

Quantum Mechanics is about RED CORAL

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Abstract

Probability amplitude waves described by quantum mechanics are globally coherent relativistically space-like space-time structures, having no internal causal links. In this sense these non-classical structures are anticipated by the classical ontology of special relativity, which also delineates (in a completely different way) space-like non-causal intervals in space-time. I discuss and distinguish two broad classes of *globally coherent relativistically space-like space-time structure*, and suggest a parsimonious extension of the core ontology of physics to include both of them (rather than only one as at present), as a way of resolving some puzzles in the interpretation of quantum mechanics, and motivating its formalism. I show how this ontological extension immediately yields well known formal features usually postulated ad-hoc, such as separability of the wave function, and explains the puzzling absolute nature of time in quantum mechanics. The curious fact that the ontologies of both special relativity and quantum mechanics delineate (in strikingly different ways) strictly non-causal intervals in space-time suggests the existence of an explanatory “last common ancestor” ontology, at a higher level of abstraction than the ontologies of either. I briefly sketch such an ontology, consisting of world self-mappings, in terms of which there are two distinct concepts of time, intrinsic and extrinsic, which together play an abstract identity role in the world, and which suggest part of the meaning of quantum mechanics, and of the non-causal coherence of its non-classical waves, concerns extrinsic structure, in particular extrinsic time.

Introduction

The concept of a globally *COherent Relativistically spAce-Like* (CORAL¹) space-time structure², in which structural features separated by relativistically space-like intervals are strictly coordinated over time, seems at first sight paradoxical³: since the relativistically spatially separated space-time patches of CORAL cannot physically interact (causality cannot propagate faster than light speed), they cannot mutually cooperate in maintaining a coherent whole. Yet there is fairly good evidence we are embedded in macroscopic CORAL, and conclusive evidence we are made of microscopic CORAL: the expanding universe is relativistically space-like in extent, and yet its expansion appears to have been globally coherent since inception (presumably thanks largely to inflation), and thus appears to be macroscopic CORAL; microscopically, the probability amplitude wave structures described by

¹ I will use the CORAL mnemonic both adjectivally (“globally coherent relativistically space-like-”), and to refer to CORAL structure(s) as simply “CORAL”.

² Modulo a definition of “relativistically space-like structure”: I take a *relativistically space-like structure* to consist in a collection of space-time points, in which the interval between any pair of points in the collection is relativistically space-like.

³ Modulo a definition of “globally coherent”: For example, the global coherence of an extended rigid body R travelling at velocity v is encapsulated by $R(x+vt, t) = R(x, 0)$; the global coherence of a wagging one dimensional sinusoidal wave structure S wagging at w radians per second is encapsulated by $S(wt, 0) = S(0, wt)$. As far as I know there is no generally accepted definition of this concept: I provisionally define a relativistically space-like space-time structure to be *globally coherent*, if the states of the structure at any two of its points are smoothly functionally dependent, where the functional dependence is parameterised by time as measured in a reference frame which assigns the same time to each point.

quantum mechanics, confirmed by experiment to an extraordinary level of precision, are clearly of relativistically space-like extent, yet micro-globally cohere in space-time⁴.

On the other hand, while waves of numerically complex-valued probability amplitude are CORAL structures in that they have relativistically space-like extent and evolve coherently, it is not clear in what sense they physically exist. And while the expanding universe clearly does exist, it is not certain that it is a CORAL structure: is its expansion only apparently and accidentally coherent in space-time from our limited perspective?

Moreover there are clearly utterly mundane examples of CORAL structure, which calls into question the specificity and usefulness of the proposed concept: travelling periodic waves - such as for example ocean and violin-string waves - are CORAL, since the space-time trajectories of relativistically spatially separated parts of these structures are mutual exact mathematical functions of each other; the surface of an inflating balloon is another CORAL structure, in this sense; the classical "rigid body" of pre-relativistic mechanics would be another example of a CORAL structure if it really existed (push it anywhere and the whole structure responds immediately); laminar flow of a fluid is CORAL ; it is easy in abstract to create periodic CORAL - simply iteratively extend and entrain a spatial unit (one period of a periodic function), to potentiate a globally coherent oscillating periodic structure with arbitrary relativistically space-like extent.

Thus we need to distinguish different kinds of CORAL: phenomena such as ocean or violin-string waves arise from one or multiple time-like initial causal impulses (such as wind gusts and violin bowings), which assemble and entrain a seed structure in a time-like way, such that a spatially extended region is potentiated for globally coherent further evolution; each patch of the extended potentiated region then evolves subsequently also in a causal time-like way; but since the separate time-like evolution of all the patches is governed by the same simple local rules, the evolving periodic wave-structure maintains an appearance of global coherence. The combination of time-like causally entrained and potentiated initial conditions, and parallel law-like causal evolution of each patch of the potentiated region, as it were weaves together the *appearance* of a globally coherent structure from multiple time-like threads: I will refer to this as Woven-tHreads-from-Initial-conditions-and-Temporally-Entrained (WHITE) CORAL.

That the global coherence of WHITE CORAL is only apparent becomes clear when we try interacting locally with some part of it: we will find the effect of any such interaction propagates to the rest of the structure in a time-like causal manner, and its global coherence is lost ; for example, we could drop a small pebble onto an ocean wave - the ripples will propagate causally to the rest of the structure in a time-like way; we could interact with the laminar flow of a fluid - the flow will become turbulent at the point of interaction and the turbulence will propagate downstream; the previous global coherence identities linking the end-points of relativistically space-like intervals no longer hold.

But are there in fact any types of CORAL, other than WHITE CORAL ? That is: is there any such thing as a globally coherent relativistically space-like space-time structure, with which we *cannot* interact

⁴ Consider for example the abstract balloon shaped orbital clouds surrounding the atomic nucleus, in which electrons may be found with varying probability density: while these structures certainly evolve over time (in accordance with the postulates and resulting equations of quantum mechanics), they do so as entire coherent relativistically-space-like wholes; they do not evolve over time in anything like the way structures with internal relativistically time-like causal connections do - such as real clouds, which are the union of many time-like component world-lines; neither are they assembled in anything like the way some kinds of virtual or mathematical cloud might be - as a blurred average of a single entity tracing an orbital space-time world-line at high speed.

locally ? Such hypothetical structures are in abstract rigid bodies⁵ (with which we cannot interact purely locally because any interaction immediately propagates globally to the entire structure), and so I will refer to them as Rigid Extended Domains of (RED) CORAL. So: is there any RED CORAL in the world? Yes ! Clearly, the complex-valued probability amplitude wave structures described by quantum mechanics are RED CORAL: they are space-time structures of relativistically space-like extent, with no internal time-like causal connections, with which it is *not* possible to interact locally, yet which are globally coherent. While we can easily superimpose a second “pebble” probability wave on top of any given probability amplitude wave structure, this is not in any way a locally propagated interaction: the global coherence of both the original wave, the superimposed “pebble” wave, and the resultant combined wave is maintained throughout. It is not possible or even meaningful to think of introducing some kind of impulse at a specific point in space-time, which causally influences the rest of a quantum probability amplitude in a time-like manner, as is possible with a macroscopic ocean wave.⁶

