Deeper insight on Existing and New Wave Equations in Quantum Mechanics

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Abstract

In this paper, we discuss in brief the most common wave equations in quantum mechanics and some recent development in wave mechanics. We also present two new quantum wave mechanics equations based on the Compton momentum. We have good reasons to think that the standard momentum is a mathematical derivative of the more fundamental Compton momentum. This will hopefully simplify interpretations of quantum mechanics significantly, our new relativistic wave equations look promising, but need further investigations on what they predict. Still, we feel it is important for the physics community to be aware of this way of looking at quantum mechanics in new light. This is not in conflict with existing equations; instead this is in addition to the collection of existing wave equations. We also prove mathematically that if one satisfies our new relativistic energy Compton momentum relation, one automatically also satisfies the standard relativistic energy momentum relation. They are two sides of the same coin, where the relations to the Compton wave likely represent the deeper reality, so we have reasons to think our new wave mechanics addresses a deeper level of wave mechanics than the existing conception.

Key Words: quantum mechanics, de Broglie wavelength, Compton wavelength.

1 Standard momentum and the de Broglie wave

The relativistic de Broglie wave [1, 2] is given by

$$\lambda_b = \frac{h}{mv\gamma} \tag{1}$$

where h is the Planck constant, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, and v is the velocity of the mass. An important note is that the de Broglie wave is not mathematically defined for a rest-mass particle, as setting $v = 0$ means we are dividing by zero. In addition, if we let v be close to zero, then the de Broglie wave converges towards infinity. Next we solve the de Broglie relation with respect to momentum; this gives

$$p = mv\gamma = \frac{h}{\lambda_b} \tag{2}$$

This means that the momentum is not defined for a rest-mass particle, since λ_b is not defined for a rest-mass particle. This is somewhat new, as modern physics directly and indirectly assumes that the momentum is simply zero when $v = 0$. For any $v > 0$, the formula gives the correct momentum, but again for a rest-mass particle, the standard momentum is not defined.

It seems this has passed the discussion among physicists. We think the likely reason is that the standard momentum was suggested long before the relativistic momentum and the de Broglie matter wave was conceived. The idea that momentum is mass times velocity was suggested in 1721 by John Jennings [3]. Jennings said that momentum is the quantity of matter multiplied by the velocity, which is the standard momentum: $p \approx mv$, which holds when $v << c$. However, we will claim it is not valid for $v = 0$. The momentum suggested by Jennings came long before the development of relativity theory. So, the relativistic momentum $p = mv\gamma$ was probably derived first by Einstein in 1905.

The de Broglie wave was a hypothesis set out by de Broglie in 1923/1924. As it had been shown that light has a particle-wave duality, de Broglie then speculated that matter had the same characteristics, so he assumed the matter wave was given by $\lambda_b = \frac{h}{p} = \frac{h}{mv\gamma}$. That is, the de Broglie wave was derived from the momentum. The fact that something was understood later does not mean that is less fundamental; on the contrary, since we live so far from the quantum world in our everyday lives, physics has mostly developed from the top down. Therefore, we have come up with rules and formulas for macroscopic objects and observations first, then later understood their connection to the particle and the quantum worlds. So, once the quantum world is established, we can just as well derive such things as momentum from there. The point is that the momentum, from a quantum perspective, must be given by the formula 2 and we have shown that this means that neither the de Broglie wave nor the standard momentum formula are valid when $v = 0$. As we will see, this is very important for quantum physics, as all the quantum wave equations of modern physics will be impacted.

1.1 The Compton wave and the new Compton momentum

Around the same time as de Broglie introduced the hypothesis of his matter wave, Compton [4] calculated and indirectly measured what today is known as the Compton wave. The relativistic Compton wave of an electron is given by[1]

$$\lambda_c = \frac{h}{mc\gamma} \tag{3}$$

First of all, here we see there are no issues with $v = 0$, as this just means that $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{0^2}{c^2}}} = 1$. That is, the Compton wave, unlike the de Broglie wave, is mathematically well defined for any velocity of $v < c$. See also how to derive the Compton wave for any mass without knowledge of h [7].

