Rethinking Mass, Energy, Momentum, Time, and Quantum Mechanics

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Abstract

In this paper, we discuss in brief the most common wave equations in quantum mechanics and some recent development in wave mechanics. We also present two new quantum wave mechanics equations based on the Compton momentum. We also question the idea that energy and mass are scalars, and we claim they are vectors instead. We have good reasons to think that the standard momentum is a mathematical derivative of the more fundamental Compton momentum. This will hopefully simplify interpretations of quantum mechanics significantly; our new relativistic wave equations look promising, but need further investigation into what they predict. This way of looking at quantum mechanics in new light is not in conflict with existing equations, but they are supplemental to the collection of existing wave equations. We prove mathematically that if one satisfies our new relativistic energy Compton momentum relation, one also satisfies the standard relativistic energy momentum relation automatically. They are two sides of the same coin, where the relations to the Compton wavelength likely represent the deeper reality, so we have reasons to think our new wave mechanics addresses a deeper level of understanding than the existing conception. We also look at our new wave equation in relation to hydrogen-like atoms; we follow the "standard approach" used for the Schrödinger equation of putting it in polar coordinate form and, by change of variables, finding three ODEs and their solutions. We give a table summary of our new ODEs and their solutions compared to the well-known solutions of the Schrödinger equation.

Key Words: quantum mechanics, de Broglie wavelength, Compton wavelength.

1 Momentum, Energy and Mass: Scalars or Vectors? Think Again!

Before we go into an analysis of quantum mechanics, we need to take a close look on whether momentum, energy and mass are scalars or vectors. In modern physics, mass is considered a scalar; the same is the case with energy, while momentum is a vector. Here we will evaluate the assumptions behind why this is so. To our own surprise, it seems that mass and energy should also be vectors and not scalars. If so, this has implications for quantum mechanics, and also for our understanding of fundamental particles and time. First, several questions must be addressed in detail.

1.1 Why is momentum a vector?

The relativistic momentum for a particle with mass is considered to be

$$p = mv\gamma = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1}$$

Since the mass m, and the speed of light c are considered scalars, then it must be the velocity of the particle v that pushes modern physics to interpret momentum as a vector. This corresponds to the standard view and is even easier to see when $v << c$, as we can then approximate the momentum using the first term of a Taylor expansion, and get the well-known momentum formula

$$p \approx mv \tag{2}$$

Again, since the mass is considered a scalar, the momentum is considered a vector because the mass is multiplied by velocity. We can write the momentum as the well-known three-momentum, where it is very clear from the notation that the velocity causes the momentum to become a vector:

$$\boldsymbol{p} = (p_x, p_y, p_z) = m\gamma\boldsymbol{v} \tag{3}$$

where $\boldsymbol{v} = v_x + v_y + v_z$ is the particle's three-velocity

On the other hand, the well-known kinetic energy formula is $E_k \approx \frac{1}{2}mv^2$, when $v << c$. In the kinetic energy formula, the velocity is squared, and a vector multiplied by itself (dot product) is a scalar, so the energy must be a scalar. However, that the momentum is a vector and energy is a scalar is based on the assumption that the mass is a scalar, something we will get back to soon.

In modern physics, it is also assumed that photons have momentum, despite the fact that photons are considered to be massless. In other words, we cannot use the standard momentum formula $p = mv\gamma$ for photons, as is naturally well known. First, we would have no mass to put into the formula, although we could argue that the imaginary or equivalent mass of the photon was $m = E/c^2$. However, we still could not use the relativistic momentum formula, as inputting $v = c$ would mean we needed to divide by zero, which is mathematically undefined. This is why there is a separate formula for the momentum of photons, which is given by

$$p = \frac{h}{\lambda} \tag{4}$$

where h is the Planck constant and λ is the wavelength of the photon. The Planck constant is just a constant and must be a scalar, and a wavelength is normally considered to be a scalar. So, does this mean that the photon momentum, unlike the standard momentum, is a scalar and not a vector? Further, since the photon momentum is considered to be a vector, our only choice is to consider the photon wave as a vector. We have to be careful here distinguishing between the photon wave and the photon wavelength, as the photon wave has a magnitude, which is its length, but it also has a direction in space. It would be better to reformulate it more precisely, as we suggest below

$$\mathbf{p} = (p_x, p_y, p_z) = \frac{h}{\boldsymbol{\lambda}} = \left(\frac{h}{\lambda_x}, \frac{h}{\lambda_y}, \frac{h}{\lambda_z}\right) \tag{5}$$

where $\lambda_x + \lambda_y + \lambda_z = \lambda$, and $\lambda = \|\boldsymbol{\lambda}\|$.

It is well-known that the photon momentum is considered a vector; more precisely the so-called four-momentum is given by

$$\mathbf{p} = (p^t, p^x, p^y, p^z) = \left(\frac{E}{c}, \frac{Ev_x}{c^2}, \frac{Ev_y}{c^2}, \frac{Ev_z}{c^2}\right) \tag{6}$$

where $v_x^2 + v_y^2 + v_z^2 = c^2$. Here one gets the impression that the photon momentum is indeed caused by velocity, since energy here is considered a scalar. The terms $\frac{Ev_x}{c^2}$, $\frac{Ev_y}{c^2}$, $\frac{Ev_z}{c^2}$ give the impression that the photon velocity makes the photon momentum a vector. But the speed of light is considered a scalar, so where did the v_x and v_y and v_z suddenly come from? Can we just introduce the velocity of light rather than the speed of light when we need to turn energy into a vector? This may seem more like a mathematical trick than a well-founded theory, but let?s explore the idea further. Based on the premise that the photon momentum is a vector due to the four-velocity of light, then we could even turn energy into a vector by simply claiming we have

$$\mathbf{E} = (E^t, E^x, E^y, E^z) = \left(E, \frac{Ev_x}{c}, \frac{Ev_y}{c}, \frac{Ev_z}{c}\right) \tag{7}$$

where $v_x^2 + v_y^2 + v_z^2 = c^2$. This is shown simply to highlight that it could be considered a mathematical trick to turn the photon momentum $p = \frac{h}{\lambda}$ into a vector by suddenly introducing light velocity. We think it is more sound to assume that the photon wave has a direction in space in addition to a magnitude, so that we simply have $\boldsymbol{p} = \frac{h}{\boldsymbol{\lambda}}$.

1.2 Mass a Scalar or a Vector?

Assume the smallest possible particle is a spherical indivisible particle, as shown in Figure 1. It clearly has no direction in space since it is perfectly spherical, so it must be a scalar and not a vector. In standard physics, elementary particles are point particles, but they also have wave-particle duality. It is also assumed that the matter wave, which has a length equal to the de Broglie wavelength spread outs symmetrically in all directions. So, standard physics clearly assumes that a rest-mass is a scalar.

