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In search of the fourth dimension of space
The Galaxy Epoch

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ABSTRACT

This speculation is the logical continuation of the previous one [viXra:2006.0202] which presents a mathematical construction where Special Relativity is inferred as a very close approximation. Its geometry finds an application in the calculations of Galactic Recession: those calculations are confirmed here.

In this subsequent formulation an explanation is sought as to why, in this "empty" space, a ray of light is bound to move on the 4-sphere surface. Then, we proceed by building a physical model, in which that empty space is filled with matter and radiation, and we try to check for any flaws.

Given the constant expansion speed hypothesized for the universe, no redshift is due to expansion itself. Here the Cosmological redshift is Gravitational or Doppler.

The involvement of the fourth spatial dimension is inevitable. Nevertheless, in our reference frame, the planned result is a usual (*x, y, z, t*) curved space-time to be verified.

In this conjecture the surface of the 4-sphere (like a kind of bubble expanding over time) goes through a continuum of states of equilibrium in which an internal pressure exerted by a radiation, in a reversible adiabatic expansion resulting from the Big Bang, balances the cohesion of the Universe whose surface tension is due solely to the pressure of the Cosmic Background Radiation.

Resulting model is an approximation for the Galaxy Epoch and it is based to Einstein's solution for weak fields. You can use this only in the context of observable universe, during the lasts 10 billion years.

The exact model, also applicable to the earlier periods and to the Radiation Era, results difficult. Mathematical complexities can perhaps be overcome by exploiting the isotropy of the coordinates but, before proceeding, all the hypotheses must be verified by analyzing the theory of the Big Bang in its detail.

We briefly summarize what was previously said:

a) Our universe lies on a 4-sphere surface $x_1^2 + x_2^2 + x_3^2 + x_4^2 = c^2t^2$ where radius is $r = ct$ with c as light velocity and t as time elapsed from Big Bang.
b) Radial velocity $v_r = c$ is constant except during the initial period.
c) Also tangent velocity $v_t = ctd\Theta/dt = c$ is constant over time. Galactic redshift is due to Doppler effect. [3]
d) Our relativistic time-like zone is a portion of space delimited, in every direction, by an arc of length $ct\Theta\ with\ \Theta = 1\ rad$.

STILL ASSUMPTIONS

A ray of light can travel an entire great circle and return to the starting point (much forward in time). However, we cannot detect in any way a radiation from a galaxy outside the relativistic cone of light (a bit like what happens to an observer who cannot capture any photon from a black hole). Whatever the frame of reference, only radiation emitted by objects belonging to one's own time-like zone can be detected. These photons continue to go round in circles along a geodesic. Outside the limits of the observable universe (at angular distances Θ > 1 in every direction) the light ray, during the entire route, cannot meet anything because everything flies ahead at faster speeds. In vacuum it cannot be deflected or absorbed in any way.

Up to now, no hypothesis has been made on the "empty" space delimited by this geometry. To proceed, the fourth dimension of space is involved. We place the Big Bang at the center of the 4-sphere and assume that all the primordial *ylem* (hot plasma), launched away by a giant explosion, was blocked onto a sort of event horizon. There remained, squeezed on the surface, nothing could get out and subsequently also the scattering photons. Over time, reactions took place and cooling changed the conditions. The event horizon somehow shrank, radiation was released and expansion begun.

THE SPECIAL RELATIVITY APPROXIMATION

At "time of last scattering", after the of Recombination era [4], relic photons were released and traveled along 4-sphere's surface arcs as geodesics. This radiation has not disappeared, it is still present today as Cosmic Background Radiation (CBR) [1] providing the "vacuum" with sufficient energy and pressure that, in a homogeneous space, still provide the gravity to maintain these geodesics. The last statement derives from the equivalence between mass and energy. As we saw in the previous article, the flat space of Special Relativity enters the context of this curved surface.

From the assumptions made previously, at "time of last scattering" expansion velocity was null. In absence of relative motion, rays, started from any point on the surface, can reach any other point. Since then, expansion resumes, maintaining a constant speed.

The subsequent constancy of radial velocity $v_r = c$, hypothesized in the previous article, implies that also tangent velocity $v_t = ctd\Theta/dt = c$ does not change over time. This is valid for the whole period in which gravity has maintained these geodesics, that is, for the whole period concerned.

Let's write the geodesic equation with reference to 4-sphere geometry:

$$ct\frac{d\Theta}{dt} = c \quad or \quad \frac{d\Theta}{dt} = \frac{1}{t} \quad and \quad \frac{t_2}{t_1} = e^{\Theta} \quad for\ every\ \Theta$$

Knowing the angle Θ we can easily get the time the ray started: $t_{past} = t_{today}e^{-\Theta}$.

From the interval of flat space-time $ds^2 = -dx_1^2 - dx_2^2 - dx_3^2 + c^2dt^2$ we put $dx_1^2 + dx_2^2 + dx_3^2 = (ctd\Theta)^2$

With $ds^2 = 0$ for a light-like interval, we obtain $ctd\Theta = cdt$ that is the geodesic equation $v_t = ctd\Theta/dt = c \ \ and \ \ d\Theta = dt/t$.

Today the arc approximated here by a segment has a curvature of $2.40 * 10^{-4}Mpc^{-1} = 7.77 * 10^{-27}m^{-1}$: Special Relativity is a very close approximation for this curved surface.