That quantum mechanics is about RED CORAL is thus already known (albeit an unknown known ?). Yet while the content of “*quantum mechanics is about RED CORAL*” is de facto not new, the manifesto remains interesting in a number of ways. Firstly, it provides a fresh perspective on the incompleteness of the formalism of quantum mechanics, and on some of the ensuing confusion. From the manifesto’s point of view, the formalism *must* be incomplete because, while it is clear empirically that quantum mechanics is specifically about RED CORAL, its mathematical formalism, which evolves initial space-like states in a time-like way, describes both RED *and* WHITE CORAL: the formalism itself has nothing to say about the provenance of the input initial state, only that given that state it knows how to evolve it, and hence describes in principle not only waves with causal time-like provenance with which one can interact locally (such as ocean and violin-string waves) and which only *appear* to have global coherence; but also waves of complex-valued probability amplitude which, despite their waviness, behave counter-intuitively like rigid bodies in that we cannot initiate local temporally propagated causal interactions with them, and for which we have no plausible time-like constructive explanation of their initial state (no quantum equivalent of violin bows, gusts of wind, or of entrainment of an underlying medium such as a violin string or body of open water).

It is not that the mathematical formalism of quantum mechanics is *wrong* in providing a framework which describes both RED *and* WHITE CORAL: just that this lack of specificity sets us up to commit a syllogistic fallacy⁷. From the truth of the premises that (1) the formalism accurately describes WHITE CORAL and (2) the formalism accurately describes complex waves of probability amplitude, we are tempted to infer incorrectly that complex waves of probability amplitude *must* be examples of WHITE CORAL (and that it is therefore necessary to elaborate quantum mechanics formalism, in what turns out to be probably the wrong direction, to be more specific for WHITE CORAL - for example by seeking to delineate some kind of local causally-threaded substructure such as “particles with guide waves”, or various hidden variable proposals).

Secondly, if a non-classical theory “Q” (such as pre-relativistic⁸ quantum mechanics), being master of its own staunchly independent domain of postulates and ontology, turns out to be about a kind of thing (strictly non-causal space-time intervals) defined by a completely different and equally independent-minded classical theory “R” (such as special relativity), with *it’s* own postulates and

⁵ An alternative metaphor would be that of an incompressible fluid, however this is a much more complex and specific concept than is appropriate at this point.

⁶ Also because in the microscopic case it is in a sense “waves all the way down”: there are no pebbles to throw at such a wave, only more such waves to throw at such a wave !

⁷ The fallacy of the undistributed middle

⁸ Dirac combined the postulates and ontology of special relativity and quantum mechanics, to develop a relativistic quantum theory. The point being made here however is, these two theories seem to be ontologically aware of each other in advance of that synthesis.

ontology, then either there is some deeper connection between the postulates and ontology of “Q” and “R” , or else we have on our hands an astonishing coincidence: why should the non-classical machinery of (pre relativistic) quantum mechanics turn out to be so specifically about a structure (RED CORAL), that is framed by the completely independent classical machinery of special relativity ?

Thirdly it seems possible that, by moving the paradoxical relativistically-space-like-non-causal-yet-globally-coherent (in a RED way) nature of the structures described by quantum mechanics fully center stage, as first rank members of the ontology of the world, we approach the theory from a different angle with a refreshing and more intuitive perspective, having the potential to motivate some of it’s ontology, postulates and mathematical paradigms, and side-step puzzles of interpretation. Part of psychological preparation for this fresh perspective, is to reflect on what an odd kind of thing a “rigid extended domain of relativistically space-like, yet globally coherent” structure is going to have to be⁹: any interactions with RED CORAL are , by definition, such that to interact with a part is to interact with the whole.

For example, the *a priori* impossibility of local interaction with some specific region of a RED CORAL structure, implies that it cannot be conventionally scanned or imaged, since any conventional scanning beam (using photons, electrons or other) must interact either with the entire structure, or not at all: the beam cannot interact locally with some sub-region of a RED CORAL and therefore we cannot collate an image of the structure from reflections or scatterings off different sub-regions, as those would be *local* interactions¹⁰. The only conceivable way to image RED CORAL is “destructively”, by probing it with a beam which, on encounter, interacts globally and causes the entire structure to collapse: if we can somehow then make identical copies of a RED CORAL target, and probe each copy with a differently aligned beam, and plot the points at which the beam interacts globally with, and collapses, the target, we could build up an image of it.

Yet this is just an odd description of paradigmatic quantum measurement scenarios, such as the double-slit experiment: in which the RED CORAL structure we are trying to “image” is a photon (or electron etc.) “passing through” *both* slits of the device ; the identical copies are the successive photons we send through; the “differently aligned destructive beams” used to destructively image the target structure are simply all the particles comprising the surface of the receiving screen, which destructively interact with each successive copy of the RED CORAL structure and “collapse” it; we gradually build up an image of the structure of the RED CORAL, one collapse at a time; the famous interference fringes that result are really a *picture* of RED CORAL, obtained by an unconventional destructive imaging technique.

But how exactly are we to causally embed a *globally* coherent structure into a *locally* causal environment ? On the one hand it’s external environment *geometrically* impinges the internal space occupied by a RED CORAL structure at myriad different positions and from many different orientations: and yet on the other it must *causally* encounter the structure as though it were a single point, due to its global coherence. Conversely, considered from the point of view of the embedded globally coherent structure itself, on the one hand it geometrically impinges the external world at all the different locations and orientations on its periphery and thus has multiple views of that world:

⁹ Indeed it is a much stranger kind of thing than any of the strange things we already know are described by quantum theory. But as reward for that preparation we might find quantum solace in the realisation that a seemingly existentially impossible thing - a “rigid extended domain of coherent relativistically-space-like structure” - turns out to make an honest living as merely a strange kind of thing - waves of complex-valued probability amplitude whose existential status is unclear.