Next, if we follow a similar approach to the one we used for the de Broglie wave, we get

$$p_t = mc\gamma = \frac{h}{\lambda_c} \tag{4}$$

where λ_c here is the Compton wavelength rather than the de Broglie wavelength as in standard momentum, when derived/understood from the quantum entities. We use the symbol p_t to distinguish this from standard momentum p. We can call p_t the Compton momentum, or as we soon will understand total Compton momentum.

This is what we will call the Compton momentum and it is a new type of momentum recently introduced by Haug [5]. Unlike the standard momentum, this momentum is well defined for $v = 0$ as well, since the Compton wavelength λ_c is well defined for $v = 0$. Also it does not have strange properties, such as going towards infinity when v is close to zero, see for example [8]. The Compton wave is always on the scale of the atomic quantum realm (very short compared to anything microscopic).

Further, it is important to note that we can always find the de Broglie wave from the Compton wave; we have $\lambda_b = \lambda_c \frac{c}{v}$. So, if we know the Compton wave, we can calculate the de Broglie wave. The same is true with the standard momentum (the de Broglie momentum); it can always be calculated from the Compton momentum as $p = p_t \frac{v}{c}$.

Why should there be two wavelengths linked to matter? And why should we have two types of momentum? We will suggest that the standard momentum and the de Broglie wave only are derivatives of the true matter wave and the true and deeper physical momentum, namely what we call Compton momentum. If we should connect the standard energy definition to the Compton momentum, we simply get

$$E = \mathbf{p}_t c \tag{5}$$

That is, the total energy is equal to the Compton momentum multiplied by the speed of light; this is a new (additional) relativistic energy momentum relation and means we can also derive a new quantum mechanical wave equation from this new relation, which will be the relation between energy and the Compton momentum.

It is worth mentioning that the standard momentum is never observed directly – it is a mathematical construct. First, assume we have a brass ball; we can measure its relative weight relative to one kg and then find its mass relative to one kg. Second, we can put this brass ball in motion. We can then measure its velocity, but we cannot directly observe mv. What we can observe is the impact from its kinetic energy. We can drop a brass ball, for example, and measure its velocity just before it hit a brick of "soft" clay. Most of the kinetic energy will then be used to make an indent in the clay. Gravesande [9] did this and confirmed that experimentally the kinetic energy was a function of v^2 and not just of v. At that time, the question of whether the kinetic energy was a function of v or v^2 had been a debate among leading physicists for many years. And at least when $v << c$, the kinetic energy is a

[1] Actually this is a relativistic extension of Compton's work, see [5, 6].

function of v^2. So, indeed we can measure the kinetic energy of a moving body, and the mass of a body easily when it is at rest, and we can easily measure the velocity of a body, but we cannot measure mv; this is a mathematical entity, that is, however, linked to real observable entities, so it can be very useful. What about our new Compton momentum? Can it be observed more directly? It should be possible because, as we have explained here, we think the standard momentum is a derivative of the true (more real) momentum.

First, looking at our new Compton momentum: when $v = 0$, we get

$$\mathbf{p}_t = mc\gamma = mc \tag{6}$$

In other words, we get a rest-mass momentum that is mc. This is not easy to observe, as it is an embedded momentum, a rest-mass momentum. This may sound strange, as we are not used to thinking of rest-mass momentum, and some may even say that this is impossible, as momentum is related to something that is moving. However, that is the standard momentum that indeed only is defined for something that is moving. This is nothing more strange than rest-mass energy. If the rest-mass momentum is $p_r = mc$, then we must also have what we can call a kinetic momentum, and this must be

$$\mathbf{p}_k = \mathbf{p}_t - \mathbf{p}_r = mc\gamma - mc \tag{7}$$

This formula holds for any $v < c$. It would require advanced laboratory equipment to test this when v is significantly close to the speed of light c. However, when $v << c$ what do we expect to observe? When $v << c$, we can approximate the formula above with the first term of a Taylor expansion, and we then get

$$\mathbf{p}_k \approx \frac{1}{2} m \frac{\mathbf{v}^2}{c} \tag{8}$$

That is, our Compton momentum is a function of v^2 and not of v, so it is kinetic energy divided by c. Our Compton momentum is exactly the same function of v as kinetic energy, but it is simply our standard energy definition divided by c, a significant finding. Our Compton momentum is observable through measurements of impacts, while standard momentum is not, as it is a function of v and not v^2. This supports our view that our newly defined Compton momentum is the real momentum and that the standard momentum is a derivative of this momentum. While the reader may not wish to take this for granted, it will be helpful to be open to the thought that there can be a momentum (or another term could be chosen and defined) that is linked to the Compton wavelength, and next we will look at the Relativistic Energy Momentum Relation in more detail.