In the context of the Principle of Equivalence, get easily the *proper coordinates* for ourselves as observer, marks a positive point for the 4-sphere hypothesis.

ON THE EDGE OF PHYSICS AND BEYOND: SPHERES, BUBBLES, WORK AND ENERGY

I find that the 4-sphere surface is an interesting entity that we can find also in the interior solution of the Schwarzschild metric, as its space-time geometry. Then, you could think that, in extreme physical conditions, fluids can settle in this geometry and that, when conditions cease, this geometry may be preserved in a following expansion.

What we add to doing here is to extend physical laws to a four-dimensional space with the aim of verifying later the results in our local frame of reference. We had already done this previously, hypothesizing a fourth dimension and then obtaining, as result, the Special Relativity of our space. Note that II Postulate of Relativity has been inferred here not as a limit of an ad hoc chosen metric. Since it is not a postulate here, there is no reason to apply it outside our space. It should not, in general, be applied to the whole 4-sphere geometry.

Current cosmology accepts an *origin* for time and, referring to the Big Bang, it speaks of a "*singularity*". We have not changed philosophy too much, here, if we replace the concept of "*singularity*" with a point in another dimension that cannot ever be reached and measured by us. About, instead, the relativity of reference frames between dimensions, there are no violations until we do not try to compare values.

The sphere and the bubble have a symmetry that lends themself to be easily generalized. We can think of the 4-sphere surface as a bubble where the cohesion force is due not to a surface tension [2] but to gravity. Because of its high discontinuity in space, mass from matter should be irrelevant for great values of r. Pressure from radiation, instead, may be essential.

In this generalization, we hypothesize a 4-sphere that expands over time because of an explosion at its center:

1. About the kinetic energy, with a constant expansion speed, $\Delta E_k = 0$.
2. Referring to the 4-sphere surface a work E_γ is done by gravity acting like a surface tension: the cohesion force of the surface is $\gamma = f(r)$.
3. We cannot be sure that transformations are adiabatic: heat could flow out from surface through some mechanism like thermal radiation or something else.
4. About the pressure gradient on the bubble Δp_{4-dim} we assumed a null external pressure so that no additional work is done by volume expansion. By analogy with the surface tension we put $\gamma dS_{4-sphere}$ for the work done by the cohesion forces. The equilibrium relation, then, could take the form: $p_{4-dim}(t)dV_{4-sphere} = \gamma(t)dS_{4-sphere}$.
5. Equilibrium is maintained in expansion. If $p_{4-dim}(t) = f(\gamma)$ then the equality must hold for every value of $r = ct$. The continuous succession of states of equilibrium over time suggests a reversible expansion.

Check the conservation of energy of the universe is not obvious but here we are only looking for a criterion to establish which physical entities are involved in our balances and if it is possible to exclude the concept of absolute space. For this purpose, we say that energy is conserved if it holds locally for the proper energy in any region of space.

Referring to the Galaxy epoch, after an initial period, some internal pressure p_{4-dim} left after the explosion. With reference to our universe and considering the cohesion energy E_γ as part of its Internal Energy U_{Univ} we have:

$\Delta U_{Univ} = q - w$. Both w and q are negative, w is work done by pressure p_{4-dim}, q is the heat given up:

$$dU_{Univ} = dE_m + dE_r + \gamma dS_{4-sphere} = q - w$$

where E_m is energy from matter, E_r from radiation.

We can write:

$$dE_m + dE_r + \gamma dS_{4-sphere} = q - w \quad \text{but} \quad \gamma dS_{4-sphere} = -w \quad \text{so} \quad dE_m + dE_r = q.$$

If ρ is the density of radiation, $V = S_{4-sphere}$ and $E_r = \rho V h\nu$ (where $h\nu$ is the energy of a photon) then:

$$dE_m + dE_r = c^2 dm + (V + dV)(\rho + d\rho)(h\nu + hd\nu) - \rho V h\nu = c^2 dm + \rho V h\delta\nu + d(\rho V)h\nu$$

but $-c^2 dm = d(\rho V)h\nu$ (from the mass-energy equivalence) and the result is $\rho V h\delta\nu = q$.

In our assumption cosmological redshift is of a gravitational type. If we exclude other causes for the redshift there is no exchange of energy. We would have to ask ourselves how we should reason if the expansion of the universe did not happen at constant speed but this is not the case.

Then:

$$dU_{Univ} = \gamma dS_{4-sphere} = -w$$

Assuming that energy is not conserved could reintroduce the concept of absolute space. However, if we accept a work w from an adiabatic expansion for the explosion, then for the energy balance it would be: $U_{4-sphere} = const$ favoring the idea, stated above, of a Schwarzschild geometry acquired by fluids in extreme physical conditions and subsequently preserved for our Universe.

Isotropy, homogeneity, circular path for radiation and lack of energy conservation are the essential conditions for this speculation.

To avoid collapsing, the cohesive force of the 4-sphere surface needs to be balanced by another force. The lack of energy conservation, is only due to the work w, and we have two possible conjectures to proceed:

a) $U_{4-sphere} = const$. A residual radiation propagates from the center of the 4-sphere, origin of the Big Bang, in a radial direction, exerting some form of pressure on the inside of the surface. The 4d state equation of its adiabatic expansion is unknown. It seems reasonable to put $U_{4-sphere} = const$.
b) $U_{Univ} = const$. Some non-directly measurable form of energy w, belonging to our universe, opposes radiation pressure. In this way U_{Univ} is conserved and we can consider the Universe as a single physical entity.

