{3}(\varepsilon^{22} - \varepsilon^{33}) & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \end{bmatrix}
\end{aligned}
\tag{187}
$$

Hence if at least one $\varepsilon^{ii}(i > 0)$ is $\varepsilon^{ii} \neq \varepsilon^{i\pm1 i\pm1}$ then $[\hat{\epsilon}, A] \neq 0$ will be satisfied.

By utilizing the relationship (182), we can define the operator $\hat{0} \equiv \hat{\epsilon}$ that satisfies $\mathrm{Tr}(\hat{\varepsilon}) = \Sigma\varepsilon^{ii} = 0$ because $\hat{\epsilon}$ satisfies $\mathrm{Tr}(\hat{\varepsilon}) = \Sigma\varepsilon^{ii} = \epsilon$ like (186). Therefore $\hat{0} \equiv \partial_\mu A^\mu(\mathbf{x}, t)$ satisfies (174) as the conventional Lorentz gauge

As described in single photon calculation, even if the number of photon becomes 0 due to interference, the invisible photons are there at the space-time due to the interference of the opposite phase waves.

Even if the photon number becomes 0 due to interference, there is the invisible photons at the space-time point due to the interference of waves with opposite phases. It is necessary to identify that space with a space-time of the phase state such that $f(\theta) = 0$, and it cannot be set to 0 as an empty space-time.

Similarly, considering the right side of the Lorentz condition $\partial_\mu A^\mu = 0$ as empty 0 and quantizing it as 0 will ignore the space-time that has some geometrical arrangement.

Since classical Lagrangian density (163) assumes free space which is an ideal vacuum with no geometrical arrangement, the same approach will be applicable by introducing a phase into A_0 that includes the existence of the geometrical arrangement.

VII. SUMMARY

Probability interpretation, which is the basic concept of quantum theory, has been considered essential in order to match the observation results of quantum phenomena with theory. However, the probability interpretation had not been able to provide the picture of objective physical reality like classical physics and gave rise to the famous paradox that an indivisible particle is in a quantum superposition state divided into two or more paths instead of objective physical reality.

In addition, the probability interpretation asserts that the divided two particle in EPR state have a correlation even if they are far apart, and one of pair is suddenly changed from probability existence to a particle as an objective physical reality at the very moment when the other of pair is observed. This correlation leads to a physical phenomenon that exceeded the speed of light that contrary to relativity. However, a technology called quantum teleportation has been tried to develop by applying the physical phenomena that exceed the speed of light.

In this paper, we have discarded the probability interpretation that conflicts with common sense and proposed an alternate interpretation of the quantum theory using objective physical reality, which reproduces the same observation results as the conventional standard quantum theory.

For the alternate interpretation in single photon interference and EPR correlation, we have used an indefinite metric associate with covariant quantization of Maxwell's equations. Although some mathematical procedures have been developed to remove it because the indefinite metric contradicts the probability interpretation, in this paper, we pay attention to the fact that the indefinite metric is inevitably required from the theory, and have approved that the probability interpretation should be removed instead. Therefore we have incorporated the indefinite metric into the calculations of single photon and EPR correlation as it is. Then by using covariant description and simple calculation method, we have shown when the space is divided into two paths, the scalar potential, which is the source of the indefinite metric, forms an oscillatory field by the division. We have clarified the objective physical realities that the single photon and photon pair having opposite polarizations move in the oscillatory field which occurs the interferences and correlations, and shown that the calculation results agree with the conventional standard quantum theory. As for single electron interference and uncharged particle interference, we have shown the geometrical phase so-called Berry's phase which can be formally identified with scalar potential can generate the interference.

For EPR correlation, various reports have been made that suggest there exists a long-range correlation beyond the causality.[19–21,46,73–75] We believe that these reports can be explained by the oscillatory field of the scalar potential caused as the result of path division and the indefinite metric in this paper.

We have also shown that the number of space division in "initial vacuum" or "final vacuum" corresponds to 1 (no division) or ∞ respectively by generalizing the number of the division of "real vacuum" from two into an arbitrary number. Then we have clarified that the oscillatory field due to the scalar potential is formed when some geometrical arrangement come into in the "initial vacuum".

Furthermore, we have shown that the zero-point energy fluctuates due to the oscillatory field of the scalar potential, the removal of the infinite zero-point energy which was difficult for conventional quantum theory and the Casimir effect originating from zero-point energy can be calculated.