When it comes to macroscopic objects, we can agree that if a parked car is sitting in a given direction, it can be better described by a vector than a scalar. However, a ball lying on the ground is symmetrical and is a scalar. Still, we could imagine that the building blocks of the ball were many oval shaped particles, and then the building blocks of the ball would be vectors. What we are interested in here is whether the most fundamental particles are vectors or scalars. In our model, the ultimate particle is indeed indivisible [2], but this is a particle that always travels at velocity c, except when colliding with other particles. This particle makes up both energy and mass; when it moves, and it moves with velocity c, then it is what we call energy. So, if the particle in Figure one moves relative to the observer, then it is a vector. That is, energy is a vector. A mass in our model is two colliding indivisible particles, this is illustrated in Figure 2. The ultimate mass is two indivisible particles, and this structure is not symmetrical; it has a direction in space, so it is a vector. In our model, both energy and mass are vectors at the quantum level, which is unlike standard physics, where mass and energy are scalars.

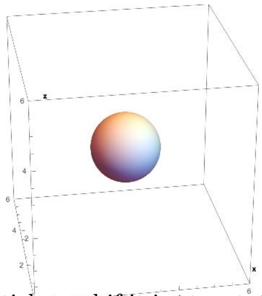

Figure 1: The figure shows one indivisible particle, and if it is at rest, it is a scalar. However, indivisible particles that are not colliding are energy that moves at speed c, and since the motion is in a direction, they are vectors.

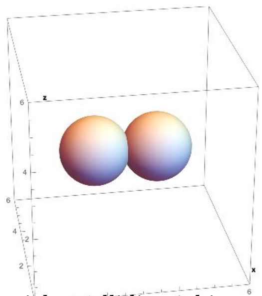

Figure 2: The figure shows two indivisible particles colliding; this structure is what we call mass, and it can simply be described as a vector.

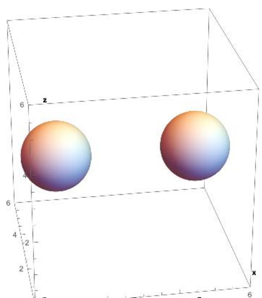

Figure 3: The figure shows two indivisible particles traveling after each other with a distance center to center equal to the Compton wavelength; this is a vector.

1.3 Mass, Energy, and Time as Scalars or as Vectors?

In standard physics, a fundamental particle is both a point particle, with no spatial dimensions, and at the same time it is a wave-particle duality. That is, all masses or at least elementary particles have also a matter wave, normally it is assumed this wave extends out isotopically in all directions of space from the "center" of the particle; in that case, the particle is a scalar. However, in our view, this is nothing more than a hypothesis. We will suggest that the wave of a fundamental particle has a direction in space. A larger mass consisting of many elementary particles will be a cloud of such particles with waves going in all directions, so a large mass is likely a scalar from this point of view, while an elementary particle, in our view, is a vector.

The Compton wavelength formula [1] is given by

$$\lambda_c = \frac{\hbar}{mc} \tag{8}$$

Solved with respect to mass this gives

$$m = \frac{h}{\lambda_c}\frac{1}{c} \tag{9}$$

This simple formula can describe any mass in terms of kg. The Planck constant is a scalar, so is the speed of light. The wavelength is a magnitude, but we will claim the Compton wavelength itself is a vector. The Compton wavelength is the distance center to center between indivisible particles, as described by Haug [2], but the Compton wavelength is a vector. If this is the case, then the mass is a vector of the form

$$\boldsymbol{m} = (m_x, m_y, m_z) = \frac{h}{\boldsymbol{\lambda}_c}\frac{1}{c} = \left(\frac{h}{\lambda_x}\frac{1}{c}, \frac{h}{\lambda_y}\frac{1}{c}, \frac{h}{\lambda_z}\frac{1}{c}\right) \tag{10}$$

where $\lambda_x + \lambda_y + \lambda_z = \lambda$, where $\lambda_c = \|\boldsymbol{\lambda}_c\|$. Now relativistic energy will be a vector, because the mass is a vector. The standard momentum for something with rest-mass will no longer be a vector, but actually a scalar, since we have $p = \boldsymbol{m} \cdot \boldsymbol{v}$. However, one can discuss how velocity can be a vector. The velocity is nothing physical and it has no direction in space; it is the mass standing still or moving that has a direction in space. However, we will keep the standard momentum as a scalar; this gives no problems for the consistency of the relativistic energy momentum relation

$$||\boldsymbol{E}||^2 = p^2c^2 + ||\boldsymbol{m}||^2c^4 \tag{11}$$

So here we end up with only scalars in our modified standard relativistic energy momentum relation. This may seem inconsistent at this stage, as we now have a photon momentum that is a vector, and a momentum of particles with rest-mass that is a scalar. However, as we have suggested in several papers [2], the standard momentum is a derivative of what we consider the real momentum, or what we call a Compton momentum. The Compton momentum is given by

$$\boldsymbol{p}_t = \boldsymbol{m}c\gamma \tag{12}$$

and can be derived from the Compton wavelength of matter, while the standard momentum is linked to the de Broglie wavelength. In other words, we have

$$\boldsymbol{p}_t = (p_x, p_y, p_z) = \boldsymbol{m}c\gamma = (m_xc\gamma, m_yc\gamma, m_zc\gamma) \tag{13}$$

and for energy we will have

$$\boldsymbol{E} = (E_x, E_y, E_z) = \boldsymbol{m}c^2\gamma = (m_xc^2\gamma, m_yc^2\gamma, m_zc^2\gamma) \tag{14}$$

However, we recently have shown [2] that one can define a new energy unit simply by dividing the standard energy unit by c. We see then that the new energy is identical to our Compton momentum. There is actually no need for both momentum and energy, and we can naturally call this new unit whatever we want: the Compton momentum, since it is derived from the Compton wavelegth?s relation to matter, or we could call it energy. This will simplify physics and make it more consistent.

We will even suggested that time is a vector rather than a scalar; that is, we suggest that

$$\boldsymbol{t} = (t_x, t_y, t_z) \tag{15}$$

This leads to a 6-dimensional space-time, where the three space dimensions will be directly linked to three time dimensions. Observations in space are directly linked to observations in time, so the three space dimensions and the three time dimensions will be two sides of the same coin. That is, we have $t = \sqrt{t_x^2 + t_y^2 + t_z^2}$. We actually introduced this in our collision space-time paper, but we said little about it. Now it seems clear to us that our unified theory is likely fully compatible with a such a 6-dimensional space-time, or as I would prefer to call it, double-3D, as the three space dimensions are directly linked to the three time dimensions. We will discuss this is greater depth later.

Returning to velocity, we are not convinced on whether the velocity should be a scalar or a vector. However, does not seems to be important, as we no longer need the standard momentum since the Compton momentum is closer to reality. The Compton momentum and kinetic energy are, in our view, clearly vectors that are readily observable. They are directly linked to the impact from moving objects in collisions. For example, when a car hits another car, it is of great importance what direction it came from, and the damage from the impact is directly proportional to v^2 and not to v, when $v << c$. There are no direct observations of the standard momentum; it is simply a derivative of the Compton momentum. The standard momentum does not exist physically, as it not can be observed physically, but can only be derived from observations. The same is true for v in isolation; one cannot observe a velocity without having a moving object. Since we are working with physics and not just math or geometry, one can ask if a vector in fundamental physics actually must be something that has a physical extension in real space. With physical extension in space we are not only talking about a solid object, since solid objects are not necessarily that solid at the quantum level, but we are talking about something that can actually be measured, which include interactions with energy. Assume that I have a velocity v; the velocity itself is nothing and cannot be observed, but a particle moving at velocity v has a direction in space that can be measured through kinetic energy (collision), or the Compton momentum, but not by standard momentum or v.