¹⁰ Thus, while a RED CORAL structure has a definite extent and shape, and we might see no harm in mentally visualising that shape and depicting it in diagrams, these visualisations are deeply deceptive, in that they implicitly convey, as well as the shape itself, the possibility of local interaction with the structure: they do not do justice to how necessarily strange and exotic a globally coherent structure has to be.

and yet on the other it must causally encounter the external world from just a single viewpoint, due to its global coherence. One (I conjecture, the only) way we can satisfy these two seemingly contradictory requirements, is to consider a RED CORAL structure to be a *superposition* of myriad *potential* local views and causal encounters, which in some way then collapses to a single *actual* causal encounter, which of course is just the familiar framework of quantum mechanics: that framework thus reflects one of the only conceivable ways open to nature, to consistently embed a *globally coherent* relativistically space-like structure into a *locally causal* environment.

Probing the *a priori* implications of RED CORAL further we can ask how one might from first principles conceive of doing *mechanics* with a RED CORAL structure: how to describe its current state, and evolve that to another? what sort of mathematical structures might be needed for state description and evolution? Remembering all the while that by definition RED CORAL structure has no internal time-like causal levers that we can manipulate with our mathematics, and that formally it behaves like a rigid body.

Starting with nothing more in hand than these curious questions, and the ontological premise that all parts of a RED CORAL structure must evolve “as one”, we can predict the mathematical form used to evolve it will be spatio-temporally “separable” *a priori*: since time must supervene globally over the entirety of a RED CORAL structure (. . . it cannot intervene locally to influence only some part of the structure since there are no internal temporal causal links available to propagate the intervention to the remainder. . .), then it *must* combine multiplicatively with the structure’s spatial description. Thus if $\Psi(x, t)$ is some function purporting to describe evolving RED CORAL, it *must* factorise into distinct spatial and temporal factor-functions:

$$\Psi(x, t) \equiv \psi(x) \varphi(t)$$

Separability of the wave function $\Psi(x, t)$ is invariably pragmatically *assumed* ad-hoc (with justification along the lines of: “*we assume the solution of the wave equation will take this separable form, since if we don’t formally constrain the solution in this way, it is not clear how to obtain a solution*”); but the RED CORAL manifesto offers a deeper yet still intuitive reason for separability.

Furthermore, since the proposed canonical mathematical expression of the *global coherence* of a RED CORAL structure is in terms of an inertial frame relative to which the end-points of relativistically space-like spanning intervals have the same time coordinate, it follows that the temporal ontology underlying a function $\Psi(x, t)$ describing the evolution of RED CORAL will involve what *appears* to be pre-relativistic “absolute time”. This odd “absolute” guise of the temporal ontology of quantum mechanics is a well known puzzle, but from the point of view of the RED CORAL manifesto there is a simple explanation: time in quantum mechanics is *not* absolute (in a Newtonian sense), it is just that it is relative to a special inertial frame in which the global coherence of the structure it describes is natively defined.

It also follows that, whereas the dynamical state of a non-RED-CORAL ensemble woven from causal time-like threads - for example a collection of particles - consists of the aggregation of the dynamic states (e.g. momenta) of its parts, the dynamical state of monolithic RED CORAL, which lacks internal time-like causal structure, cannot be any such thing, but rather must be entirely encoded in its spatial structure as some (purely spatial) function $\psi(x)$; and we can expect the mathematical formalism best suited to representing and extracting that encoding may be strikingly different to what is best suited for handling ensembles of particles; and since Fourier analysis provides an infinite dimensional basis for any such predicted encoding function $\psi(x)$, we can expect that infinite dimensional state vectors will provide a natural framework for encoding the dynamical state of RED CORAL; and that we find (as is *postulated* by quantum mechanics) we need *operators* acting on those states to extract and represent dynamical properties such as momentum.

Thus from a RED CORAL point of view, the distinctive mathematical framework of quantum mechanics, so radically different from its classical predecessor-mechanics' frameworks, is not merely as it appears a matter of ad-hoc postulation, but rather a natural consequence of the bestiary of the world, which happens to include a strange kind of beast foretold in principle by special relativity: *rigid extended domains of globally coherent relativistically space-like space-time structure.*

Suppose there is indeed some deeper connection between the postulates and ontology of Q and R, explaining the odd fact that Q turns out to be about a strange kind of thing (non-causal space-like intervals and RED CORAL) framed by R. Here "deeper" means, a more abstract "last common ancestor ontology" of the ontologies of Q and R: the terms in the ancestor will involve a higher level of abstraction¹¹ than the terms in the descendants Q and R, and will thus enable us to pose statements about the *meaning* of those less-abstract descendants. In the rest of this essay I sketch a simple candidate abstract ancestry for the ontologies of Q and R, the existence of which is predicted by the truth that quantum mechanics is about RED CORAL.

Abstract Intrinsic and Extrinsic Concepts of Time

Intrinsic Time

Consider a static purely space-like world U_0 , in which nothing can happen, and the question of how to augment U_0 so as to allow "events" to happen. I will assume events are conservative inasmuch as they conserve U_0 's potential to experience further events: things happen in/to U_0 , but U_0 itself endures. This implies an event in U_0 represents, in abstract, a *self mapping* of U_0 back to itself after which it's configuration is the same in some essential way as before the event, leaving U_0 potentiated for a further recursive event-self-mapping, and so on: in other words, each event exploits one or more symmetries of U_0 . Let U_S denote a world enduring indefinitely across such conservative events - i.e. the result of augmenting U_0 with an intrinsic symmetry structure.

We can describe hypothetical sequences of self-mapping events e, f, g, \dots happening in/to U_S like this:

$$U_S : e \circ f \circ g \circ \dots$$

which we read as *e happened in/to U_S , then f happened to U_S etc;*¹² I will refer to such self-mappings of a structure U_S , as its *intrinsic* symmetries, and to the collection of all its intrinsic symmetries as its *intrinsic structure*.

¹¹ The odd RED CORAL intersection of Q and R is evidence there should exist meaningful and substantive things to be said at a higher level of abstraction than either of these theories, and perhaps roughly at the level of abstraction attempted here. In some ways this is not as interesting a thesis as its negation - that there do not exist any meaningful or substantive things to be said about physical theory at such a higher level of abstraction. The present essay could potentially establish the implicit abstraction thesis by demonstrating at least one such both meaningful and substantive statement: but failure to do so will not establish its negation (since somebody else may already have provided the confirming example, or will do so in the future). Trying to imagine what sort of reality we inhabit, assuming the negation is true - there is little or no abstraction to be had along these lines - is an interesting exercise: indeed much more interesting than imagining the affirmative case; if the negation is true, then some kind of oddly concrete and illiberal reality seems hinted at (illiberal, because abstraction always requires degrees of freedom in what is abstracted from, and conversely degrees of freedom always mean there is abstraction to be had).

¹² Note this kind of description is constructed in, and assumes the existence of, a realm *external* to U_S , it has no existence or meaning *inside* U_S ; in this context the word "then" expresses notational extension in this realm, rather than temporal extension within U_S .