Conventionally, there is an argument that the zero-point energy is simply subtracted to erase in engineering applications. By using an analogy from an electric circuit, we have discussed that this kind of approach corresponds to the signal of AC coupling which subtract DC bias voltage (current). And a real physical phenomenon corresponds to the electric circuit of which bias voltage (current) is not DC component but AC component, which causes interference with the AC component of the signal.

In addition, we have clarified that the space has symmetry when the number of division of the space is 1 (no division) or ∞ and at other number of division the oscillatory field of the scalar potential is generated which breaks the symmetry.

Moreover, we have discussed the generalization of a single particle interference and suggested that the neutrino oscillation may be caused by the self-interference fluctuation due to the oscillatory field of the physical quantity corresponding to the geometrical phase.

We have discussed the origin of the scalar potential, which is the essence of this paper, by separating Maxwell's equations into two equations associate with matter and vacuum and shown the picture that vacuum create and annihilate photons as the objective physical reality.

Finally, we have discussed the contradiction between Lorentz condition as an operator and commutation relation in covariant canonical quantization of Maxwell's equations, and proposed the alternate method that can avoid the contradiction by introducing the extended Lorentz gauge.

Despite the different interpretation and calculation methods, both the conventional probability and the alternate interpretation in this paper give the same calculation results. This can be regarded as a one-to-one correspondence of the inverse problem. The conventional standard quantum theory is not a systematic representation of physical observations (outcome) by mathematical expression using objective physical reality, but the assumption replaced by non-physical state (pseudo-reality) called probability interpretation, because it was initially difficult to explain the observation result (outcome) by objective physical reality. For example, we can estimate the sound (outcome) from the shape and state of the musical instrument (reality). On the contrary, from the sound (outcome), we can estimate some of sound sources, for example, an electric sound devices (pseudo-reality), in addition to the musical instrument (reality) that actually makes the sound. Even if the observations (sounds or outcomes) are identical, the true reality must be unique.

The alternate interpretation seems to be superior to probability interpretation because intuitively reasonable and exactly follows from Maxwell's equations, relativity, scattering problem of Schrödinger equation with geometrical phases and the indefinite metric derived from covariant quantization and gives the same result as the conventional standard quantum theory that reproduces the experimental results.

Despite the reproduction of such identical results and the theoretical derivation, no new concept has been introduced. The conventional concepts such as state vectors, operators as physical quantities, commutation relations and gauge transform remain unchanged. This seems to be evidence that such concepts of conventional quantum theory is in the theoretically correct direction. However the probability interpretation and applications based on it seems to result in pseudoscience, in addition, mathematical procedures associated with it might be extra things for physics.

From the alternate interpretation, the incompleteness of quantum theory, which has been alerted by A. Einstein, might originate from lack of introduction of indefinite metric and geometrical phase as an objective physical reality.

As A. Einstein continued to insist[5], we also insist that quantum theory should be debated and reconstructed using an objective physical reality without probability interpretation.

REFERENCES

[1]M. Morimoto, "Unobservable potentials to explain single photon and electron interference," Journal of Computational and Theoretical Nanoscience **14**, 4121–4132 (2017), arXiv:1709.04103 [physics.gen-ph].

[2]M. Morimoto, "Unobservable potentials to explain a quantum eraser and a delayed-choice experiment," Progress In Electromagnetics Research M **62**, 111–122 (2017), arXiv:1711.09007 [physics.gen-ph].

[3]M. Morimoto, "Consideration of Covariant Quantization of Electromagnetic Field," arXiv e-prints , arXiv:1806.02175 (2018), arXiv:1806.02175 [physics.gen-ph].

[4]J. D. Trimmer, "The present situation in quantum mechanics: A translation of Schrödinger's "cat paradox" paper," Proceedings of the American Philosophical Society **124**, 323–338 (1980).

[5]A. Einstein, B. Podolsky, and N. Rosen, "Can quantum-mechanical description of physical reality be considered complete' ?" Phys. Rev **47**, 777–780 (1935).

[6]D. Bohm, "A suggested interpretation of the quantum theory in terms of "hidden" variables. i," Phys. Rev. **85**, 166–179 (1952).

[7]D. Bohm, "A suggested interpretation of the quantum theory in terms of "hidden" variables. ii," Phys. Rev. **85**, 180–193 (1952).