2 Standard Momentum and the de Broglie Wavelength

The relativistic de Broglie wavelength [3] is given by

$$\lambda_b = \frac{h}{mv\gamma} \tag{16}$$

where h is the Planck constant, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, and v is the velocity of the mass. An important note is that the de Broglie wavelength is not mathematically defined for a rest-mass particle, as setting $v = 0$ means that we are dividing by zero. In addition, if we let v be close to zero, then the de Broglie wavelength converges towards infinity. Next we solve the de Broglie relation with respect to momentum; this gives

$$p = mv\gamma = \frac{h}{\lambda_b} \tag{17}$$

This means that the momentum is not defined for a rest-mass particle, since λ_b is not defined for a rest-mass particle. This is somewhat new, as modern physics directly and indirectly assumes that the momentum is simply zero when $v = 0$. For any $v > 0$, the formula gives the correct momentum, but again for a rest-mass particle, the standard momentum is not defined.

It seems this does not appear in the discussion among physicists. We think the likely reason is that the standard momentum was suggested long before the relativistic momentum was conceived. The idea that momentum is mass times velocity was suggested by Newton in the Principia in 1686 and in 1721 by John Jennings [4]. Jennings said that momentum is the quantity of matter multiplied by the velocity, which is the standard momentum: $p \approx mv$, which holds when $v << c$. However, we will claim it is not valid for $v = 0$. Also, Newton mentioned momentum in the Principia, but it was less clearly defined than Jennings; the last version of Principia was published several years after Jennings work[1] The momentum suggested by Jennings came long before the development of relativity theory. So, the relativistic momentum $p = mv\gamma$ was probably derived first by Einstein in 1905.

The de Broglie wavelength was a hypothesis set out by de Broglie in 1923. As it had been shown that light has a particle-wave duality, de Broglie then speculated that matter had the same characteristics, so he assumed the matter wave was given by $\lambda_b = \frac{h}{mv\gamma}$. That is, the de Broglie wavelength was derived from the momentum. The fact that something was understood later does not mean that is less fundamental; on the contrary, since we live so far from the quantum world in our everyday lives, physics has mostly developed from the top down. Therefore, we have come up with rules and formulas for macroscopic objects and observations first, then later understood their

[1]The exact history of momentum would require a detailed historical study of all versions of the Principia.

connection to the particle and the quantum worlds. So, once the quantum world is established, we can just as well derive such things as momentum from there. The point is that the momentum, from a quantum perspective, must be given by the formula 17 and we have shown this means that neither the de Broglie wavelength nor the standard momentum formula are valid when $v = 0$. As we will see, this is very important for quantum physics, as all the quantum wave equations of modern physics will be impacted.

3 The Compton Wavelength and the New Compton Momentum

Around the same time as de Broglie introduced the hypothesis of his matter wave, Compton [1] calculated and indirectly measured what today is known as the Compton wavelength. The relativistic Compton wavelength of an electron is given by[2]

$$\lambda = \frac{h}{mc\gamma} \tag{18}$$

First of all, here we see there are no issues with $v = 0$, as this just means that $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{0^2}{c^2}}} = 1$. That is, the Compton wavelength, unlike the de Broglie wavelength, is mathematically well defined for any velocity of $v < c$. See how to derive the Compton wavelength for any mass without knowledge of h in [6].

Next, if we follow a similar approach to the one we used for the de Broglie wavelength, we get

$$p_t = mc\gamma = \frac{h}{\lambda} \tag{19}$$

This is what we will call the total Compton momentum and it is a new type of momentum recently introduced by Haug [2]. Unlike the standard momentum, this momentum is well defined for $v = 0$ as well, since the Compton wavelength λ is well defined for $v = 0$. Also, it does not have strange properties, such as going towards infinity when v is close to zero, see [7], for example. The Compton wavelength is always on the scale of the atomic quantum realm (very short compared to anything microscopic).

Further, it is important to note that we can always find the de Broglie wavelength from the Compton wavelength; we have $\lambda_b = \lambda \frac{c}{v}$. So, if we know the Compton wavelength, we can calculate the de Broglie wavelength. The same is true with the standard momentum (the de Broglie momentum); it can always be calculated from the Compton momentum as $p = p_t \frac{v}{c}$.

Why should there be two wavelengths linked to matter? And why should we have two types of momentum? We will suggest that the standard momentum and the de Broglie wavelength only are derivatives of the true matter wave and the true and deeper physical momentum, namely what we call Compton momentum. If we should connect the standard energy definition to the Compton momentum, we simply get

$$E = p_t c \tag{20}$$

That is, the total energy is equal to the Compton momentum multiplied by the speed of light; this is a new (additional) relativistic energy momentum relation and means we can also derive a new quantum mechanical wave equation from this new relation, which will be the relation between energy and the Compton momentum.

It is worth mentioning that the standard momentum is never observed directly – it is a mathematical construct. First, assume we have a brass ball; we can measure its relative weight relative to one kg and then find its mass relative to one kg. Second, we can put this brass ball in motion. We can then measure its velocity, but we cannot directly observe mv. What we can observe is the impact from its kinetic energy. We can drop a brass ball, for example, and measure its velocity just before it hit a brick of "soft" clay. Most of the kinetic energy will then be used to make an indent in the clay. Gravesande [8] did this and confirmed that experimentally the kinetic energy was proportional to v^2 and not just of v. At that time, the question of whether the kinetic energy was a function of v or v^2 had been a debate among leading physicists for many years. And at least when $v << c$, the kinetic energy is a function of v^2. So, indeed we can measure the kinetic energy of a moving body, and the mass of a body easily when it is at rest, and we can easily measure the velocity of a body, but we cannot measure mv; this is a mathematical entity, that is, however, linked to real observable entities, so it can be very useful. What about our new Compton momentum? Can it be observed more directly? It should be possible because, as we have explained here, we think the standard momentum is a derivative of the true (more real) momentum.

First, looking at our new Compton momentum: when $v = 0$, we get

[2] Actually this is a relativistic extension of Compton's work, see [2, 5].

$$p_t = mc\gamma = mc \tag{21}$$

In other words, we get a rest-mass momentum that is mc. This is not easy to observe, as it is an embedded momentum, a rest-mass momentum. This may sound strange, as we are not used to thinking of rest-mass momentum, and some may even say that this is impossible, as momentum is related to something that is moving. However, that is the standard momentum that indeed only is defined for something that is moving. This is nothing more strange than rest-mass energy. If the rest-mass momentum is $\tilde{p}_r = mc$, then we must also have what we can call a kinetic momentum, and this must be

$$p_k = p_t - p_r = mc\gamma - mc \tag{22}$$

This formula holds for any $v < c$. It would require advanced laboratory equipment to test this when v is significantly close to the speed of light c. However, when $v << c$ what do we expect to observe? When $v << c$, we can approximate the formula above with the first term of a Taylor expansion, and we then get

$$p_k \approx \frac{1}{2}m\frac{v^2}{c} \tag{23}$$

That is, our Compton momentum is a function of v^2 and not of v, so it is kinetic energy divided by c. Our Compton momentum is exactly the same function of v as kinetic energy, but it is simply our standard energy definition divided by c, a significant finding (that we will come back to in a new update of this paper). Our momentum is observable through measurements of impacts, while standard momentum is not, as it is a function of v and not v^2. This supports our view that our newly defined Compton momentum is the real momentum and that the standard momentum is a derivative of this momentum. While the reader may not wish to take this for granted, it will be helpful to be open to the thought that there can be a momentum (or another term could be chosen and defined) that is linked to the Compton wavelength, and next we will look at the Relativistic Energy Momentum Relation in more detail.