While by assumption U_S is now the kind of enduring world-structure in which (“conservative”) events may happen, it is still static, yet it is not even meaningful to say so: we cannot yet say “*nothing happened in U_S* ”, since this kind of statement presupposes some concept of *time*. One possibility is to augment the *intrinsic* structure of U_S , with a new self-mapping, “*tick*”, which we will choose to designate as temporal, and which can mix freely on equal terms with its sibling intrinsic symmetries. We can then express “nothing happened to U_S ”, like this :

$$U_S : tick \circ tick \circ tick \circ \dots$$

; we can mix the new intrinsic symmetry with others in more eventful sequences such as

$$U_S : tick \circ e \circ tick \circ tick \circ f \circ \dots$$

which we read as “*nothing happened in/to U_S , then e happened, then there was a “longer” (more ticks) interval of nothing happening, then f happened . . .*”. I will refer to this as an *intrinsic* concept of time.

We might be tempted to think of the new intrinsic *tick* self-mapping as in a sense “*snapshotting and identically returning U_S unchanged*”, thus assigning *tick* the abstract role of an *algebraic identity*, I . Indeed it is a reasonable conjecture that pre-relativistic concepts of time *have* usually implicitly equated, in the limiting case of an *infinitesimal tick*, the *tick* self-mapping with what amounts to an identity self-mapping: in the limit of an infinitesimally short time duration, an *infinitesimal tick* does in a sense “*snapshot the world and return it*”, if one has in mind time as the kind of universal absolute clock of Newtonian cosmology. But, at least at this level of abstraction, one could equally well on that basis equate a limiting infinitesimal slice of any of the other (e.g. spatial) self-mappings of U_S , with I , so it is a fallacy to at this stage think *specifically* of the *tick* self-mapping in this snap-shotting-in-the-infinitesimal-limit kind of way. Therefore provisionally:

$$U_S : tick \neq U_S : I \quad (\text{“tick” and “I” are not necessarily the same (?)})$$

Extrinsic Time

Consider an “iterating-automorphism” temporal operator *Tick*, iterating a spatial self-mapping event Δx of U_S , yielding a resultant spatial self mapping X :

$$Tick(\Delta x) \equiv \Delta x \circ \Delta x \circ \Delta x \circ \dots \circ \Delta x = \Delta x^T \rightarrow X \quad [1]$$

Tick says “*take whatever self-mapping of U_S I am given (Δx in this case), and recursively compose it with itself a certain measure of times T , to obtain a new resultant self-mapping of U_S* ”. *Tick* is an *extrinsic* symmetry, inasmuch as it is a higher-level *self-mapping of the self-mappings* of U_S , rather than a self-mapping of U_S itself: i.e. roughly, it acts as an automorphism of the intrinsic structure of U_S . (I will abbreviate this by saying higher-level mappings such as *Tick*, self map “ $\&U_S$ ” rather than “ U_S ”). This leads to a concept of *extrinsic time*, as an abstract logarithm of the self-mappings of U_S in the sense that, in the above example,

$$T = abstractlog(X) \text{ (to the base } \Delta x)$$

Clearly this *extrinsic* concept of time is close to the way time is actually measured: let X be the event of the second-hand of a clock making a certain number of revolutions T , and Δx be the event of the second-hand taking one revolution: we assign a temporal measure of T to X , precisely when, abstractly,

$$\text{Tick}(\Delta x) \equiv \Delta x^T$$

$$= \Delta x + \Delta x + \Delta x + \dots + \Delta x = T\Delta x \rightarrow X$$

(substituting arithmetic "+", and "additive log" (i.e. division) for abstract "o" and abstract log)

(This abstraction also more broadly underlies the mathematical description of periodic phenomena in general: let Δx be a single period of a sinusoidal wave, then the functional extension of this single period is at heart an extrinsic construction - "take whatever form I am given and iterate it"; thus suggesting the formal algebraic template of exponentiation as in [1]; thus also motivating the fundamental linkage between exponentiation and periodic functions represented by the identity $e^{i\theta} = \cos(\theta) + i \sin(\theta)$)

On the other hand, it is also close to the way distance is measured: we assign a distance D to some increment X , precisely because

$$\Delta x + \Delta x + \Delta x + \dots + \Delta x = D\Delta x \rightarrow X \text{ (again substituting "+" for "o" etc.)}$$

While this notion of *extrinsic* time - a higher level self-mapping of self-mappings of the world - is closer to what we actually measure, the concept of *intrinsic* time - a self-mapping of the world itself - is perhaps closer to our intuitive concept of "time" as an extra degree of freedom, or dimension, for the world: just as we can locate an object using 3 spatial coordinates so can we (surely ?) locate an event in space and time using 4 coordinates; and this intrinsic more geometric notion of time has nothing specifically to do with events in the world, as does the extrinsic notion.

Time

The argument of Appendix 1 below ("groups can't laugh at themselves") reminds us that, in the case of a collection of self-mappings satisfying the group axioms, there is only one structure-preserving *self-mapping*, that is also a structure preserving *self-mapping of self-mappings* : the identity I ¹³. In other words, only the identity plays both intrinsic (symmetric self-map of U_S) and extrinsic (symmetric self map of $\&U_S$) abstract structural roles. This suggests exploring the idea that *Time*, which as we have seen *can* be refracted into both intrinsic and extrinsic abstractions, *does* in fact play an abstract structural role in the world akin to that of the identity. I will start by noting that the phenomenon of light, considered as a self-mapping of the world, is also refractable into intrinsic and extrinsic abstractions, suggesting an isomorphism between *Time*, and the phenomenon of light, and an explanation of the meaning of the special role that light and the speed of light, plays in the structure of the world as revealed by special relativity.

In the years immediately preceding the special theory of relativity, light was understood as part of the *intrinsic symmetry* structure of the natural world, that is, as a self-mapping of the stuff of the world itself: specifically, a self-mapping of luminiferous ether. Light had been shown to have wave-like properties such as diffraction; all known waves at the time were self-mappings of some medium such as a body of water, or air, or string or spring under tension; therefore it was reasonable to infer an analogous medium, disturbance of which corresponded to the propagation of light waves.

¹³ But *why* is this the case ? Here we can offer the explanation proposed in Appendix 1: the intrinsic structural richness of the collection of symmetry self-mappings of the world-in-abstract U_S (similar if not identical in richness to that of a mathematical group) means these mappings are in a sense "fully subscribed", with no degrees of freedom spare to partake in further extrinsic structural formulations; thus we can't naively collapse intrinsic and extrinsic time into a single concept of time; and this is part of the meaning of special relativity. (One could envisage reprising this explanation in other contexts: a kind of meta-symmetric maximum-symmetry-occupancy rubric).