2 Relativistic Energy Momentum Relation

The standard relativistic energy momentum of Einstein [10] is given by

$$E^2 = \mathbf{p}^2 c^2 + m^2 c^4 \tag{9}$$

the standard momentum and this relation have played a central role in developing the well-known quantum mechanical wave equations. If our analysis is correct and the standard momentum (de Broglie momentum) not is valid for rest-mass particles and further, if it is a mathematical derivative of the Compton momentum, then any quantum mechanical wave equation derived from it will be a wave equation linked to a derivative and all such quantum mechanical wave equations will probably also not be valid for rest-mass particles.

Our new relativistic energy momentum relation is

$$E = \mathbf{p}_t c = \mathbf{p}_k + mc^2 \tag{10}$$

When deriving a quantum mechanical wave equation consistent with this, the equation should also be valid for rest-mass particles, and should also be more directly linked to the depth of the reality, and therefore will likely be easier to interpret.

This means we are claiming that there are two types of relativistic energy momentum relations: one related to the standard momentum, that is linked to the de Broglie wave, and one that is linked to the Compton momentum, that is linked to the Compton wave. These two energy momentum relations are two sides of the same coin, so to speak, as shown in the derivation below. We have decided to show it rigorously line by line, as the connection is important to understand.

$$\begin{aligned}
E^2 &= p^2c^2 + m^2c^4 \\
E^2 &= p_0^2c^2\gamma^2 + m^2c^4 \\
E^2 &= m^2v^2c^2\gamma^2 + m^2c^4 \\
E^2 &= m^2c^4v^2/c^2\gamma^2 + m^2c^4 \\
E^2 &= m^2c^4\left(\frac{v^2}{c^2} - 1\right)\gamma^2 + m^2c^4\gamma^2 + m^2c^4 \\
E^2 &= \frac{m^2c^4\left(\frac{v^2}{c^2} - 1\right)}{1 - \frac{v^2}{c^2}} + m^2c^4\gamma^2 + m^2c^4 \\
E^2 &= \frac{-1 \times m^2c^4\left(\frac{v^2}{c^2} - 1\right)}{-1 \times \left(1 - \frac{v^2}{c^2}\right)} + m^2c^4\gamma^2 + m^2c^4 \\
E^2 &= -m^2c^4 + m^2c^4\gamma^2 + m^2c^4 \\
E^2 &= m^2c^4\gamma^2 \\
E &= mc^2\gamma \\
E &= p_tc = p_kc + mc^2 \qquad (11)
\end{aligned}$$

This means that if we build wave equations that satisfy the relativistic energy Compton momentum relation, then we automatically satisfy the standard relativistic energy momentum relation and also the other way around. However, as we soon will see, this gives us two new relativistic wave equations and will enable us to look at existing problems from a new angle. This angle appears to be more directly linked to physical reality, as we have claimed the de Broglie wave is a mathematical derivative of the Compton wave. Although we do not expect anyone to take this for granted, we think it is interesting enough to warrant considerable further study and investigation outside of what we are able to cover in this paper.

Next, we will shortly discuss a series of well-known quantum mechanical wave equations and also show a few new quantum mechanical wave equations.

3 Two New Relativistic Wave Equations

We have that

$$E = \mathbf{p}_tc \qquad (12)$$

This can be rewritten as

$$E = \mathbf{p}_tc - mc^2 + mc^2 = \mathbf{p}_kc + mc^2 \qquad (13)$$

where $\mathbf{p}_t = \frac{mc}{\sqrt{1-\frac{\mathbf{v}^2}{c^2}}}$ and $\mathbf{p}_k = \frac{mc}{\sqrt{1-\frac{\mathbf{v}^2}{c^2}}} - mc$, in other words, the total and kinetic Compton momentum. From this we get the following quantum wave equation, using energy operator: $\hat{E} = i\hbar\frac{\partial}{\partial t}$, and kinetic Compton momentum operator: $\hat{\mathbf{p}}_k = -i\hbar\nabla$, we get