4 Relativistic Energy Momentum Relation

The standard relativistic energy momentum of Einstein [9] is given by

$$E^2 = p^2c^2 + m^2c^4 \tag{24}$$

the standard momentum and this relation have played a central role in developing the well-known quantum mechanical wave equations. If our analysis is correct and the standard momentum (de Broglie momentum) not is valid for rest-mass particles and further, if it is a mathematical derivative of the Compton momentum, then any quantum mechanical wave equation derived from it will be a wave equation linked to a derivative and all such quantum mechanical wave equations will probably also not be valid for rest-mass particles.

Our new relativistic energy momentum relation is

$$E = p_tc = p_k + mc^2 \tag{25}$$

When deriving a quantum mechanical wave equation consistent with this, the equation should also be valid for rest-mass particles, and should also be more directly linked to the depth of the reality, and therefore will likely be easier to interpret.

This means we are claiming that there are two types of relativistic energy momentum relations: one related to the standard momentum, that is linked to the de Broglie wavelength, and one that is linked to the Compton momentum, that is linked to the Compton wavelength. These two energy momentum relations are two sides of the same coin, so to speak, as shown in the derivation below. We have decided to show it rigorously line by line, as the connection is important to understand.

Next, we will shortly discuss a series of well-known quantum mechanical wave equations and also show a few new quantum mechanical wave equations.

$$
\begin{aligned}
E^2 &= p^2c^2 + m^2c^4 \\
E^2 &= p_0^2c^2\gamma^2 + m^2c^4 \\
E^2 &= m^2v^2c^2\gamma^2 + m^2c^4 \\
E^2 &= m^2c^4v^2/c^2\gamma^2 + m^2c^4 \\
E^2 &= m^2c^4\gamma^2 - m^2c^4\left(1 - \frac{v^2}{c^2}\right)\gamma^2 + m^2c^4 \\
E^2 &= m^2c^4(v^2/c^2 - 1)\gamma^2 + m^2c^4(v^2/c^2)\gamma^2 + m^2c^4 \\
E^2 &= m^2c^4\gamma^2 - m^2c^4 + m^2c^4 \\
E^2 &= m^2c^4\gamma^2 \\
E &= mc^2\gamma \\
E &= \frac{pc^2}{v} \\
E &= p_tc = p_k + mc^2
\end{aligned} \tag{26}
$$

This means that if we build wave equations that satisfy the relativistic Compton momentum relation, then we automatically satisfy the standard relativistic energy momentum relation and also the other way around. However, as we soon will see, this gives us two new relativistic wave equations and will enable us to look at existing problems from a new angle. This angle appears to be more directly linked to physical reality, as we have claimed the de Broglie wavelength is a mathematical derivative of the Compton wavelength. Although we do not expect anyone to take this for granted, we think it is interesting enough to warrant further study and investigation outside of what we are able to cover in this paper.

Next, we will shortly discuss a series of well-known quantum mechanical wave equations and also show a few new quantum mechanical wave equations.

5 Two New Relativistic Wave Equations

We have that

$$\boldsymbol{E} = \mathbf{p}_tc \tag{27}$$

This can be rewritten as

$$\boldsymbol{E} = \mathbf{p}_kc + \boldsymbol{m}c^2 \tag{28}$$

where $\mathbf{p}_k = \frac{\boldsymbol{m}c}{\sqrt{1-\frac{||\boldsymbol{v}||^2}{c^2}}} - \boldsymbol{m}c$, in other words, the kinetic Compton momentum. From this we get the following quantum wave equation, using energy operator: $i\hbar\nabla_t = i\hbar\frac{\partial}{\partial t_x} + i\hbar\frac{\partial}{\partial t_y} + i\hbar\frac{\partial}{\partial t_z}$, and kinetic Compton momentum operator: $-i\hbar\nabla = i\hbar\frac{\partial}{\partial x} + i\hbar\frac{\partial}{\partial y} + i\hbar\frac{\partial}{\partial z}$, we get

$$i\hbar\nabla_t\Psi = \left(-i\hbar c\nabla + mc^2\right)\Psi \tag{29}$$

This we can rewrite as

$$
\begin{aligned}
i\nabla_t\Psi &= \left(-ic\nabla + \frac{mc^2}{\hbar}\right)\Psi \\
i\nabla_t\Psi &= \left(-ic\nabla + \frac{c}{\bar{\lambda}}\right)\Psi
\end{aligned} \tag{30}
$$

This is relativistic quantum wave equation[3] consisting of a first order PDE where ∇ is the operator linked to the kinetic Compton momentum and ∇_t an energy operator. It has nice properties, as time and spatial dimensions

[3] We actually put out a rough working draft paper with this equation in December 28, 2018 [10], and also we did not look into it much before we adjusted the working paper a few days later to center on a wave equation with focus on total Compton momentum rather than kinetic, something we soon will return to later. The wave equation we put out at that time likely had a sign error due to the fact that we had the wrong sign on the energy operator; the equation we presented then was $-i\frac{\partial\psi}{\partial t} = \left(-ic\nabla + \frac{mc^2}{\hbar}\right)\psi$

are on the same order (first order), unlike the Schödinger equation where the time and spatial dimension are on a different order in terms of derivatives. It is well known that the Schödinger equation, therefore, is not Lorentz invariant. Our new relativistic wave equation, which is fully rooted in the Compton wavelength, is, on the other hand, also mathematically Lorentz invariant, except at the Planck scale.

Our new relativistic quantum equation has quite a different plane wave solution than the Klein–Gordon and Schrödinger equations, but at first glance it looks exactly the same:

$$\psi = e^{i(kx-\omega t)} \tag{31}$$

However, in our theory, we should have $k = \frac{2\pi}{\lambda}$, where λ is the relativistic Compton wavelength and not the de Broglie wavelength, as in standard wave mechanics. We can rewrite the plane wave solution as

$$e^{i\left(\frac{p_k}{\hbar}x-\frac{E}{\hbar}t\right)} \tag{32}$$

where p_k is the total kinetic Compton momentum, as defined earlier. So, our quantum wave function is rooted in the Compton wavelength instead of the de Broglie wavelength. For the formality of it, we look at the momentum and energy operators and see that they are correctly specified

$$\frac{\partial\psi}{\partial x} = \frac{ip_k}{\hbar}e^{i\left(\frac{p_k}{\hbar}x-\frac{E}{\hbar}t\right)} \tag{33}$$

This means the momentum operator must be

$$\hat{p}_k = -i\hbar\nabla\cdot \tag{34}$$

and for energy we have

$$\frac{\partial\psi}{\partial t_x} = \frac{-iE}{\hbar}e^{i\left(\frac{p_k}{\hbar}x-\frac{E}{\hbar}t\right)} \tag{35}$$

and this gives us a time operator of

$$\hat{E} = -i\hbar\nabla_t \tag{36}$$

The momentum and energy operators are the same as under standard quantum mechanics.