On the other hand relativity's postulation that the velocity of light is the same to all observers whatever their state of unaccelerated motion, amounts to saying that light also behaves as though it were part of the higher-level *extrinsic* structure of the world. Special relativity says, in abstract, that light is a higher-level mapping like this: *take some event structure of the world (which we will call an observer moving with any unaccelerated velocity), and map this to a new event structure consisting of the original structure together with a light beam emitted in a certain direction with a certain velocity c ; and conversely, it is not entirely like this : take some piece of the world itself - such as a piece of luminiferous ether - and wiggle it up and down at a certain frequency so that it propagates through the ether with a certain velocity c relative to that medium.*

This higher-level way of framing the phenomenon of light, in which we refer to events in the world rather than to the stuff of the world itself, is also structure preserving inasmuch as, if we take a collection of observers in various different states of unaccelerated motion, and remap them all in this way, the invariance of the speed of light means we have in a sense done the same thing to all of them: adjoining an emitted light beam to each is to adjoin exactly the same thing to each, as though we had painted the same white dot on each of their foreheads. The constant light speed postulate thus says that the phenomenon of light is in some way a higher-level abstraction: a structure preserving *self-mapping of the self-mappings of the world*; and not just an intrinsic self-mapping of some ethereal stuff-of-the-world.

Yet, according to a very broad and deep naturalist premise subtending this and all other phenomena in the world, we must refrain from classifying light as a *supernatural* phenomenon: by this premise, light is a material event in the material world and hence *must* also in some sense be part of its *intrinsic* structure - i.e. some kind of self-mapping of the stuff of the world. It is this "clash" between the intrinsic and extrinsic personalities of the phenomenon of light that is at the heart of the affront to intuition represented by special relativity: light's *intrinsic* personality, conferred by the fundamental and very reliable naturalist premise that it is a material phenomenon, clashes with its higher level *extrinsic* personality, encapsulated in the core postulate of special relativity, and forced on us by a requirement for logical consistency (for example consistency in Maxwell's electromagnetic framework) , and empirical evidence (such as Michelson–Morley). (The theorem of Appendix 1 explains clearly why these roles must clash so viscerally: at a high level of abstraction, no self-mapping of any structure with the richness of symmetry that the world appears to have, apart from the identity mapping, can consistently play *both* intrinsic and higher level extrinsic roles)

There are a number of hints that suggest this way of thinking about the role of light at a higher level of abstraction, as a self-map of both U_S and $\&U_S$, and hence (via the Appendix) as reprising a role as in some sense an identity for both U_S and $\&U_S$, may have promise. Proceeding from the right hand side of the proposed abstract equation " $light \equiv I$ " , and heading as opportunity presents toward the left hand side: as an *intrinsic* self-map of U_S , I should be a kind of *event*, not simply absence-of-any-event: it is if anything one of the infinite continuum of self-mappings e, f, g, \dots of U_S , but one with special properties conferring a formal identity role; for example it must be "irreducible" inasmuch as, unlike other events, it cannot be split¹⁴:

$$U_S : I \neq U_S : e \circ e \quad \forall e \neq I \tag{2}$$

¹⁴ The physical meaning of this abstract irreducibility, is the irreducibility of light speed: i.e. photons do not accelerate to light speed through intermediate velocities

Also, considered from the point of view of the event I ¹⁵, it must somehow be the case that, despite my occurrence (as I), “nothing else appears to change” in U_S : there are no finite spatial or temporal increments of U_S relative to me; if this were *not* the case, then we could split I into sub-events representing “smaller” increments in the rest of the world than from the point of view of I , in contradiction of [2]; and furthermore this is intuitively what we expect of an identity - such an event should in a sense, from its own point of view “*snapshot the world and return it*”.

This is the first hint that part of the meaning of special relativity, lies in it’s assigning the abstract role of an identity self-map of the world, to the event of a massless particle (such as a photon) moving at light-speed: relativity requires that, from the point of view of such an event, the spatial and temporal extent of the rest of the world shrinks to zero; from the light-speed-event’s point of view the world has zero length in the direction of travel and (consistently), no time passes there; and these are precisely what an event playing an intrinsic formal abstract identity role should see¹⁶.

Consider next, formally, the abstract *extrinsic* role of the identity I : in this role I is the only self-mapping of U_S that is also an automorphic structure-preserving self-map of $\&U_S$; i.e. a self-mapping of the self-mappings e, f, \dots of U_S , satisfying

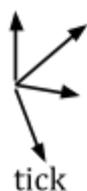
$$I(e) \circ I(f) = I(e \circ f) \tag{3}$$

But wherever we can write [3] we can also write the dual:

$$e(I) \circ f(I) = (e \circ f)(I) \tag{4}$$

in which the e, f events can be considered to be structure-preserving self mappings of I ; yet by definition e, f, \dots are structure-preserving self-maps of the world U_S ; implying I is isomorphic with at least part of the fundamental invariant symmetry structure of U_S being conserved by self-mappings e, f, \dots . This is the second hint of light’s abstract identity self-mapping role, since light is indeed part of the core invariant intrinsic symmetry structure of the world, as encapsulated in abstract in [4] (and first revealed in detail by Maxwell’s electromagnetic theory).

The third hint relates to a curious role light events play in the world as revealed by special relativity, involving what I will call *phenomenological labelling*.



¹⁵ What do I mean by “considering the self-mapping represented by an event in U_S from that event’s point of view”: suppose I am an infinitesimal event e which consists in exchanging my location, with a location 1 millimeter away in some direction: then from my point of view all of U_S has been self-mapped to different positions, 1 millimeter away from where they started; i.e. I mean a change in reference frame. The point being made is that from the point of view of an event having a formal role of an identity for U_S , the self mapping induced on U_S by my frame shift that I perceive must be a strange one, in which no (finite) spatial or temporal increments occur.

¹⁶ The contraction of relative space and time to zero from the point of view of an observer at the speed of light is a *theorem* of special relativity, not a postulate: however this argument suggests the possibility of an alternative exposition in which this would be *postulated*, as part of a postulate concerning the existence of events playing an abstract role akin to an “identity self map” of the world.

Figure 1: My 4-dimensional world (pre-relativity) (no identity self-map)

Figure 1 depicts¹⁷ how pre-relativistic physics (and each of us day to day, avocationally) assumes we can *nominatively label* our 4-dimensional world: into distinct regions of *up/down-like-increment*, *left/right-like-increment*, *forward/backward-like-increment* and *past/future-like-increment*: the labelling is *nominative*, because we can choose completely different labels without any real phenomenological consequence - our labelling is purely conventional.