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-ci\hbar\nabla + mc^2\right)\psi \qquad (14)$$

This we can rewrite as

$$\begin{aligned}
i\frac{\partial\psi}{\partial t} &= \left(-ic\nabla + \frac{mc^2}{\hbar}\right)\psi \\
i\frac{\partial\psi}{\partial t} &= \left(-ic\nabla + \frac{c}{\bar{\lambda}_c}\right)\psi \qquad (15)
\end{aligned}$$

This is relativistic quantum wave equation[2] consisting of a first order PDE where $-i\hbar\nabla$ is the operator linked to the kinetic Compton momentum. It has nice properties, as time and spatial dimensions are on the same order (first order), unlike the Schödinger equation where the time and spatial dimension are on a different order in terms of derivatives. It is well known that the Schödinger equation, therefore, is not Lorentz invariant. Our new relativistic wave equation, which is fully rooted in the Compton wave, is, on the other hand, also mathematically Lorentz invariant.

Our new relativistic quantum equation has quite a different plane wave solution than the Klein–Gordon and Schrödinger equations, but at first glance it looks exactly the same:

$$\psi = e^{i(kx-\omega t)} \tag{16}$$

But in our theory, we should have $k = \frac{2\pi}{\lambda_c}$, where λ_c is the relativistic Compton wavelength and not the de Broglie wavelength, λ_b, as in standard wave mechanics. We can rewrite write the plane wave solution as

$$e^{i\left(\frac{p_k}{\hbar}x-\frac{E}{\hbar}t\right)} \tag{17}$$

where p_k is the total kinetic Compton momentum, as defined earlier. So, our quantum wave function is rooted in the Compton wavelength instead of the de Broglie wavelength. For the formality of it, we look at the momentum and energy operators and see that they are correctly specified

$$\frac{\partial\psi}{\partial x} = \frac{i\mathbf{p}_k}{\hbar}e^{i\left(\frac{\mathbf{p}_k}{\hbar}x-\frac{E}{\hbar}t\right)} \tag{18}$$

This means the momentum operator must be

$$\hat{\mathbf{p}}_k = -i\hbar\nabla \tag{19}$$

and for energy we have

$$\frac{\partial\psi}{\partial t} = \frac{-iE}{\hbar}e^{i\left(\frac{p_k}{\hbar}x-\frac{E}{\hbar}t\right)} \tag{20}$$

and this gives us a time operator of

$$\hat{E} = i\hbar\frac{\partial}{\partial t} \tag{21}$$

The momentum and energy operators are the same as under standard quantum mechanics.

3.1 Wave equation based on operator on total Compton momentum

We also get a relativistic quantum wave equation linked to the total momentum instead of the kinetic momentum, this is because

$$E = \mathbf{p}_k c + mc^2 = \mathbf{p}_t c \tag{22}$$

This gives

$$i\hbar\frac{\partial\psi}{\partial t} + ci\hbar\nabla\psi = 0 \tag{23}$$

which can be simplified to (see also [5] that basically gives the same wave equation, but we suspect with a sign error.)

$$\frac{\partial\psi}{\partial t} + c\nabla\psi = 0 \tag{24}$$

That is, we have two new quantum mechanical wave equations. Also in this case it has quite a different plane wave solution than the Klein–Gordon and Schrödinger equations, but at first glance it looks exactly the same:

$$\psi = e^{i(kx-\omega t)} \tag{25}$$

[2] We actually put out a rough working draft paper with this equation in December 28, 2018 [11], and also we did not look into it much before we adjusted the working paper a few days later to center on a wave equation with focus on total Compton momentum rather than kinetic, something we soon will return to later. The wave equation we put out at that time likely had a sign error due to the fact that we had the wrong sign on the energy operator; the equation we presented then was $-i\frac{\partial\psi}{\partial t} = \left(-ic\nabla + \frac{mc^2}{\hbar}\right)\psi$