In any case the 4-sphere surface model can survive as a curvature for space-time.

Here are some hypothetical calculations regarding choice a).

Assuming zero for variable t at the beginning of the expansion (after the last scattering), it follows ($scat$ stays for "relative to last scattering"):

1. from $p(t)dV = \gamma(t)dS$ it follows $p(t) = 3\gamma(t)/ct$
2. but $\gamma(t) = \rho/3$ where the latter is the is the expression for the pressure of a disordered radiation of density ρ
3. we put $\rho = (\rho_{scat}S_{scat}/S)/ct$ for the CBR density, decreasing with S and redshift z as $(ct)^{-4}$
4. the result is $p(t) = \rho/ct = p_{scat}/(ct)^5 = aV^{-5/4}$ where a is constant.

The state equation of a 3d reversible adiabatic expansion for radiation is $PV^{4/3} = const$. Here for the above 4d expansion we obtained $PV^{5/4} = const$. Accepting this result as a 4d reversible adiabatic expansion would also confirm radiation pressure as the only cohesive force γ.

We opt for choice a).

The purpose of these calculations is only to describe qualitatively, but using a language that we know, the functioning of this model. Referring to the Galaxy epoch, the 4-sphere hypothesis incudes that:

a) The surface of the 4-sphere (like a kind of bubble expanding over time) goes through a continuum of states of equilibrium in which an internal pressure by a radiation, in a reversible adiabatic expansion resulting from the Big Bang, balances the cohesion of the Universe whose surface tension is due solely to the pressure of the Cosmic Background Radiation.
b) There is no exchange of energy due to the cosmological redshift of radiation: it is of gravitational origin. Then, for the energy balance of the whole 4-sphere, it would be: $U_{4-sphere} = const$ without heat exchange between the surface and the inside.
c) It is not reintroduced the concept of absolute space.

It is clear that, if we want to further develop the model, all hypotheses of this speculation must be questioned by analyzing the details of the Big Bang theory. From the macroscopic point of view, the conservation of the energy for the whole 4-sphere is an interesting conjecture. What remains to analyze is its inner, mysterious part.

ABOUT ASSUMING A METRICAL TENSOR

By relating time to the 4th spatial dimension we obtain the usual curved space-time. After this, we no longer need the equation of the surface: $x_1^2 + x_2^2 + x_3^2 + x_4^2 = c^2t^2$. As we will see later, fourth dimension of space x_4 will appear again in a mathematical context but no longer in physics.

The generic procedure to get the metric of 4-sphere curved space-time sems extremely complex in a Cartesian reference frame.

The solution is not even simplified using polar coordinates:

1. Let's choose a reference frame based on a radius $r = ct$ as time coordinate and on three angles θ, φ, ψ as space coordinates $(0, 2\pi)$. As reference points, unfortunately, we cannot choose known stars as "Alpha Ursae Minoris – Polaris" or "Delta Orionis – Mintaka" on the Orion's Belt. This because of their proximity to us.
2. The three coordinates on the surface are given by the angles θ, φ, ψ where the first two are the equivalent of Longitude and Colatitude (using zenith angle $= 90^0 -$ Latitude) and where we will call the third "Universe Height". Astronomic Celestial coordinate Declination and Right ascension are relative to our observable universe, here Universe Colatitude and Longitude refers to the whole 4-sphere. As convention we indicate a point P as $P(\varphi, \theta, \psi)$, with Colatitude before Longitudes.
3. Let's establish a position $P_N(0, 0, 0)$ for the "North pole" of our 4-sphere. Since all the points on the surface are equivalent, we can choose "Ursa Major GN-108036". Then we chose a Prime Meridian $P_{M0}(undef, 0, undef)$, passing through some other known point in space (say passing through "Sculptor A2744 YD4"). Note that all points $P_{EM-}(\pi/2, 0, undef)$ on the Universe Equator are out of our observable universe. A third point

$P_{EM}(\pi/2, 0, \pi/2)$ is at Universe Height $\pi/2$ on the Universe Equator, at $\pi/2$ from P_N measured on Prime Meridian.

The corresponding Cartesian coordinate can be useful:

1. $x_1 = ct\sin(\psi)\sin(\varphi)\cos(\theta)$
2. $x_2 = ct\sin(\psi)\sin(\varphi)\sin(\theta)$
3. $x_3 = ct\sin(\psi)\cos(\varphi)$
4. $x_4 = ct\cos(\psi)$

Note that θ, φ are the Longitude and Colatitude of the sphere.