6 Wave Equation Based on Operator on Total Compton Momentum

We also get a quantum wave equation linked to the total momentum instead of the kinetic momentum. This is because

$$\boldsymbol{E} = \boldsymbol{p}_k c + \boldsymbol{m}c^2 = \boldsymbol{p}_t c \tag{37}$$

which gives

$$i\hbar\nabla_t\Psi + i\hbar c\nabla\Psi = 0 \tag{38}$$

which can be simplified to (see also [2] that basically gives the same wave equation, but we suspect with a sign error.)

$$\nabla_t\Psi + c\nabla\Psi = 0 \tag{39}$$

That is, we have two new quantum mechanical wave equations. The plane wave solution to this wave equation should be the same as in the other wave equation above. This is mathematically identical to the advection equation, but the advection equation is not used in quantum mechanics, so even if it is not a new PDE from a mathematical point of view, in our view, this can be used as a relativistic quantum mechanical equation when we root our quantum mechanics in the Compton wavelength rather than the de Broglie wavelength.

Also ,in this case it has quite a different plane wave solution than the Klein–Gordon and Schrödinger equations, but at first glance it looks exactly the same:

$$\psi = e^{i(kx-\omega t)} \tag{40}$$

However, here $k = \frac{2\pi}{\lambda}$, where λ is the relativistic Compton wavelength and not the de Broglie wavelength, as in standard wave mechanics. We can rewrite the plane wave solution as

$$e^{i\left(\frac{p_t}{\hbar}x - \frac{E}{\hbar}t\right)} \tag{41}$$

where p_t is the total Compton momentum, as defined earlier. So, our quantum wave function is rooted in the Compton wavelength instead of the de Broglie wavelength. For the formality of it, we look at the momentum and energy operators and see that they are correctly specified

$$\frac{\partial\psi}{\partial x} = \frac{ip_t}{\hbar}e^{i\left(\frac{p_t}{\hbar}x - \frac{E}{\hbar}t\right)} \tag{42}$$

This means the momentum operator must be

$$\hat{p}_t = -i\hbar\nabla\cdot \tag{43}$$

and for energy we have

$$\frac{\partial\psi}{\partial t} = \frac{-iE}{\hbar}e^{i\left(\frac{p_t}{\hbar}x - \frac{E}{\hbar}t\right)} \tag{44}$$

and this gives us a time operator of

$$\hat{E} = -i\hbar\frac{\partial}{\partial t} \tag{45}$$

That is, the momentum and energy operators are the same as are used in already established quantum mechanics.

7 The Klein–Gordon Equation

Another well-known relativistic quantum equation is the Klein–Gordon equation, which is given by

$$E^2 = \boldsymbol{p}\cdot\boldsymbol{p}c^2 + m^2c^4 \tag{46}$$

where $\mathbf{p}$ is the relativistic (de Broglie) momentum. When replacing E and p with their energy, $i\hbar\frac{\partial}{\partial t}$, and momentum operator, $i\hbar\nabla$, we get the following wave equation

$$\begin{aligned}
i^2\hbar^2\frac{\partial^2\psi}{\partial t^2} - i^2\hbar^2c^2\nabla^2\psi - m^2c^4\psi &= 0 \\
-\hbar^2\frac{\partial^2\psi}{\partial t^2} + \hbar^2c^2\nabla^2\psi - m^2c^4\psi &= 0 \\
\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi &= 0
\end{aligned} \tag{47}$$

The last line is how the Klein–Gordon equation is often presented. Since the reduced Compton wavelength is given by $\bar{\lambda} = \frac{\hbar}{mc}$, we can replace $\frac{m^2c^2}{\hbar^2}$ in the equation above with $\frac{1}{\bar{\lambda}^2}$ and we get

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi + \frac{1}{\bar{\lambda}^2}\psi = 0 \tag{48}$$

The Klein–Gordon equation is indirectly liked to the de Broglie momentum (standard momentum), $\mathbf{p} = m\mathbf{v}\gamma$, which we claim is a derivative of the real momentum, the Compton momentum. The Klein–Gordon equation is therefore unnecessarily complex. Yet, it cannot be simplified further if we want a relativistic wave equation from the de Broglie momentum. The formula is likely not valid for a rest-mass particle, since it is derived from the de Broglie momentum.

The Klein–Gordon equation is often written as

$$(\Box + \mu^2)\psi = 0 \tag{49}$$

where $\Box$ is the d'Alembert operator: $\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$, and $\mu = \frac{mc}{\hbar} = \frac{1}{\bar{\lambda}^2}$. Do not let unfamiliar notation stop you from exploring the mysteries of quantum mechanics.

The Klein–Gordon equation is only valid for spin zero particles, and the only particle detected it is valid for is, therefore, the Higgs boson. The Klein–Gordon equation has unwanted properties, i.e., it allows negative probabilities, which is what motivated Dirac to come up with another relativistic wave equation.

8 The Schrödinger Equation

We have

$$\begin{aligned} E &= E_k + mc^2 \\ E &= mc^2\gamma - mc^2 + mc^2 \end{aligned} \tag{50}$$

when $v << c$ we can approximate the kinetic energy from the first term of a Taylor series expansion with $E_k \approx \frac{1}{2}mv^2$, this gives

$$\begin{aligned} E &\approx \frac{1}{2}mv^2 + mc^2 \\ E &\approx \frac{p^2}{2m} + mc^2 \end{aligned} \tag{51}$$

If we now use a momentum operator of $i\hbar\nabla$ and an energy operator of $i\hbar\frac{\partial}{\partial t}$, we can write this as a wave equation, something giving us the Schödinger [11] equation

$$i\hbar\frac{\partial\psi}{\partial t} \approx \left(\frac{i^2\hbar^2}{2m}\nabla^2 + mc^2\right)\psi \tag{52}$$

This we can rewrite further

$$\begin{aligned} i\hbar\frac{\partial\psi}{\partial t} &\approx \left(\frac{-\hbar^2}{2m}\nabla^2 + mc^2\right)\psi \\ i\frac{\partial\psi}{\partial t} &\approx \left(\frac{-\hbar}{2m}\nabla^2 + \frac{mc^2}{\hbar}\right)\psi \\ i\frac{\partial\psi}{\partial t} &\approx \left(\frac{-\bar{\lambda}c}{2}\nabla^2 + \frac{c}{\bar{\lambda}}\right)\psi \end{aligned} \tag{53}$$

Note that when rewritten this way, there is no Planck constant in the Schrödinger equation in this form. Also note that the imaginary number does not go away as it does in the Klein–Gordon equation. More important is that the Schrödinger equation does not treat the time (energy) on equal footing with the spatial dimension (momentum). This is because we have first- versus second-order derivatives in the same equation. This is unlike the Klein–Gordon equation, where everything is on second order, and also unlike the Dirac equation, where all is first order. Our new wave equations are both first order and note again that the Schequation does not have time and the spatial dimension of the same order.