However, here $k = \frac{2\pi}{\lambda_c}$, where λ_c is the relativistic Compton wavelength and not the de Broglie wavelength, as in standard wave mechanics. We can rewrite the plane wave solution as

$$e^{i\left(\frac{\mathbf{p}_t}{\hbar}x - \frac{E}{\hbar}t\right)} \tag{26}$$

where $\mathbf{p}_t$ is the total Compton momentum, as defined earlier. So, our quantum wave function is rooted in the Compton wavelength instead of the de Broglie wavelength. For the formality of it, we look at the momentum and energy operators and see that they are correctly specified

$$\frac{\partial \psi}{\partial x} = \frac{i\mathbf{p}_t}{\hbar} e^{i\left(\frac{p_t}{\hbar}x - \frac{E}{\hbar}t\right)} \tag{27}$$

This means the momentum operator must be

$$\hat{\mathbf{p}}_t = -i\hbar\nabla \tag{28}$$

and for energy we have

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar} e^{i\left(\frac{p_t}{\hbar}x - \frac{E}{\hbar}t\right)} \tag{29}$$

and this gives us a time operator of

$$\hat{E} = i\hbar\frac{\partial}{\partial t} \tag{30}$$

That is, the momentum and energy operators are the same as are used in already established quantum mechanics.

4 The Klein–Gordon Equation

Another well-known relativistic quantum equation is the Klein–Gordon equation (see for example [12]), which is given by

$$E^2 = \mathbf{p}^2c^2 + m^2c^4 \tag{31}$$

where $\mathbf{p}$ is the relativistic (de Broglie) momentum. When replacing E and $\mathbf{p}$ with their energy, $\hat{E} = i\hbar\frac{\partial}{\partial t}$, and momentum operator, $\hat{\mathbf{p}} = -i\hbar\nabla$, we get the following wave equation

$$\begin{aligned} i^2\hbar^2\frac{\partial^2\psi}{\partial t^2} - i^2\hbar^2c^2\nabla^2\psi - m^2c^4\psi &= 0 \\ -\hbar^2\frac{\partial^2\psi}{\partial t^2} + \hbar^2c^2\nabla^2\psi - m^2c^4\psi &= 0 \\ \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi &= 0 \end{aligned} \tag{32}$$

The last line is how the Klein–Gordon equation is often presented. Since the reduced Compton wave is given by $\bar{\lambda}_c = \frac{\hbar}{mc}$, we can replace $\frac{m^2c^2}{\hbar^2}$ in the equation above with $\frac{1}{\bar{\lambda}_c^2}$ and we get

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi + \frac{1}{\bar{\lambda}_c^2}\psi = 0 \tag{33}$$

The Klein–Gordon equation is indirectly liked to the de Broglie momentum (standard momentum), $\mathbf{p} = m\mathbf{v}\gamma$, which we claim is a derivative of the real momentum, the Compton momentum. The Klein–Gordon equation is therefore unnecessarily complex. Yet, it cannot be simplified further if we want a relativistic wave equation from the de Broglie momentum. The formula is likely not valid for a rest-mass particle, since it is derived from the de Broglie momentum.

The Klein–Gordon equation is often written as

$$(\Box + \mu^2)\psi = 0 \tag{34}$$

where $\Box$ is the d'Alembert operator: $\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$, and $\mu = \frac{mc}{\hbar} = \frac{1}{\bar{\lambda}_c^2}$. Do not let unfamiliar notation stop you from exploring the mysteries of quantum mechanics.

The Klein–Gordon equation is only valid for spin zero particles, and the only particle detected it is valid for is therefore the Higgs boson. The Klein–Gordon equation has unwanted properties such as it allows negative probabilities, which is what motivated Dirac as well as Schrödinger to come up with alternative wave equations.

5 The Schrödinger Equation

We have

$$\begin{aligned} E &= E_k + mc^2 \\ E &= mc^2\gamma - mc^2 + mc^2 \end{aligned} \tag{35}$$

when $v << c$ we can approximate the kinetic energy from the first term of a Taylor series expansion with $E_k \approx \frac{1}{2}mv^2$, this gives

$$\begin{aligned} E &\approx \frac{1}{2}mv^2 + mc^2 \\ E &\approx \frac{\mathbf{p}^2}{2m} + mc^2 \end{aligned} \tag{36}$$

If we now use a momentum operator of $\hat{p} = -i\hbar\nabla$ and an energy operator of $\hat{E} = i\hbar\frac{\partial}{\partial t}$, we can write this as a wave equation, something giving us the Schödinger [13] equation

$$i\hbar\frac{\partial\psi}{\partial t} \approx \left(\frac{-i^2\hbar^2}{2m}\nabla^2 + mc^2\right)\psi \tag{37}$$