Also are useful the 4-vector $\boldsymbol{r} =$
$(ct\sin(\psi)\sin(\varphi)\cos(\theta),\quad ct\sin(\psi)\sin(\varphi)\sin(\theta),\quad ct\sin(\psi)\cos(\varphi),\quad ct\cos(\psi))$

and its derivatives ($t = \text{const}$ on the surface):

1. $\mathbf{r}_\theta = (-ct\sin(\psi)\sin(\varphi)\sin(\theta),\ ct\sin(\psi)\sin(\varphi)\cos(\theta),\quad 0,\quad 0)$
2. $\mathbf{r}_\varphi = (ct\sin(\psi)\cos(\varphi)\cos(\theta),\ ct\sin(\psi)\cos(\varphi)\sin(\theta),\quad -ct\sin(\psi)\sin(\varphi),\quad 0)$
3. $\mathbf{r}_\psi = (ct\cos(\psi)\sin(\varphi)\cos(\theta),\ ct\cos(\psi)\sin(\varphi)\sin(\theta),\quad ct\cos(\psi)\cos(\varphi),\ -ct\sin(\psi)))$

These are 4-vectors of a Euclidean space: for us, there is the inner product and the angles it defines.

The three inner products are all equal to zero: $\mathbf{r}_\theta \cdot \mathbf{r}_\varphi = \mathbf{r}_\varphi \cdot \mathbf{r}_\psi = \mathbf{r}_\theta \cdot \mathbf{r}_\psi = 0$: they are orthogonal.

Once the arc ξ between two points, P_1 *with vector* $\mathbf{r}_1$ and P_2 *with vector* $\mathbf{r}_2$, has been calculated:

$$\xi = ct\,|\arccos\,(\frac{1}{c^2t^2}\mathbf{r}_1 \cdot \mathbf{r}_2)|$$

we have again the same light geodesic referring to the arc of great circle: $d\xi = dt/t$.

Once saw the variables to use, it seems hard to set up the latter relation. Space and time variables are tightly coupled: it is not at all obvious to formulate a covariant expression for this space-time interval:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

AN APPROXIMATE SOLUTION FOR THE GALAXY EPOCH
FROM EINSTEIN'S WEAK FIELDS

The very small curvature of space in our present period is the confirmation of a current weak gravitational field. We can resume the analysis with the previously described coordinates $x^\mu = \varphi,\ \theta,\ \psi,\ ct$: We look for a model that approximates an almost flat space-time in a neighborhood of any point on the surface. From this part of the whole we expect to derive the field equation for the present and to apply it back in time so that we can observe rays of light from the most distant galaxies.

We have already seen before that, for each point $P(\varphi, \theta, \psi)$, the tangents to Colatitude, Longitude and Height are orthogonal: the angles between the coordinates φ, θ, ψ are always $\pi/2$. Then the differential arc is:

$$c^2t^2d\xi^2 = c^2t^2\sin^2(\psi)d\varphi^2 + c^2t^2\sin^2(\psi)\sin^2(\varphi)d\theta^2 + c^2t^2d\psi^2$$

If the vectors $\mathbf{e}_\varphi, \mathbf{e}_\theta, \mathbf{e}_\psi$ can be assumed as an orthogonal covariant basis of this space we note that, with the 4-sphere radius $\boldsymbol{r} = ct\,\mathbf{e}_t$, the basis $\mathbf{e}_t$ for our time coordinate is orthogonal to the previous ones too (so it had to be on the basis of the Principle of Equivalence).

For the basis $\mathbf{e}_\varphi, \mathbf{e}_\theta, \mathbf{e}_\psi, \mathbf{e}_t$, a double angle rotation on ψ and φ is function of the current values of ψ_0 and φ_0

$$f_\psi = \sin(\psi)/\sin(\psi_0) \quad \text{and} \quad f_\varphi = \sin(\varphi)/\sin(\varphi_0)$$

and it is given by:

$$\mathbf{C} = \begin{bmatrix} f_\psi & 0 & 0 & 0 \\ 0 & f_\psi f_\varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with the metric tensor $g_{\mu\nu}$ invariant for the transformation $g'_{\mu\nu} = \mathbf{C}\, g_{\mu\nu}\mathbf{C^{-1}} = g_{\mu\nu}$, as we will see.

To get a simple solution we have to eliminate the tightly coupling of space and time assuming the adimensional variables:

$$\text{space } x'^\mu = \frac{x^\mu}{ct} \quad \mu = 1{,}3\,, \qquad \text{time } x'^4 = \ln(t) \qquad \text{and} \qquad ds' = \frac{ds}{ct}$$

All points are equivalent, to simplify we choose the point at the Universe Equator in $P_{EM}(\pi/2, 0, \pi/2)$, then what remains is $d\xi^2 = d\varphi^2 + d\theta^2 + d\psi^2$ that leads to the usual light geodesic: $d\xi = dt/t$.

Now let's solve the following field equation:

$$\frac{8\pi G}{c^4} T_\mu^\nu = R_\mu^\nu - \frac{1}{2} R g_\mu^\nu$$

to get the tensor $g_{\mu\nu}$ for the adimensional interval $ds'^2 = g_{\mu\nu}dx'^\mu dx'^\nu$

As an expression for volume we put $V = 2\pi^2c^3t^3$ for 4-sphere surface and, for the previous assumptions about gravity, $\rho_0 V_0 \simeq \rho V$ is constant over time. [*] We can, then, calculate mass (or energy) density and volume at present time. Moreover ρ_0 can be considered the density of a perfect fluid composed of a mix of matter and radiation.

The analysis begin with the Einstein's solution for weak fields $g_{\mu\nu} = h_{\mu\nu} + \delta_{\mu\nu}$ where $\delta_{\mu\nu}$ are the constant Galilean values for Special relativity and $h_{\mu\nu}$ are small correction terms. $\mathcal{E}_{0r}$ and $c^2\rho_{0m}$ are respectively the current energy density of radiation and matter. The surface cohesive force of this model is attributable to uniform radiation pressure p.