[8]X. Oriols and J. Mompart, "Overview of Bohmian Mechanics," ArXiv e-prints (2012), arXiv:1206.1084 [quant-ph].

[9]P. Pylkkänen, B. J. Hiley, and I. Pättiniemi, "Bohm's approach and Individuality," ArXiv e-prints (2014), arXiv:1405.4772 [quant-ph].

[10]D. Durr, S. Goldstein, T. Norsen, W. Struyve, and N. Zanghi, "Can Bohmian mechanics be made relativistic?" Royal Society of London Proceedings Series A **470**, 30699 (2013), arXiv:1307.1714 [quant-ph].

[11]V. Allori, S. Goldstein, R. Tumulka, and N. Zanghi, "Predictions and Primitive Ontology in Quantum Foundations: A Study of Examples," ArXiv e-prints (2012), arXiv:1206.0019 [quant-ph].

[12]G. Dennis, M. A. de Gosson, and B. J. Hiley, "Fermi's ansatz and bohm's quantum

potential," Physics Letters A **378**, 2363 – 2366 (2014).

[13]M. Arndt and K. Hornberger, "Testing the limits of quantum mechanical superpositions," Nature Physics. **10**, 271–277 (2014).

[14]J. Grond, U. Hohenester, J. Schmiedmayer, and A. Smerzi, "Mach-zehnder interferometry with interacting trapped bose-einstein condensates," Phys. Rev. A **84**, 023619 (2011).

[15]G. D. McDonald, H. Keal, P. A. Altin, J. E. Debs, S. Bennetts, C. C. N. Kuhn, K. S. Hardman, M. T. Johnsson, J. D. Close, and N. P. Robins, "Optically guided linear mach-zehnder atom interferometer," Phys. Rev. A **87**, 013632 (2013).

[16]G. Haack, M. Moskalets, J. Splettstoesser, and M. Büttiker, "Coherence of single-electron sources from mach-zehnder interferometry," Phys. Rev. B **84**, 081303 (2011).

[17]S. Ngo Dinh, D. A. Bagrets, and A. D. Mirlin, "Analytically solvable model of an electronic mach-zehnder interferometer," Phys. Rev. B **87**, 195433 (2013).

[18]M. Morimoto, "Proposal for a plasmonic mach-zehnder modulator utilizing quantum interference effect," Selected Topics in Quantum Electronics, IEEE Journal of **19**, 14–20 (2013).

[19]A. Aspect, P. Grangier, and G. Roger, "Experimental tests of realistic local theories via bell's theorem," Physical review letters **47**, 460 (1981).

[20]A. Aspect, P. Grangier, G. Roger, *et al.*, "Experimental realization of einstein-podolsky-rosen-bohm gedankenexperiment: a new violation of bell's inequalities," Physical review letters **49**, 91–94 (1982).

[21]A. Aspect, J. Dalibard, and G. Roger, "Experimental test of bell's inequalities using time-varying analyzers," Physical review letters **49**, 1804 (1982).

[22]S. P. Walborn, M. O. Terra Cunha, S. Pádua, and C. H. Monken, "Double-slit quantum eraser," Phys. Rev. A **65**, 033818 (2002).

[23]V. Jacques, E. Wu, F. Grosshans, F. m. c. Treussart, P. Grangier, A. Aspect, and J.-F. m. c. Roch, "Delayed-choice test of quantum complementarity with interfering single photons," Phys. Rev. Lett. **100**, 220402 (2008).

[24]S. Longhi, "Classical and quantum interference in multiband optical bloch oscillations," Phys. Rev. B **79**, 245108 (2009).

[25]A. Danan, D. Farfurnik, S. Bar-Ad, and L. Vaidman, "Asking photons where they have been," Phys. Rev. Lett. **111**, 240402 (2013).

[26]P. L. Saldanha, "Interpreting a nested mach-zehnder interferometer with classical optics,"

Phys. Rev. A **89**, 033825 (2014).

[27]R. Bach, D. Pope, S.-H. Liou, and H. Batelaan, "Controlled double-slit electron diffraction," New Journal of Physics **15**, 033018 (2013).

[28]C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill, 1985).

[29]S. N. Gupta, "Theory of longitudinal photons in quantum electrodynamics," Proceedings of the Physical Society. Section A **63**, 681–691 (1950).