This we can rewrite further

$$\begin{aligned} i\hbar\frac{\partial\psi}{\partial t} &\approx \left(\frac{-\hbar^2}{2m}\nabla^2 + mc^2\right)\psi \\ i\frac{\partial\psi}{\partial t} &\approx \left(\frac{-\hbar}{2m}\nabla^2 + \frac{mc^2}{\hbar}\right)\psi \\ i\frac{\partial\psi}{\partial t} &\approx \left(\frac{-c\bar{\lambda}_c}{2}\nabla^2 + \frac{c}{\bar{\lambda}_c}\right)\psi \end{aligned} \tag{38}$$

Note that when rewritten this way, there is no Planck constant in the Schrödinger equation in this rewritten form. Also note that the imaginary number does not go away as it does in the Klein–Gordon equation. More important is that the Schrödinger equation does not treat the time (energy) on equal footing with the spatial dimension (momentum). This is because we have first, versus second-order derivatives in the same equation. This is unlike the Klein–Gordon equation, where everything is on second order, and also unlike the Dirac equation, where all is first order. Our new wave equations are both first order and therefore time and the spatial dimension of the same order.

6 Dirac Equation

The Klein–Gordon equation was not suitable to model the electron, as it only allows for zero spin, while electrons are $\frac{1}{2}$ spin fermions. Also, the fact that Klein–Gordon allows such things as negative probabilities pushed Dirac [14] out to develop a new relativistic wave equation, that is given by

$$i\hbar\frac{\partial\psi}{\partial t} - \left(c\sum_{i=n}^{3}\alpha_n\mathbf{p}_n - \beta mc^2\right)\psi = 0 \tag{39}$$

this can be rewritten as

$$i\frac{\partial\psi}{\partial t}-\left(\frac{c}{\hbar}\sum_{i=n}^{3}\alpha_n\mathbf{p}_n-\beta\frac{c}{\bar{\lambda}_c}\right)\psi=0 \tag{40}$$

So, there is a Compton wave embedded in the Dirac equation, but the momentum operator is operating on the standard momentum that is linked to the de Broglie wave.

7 Summary

Table 1 shows a summary of three well-known wave equations in quantum mechanics, as well as two new ones. The three older equations are all rooted in standard momentum and therefore the de Broglie wavelength. In addition, these equations contain the Compton wavelength embedded in their last term, but the momentum they are deriving their equations from involve the de Broglie equivalent momentum. The standard momentum is not defined for $v=0$, that is to say, for rest-mass particles, so we suggest that these three traditional wave equations likely not are valid for rest-mass particles. In addition, these three wave equations are, to a large degree, modeling mathematical derivatives of reality rather than the deeper reality because the de Broglie momentum is a derivative of the more fundamental Compton momentum. The two new wave equations are linked directly to the Compton momentum; therefore, they are simpler and also hold for $v=0$.

	Wave equations:	**Momentum operator on :**	**Comments :**
Klein–Gordon (Spin 0)	$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}-\nabla^2\psi+\frac{m^2c^2}{\hbar^2}\psi=0$	de Broglie momentum	Relativistic
deeper level	$\frac{\partial^2\psi}{\partial t^2}-c^2\nabla^2\psi+\frac{c^2}{\bar{\lambda}_c^2}\psi=0$		
Dirac (Spin 1/2)	$i\hbar\frac{\partial\psi}{\partial t}-\left(c\sum_{i=n}^{3}\alpha_n\mathbf{p}_n-\beta mc^2\right)\psi=0$	de Broglie momentum	Relativistic
deeper level	$i\frac{\partial\psi}{\partial t}-\left(\frac{c}{\hbar}\sum_{i=n}^{3}\alpha_n\mathbf{p}_n-\beta\frac{c}{\bar{\lambda}_c}\right)\psi=0$		
Schrödinger	$i\frac{\partial\psi}{\partial t}\approx\left(\frac{-\hbar}{2m}\nabla^2+\frac{mc^2}{\hbar}\right)\psi$	de Broglie momentum	Non-relativistic
deeper level	$i\frac{\partial\psi}{\partial t}\approx\left(\frac{-c\bar{\lambda}_c}{2}\nabla^2+\frac{c}{\bar{\lambda}_c}\right)\psi$		
Haug-1: (Spin ?)	$i\frac{\partial\psi}{\partial t}=\left(-ic\nabla+\frac{mc^2}{\hbar}\right)\psi$	Kinetic Compton momentum	Relativistic
deeper level	$i\frac{\partial\psi}{\partial t}=\left(-ic\nabla+\frac{c}{\bar{\lambda}_c}\right)\psi$		
Haug-2: (Spin ?)	$\frac{1}{c}\frac{\partial\psi}{\partial t}+\nabla\psi=0$	Total Compton momentum	Relativistic