Now we remember the precedent qualitative description of the 4-sphere model in which we put the relation $\Delta U_{Univ} = -w$. The latter will be used in the next calculation in which, being $T^{\mu\nu}$ a tensor isotropic in space (perfect fluid), result does not change if we pass from our adimensional coordinates to the Galilean ones. The same goes for $g_{\mu\nu}$ and all the equation:

$$\frac{8\pi G}{c^4} T_\mu^\nu = R_\mu^\nu - \frac{1}{2} R g_\mu^\nu$$

whose solution in Galilean coordinates returns a tensor $g_{\mu\nu}$ which is invariant with respect to our adimensional reference frame.

With $\Delta U_{Univ} = -w$, for an infinitesimal element of volume δV, we have a work $-\delta w$ done by internal pressure so as to satisfy the relationship:

$$\frac{\partial (c^2 \rho \delta V)}{\partial t} dt = \delta w - p \frac{\delta V}{\partial t} dt$$

in which matter, in the form of discontinuities in mass distribution, has no rule. You can eliminate it from the Stress Energy tensor.

From the equilibrium condition the right member is zero, then the Stress Energy tensor for this disordered radiation is:

$$T^{\mu\mu} = 0 \quad \mu = 1,3 \ , \ T^{44} = \frac{G\mathcal{E}_r}{c^4} \quad and \quad T_\mu^\nu = \frac{G\mathcal{E}_r}{c^4} \delta_\mu^\nu$$

With the quantities $h_\mu^\lambda = \delta^{\lambda\alpha} h_{\mu\alpha}$ and $h = \delta^{\lambda\alpha} h_{\alpha\lambda}$, the field equation result

$$\left(h_\mu^\nu - \frac{1}{2} \delta_\mu^\nu h \right) = 4 \int \frac{T_\mu^\nu}{r} dV = \frac{G}{c^4 ct} \delta_\mu^\nu \int \mathcal{E}_r \, dV$$

We put $r = ct = const$ over V because the "interesting point" of the Einstein's solution, here is any point in time of the 4-sphere surface.

Integrating on V, after calculating the quantity $\mathcal{E}_{0r} V_0 = \int \mathcal{E}_r \, dV \simeq const$, we get:

$$\left(h_\mu^\nu - \frac{1}{2} \delta_\mu^\nu h \right) = \frac{4GE_r}{c^4 ct} \delta_\mu^\nu \quad \mu = \nu \qquad 0 \quad \mu \neq \nu \qquad E_r = \mathcal{E}_{0r} V_0 \simeq const$$

We can see that $h_{\mu\nu} = \delta_{\mu\nu} h_0$. Values of h_μ^λ are all equals, say to $4h_0$, and with $h = 4h_0$ then follows:

$$h_0 = -\frac{2GE_r}{c^4 ct}$$

and the adimensional space-time interval is

$$ds'^2 = \delta_{\mu\nu}(1 + h_0) dx'^\mu dx'^\nu$$

but coordinates are isotropic, that is all points of space are equivalent, so the latter expression holds for all spatial rotations, in this case the above rotation $\mathbf{C} \, \delta_{\mu\nu}(1 + h_0) \, \mathbf{C}^{-1}$. Moreover, now that we have the solution, we can return to the correct dimensions by multiplying both members by $(ct)^2$:

$$ds^2 = -\, c^2t^2(1 + h_0)\sin^2(\psi)d\varphi^2 - c^2t^2(1 + h_0)\sin^2(\psi)\sin^2(\varphi)d\theta^2 - c^2t^2(1 + h_0)d\psi^2 + (1 + h_0)c^2dt^2$$

where variables are Universe Colatitude φ, Universe Longitude θ, Universe Height ψ and the natural logarithm of time elapsed from Big Bang $ln(t)$. The latter appears in the usual differential form of the time dt.

Let's do some calculation:

Calculation for h_0. (We assume that mass E_r is constant over time)

- Today energy density of CBR $\mathcal{E}_{0r} = 4.02 * 10^{-14}\, J\, m^{-3}$ [**]
- Constant over time, energy $E_r = \mathcal{E}_{0r}V = 2\pi^2r^3\mathcal{E}_{0r} = 1.69 * 10^{66}\, J$
- Constant $h_0 = 2.94 * 10^6\, ly$

Verification of the gravitational redshift relative to the time when ray of light started from the farthest galaxy.

- The expansion speed c is constant over time. In the absence of other factors, it means that the distance, measured from source and receiver, between two successive wave crests does not change over time. <u>There is no redshift due to the expansion itself</u>.
- In absence of an angle ξ , that gives the Doppler effect, the redshift is the quotient between the proper times of receiver and transmitter, as for the Schwarzschild metric:

$$1 + z = \frac{\sqrt{1 - \frac{2GE_r}{c^5 t_{today}}}}{\sqrt{1 - \frac{2GE_r}{c^5 t_{early}}}}$$

For a galaxy at its maximum distance (Θ ≃ 1), $t_{Max} \simeq\ 5 * 10^9\ years$ value is $z = 1.86 * 10^{-4}$. [***]

The latter value is the confirmation that throughout the Galaxy Epoch gravity remained negligible. The Einstein's model for weak fields has been correctly applied.

