Local Gravity Viewed as the Gradient of Time Dilation

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Abstract

Gravitational time dilation can be expressed as a function of the gravitational acceleration caused by a central mass. The function in this form coincides with the usual view that gravity causes time dilation.

This paper develops a companion function that expresses local gravitational acceleration as the gradient of local time dilation. The function in this form coincides with the view that the gradient of time dilation causes gravity. This view reverses the usual cause and effect association.

The relationship between gravity and time dilation normally uses a hypothetical far distant reference clock. The removal of this linkage to a distant reference clock may help facilitate consideration of theoretical modifications to Newtonian gravity.

Local Gravity as the Gradient of Time Dilation

An equation[1] commonly used for gravitational time dilation outside a non-rotating spherically symmetric object is :

$$\frac{\mathrm{d}t_{\mathrm{r}}}{\mathrm{d}t_{\mathrm{far}}} = \left[1 - \frac{2\mathrm{GM}}{\mathrm{c}^2 r}\right]^{\frac{1}{2}} \tag{1}$$

where $\mathrm{d}t_{\mathrm{r}}$ is the rate of time passage at distance r and $\mathrm{d}t_{\mathrm{far}}$ is the rate of time passage at a hypothetical far distance. Note that standard formulas may omit the differential form of t, but it is implied.

Define $\tau_{\mathrm{ref}}\,(r)$ as the time dilation ratio at radius r with respect to time at $r = r_{\mathrm{ref}}$:

$$\tau_{\mathrm{ref}}(r) = \frac{dt_{\mathrm{r}}}{dt_{\mathrm{ref}}} \tag{2}$$

This paper uses reference labels for distances from a mass M; where *near* refers to just beyond the region where quantum relationships prevail, *local* refers to any arbitrary local position, and *far* refers to a hypothetical far distance.

Equation (2) can be used to convert $\tau_{\mathrm{ref1}}\,(r)$ to $\tau_{\mathrm{ref2}}\,(r)$:

$$\tau_{\mathrm{ref2}}(r) = \tau_{\mathrm{ref1}}(r) \cdot \tau_{\mathrm{ref2}}(r_{\mathrm{ref1}}) \tag{3}$$

note that the conversion factor $\tau_{\mathrm{ref2}}(r_{\mathrm{ref1}})$ is a constant.

Express the left side of equation (1) in terms of $\tau_{\mathrm{local}}(r)$:

$$\frac{\mathrm{d}t_{\mathrm{r}}}{\mathrm{d}t_{\mathrm{far}}} = \tau_{\mathrm{far}}(r) = \tau_{\mathrm{local}}(r) \cdot \tau_{\mathrm{far}}(r_{\mathrm{local}}) \tag{4}$$

Substitute $\tau_{\mathrm{local}}(r) \cdot \tau_{\mathrm{far}}(r_{\mathrm{local}})$ for the left side of equation (1), and square both sides :

$$\tau^2_{\mathrm{local}}(r) \cdot \tau^2_{\mathrm{far}}(r_{\mathrm{local}}) = 1 - \frac{2\mathrm{GM}}{\mathrm{c}^2 r} \tag{5}$$

Take the derivative of equation (5) :

$$2\,\tau_{\mathrm{local}}(r) \cdot \frac{\mathrm{d}(\tau_{\mathrm{local}}(r))}{\mathrm{d}r} \cdot \tau^2_{\mathrm{far}}(r_{\mathrm{local}}) = \frac{2\mathrm{GM}}{\mathrm{c}^2 r^2} \tag{6}$$

Divide both sides of equation (6) by $2\,\tau^2_{\mathrm{far}}(r_{\mathrm{local}})$ and multiply by c^2 :

$$\mathrm{c}^2 \cdot \tau_{\mathrm{local}}(r) \cdot \frac{\mathrm{d}(\tau_{\mathrm{local}}(r))}{\mathrm{d}r} = \tau^{-2}_{\mathrm{far}}(r_{\mathrm{local}}) \cdot \frac{\mathrm{GM}}{r^2} \tag{7}$$

Substitute $g(r)$ for $\frac{\mathrm{GM}}{r^2}$ in equation (7) :

$$\underbrace{\mathrm{c}^2 \cdot \tau_{\mathrm{local}}(r) \cdot \frac{\mathrm{d}(\tau_{\mathrm{local}}(r))}{\mathrm{d}r}}_{\substack{\text{(a.)}\\ \text{local}\\ \text{field}\\ \text{strength}\\ \text{of gravity}}} \quad = \quad \underbrace{\tau^{-2}_{\mathrm{far}}(r_{\mathrm{local}})}_{\substack{\text{(b.)}\\ \text{inverse}\\ \text{squared}\\ \text{time}\\ \text{dilation}}} \quad \cdot \quad \underbrace{g(r)}_{\substack{\text{(c.)}\\ \text{acceleration}\\ \text{due to}\\ \text{local}\\ \text{gravity}}} \tag{8}$$

Equation (8) requires (c.), the acceleration due to local gravity, be increased by a factor equal to (b.), the inverse squared of time dilation, in order to produce (c.), a field strength of gravity, that maintains its magnitude as it propagates outward.

In plain terms, the gravity must increase in regions of time dilation to offset the slower rate of time and still transfer the full field strength. The effect of this at normal distances would usually be undetectable.

Equation (8) provides a hint of how modified gravity might occur. The gradient of time dilation at cosmological distances would need to be appreciable compared to the field strength of gravity. With conventional views one might consider this unlikely to occur. Other views are possible.

References

[1]Wikipedia - https://en.wikipedia.org/wiki/Gravitational_time_dilation