[30]K. Bleuler, "Eine neue methode zur behandlung der longitudinalen und skalaren photonen," Helv. Phys. Acta **23**, 567–586 (1950).

[31]N. Nakanishi, "Indefinite-Metric Quantum Field Theory," Progress of Theoretical Physics Supplement **51**, 1–95 (1972), https://academic.oup.com/ptps/article-pdf/doi/10.1143/PTPS.51.1/23577325/51-1.pdf.

[32]R. Loudon, *The quantum theory of light*, 2nd ed. (Oxford University Press, 1983).

[33]Y. Aharonov and D. Bohm, "Significance of electromagnetic potentials in the quantum theory," Phys. Rev. **115**, 485–491 (1959).

[34]T. T. Wu and C. N. Yang, "Concept of nonintegrable phase factors and global formulation of gauge fields," Phys. Rev. D **12**, 3845–3857 (1975).

[35]R. P. Feynman, R. B. Leighton, and M. L. Sands, *The Feynman Lectures on Physics Vol.II, III* (Addison-Wesley, 1965).

[36]E. Karlson and R. E. Peierls, "The eigenvalues of the charge operator," Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences **230**, 382–389 (1955), https://royalsocietypublishing.org/doi/pdf/10.1098/rspa.1955.0140.

[37]J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, 1941).

[38]Y. Aharonov and J. Anandan, "Phase change during a cyclic quantum evolution," Phys. Rev. Lett. **58**, 1593–1596 (1987).

[39]J. Anandan and Y. Aharonov, "Geometry of quantum evolution," Phys. Rev. Lett. **65**, 1697–1700 (1990).

[40]J. Samuel and R. Bhandari, "General setting for berry's phase," Phys. Rev. Lett. **60**, 2339–2342 (1988).

[41]M. V. Berry, "Quantal phase factors accompanying adiabatic changes," Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences **392**, 45–57 (1984), https://royalsocietypublishing.org/doi/pdf/10.1098/rspa.1984.0023.

[42]A. Tonomura, "Experimental confirmation of the aharonov-bohm effect by electron holog-

raphy," Proc. Int. Symp. Foundation of Quantum Mechanics, Tokyo , 18–33 (1983).

[43]P. K. Rachel Hillmer, "A do-it-yourself quantum eraser," Scientific American , 90–95 (2007).

[44]J. BELL, "On the einstein-podolsky-rosen paradox," Physics **1**, 195–200 (1964).

[45]J. S. BELL, "On the problem of hidden variables in quantum mechanics," Rev. Mod. Phys. **38**, 447–452 (1966).

[46]J. Handsteiner, A. S. Friedman, D. Rauch, J. Gallicchio, B. Liu, H. Hosp, J. Kofler, D. Bricher, M. Fink, C. Leung, A. Mark, H. T. Nguyen, I. Sanders, F. Steinlechner, R. Ursin, S. Wengerowsky, A. H. Guth, D. I. Kaiser, T. Scheidl, and A. Zeilinger, "Cosmic bell test: Measurement settings from milky way stars," Phys. Rev. Lett. **118**, 060401 (2017).

[47]S. Pancharatnam, "Generalized theory of interference, and its applications," Proceedings of the Indian Academy of Sciences - Section A **44**, 247—262 (1956).

[48]R. L. Jaffe, "Casimir effect and the quantum vacuum," Phys. Rev. D **72**, 021301 (2005).

[49]E. Myers, "Interpretation of the energy of the vacuum as the sum over zero-point energies," Phys. Rev. Lett. **59**, 165–168 (1987).

[50]S. K. Lamoreaux, "Demonstration of the casimir force in the 0.6 to 6μm range," Phys. Rev. Lett. **78**, 5–8 (1997).

[51]U. Mohideen and A. Roy, "Precision measurement of the casimir force from 0.1 to 0.9 μm," Phys. Rev. Lett. **81**, 4549–4552 (1998).

[52]I. Brevik, V. N. Marachevsky, and K. A. Milton, "Identity of the van der waals force and the casimir effect and the irrelevance of these phenomena to sonoluminescence," Phys. Rev. Lett. **82**, 3948–3951 (1999).

[53]W. Arnold, S. Hunklinger, and K. Dransfeld, "Influence of optical absorption on the van der waals interaction between solids," Phys. Rev. B **19**, 6049–6056 (1979).