<u>Accepting a negligible error, Galactic redshift can always be calculated as Doppler redshift</u>.

[*] = We assume that the mass of matter does not change from past. About the energy of radiation, its constancy, as an approximation over the range of time in question, is due to the Weak Fields hypothesis.

[**] = See later in the paragraph USING 4-SPHERE FORMULAS.

[***] = Here, for the age of the universe, the time used $t = 1.36 * 10^{10}\ years$ is different from the value of other models as the Lambda-CDM. [7] However, a verification regards the time elapsed from the Big Bang is possible, through a simple calculation on the observed Hubble constant:

Hubble's recessional velocity $H = 72\ Km\ s^{-1}\ Mpc^{-1}$

Calculated $\Theta_{1\,Mpc} = H/c = 2.4 * 10^{-4}\,rad$
Time elapsed from Big Bang $t_{now} = 1/c\Theta_{1\,Mpc} = 3.26 * 10^{6}/c\Theta_{1\,Mpc} = 1.36 * 10^{10}\,years$
Corresponding time from Lambda-CDM $t = 1.37 * 10^{10}\,years$

USING 4-SPHERE FORMULAS

Observing the solution found for the Galaxy Epoch, one realizes that this could not be a good candidate for the complete solution: fields became too strong as radiation energy increase. Concluding, we have provided only a part of the solution and without the rest we cannot proceed furthermore with the earlier periods. We need the exact model to move on a space-time context, so we can use the physics we know.

In fact, looking at the surface equation $x_1^2 + x_2^2 + x_3^2 + x_4^2 = c^2t^2$ we can immediately see that the presence of the fourth spatial dimension x_4 leads us to a dead end. Previously we made some rough energy balance and some hypothetical calculation but nothing more. Variable x_4 cannot be used in any law of physics and, in the moment, about it is impossible to make any logical reasoning. The absence of a law on x_4 does not allow us to predict changes on the evolution of the universe over time.

Only the surface formulas can be used:

$V = 2\pi^2c^3t^3$ $\quad M = (\rho_r + \rho_m)2\pi^2c^3t^3$ $\quad$ where ρ_r, ρ_m are the densities of radiation and matter and M is the total mass.

As an example, we calculate the mass $M_r = \rho_r 2\pi^2c^3t^3$ equivalent to the total energy of CBR and $M_m = \rho_r 2\pi^2c^3t^3$ corresponding to the total mass of matter:

$E_{avg} = 3.83k_bT = 3.83 * 1.38 * 10^{-23}JK^{-1} * 2.7K = 1.43 * 10^{-22}J$
whereE_{avg} is the average energy of a photons (as a blackbody) [8]
$\mathcal{E}_r = aT^4 = 7.566 * 10^{-16}Jm^{-3}K^{-4} * 2.7^4K^4 = 4.02 * 10^{-14}Jm^{-3}$
where $a = 4\sigma/c$ is the radiation constant [5]
$\rho_r = \mathcal{E}_r/E_{avg} = 2.82 * 10^{8}m^{-3}$ (the number of CBR photons per cubic meter)
$M_r = \mathcal{E}_r c^{-2}2\pi^2c^3t^3 = 1.88 * 10^{49}\,Kg$
$\rho_{nH} \simeq 0.225$ hydrogen atoms m^{-3} [6]
$\rho_H = \rho_{nH}uMA/u = 0.225 * 1.00784 *1.66 * 10^{-27} = 3.76 * 10^{-28}\,Kg\,m^{-3}$ (other sources give a value of approximately $1.50 * 10^{-33}\,Kg\,m^{-3}$)
$M_m = \rho_H 2\pi^2c^3t^3 = 1.58 * 10^{52}\,Kg$

GALACTIC REDSHIFT IN COSMOLOGICAL EPOCHS: COSMIC BACKGROUND RADIATION

The assumption that at "time of last scattering" expansion velocity was null is necessary for CBR to respect the observed value of the standard deviation in its radiation temperature: T = 2.7255 ± 0.0006K. With a tolerable deviation of 0.0002 we cannot admit the presence of any Doppler effect.

During Recombination [*] and earlier, in the Radiation Era, pressure and energy density were so high that radiation itself were imprisoned. After the end of Recombination era, all radiation has been released. These relic photons reach us with the same redshift. Note that to reach us, a radiation emitted in the end of Recombination Era (380,000 years from Big Bang), traveled one or more full laps. [**]

We must then look for different models for specific eras. A first rough subdivision could be between Galaxy Epoch and "time of last scattering" of CBR:

$z = z(t)\ and\ \partial z/\partial \Theta = 0$ after release of relic photons
$z = z(\Theta)\ and\ \partial z/\partial t = 0$ in late matter dominated period

More specifically:

- After release of relic photons and throughout an initial period, gravity is strong and uniform, decreasing with time. It depends on matter and on strong radiation energy.
- During the Galaxy Epoch, close to a star, the uniform component of gravity, from radiation, is negligible compared to that generated by the star [***]. If gravity has changed since the light ray started, this may be due to a change in mass of the star or to some other reason.