Table 1: The table shows a summary of three well-known quantum mechanical wave equations derived from standard momentum (de Broglie) momentum, and two new quantum mechanical wave equations derived from the Compton momentum.

- The Schrödinger, Klein–Gordon, and Dirac equations all use a momentum operator on the standard momentum. The standard momentum is, at a quantum level, actually directly linked to the de Broglie wave. The de Broglie wave is not mathematically defined for a rest-mass particle. Second, the de Broglie wave and also the standard momentum are just mathematical derivatives of the likely much more fundamental Compton wave and what we call Compton momentum. In other words, in the traditional equations, we are taking partial derivatives of mathematical functions of reality, not of the deepest entities. This makes the Schrödinger, Klein–Gordon, and Dirac equations all very hard to interpret at a deeper level. Of course, they are "easy" enough to interpret at the surface inside the exotic zoo of terminology that has evolved in physics and quantum physics, as long as the analysis of modern physics does not go too deep. Lots of work over decades have gone into analyzing these three equations, while we are just at the doorsteps of the two new relativistic quantum equations rooted in the Compton wave.

- We have strong reasons to believe that our new quantum mechanical wave equations are better suited to understanding certain aspects of the depth of reality. They are mathematically correct for $v=0$, and they are more directly linked to the depth of reality, as we are modeling from the Compton momentum directly instead of the derivative of it, which is the de Broglie momentum and its corresponding de Broglie wave. Also, our equations are both relativistic and always of first order. This could even help to answer such questions as if quantum mechanics is consistent with Minkowski space-time, something that according to several leading physicists not is clear, see for example [15].

- There is no Planck constant in the any of the wave equations except for the Dirac equation that we will soon comment on separately. The apparent Planck constant in the Schrödinger and Klein–Gordon all cancel out against a Planck constant that is hidden in the mass. What we obtain is the Compton wavelength in the Schrödinger and Klein–Gordon equations, or more precisely, the Compton frequency is also embedded in these equations. One might think that such a line of thought is wrong, if one believed that the Planck constant is needed to find the Compton wavelength. However, this is not the case; even from the Compton 1923 paper it is clear that one can find the Compton wavelength without any knowledge of the Planck constant. Actually, one can find the Compton wave for any mass without any knowledge of any fundamental constant, see [7]. In the Dirac equation, the Planck constant will indirectly cancel out as well, as the momentum operator on the wave function returns the momentum, and the momentum embedded contains the Planck constant in the mass will then cancel out against the Planck constant we see here. In our new wave equation from the Compton momentum, the Planck constant also cancels out.

8 Conclusion

We have introduced two new relativistic wave equations that seems fully valid. They are rooted in what we can call the Compton momentum, which is linked to the Compton wavelength rather than the standard momentum, which is linked to the de Broglie wavelength. We claim the standard momentum is not defined for rest-mass particles, and also that it is likely just a derivative of the more physical Compton momentum. This has likely made the interpretation of results from today?s wave mechanics extraordinarily difficult, and we suspect one therefore has not yet reached the bottom of the quantum world. We have reasons to think that our new relativistic wave equations can give us fresh and additional insights, but we leave this up to further studies by ourselves and others to develop the ideas more fully. A series of tasks lie ahead to test this, in order to determine for which particles these two relativistic wave equations are valid, and what can they predict that we can test and check their validity against. Still, even if much work is left to do, at this stage we feel it is important for the physics community to be aware of this additional and alternative path for deriving quantum wave equations, as this may help to improve our understanding of the quantum world in the near future.

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