9 The Dirac Equation

The Klein–Gordon equation was not suitable to model the electron, as it only allows for zero spin, while electrons are $\frac{1}{2}$ spin fermions. Also, the fact that Klein–Gordon allows such things as negative probabilities pushed Dirac [12] to develop a new relativistic wave equation, which is given by

$$i\hbar\frac{\partial\psi}{\partial t} - \left(c\sum_{i=n}^{3}\alpha_n\mathbf{p}_n - \beta mc^2\right)\psi = 0 \tag{54}$$

this can be rewritten as

$$i\frac{\partial\psi}{\partial t} - \left(\frac{c}{\hbar}\sum_{i=n}^{3}\alpha_n\mathbf{p}_n - \beta\frac{c}{\bar{\lambda}}\right)\psi = 0 \tag{55}$$

So, there is a Compton wavelength embedded in the Dirac equation, but the momentum operator is operating on the standard momentum that is linked to the de Broglie wavelength.

10 Summary

Table 1 shows a summary of three well-known wave equations in quantum mechanics, as well as two new ones. The three older equations are all rooted in standard momentum and therefore in the de Broglie wavelength. In addition, these equations contain the Compton wavelength embedded in their last term, but the momentum they are deriving their equations from involve the de Broglie equivalent momentum. The standard momentum is not defined for $v = 0$; that is to say, for rest-mass particles, so we suggest that these three traditional wave equations are likely not valid for rest-mass particles. In addition, these three wave equations are, to a large degree, modeling mathematical derivatives of reality rather than the deeper reality because the de Broglie momentum is a derivative of the more fundamental Compton momentum. The two new wave equations are linked directly to the Compton momentum; therefore, they are simpler and also hold for $v = 0$.

	Wave equations:	**Momentum operator on :**	**Comments :**
Klein–Gordon (Spin 0)	$\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi = 0$	de Broglie momentum	Relativistic
deeper level	$\frac{\partial^2\psi}{\partial t^2} - c^2\nabla^2\psi + \frac{c^2}{\bar{\lambda}^2}\psi = 0$		
Dirac (Spin 1/2)	$i\hbar\frac{\partial\psi}{\partial t} - \left(c\sum_{i=n}^{3}\alpha_n\mathbf{p}_n - \beta mc^2\right)\psi = 0$	de Broglie momentum	Relativistic
deeper level	$i\frac{\partial\psi}{\partial t} - \left(\frac{c}{\hbar}\sum_{i=n}^{3}\alpha_n\mathbf{p}_n - \beta\frac{c}{\bar{\lambda}}\right)\psi = 0$		
Schrödinger	$i\frac{\partial\psi}{\partial t} \approx \left(\frac{-\hbar}{2m}\nabla^2 + \frac{mc^2}{\hbar}\right)\psi$	de Broglie momentum	Non-relativistic
deeper level	$i\frac{\partial\psi}{\partial t} \approx \left(\frac{-c\bar{\lambda}}{2}\nabla^2 + \frac{c}{\bar{\lambda}}\right)\psi$		
Haug-1: (Spin ?)	$i\nabla_t\psi = \left(-ic\nabla + \frac{mc^2}{\hbar}\right)\psi$	Kinetic Compton momentum	Relativistic
deeper level	$i\nabla_t\psi = \left(-ic\nabla + \frac{c}{\bar{\lambda}}\right)\psi$		
Haug-2: (Spin ?)	$\frac{1}{c}\nabla_t\psi + \nabla\psi = 0$	Total Compton momentum	Relativistic

Table 1: The table shows a summary of three well-known quantum mechanical wave equations derived from standard momentum (de Broglie) momentum, and two new quantum mechanical wave equations derived from the Compton momentum.

- The Schrödinger, Klein–Gordon, and Dirac equations all use a momentum operator on the standard momentum. The standard momentum is, at a quantum level, actually directly linked to the de Broglie wavelength. The de Broglie wavelength is not mathematically defined for a rest-mass particle. Second, the de Broglie wavelength and the standard momentum are just mathematical derivatives of the more fundamental Compton wavelength and what we call Compton momentum. In other words, in the traditional equations, we are taking partial derivatives of mathematical functions of reality, not of the deepest entities. This makes the Schrödinger, Klein–Gordon, and Dirac equations all very hard to interpret at a deeper level. Of course, they are "easy" enough to interpret at the surface inside the exotic zoo of terminology that has evolved in physics and quantum physics, as long as the analysis of modern physics does not go too deep.

- We have strong reasons to believe that our new quantum mechanical wave equations are better suited to understanding certain aspects of the depth of reality. They are mathematically correct for $v = 0$, and they are more directly linked to the depth of reality, as we are modeling from the Compton momentum directly instead of the derivative of it, which is the de Broglie momentum and its corresponding de Broglie wave. Also, our equations are both relativistic and always of first order.

- There is no Planck constant in the any of the wave equations except for the Dirac equation that we will soon comment on separately. The apparent Planck constant in the Schrödinger and Klein–Gordon all cancel out against a Planck constant that is hidden in the mass. What we obtain is the Compton wavelength in the Schrödinger and Klein–Gordon equations, or more precisely, the Compton frequency is also embedded in these equations. One might think that such a line of thought is wrong, if one believed that the Planck constant is needed to find the Compton wavelength. However, this is not the case; even from Compton's 1923 paper it is clear that one can find the Compton wavelength without any knowledge of the Planck constant. Actually, one can find the Compton wavelength for any mass without knowledge of any fundamental constants, see [6]. In the Dirac equation, the Planck constant will indirectly cancel out as well, as the momentum operator on the wave function returns the momentum, and the momentum embedded contains the Planck constant in the mass will then cancel out against the Planck constant we see here. In our new wave equation from the Compton momentum, the Planck constant also cancels out.

11 Solving Our New Relativistic Wave Equation for Hydrogen-like Atoms

We will here solve our wave equation for hydrogen-like atoms, using the standard approach[4] also used for the Schödinger equation. The Haug-1 wave equation is given by

$$i\hbar\nabla_t\psi = (-ci\hbar\nabla + V)\,\psi \tag{56}$$

where V is the potential energy, and $-i\hbar\nabla$ is the Compton momentum operator. The potential energy between two charges can easily be described by the Coulomb force [13]

$$V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r} = -k_e\frac{Ze^2}{r}$$

where k_e is Coulomb's constant. Further, our wave equation rewritten in polar coordinates is given by

$$E\psi = \left(-i\hbar c\left(\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\right) + V\right)\psi \tag{57}$$

where θ is the polar angle, with φ for the azimuthal angle, and $\psi = \psi(r,\theta,\phi)$.

The hydrogen atom's Hamiltonian is

$$\hat{H} = -i\hbar c\nabla - k_e\frac{Ze^2}{r} \tag{58}$$

where $-i\hbar\nabla$ is a Compton momentum operator.