We should say that (g is gravity):

$z = z(\Theta, g)\ and\ \partial z/\partial t = 0$

and, as a more reasonable assumption in the absence of other information,

$\partial g/\partial t = 0$

As long as the expansion speed remains constant, the redshift is not attributable to the expansion itself. During the Radiation Era, from the time of last scattering onward, the redshift is gravitational while in the Galaxy Epoch it is due to the Doppler effect. In between time it is of mixed type.

[*] = Time to the end Recombination Era is taken from Theory of Big Bang

[**] = We can calculate the angle traveled by relic photons to reach us $\Theta = 5/2\pi + 2.63$. You can use:

$$\Theta = \ln\left(\frac{t_{today}}{t_{past}}\right) \quad for\ every\ \Theta$$

[***] = The observed surface gravitational redshift of a massive neutron star is about $z = 0.4$

GALACTIC COORDINATES

The observable universe is a volume, on the surface of the 4-sphere, delimited in the three spatial dimensions by an arc of $\Theta = 1\ rad$. In this volume we are at the center O.

Fixed the origin for the time axis t coinciding with the Big Bang, we can use three angles as a galactic coordinate system: the position of an astronomic object A can be defined by the direction of the 4-sphere arc OA and the angle λ of this one. For the direction we can adopt the usual coordinates: Right ascension α and Declination δ. About the 4-sphere arc angle, say "Arc λ", knowing the Galactic redshift z, you have:

$\lambda = ((1+z)^2 - 1)/((1+z)^2 + 1)\ rad$
Present proper distance $s = ct_{now}\lambda$

Moving on 4-sphere surface coordinates, Colatitude, Longitude and Height, is quite complicate. Maybe it needs the aid of a computer program or some more suitable mathematical method. Here we give only some tools and a way to approach the solution:

Let's recall the coordinate in the 4-sphere space **U**: $P = P(\varphi, \theta, \psi)$:

1. $x_1 = ct\sin(\psi)\sin(\varphi)\cos(\theta)$
2. $x_2 = ct\sin(\psi)\sin(\varphi)\sin(\theta)$
3. $x_3 = ct\sin(\psi)\cos(\varphi)$
4. $x_4 = ct\cos(\psi)$

The 4-vector $\boldsymbol{r} =$
$(ct\sin(\psi)\sin(\varphi)\cos(\theta),\quad ct\sin(\psi)\sin(\varphi)\sin(\theta),\quad ct\sin(\psi)\cos(\varphi),\quad ct\cos(\psi))$

and its derivatives:

1. $\mathbf{r}_\theta = (-ct\sin(\psi)\sin(\varphi)\sin(\theta),\ ct\sin(\psi)\sin(\varphi)\cos(\theta),\quad 0,\quad 0)$
2. $\mathbf{r}_\varphi = (ct\sin(\psi)\cos(\varphi)\cos(\theta),\ ct\sin(\psi)\cos(\varphi)\sin(\theta),\quad -ct\sin(\psi)\sin(\varphi),\quad 0)$
3. $\mathbf{r}_\psi = (ct\cos(\psi)\sin(\varphi)\cos(\theta),\ ct\cos(\psi)\sin(\varphi)\sin(\theta),\quad ct\cos(\psi)\cos(\varphi),\quad -ct\sin(\psi)))$

After converting δ using zenith angle $= 90^0 -$ Declination, in the space **O** of observable universe, for a point, $U = U(\delta, \alpha, \lambda)$:

1. $y_1 = \sin(\delta)\cos(\alpha)$
2. $y_2 = \sin(\delta)\sin(\alpha)$
3. $y_3 = \cos(\delta)$
4. $y_4 = ct\lambda$

The vector $\boldsymbol{u} = (\sin(\delta)\cos(\alpha),\quad \sin(\delta)\sin(\alpha),\quad \cos(\delta))$ (with unit length)

and its derivatives:

1. $\mathbf{u}_\alpha = (-\sin(\delta)\sin(\alpha),\quad \sin(\delta)\cos(\alpha),\ 0,)$
2. $\mathbf{u}_\delta = (\cos(\delta)\cos(\alpha),\quad \cos(\delta)\sin(\alpha),\quad 0)$

Note that two stars can be nearby on **U** but distant on **O**: it complicates approximations.

An angle on the 4-sphere is given by:

$$\xi = \arccos\left(\frac{1}{c^2t^2}\mathbf{r}_1 \cdot \mathbf{r}_2\right)$$

while the one on the observable universe (that is on the 4-sphere surface, between the Earth and two star) is:

$$\gamma = \arccos\left(\mathbf{u}_1 \cdot \mathbf{u}_2\right)$$

To use Right Ascension and Declination we need the formulas effective for arcs and angles on the surface. For this purpose, given three points, we can set the 4-plane that passes through them and the center of the 4-sphere. Once got it, we have a 3-sphere so to use the Sine Theorem and other tools.

Here calculations in polar coordinates are hard so let's move on to Cartesian ones:

$x_1^2 + x_2^2 + x_3^2 + x_4^2 = c^2t^2$
$x_4 = ax_1 + bx_2 + cx_3$ (where this 4-plane passes through the North Pole and the Earth).

We have $x_1^2 + x_2^2 + x_3^2 - c^2t^2 = -(ax_1 + bx_2 + cx_3)^2$.