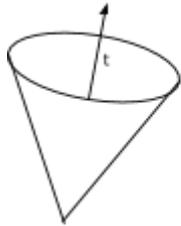


Figure 2: My 4-dimensional world (post-relativity) (identity self-map on the surface of the cone)

Figure 2 depicts how Minkowski (following Einstein) (and none of us avocationally !) find our 4-dimensional world is actually labelled, *phenomenologically* by (the phenomenon of) light: into *time-like-intervals* (inside the cone), *space-like-intervals* (outside the cone), and *light-like intervals* (the surface of the cone); it turns out that despite appearances, and completely contrary to intuition, the nominative labelling scheme of Figure 1 cannot be carried through consistently.

The identity map¹⁸ is associated with labelling because, of all the self-maps of a symmetric structure, it alone remains when the target structure is unambiguously labelled. Consider for example an equilateral triangle, with three mirror symmetries a, b, c and rotational symmetries r, r^2 : by labelling one vertex we abolish all symmetries except one of the reflections, and I ; by labelling two vertices, so the triangle is now unambiguously labelled¹⁹ - i.e. completely *asymmetric* - then we abolish *all* symmetries *except* I . Thus part of the abstract meaning of the identity self-map: it is associated with unambiguous *labelling*.

¹⁷ The depiction of the 4th (time) axis in Figure 1 is of course rhetorical only: and while it is actually easy for even a non-professional to envisage erecting in principle a 4th axis for time, and even to do a certain amount of successful informal reasoning about such a thing, it is impossible for almost anybody (including almost all professionals) (apart from perhaps a few "4-D whisperers" with rare mutations hard-wiring such an ability), to *visualise* 4 dimensions; on the other hand it is not at all easy for anybody to envisage even in principle erecting the light cone of Figure 2 (also a rhetorical depiction), and practically impossible for anybody to successfully reason informally / non-technically about it. (Perhaps though a higher level of abstraction may support successful non-technical reasoning in this area).

¹⁸ The identity element of a symmetry group belies its appearance of utmost simplicity, and is in fact an exceedingly subtle and puzzling concept. One immediate puzzle about this self-map is: since *all* the intrinsic symmetries of a group "self map the underlying object (e.g. , say, an equilateral triangle) back to itself so its configuration post-self-mapping is identical to its pre-mapping configuration, and thus is available for a further recursive self-composition of another out-going mapping", then what could be "more identical" about the self-map I , than the other self-maps ? A second puzzle about the identity: it is always one of the symmetries, of *any* object described by a group; what could possibly be common to all possible objects whose symmetries are described by groups, that the uber-symmetry I could be picking out ?

¹⁹ Note also, the abstract labelling here is "phenomenological" inasmuch as the labels are considered to be incorporated into the triangle figure itself - they don't sit outside it as extrinsic nominal descriptors: so when we add them we really (asymmetrically) graffiti the triangle itself, and hence progressively abolish its symmetries.

Light's phenomenological labelling role is prominent also in pre-Minkowskian relativity: in Einstein's original non-geometric exposition, different points in space are labelled with the same time coordinate, provided a central observer mid-way between them would receive light signals from each point at the same instant; yet the instrumentalist spirit in which light signalling is invoked (with an implicit preface "*let us drop all ontological pretensions and focus on how we actually might measure things; and we may as well use light signals to do this*") is in some ways puzzling: how can what seems to be a mere positivist feint yield such astonishing insight into the real structure of the world? Suppose we had operationally assigned time coordinates by arranging for observers to bowl (or fire, using identically compressed identical springs) cricket balls at one another, rather than signalling with light? The answer offered here is: at a higher level of abstraction, relativity's phenomenological labelling is associated with an identity self-mapping of the world; massless particles moving at light speed can play the role of I , whereas cricket balls cannot; therefore, arguments involving labelling the world with light signals are bound to be more fruitful in revealing its real structure, than those involving cricket ball signals^{20 21}.

Interestingly the Figure 2 metaphor also clearly depicts segregation of the self-mappings of the world into intrinsic and extrinsic types: intrinsic structure inside the cone; extrinsic structure outside the cone; and the self-mapping I , playing both intrinsic and extrinsic roles, at the intersection of those regions, on the cone's surface. That the light-cone-surface self-mapping plays an abstract identity role is also represented by this metaphor inasmuch as, from the point of view of a light-like self-mapping (i.e. we move from one position on the surface of the cone to another), the cone-world view does not change, apart from a change in scale; whereas for any other off-cone (i.e. non-light-like) event, our view of the world changes in a way not fixable by a change in scale. And, conversely, the nominative labelling scheme of Figure 1 cannot represent such an identity mapping: this euclidean-grid-world *always* looks different, in a way not fixable by a change in scale, when I move from one point to the other.

²⁰ The "labelling" role light plays in the standard exposition of special relativity, is reminiscent of its role as force-carrier for electro-magnetic interaction (as revealed by quantum field theory): one can think ("poetically") of the way photons carry out their force-carrying role, as being a kind of phenomenological labelling of that part of the world encompassing the interacting sub-units, so they can in a sense locate each other and work out how to interact. The argument here thus suggests the possibility of a different construction route for this kind of physics, which would start from an *a priori* abstract notion of the identity structure of self-mappings of the world, and would then go on to deduce the kinds of physical meaning such special kinds of self-mapping must have.

²¹ That a massless entity moving at the speed of light - an event in the world involving the maximum rate of change of location permissible - should be associated even abstractly with an "identity self map of the world" seems questionable, since one might expect some kind of *minimal* change from a self-map posing as an identity: we expect such a map to as it were "*snap-shot the world and return it*"; until one recalls, stationarity in the world is illusory; and even granted one were to be considered (somehow) stationary, the river of time roars on past and beneath you, and of course absolute stationarity itself *cannot* be established. Yet indeed it is true: as one approaches more closely being massless and moving close to the speed of light, the less the events of the universe do "roar on"; the spatial extent of the world in the direction of travel shrinks and the passage of time in it slows; it is as though massless particles moving at light speed are indeed some kind of fixed identity-like center orchestrating the appearance of the passage of time and extent of space for events having mass and travelling more slowly than light.

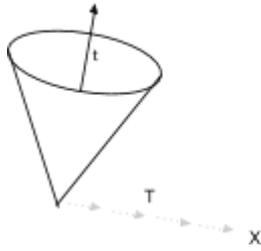


Figure 3: Space-like intervals are extrinsic structures

Extrinsically, arbitrary self-mappings of self-mappings such as *Tick* in [1] above can be constructed, which connect the origin to other space-time points, in a relativistically space-like way, as depicted in Figure 3: the light cone is also a time cone, with intrinsic time inside the cone, and extrinsic time outside the cone.