The Haug relativistic wave equation for the hydrogen atom in polar coordinates is given by

$$E\psi = \left(-i\hbar c\left(\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\right) - \frac{Ze^2}{4\pi\epsilon_0 r}\right)\psi \tag{59}$$

This we can rearrange as

$$\frac{i}{\hbar c}\left(E + \frac{Ze^2}{4\pi\epsilon_0 r}\right)\psi = -\frac{\partial\psi}{\partial r} - \frac{1}{r}\frac{\partial\psi}{\partial\theta} - \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\varphi} \tag{60}$$

Next we rely on separation of variables

$$\psi(r,\theta,\phi) = R(r)\cdot Y(\theta,\phi) \tag{61}$$

Further, since Y does not depend on r, we can move it in front of the radial derivative, which gives

$$\frac{\partial\psi}{\partial r} = \frac{\partial}{\partial r}RY = Y\frac{dR}{dr} \tag{62}$$

and we therefore have

$$\frac{i}{\hbar c}\left(E - \frac{Ze^2}{4\pi\epsilon_0 r}\right)RY = Y\frac{dR}{dr} + R\frac{1}{r}\frac{\partial Y}{\partial\theta} + \frac{R}{r\sin\theta}\frac{\partial Y}{\partial\varphi} \tag{63}$$

Next we multiply by r and divide by RY to separate the radial and angular terms:

$$\frac{r}{R}\frac{dR}{dr} + \frac{1}{Y}\frac{\partial Y}{\partial\theta} + \frac{1}{Y\sin\theta}\frac{\partial Y}{\partial\varphi} - \frac{ir}{\hbar c}\left(E - \frac{Ze^2}{4\pi\epsilon_0 r}\right) = 0 \tag{64}$$

[4]We have basically followed the well-known "standard approach" that is used to solve the Schödinger PDE equation for the hydrogen atom to solve our new PDE wave-equation. In particular, we found the webpage by Dr. Rudolf Winter at Aberystwyth University: https://users.aber.ac.uk/ruw/teach/327/hatom.php, useful in this respect.

The first and last terms only depend on r, while the two middle terms depend on the angle. We can therefore separate the equation into two ordinary ODEs. We get a radial equation that is a first order ODE:

$$\frac{r}{R}\frac{dR}{dr} - \frac{ir}{\hbar c}\left(E - \frac{Ze^2}{4\pi\epsilon_0 r}\right) - A = 0 \tag{65}$$

where the solution is

$$\begin{aligned} R(r) &= ic_1 e^{\frac{\log(r)\left(Ac\hbar - izk_e e^2\right)+iEr}{c\hbar}} \\ R(r) &= c_1 e^{\frac{\log(r)\left(iAc\hbar + zk_e e^2\right)-Er}{c\hbar}} \end{aligned} \tag{66}$$

where A must likely be a positive integer, as energy comes as $n\hbar$. And we also get a first order PDE linked to the angles:

$$\begin{aligned} \frac{1}{Y}\frac{\partial Y}{\partial \theta} + \frac{1}{Y\sin\theta}\frac{\partial Y}{\partial \varphi} + A &= 0 \\ \frac{\partial Y}{\partial \theta} + \frac{1}{\sin\theta}\frac{\partial Y}{\partial \varphi} + AY &= 0 \end{aligned} \tag{67}$$

where A is a separation constant.

The angle equation still contains terms for both φ and θ, so we need to do one more separation of variables:

$$Y(\theta,\phi) = \Theta(\theta)\cdot\Phi(\phi) \tag{68}$$

Replacing Y in the differential equation, we get

$$\Phi\frac{d\Theta}{d\theta} + \frac{\Theta}{\sin\theta}\frac{d\Phi}{d\varphi} + A\Theta\Phi = 0 \tag{69}$$

Next we isolate variables and separate terms

$$\Phi\frac{d\Theta}{d\theta} + \frac{\Theta}{\sin\theta}\frac{d\Phi}{d\varphi} + A\Theta\Phi = 0 \tag{70}$$

Next we divide by $\Theta\Phi$ to separate the radial and angular terms:

$$\frac{\sin\theta}{\Theta}\frac{d\Theta}{d\theta} + \frac{1}{\Phi}\frac{d\Phi}{d\varphi} + A\sin\theta = 0 \tag{71}$$

This we can separate into two new equations, the polar equation (colatitude)

$$\frac{\sin\theta}{\Theta}\frac{d\Theta}{d\theta} + A\sin\theta - B = 0 \tag{72}$$

and the azimuthal equation

$$\frac{1}{\Phi}\frac{d\Phi}{d\varphi} + B = 0 \tag{73}$$

The given azimuthal equation has solution

$$\Phi(\phi) = c_1 e^{-iB\phi} \tag{74}$$

The solution must be a valid solution for any angle ϕ. This means that B must be a positive integer for this to hold true; if not the value of the azimuth wave function would be different for $\phi = 0$ and $\phi = 360$

For comparison, the Schrödinger azimuthal ODE, $\frac{1}{\Phi}\frac{d^2\Phi}{d\varphi^2}+B=0$ has solution $\Phi(\phi)=c_1\cos\sqrt{B}\phi+c_2\sin\sqrt{B}\phi$, and when setting $c_1=1$ and $c_2=i$ is equal to $\Phi(\phi)=e^{i\sqrt{B}\phi}$. In the Schrödinger solution, one does a trick that we will think about more deeply later on, particularly in terms of its validity. However, for now, we note that here one is replacing B with m^2, and thereby gets $\Phi(\phi)=e^{im\phi}$, which is basically is identical to our solution if we replace i with -1. Yet, again in order to arrive at this solution from the Schrödinger equation, one has to replace the constant B with m^2, and it is important to determine where this came from mathematically. It seems to be guess work that is an adjustment (fudge) to fit observations. This will be the subject of further analysis and we welcome input on the explanation for this approach. In the solution to our azimuthal ODE, B itself is a quantum number, but in the Schrödinger solution one has to make the assumption of setting $B=m^2$.

Moving on, our polar equation is given by

$$\frac{\sin\theta}{\Theta}\frac{d\Theta}{d\theta}+A\sin\theta-B \;=\; 0 \tag{75}$$

rearranging

$$\frac{1}{\Theta}\frac{d\Theta}{d\theta}+A-\frac{B}{\sin\theta} \;=\; 0 \tag{76}$$

and solving directly, using Mathematica gives

$$\Theta(\theta)=ic_1e^{-A\theta-B\log[\cos(\theta/2)]+B\log[\sin(\theta/2)]} \tag{77}$$

However, we can alternatively rewrite the polar equation somewhat before we solve it. Substituting $P(\cos\theta):=\Theta(\theta)$ and $x:=\cos\theta$, we get

$$\begin{aligned}\frac{-\sin\theta}{\Theta}\frac{dP}{dx}+A-\frac{B}{\sin\theta} &= 0\\ \frac{\sin\theta}{\Theta}\frac{dP}{dx}-A+\frac{B}{\sin\theta} &= 0\end{aligned} \tag{78}$$

since $\sin^2\theta+\cos^2\theta=1$, we have that, $\sin\theta=\sqrt{1-\cos^2\theta}$, and we can therefore rewrite the equation above as

$$\begin{aligned}\frac{\sin\theta}{P}\frac{dP}{dx}-A+\frac{B}{\sqrt{1-\cos^2\theta}} &= 0\\ \sqrt{1-x^2}\frac{dP}{dx}-\left(A+\frac{B}{\sqrt{1-x^2}}\right)P &= 0\end{aligned} \tag{79}$$

The coefficient in the ODE is not constant, but depends on x. One possible solution seems to be

$$P(x)=ic_1e^{A\arcsin[x]-B[x]} \tag{80}$$

where A and B are positive integers; in other words, quantum numbers, see the previous discussion for why this is the case.