[54]J. Goldstone, A. Salam, and S. Weinberg, "Broken symmetries," Phys. Rev. **127**, 965–970 (1962).

[55]Y. Nambu and G. Jona-Lasinio, "Dynamical model of elementary particles based on an analogy with superconductivity. i," Phys. Rev. **122**, 345–358 (1961).

[56]Y. Nambu and G. Jona-Lasinio, "Dynamical model of elementary particles based on an analogy with superconductivity. ii," Phys. Rev. **124**, 246–254 (1961).

[57]C. N. Yang and R. L. Mills, "Conservation of isotopic spin and isotopic gauge invariance,"

Phys. Rev. **96**, 191–195 (1954).

[58] J. Klepp, S. Sponar, and Y. Hasegawa, "Fundamental phenomena of quantum mechanics explored with neutron interferometers," Progress of Theoretical and Experimental Physics **2014** (2014), 10.1093/ptep/ptu085, 082A01, https://academic.oup.com/ptep/article-pdf/2014/8/082A01/34838799/ptu085.pdf.

[59] F. Wilczek and A. Zee, "Appearance of gauge structure in simple dynamical systems," Phys. Rev. Lett. **52**, 2111–2114 (1984).

[60] Y. Aharonov and A. Casher, "Topological quantum effects for neutral particles," Phys. Rev. Lett. **53**, 319–321 (1984).

[61] T. Bitter and D. Dubbers, "Manifestation of berry's topological phase in neutron spin rotation," Phys. Rev. Lett. **59**, 251–254 (1987).

[62] D. J. Richardson, A. I. Kilvington, K. Green, and S. K. Lamoreaux, "Demonstration of berry's phase using stored ultracold neutrons," Phys. Rev. Lett. **61**, 2030–2033 (1988).

[63] S. A. Werner, R. Colella, A. W. Overhauser, and C. F. Eagen, "Observation of the phase shift of a neutron due to precession in a magnetic field," Phys. Rev. Lett. **35**, 1053–1055 (1975).

[64] H. Rauch, A. Zeilinger, G. Badurek, A. Wilfing, W. Bauspiess, and U. Bonse, "Verification of coherent spinor rotation of fermions," Physics Letters A **54**, 425–427 (1975).

[65] J. J. Sakurai and J. Napolitano, *Modern quantum mechanics; 2nd ed.* (Addison-Wesley, San Francisco, CA, 2011).

[66] R. Colella, A. W. Overhauser, and S. A. Werner, "Observation of gravitationally induced quantum interference," Phys. Rev. Lett. **34**, 1472–1474 (1975).

[67] H. Rauch, W. Treimer, and U. Bonse, "Test of a single crystal neutron interferometer," Physics Letters A **47**, 369–371 (1974).

[68] A. K. Mann and H. Primakoff, "Neutrino oscillations and the number of neutrino types," Phys. Rev. D **15**, 655–665 (1977).

[69] G. Feldman, J. Hartnell, and T. Kobayashi, "Long-baseline neutrino oscillation experiments," Adv.High Energy Phys. **2013**, 475749 (2013), arXiv:1210.1778 [hep-ex].

[70] A. Einstein, "Zur elektrodynamik bewegter körper," Annalen der Physik **322**, 891–921 (1905).

[71] A. Tonomura, T. Matsuda, J. Endo, T. Arii, and K. Mihama, "Direct observation of fine structure of magnetic domain walls by electron holography," Phys. Rev. Lett. **44**,

1430–1433 (1980).

[72] A. Tonomura, H. Umezaki, T. Matsuda, N. Osakabe, J. Endo, and Y. Sugita, "Is magnetic flux quantized in a toroidal ferromagnet?" Phys. Rev. Lett. **51**, 331–334 (1983).

[73] D. Salart, A. Baas, J. A. W. van Houwelingen, N. Gisin, and H. Zbinden, "Spacelike separation in a bell test assuming gravitationally induced collapses," Phys. Rev. Lett. **100**, 220404 (2008).

[74] Y.-H. Kim, R. Yu, S. P. Kulik, Y. Shih, and M. O. Scully, "Delayed " choice " quantum eraser," Phys. Rev. Lett. **84**, 1–5 (2000).

[75] J. F. Clauser and M. A. Horne, "Experimental consequences of objective local theories," Physical review D **10**, 526 (1974).