The Extrinsic Personality of Quantum Mechanics

I have sketched a simple abstract last-common-ancestor (of Q and R) ontology consisting of “self-mappings of the world”, in terms of which we can distinguish *intrinsic* time “*t*” (time as an additional dimensional self-mapping of world) and *extrinsic* time “*T*” (time as a *self-mapping of self-mappings* of the world), and in terms of which light-speed events in R play an abstract identity role, so that there is an isomorphism between *time*, and the phenomenon of light.

In terms of this ontology, the mathematical form of the spatial encoding of the dynamical state of RED CORAL $\psi(x)$ (called the *amplitude function* in Q, and which we have already noted above is expected *a priori* to be decomposable into periodic basis functions) is formally anticipated by the framework of special relativity, as a ramification of extrinsic time *T*, with *T* permitted to extend outside the light-cone as depicted in Figure 3. For, suppose we apply the abstract extrinsic temporal schema [1] to relativistic proper time τ , where

$$\tau^2 = t^2 - x^2 - y^2 - z^2$$

with

$$\tau^2 < 0$$

outside the light-cone. Considered as “extrinsic time”, then by [1] τ is a self-mapping of self-mappings of the world, schematically

$$\tau(\Delta x) \equiv \Delta x \circ \Delta x \circ \Delta x \circ \dots \circ \Delta x = \Delta x^\tau \equiv e^{k\tau}$$

But outside the light-cone we have

$$\tau = i \cdot s$$

where *s* is relativistic proper space

$$s^2 = x^2 + y^2 + z^2 - t^2 = -\tau^2$$

So outside the light-cone

$$\tau(\Delta x) = e^{kis} = \cos(ks) + i \sin(ks)$$

Thus interestingly, invoking the framework of just R, together with the proposed Q-R ancestor ontology of intrinsic and extrinsic time, yields a wave-like complex-valued mathematical niche for

RED CORAL, in advance of the confirming complex wave functional solutions of the amplitude equation obtained in relativistic quantum mechanics.

While the above argument that R accommodates complex-valued spatial waves in advance of Q might seem a little contrived, this latent prediction has a straightforwardly firm basis: as we have noted above, the easiest way to make RED CORAL is to take a small piece of some spatio-temporal function, and replicate it out along the spatial axis, thus yielding a structure that is globally coherent and relativistically space-like in extent; in other words, the mutual relationship between RED CORAL, and the form $e^{ix} = \cos(x) + i \sin(x)$ is “baked in” at an even higher level of abstraction than the proposed Q-R ancestor ontology of intrinsic and extrinsic self-mappings of the world; and one should not be surprised when a less abstract ontology, reprises an ontological theme of a more abstract parent.

(There are other plain hints as to Q’s extrinsic personality: consider for example it’s core modus operandi : “Take whatever self-mapping of the world (i.e. event) I am given , and assign a probability amplitude to it”; whereas classical and relativistic self-mappings operate in a sense on the stuff of the world itself (“take the world and map it onto itself”), Q , abstractly speaking, operates extrinsically, one level higher (“take whatever self-mappings of the world I am given and do things such as assign complex probability amplitudes to them, or entangle them”).

Conclusion

In trying to cater to the voracious appetites for explanation and interpretation, posed by such quantum puzzles as the “collapse of the wave function”, with only what is on offer from the limited ontological menu of fundamental physical theory, we tend to find ourselves asking, with Oliver Twist, “Please, sir, may I have some more [things in our ontology] please” ! The response usually ranges from at one extreme a stingy “No ! Just shut up and calculate”, leaving us as hungry for meaning as before, to another extreme of extravagant ontological banquets such as the many-worlds multiverse interpretation of quantum mechanics, which by contrast leave us ontologically bloated, and intellectually stranded.

By contrast, admitting RED CORAL to the fundamental ontology of the world is neither overly stingy, nor excessively sumptuous. Not only would it’s admission partially satisfy our appetite for greater understanding of the meaning and provenance of the postulates and mathematical framework of quantum mechanics, but would as well make possible new kinds of statements about puzzles such as the apparent absolute nature of the theory’s concept of time²², and the so-called collapse of the

²² If quantum mechanics is about RED CORAL, in the definition of which the relativistic proper time and proper space invariants feature, it seems reasonable to consider whether it may be worth attempting to re-base quantum mechanics formalism “natively” directly on the relativistic proper-space-time-invariants ontology, rather than in terms of its own native “x,y,z,t” ontology which as I have noted can be interpreted as corresponding to the special inertial frame in which all space-time points of the structure it describes have the same time coordinate. If that could be carried through it would provide a different perspective on the “absolute time” puzzle in quantum mechanics. The conjecture that re-basing quantum mechanics on a proper-space-time-invariants ontology is possible yields a falsifiable prediction: just as our classical dynamic and electrodynamic experience of the world (such as light-speed) is invariant across different inertial frames as postulated by Einstein so also, according to the conjecture, should be our non-classical experience of the world, because quantum mechanics (according to such an ontology and indeed intuition) “feels” relativistic proper time and space, and these are invariants across frames. Thus, for example, no conceivable variation of distinctively *non-classical* experiments such as double-slit interferometry, carried out within a lab in free fall in a gravitational field, should be able to detect the gravitational field (for example - there would be no diagnostically usable distortion of the double-slit interference pattern): just as no conceivable local classical experiment can detect the presence of a gravitational field in a freely gravitationally accelerated frame, neither (the conjecture predicts) is any non-classical strategy possible. (It is interesting to reflect that, if the quantum revolution had preceded special relativity, Einstein may have conceived of non-classical thought experiments to

wave function, in terms of the updated ontological menu. For example: the oddly absolutist personality of time in quantum mechanics is explained as being due to implicit adoption of a frame of reference in which all parts of the RED CORAL structure have the same time coordinate, because the coherence of this structure is canonically defined in that native frame; and instead of being limited to saying, of the infamous collapse, either at one extreme, “*shut up and calculate*”, or at the other “*the many-worlds multiverse bifurcates outrageously often*”, we could say “*a globally coherent relativistically space-like thing becomes a local time-like thing*”. This way of saying is all the more parsimonious inasmuch as admitting RED CORAL to our ontology only amounts to attaching a new mnemonic to something that has been there ever since the entrance of special relativity, but was lying neglected outside the light-cone.

The odd fact that the respective ontologies of classical special relativity and quantum mechanics, so radically different from each other, both delineate non-causal space-time intervals seems to be one worth noting; and the inference of a parent ontology, at a higher level of abstraction, from which these two ontologies descend, as a way of explaining the odd fact, seems a reasonable one to explore. Such an abstract parent has been sketched in the second part of this essay, involving self-mappings of the world, and intrinsic and extrinsic time. It suggests an interpretation of the coherence of RED CORAL, discussed in the first part, as being due to the evolution of an initial-condition seed in *extrinsic* time, according to the schema $\tau(\Delta x) = e^{kix}$, and an interpretation of time as fulfilling an abstract identity role in the world, isomorphic with the phenomenon of massless particles moving at light speed.