For comparison with the Schrödinger equation, based on the same principles we get the following ODE

$$(1-x^2)\frac{d^2P}{dx^2}-2x\frac{dP}{dX}+\left(A+\frac{B}{1-x^2}\right)P=0 \tag{81}$$

The coefficients in the ODE that we get from the Schrödinger equation are not constant, but depend on x. This is a differential equation known as a Legendre-type DE, with known solutions. Still, it is worth noticing that it is much more complicated to solve this ODE than the polar ODE from our new relativistic wave equation.

12 Our PDE Leads to Three ODEs

The radial equation

$$\frac{1}{R}\frac{dR}{dr} - \frac{i}{\hbar c}\left(E - k_e\frac{Ze^2}{r}\right) - \frac{AR}{r} = 0 \tag{82}$$

Further, we have the azimuthal equation

$$\frac{1}{\Phi}\frac{d\Phi}{d\varphi} + B = 0 \tag{83}$$

And the polar equation we get is

$$\frac{1}{\Theta}\frac{d\Theta}{d\theta} + A - \frac{B}{\sin\theta} = 0 \tag{84}$$

compared to Schrödinger's polar equation for hydrogen-like atoms, which is given by $\frac{\sin\theta}{\Theta}\frac{d}{d\theta}\left(\sin\frac{d\Theta}{d\theta}\right) + A - \frac{B}{\sin^2\theta} = 0$.

Table 1 summarizes our findings and also makes it easy to compare to Schrödinger's equation.

Equations:	**Schrödinger :**	**Solution :**
Radial equation	$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \frac{2\mu r^2}{\hbar}\left(E - k_e\frac{Ze^2}{r}\right) - AR = 0$	$R_\infty = c_3 e^{i\sqrt{\frac{2\mu E}{\hbar}}r} + c_4 e^{-i\sqrt{\frac{2\mu E}{\hbar}}r}$
Azimuthal equation	$\frac{1}{\Phi}\frac{d^2\Phi}{d\varphi^2} + B = 0$	$\Phi(\phi) = c_1 e^{i\sqrt{B}\phi} + c_2 e^{-i\sqrt{B}\phi}$
Polar equation Rewritten	$\frac{\sin\theta}{\Theta}\frac{d}{d\theta}\left(\sin\frac{d\Theta}{d\theta}\right) + A - \frac{B}{\sin^2\theta} = 0$ $(1\text{-x}^2)\frac{d^2P}{dx^2} - 2x\frac{dP}{dX} + \left(A + \frac{B}{1-x^2}\right)P = 0$	
Equations:	**From our new equation :**	**Solution :**
Radial equation	$\frac{r}{R}\frac{dR}{dr} - \frac{ir}{\hbar c}\left(E - zk_e\frac{Ze^2}{r}\right) - A = 0$	$R(r) = c_1 e^{\frac{\log(r)\left(Ac\hbar + izk_e e^2\right) - i\text{E}r}{c\hbar}}$
Azimuthal equation	$\frac{1}{\Phi}\frac{d\Phi}{d\varphi} + B = 0$	$\Phi(\phi) = c_1 e^{-iB\phi}$
Polar equation Rewritten	$\frac{1}{\Theta}\frac{d\Theta}{d\theta} + A - \frac{B}{\sin\theta} = 0$ $\sqrt{1-x^2}\frac{dP}{dx} - \left(A + \frac{B}{\sqrt{1-x^2}}\right)P = 0$	$\Theta(\theta) = ic_1 e^{-A\theta - B\log[\cos(\theta/2)] + B\log[\sin(\theta/2)]}$ $P(x) = ic_1 e^{A\arcsin[x] - B[x]}$

Table 2: The table shows three ODEs we get from the Schrödinger equation when written on polar coordinates, and also three ODEs we get from our new relativistic wave equation.

13 Our New Wave Equation Following the Pauli Approach

The Haug-1 wave-equation is given by

$$i\hbar\nabla_t\Psi = \left(-ci\hbar\nabla + V\right)\Psi \tag{85}$$

where V is the potential energy, and $-i\hbar\nabla$ is the kinetic Compton momentum operator.

The Hamiltonian operator in our new wave equation, when following the Pauli approach will be

$$\hat{H} = c\boldsymbol{\sigma}\cdot(\boldsymbol{p}_k - q\boldsymbol{A}) + q\phi \tag{86}$$

Due to the incorporation of the Pauli operator, the Hamilton operator is now a 2×2 matrix. The Pauli operator $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is formed by Pauli matrixes:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{87}$$

so, the Haug equation can then be rewritten as

$$i\hbar\nabla_t |\psi\rangle = -c\boldsymbol{\sigma}\cdot(\boldsymbol{p}_k - q\boldsymbol{A})|\psi\rangle - q\phi|\psi\rangle \tag{88}$$

Where $|\psi\rangle$ is a two-component spinor wave function or a column vector $|\psi\rangle = \begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}$ and $\boldsymbol{p}_k = i\hbar\nabla$ is the kinetic Compton momentum operator, $\boldsymbol{A}$ is the the magnetic vector potential, and ϕ the electric scalar potential. Equation 88 can be seen as a parallel to the Schrödinger-Pauli equation [11, 14], but this equation is relativistic, while the Schrödinger-Pauli equation is non-relativistic. This equation is also simpler, in general, as the time and space dimensions are of the same derivative order, while in the Schrödinger-Pauli equation, the time derivative is of the first order and the space dimension is of the second order (derivatives).

14 Conclusion

We have introduced two new relativistic wave equations that seems fully valid. They are rooted in what we can call the Compton momentum, which is linked to the Compton wavelength rather than the standard momentum, which is linked to the de Broglie wavelength. We claim the standard momentum is not defined for rest-mass particles, and also that it is likely just a derivative of the more physical Compton momentum. This has likely made the interpretation of results from modern wave mechanics extraordinarily difficult, and we suspect one therefore has not yet reached the bottom of the quantum world. We have reasons to think that our new relativistic wave equations can give us fresh and additional insights, but we leave this up to further studies by ourselves and others to develop the ideas more fully. A series of tasks lie ahead to test this, in order to determine for which particles these two relativistic wave equations are valid, and what can they predict that we can test and check their validity against. Still, even if much work is left to do, at this stage we feel it is important for the physics community to be aware of this additional and alternative path for deriving quantum wave equations, as this may help to improve our understanding of the quantum world in the near future.

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