This means that if a point belongs to the 3-plane: $ax_1 + bx_2 + cx_3 = 0$ and belongs to the 3-sphere: $x_1^2 + x_2^2 + x_3^2 = c^2t^2$ then it also belongs to the 4-sphere after we put

$$x_4 = ax_1 + bx_2 + cx_3.$$

About the steps to find the position of an unknown star $P_x(\varphi, \theta, \psi)$, variables must be chosen so that the point lies both on of the sphere and the plane. That gives a first condition $F(\varphi, \theta, \psi) = 0$. Note that parameters a, b, c, for the equation of the 3-plane, are not linearly independent but we need all them later to set x_4. [*]

For the whole procedure to be valid, we should demonstrate that the transformation preserves angles and distances between the three points in question. To avoid calculations, we see that the same is true in 3d when we intersect a sphere with a plane, passing through the center, to get a circle.

For triangulations of the 4-sphere we start getting coordinates of some points. We use our Earth, Ursa Major GN-108036, Sculptor A2744 YD4 and Piscis Austrinus BDF-3299:

1. Our Earth Us $P_0(\varphi, \theta, \psi)$ and $U_0(0, 0, 0)$
2. Ursa Major GN-108036 $z = 7.2$ $P_N(0, 0, 0)$ and $U_1(0.4863, 3.3003, 0.9707)$ - Boreal Hemisphere
3. Sculptor A2744 YD4 $z = 8.38$ $P_{EP-}(undef, 0, undef)$ and $U_2(-1.0405, 0.0629, 0.9775)$ – Austral Hemisphere
4. Piscis Austrinus BDF-3299 $z = 7.11$ $P_3(\varphi, \theta, \psi) = U_3(-0.9570, 5.8827, 0.9700)$ – Austral Hemisphere
5. … and so on …

We can give here the trace of a solution for our North Star Polaris. In these coordinates, it is close to the Earth:

1. Alpha Ursae Minoris – Polaris $z = 0.000055$ $U_4(0.0128, 0.6624, 0.000055)$ – Boreal Hemisphere
2. Our Earth $\mathbf{r}_0 = (a, b, c, d)$
3. Ursa Major GN-108036 $\mathbf{r}_N = (0, 0, 0, ct)$
4. Sculptor A2744 YD4 $\mathbf{r}_2 = (e, 0, f, g)$

With respect to the Earth $P_0(\varphi, \theta, \psi)$, the coordinates of Alpha Ursae Minoris – Polaris are: $P_4(\varphi + x, \theta + y, \psi + z)$ where x, y, z are unknown.

We follow these steps:

1. Define a point P_W on the direction $P_0\, P_N$ at the same distance $P_W\, P_N = P_4\, P_N$. U_W lies on the segment $U_0\, U_N$.
2. The first condition on x, y, z comes from the sphere and plane passing through $P_0\, P_N\, P_4$
3. Calculate the angle between P_N and P_4 in **O:** $\gamma = \arccos(\mathbf{u}_\mathrm{N} \cdot \mathbf{u}_4) = 0.8788$
4. Use the Sine Theorem in the triangle $P_0\, P_W\, P_4$, right in P_W: $|\arcsin(\lambda\gamma)| = \varepsilon = 0.000048$
5. Calculate the other cathetus with the Cosine theorem: $\cos\lambda = \cos\varsigma \cos\gamma$ and $\varsigma = 0.000027$

Now we abandon the 3-sphere $x_1^2 + x_2^2 + x_3^2 = c^2t^2$ and, back to the 4-sphere equation, we can solve the displacement between $P_0\, P_4$:

1. the value $\sin(\psi)\sin(\varphi)\,\Delta\theta$ is equal to ε.
2. the value $\sin(\psi)\,\Delta\varphi$ is equal to ς.

[*] - Since for the North Pole we arbitrarily assumed $x_4 = 0$, it is not strange that all the points are constructed in the same way and all satisfy the condition of coplanarity on x_4. In this construction, we can reasonably think that, for every three points of the 4-sphere, passes a sphere that preserves angles and distances between them.

CURIOSITIES AND FEATURES OF THE MODEL

A ray of light, which travels the most recent circle and reaches us after a rotation of 2π, had an age of 25.4 million years when started. In that period and before no stars still exist. No images may overlap, nor ghost images exist and we never could ask ourselves if the ray had traveled an arc Θ or a Θ + 2nπ one.

I wanted to present this model even if incomplete. In my opinion, it fully explains the isotropy and homogeneity of the universe, as well as it provides a circular path for CBR and radiation in general. It is also totally consistent with all the concepts expressed by relativity, giving a coherent answer for the most distant galaxies: *In this geometry, at all times, galaxies never cross the relativistic light cone.*

The 4th spatial dimension does not imply reintroducing the concept of an absolute space. The hypothesized conservation of the energy for the whole 4-sphere pushes us to analyze its internal, mysterious part.

References from Wikipedia:

[1] - https://en.wikipedia.org/wiki/Cosmic_background_radiation

[2] - https://en.wikipedia.org/wiki/Surface_tension

[3] -https://en.wikipedia.org/wiki/Redshift

[4] - https://en.wikipedia.org/wiki/Recombination_(cosmology)

[5] - https://en.wikipedia.org/wiki/Stefan%E2%80%93Boltzmann_constant

[6] - https://en.wikipedia.org/wiki/Friedmann_equations

[7] - https://en.wikipedia.org/wiki/Lambda-CDM_model

[8] - https://en.wikipedia.org/wiki/Planck%27s_law