There is ultimately a deeper physical explanation for the formalism of quantum mechanics, than the usual historical/sociological one: while it may have started out as a resigned brute instrumentalist adoption of postulates to do with complementarity and the probabilistic nature of reality, in fact the formalism reflects the only way open to nature, to embed globally coherent structures in a local causal context.

Appendix 1: Groups Can't Laugh At Themselves (all that much)²³

Suppose a group element ϕ , one of the collection of intrinsic symmetries described by the group, could also play a higher level extrinsic role inasmuch as it induced a structure-preserving self mapping of the group onto itself. Then for any group elements a, b , for ϕ to preserve structure (i.e. for it to act as an automorphism) we must have

$$(a \circ \phi) \circ (b \circ \phi) = (a \circ b) \circ \phi$$

and by associativity

$$(a \circ \phi \circ b) \circ \phi = (a \circ b) \circ \phi$$

and by cancellation and associativity

$$(a \circ \phi) \circ b = a \circ b$$

and again by cancellation

$$a \circ \phi = a$$

understand the relativistic structure of space-time, such as . . . conducting a double-slit interferometer experiment while riding on a light beam ?)

²³ Automorphisms and homomorphisms of an algebraic structure such as a group, represent and in a sense “talk about” features of its structure, in the same way a cartoon represents aspects of its subject: by preserving certain broad structural features, under a mapping - hence the name of this humble theorem.

But since the group identity is unique, we have

$$\phi = I$$

This argument and result explains why group automorphisms – such as “raising to a power”, “taking the inverse of”²⁴, and the “inner automorphism” form²⁵ $\phi \circ x \circ \phi^{-1}$ - are always *extrinsic forms*, and never part of the intrinsic symmetry structure of the group - i.e. never individual group elements.

The above argument *shows* a group “cannot laugh at itself” in a non-trivial way, but does not explain *why* this is so. So, *why not*²⁶? Perhaps because the structural richness of groups (which includes features such as associativity, unique identity, unique inverse) means their elements are in a sense “fully subscribed”, with no degrees of freedom spare to partake in further structural formulations. (Conversely, in less well organised structures, elements are not fully subscribed and one finds then a single element can induce an automorphism).

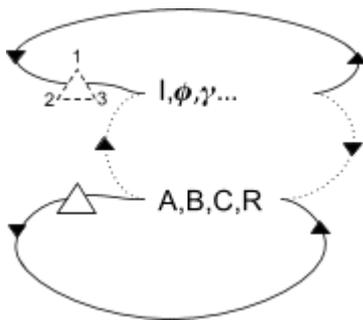


Figure 4

²⁴ These are both structure-preserving automorphisms for commutative groups.

²⁵ This is a universal structure-preserving automorphism for *all* groups.

²⁶ Proving a mathematical statement is true rarely explains satisfactorily *why* it is true. Explanations of *why* results are true are rarely given in mathematics, perhaps because it is difficult to formalise explanation, while formalisation of proof is more straightforward (even if actually obtaining one of them is often not !). Indeed it is not immediately clear what “explaining why” means in mathematics, and whether there is anything more to “explanation” than “following a proof”. Nevertheless it is a shame verbal explanations are not more often attempted in texts on or introductions to mathematical subjects such as group theory. For example the inner automorphism form is usually introduced without explanation or motivation, as an oddly arbitrary and pointless formula, yet there is an easily accessible explanation of why automorphisms *must* be forms, as shown above; which begs, though, the question of why *any* automorphism at all is of interest; the answer being that in a sense an automorphism is a kind of introspective cartoon. Of course from the formalist point of view there is no need for any motivation of any formulae, apart from an occasional pragmatic nod to utility, but this leads to an unexpected difficulty: if mathematics is a purely formal game-like intellectual creation, then the fact it mirrors and predicts so well the structure of the physical world, is an unlikely coincidence and mystery. That the strong correspondence of much of pure mathematics with physical reality, is so often adjudged mysterious and miraculous if not divine, is thus in part due to an overly formalist philosophical stance.

Figure 4 depicts the proposed intrinsic/extrinsic²⁷ meta-symmetric abstraction, as applied to a simple case of the symmetries of an equilateral triangle (reflections A,B,C and the rotation R). In the lower “intrinsic” domain, the reflections A,B,C and the rotation R , map the triangle onto itself; the middle domain depicts structure extrinsic relative to the lower domain, with the identity I in its guise as an extrinsic form, and other forms (such as exponentiation, inverse, inner-automorphism and other extrinsic formulae - signified by $\phi, \gamma \dots$), self-mapping the collection of symmetries A,B,C,R (rather than the triangle itself). (In the top domain there is a *labelled* triangle since this is in a sense what is self-mapped by the identity in its guise as an intrinsic symmetry - as noted in the text, the identity is the only symmetry remaining when the triangle is unambiguously (i.e. asymmetrically) labelled).

²⁷ “transcendent structure” possibly better captures the extrinsic arm of this distinction (and perhaps “immanent structure”, the intrinsic arm), and conforms fairly well to a commonly used philosophical idiom conveying “going beyond or outside” (which is the context here), but has overly grand connotations. *Free structure* is another possibility, using a more mathematically, less philosophically, motivated terminology, while still retaining a philosophical stance. For example Cromwell and Fox provide the following refreshingly philosophically sophisticated introduction to free groups in (1), in terms of different “realms”. They write:

“a group G is determined if there is given a set of elements g_1, g_2, \dots , called generators, that generate the group, and a set of equations $f_1(g_1, g_2, \dots) = 1, f_2(g_1, g_2, \dots) = 1, \dots$, called defining equations or defining relations, that have the property that every true relation that subsists among the elements g_1, g_2, \dots is an algebraic consequence of the given equations. Now from a stricter point of view this procedure is somewhat vague in that the left-hand sides of the equations do not have a true existence. What kind of an object is $f_i(g_1, g_2, \dots)$? It cannot be an element of G, for if it were G, it would have to be the identity element I. In order to write down such equations we must postulate the existence of some realm in which $f_i(g_1, g_2, \dots)$ has an independent existence. . . [that realm being the realm of the free group]”.

However the phrase *free structure* would also invoke the mathematical machinery of equivalence relations between the “free expressions”, part and parcel of the notion of free groups and other free algebraic structures, and this is not wanted here. We thus need something less “philosophical” than “transcendent structure”, but just slightly more abstract and less algebraically specific than “free structure”, hence the suggested extrinsic/intrinsic terminology.

References:

- (1) Cromwell R.H., Fox R.H., Introduction to Knot Theory (1977) Ginn and Company, Boston.