Prime and Twin Prime Theory

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Abstract

In this article, we use method of a modified sieve of Eratosthenes to prove the prime and the twin prime theory.

We use p_i for all the primes, 2,3,5,7,11,13,....., i=1,2,3,.....,

If a prime pair (p_m, p_{m+1}) is a twin prime, then it can be written as (6k-1, 6k+1) for some k.

Let $p_m\sharp = \prod_{i=1...m} p_i$,

Theorem 1, When sieve upto p_m, the total number of the remaining numbers { R_j^m}, inside of (0,$p_m\sharp$) is $\prod_{i=1...m}(p_i - 1)$,

We can generate the remaining numbers for period (0, $p_{m+1}\sharp$) when sieve upto p_{m+1} by sequence of the remaining numbers { R_j^m}, inside of (0,$p_m\sharp$) as following;

{ R_j^m}, { $p_m\sharp$ +R_j^m}, {2x $p_m\sharp$ +R_j^m},..., { (p_{m+1}-1)x $p_m\sharp$ +R_j^m},

and then taking out the terms of { $p_{m+1} \times R_j^m$},

Obviously the total number of the remaining numbers when sieve upto p_{m+1} in the period of (1,$p_{m+1}\sharp$) is,

$\prod_{i=1...m}(p_i - 1)$ x p_{m+1} - $\prod_{i=1...m}(p_i - 1) = \prod_{i=1...m+1}(p_i - 1)$, For m=2, the remaining numbers in (0, 6), are 1 and 5 when sieve upto 3, the period of (0,6) is the building blocks for the period (0, 30), and the remainging numbers 1 and 5 are the basic numbers to generate all the remainging numbers in period (0, 30) when sieve upto prime number 5. It can be seen from the following, blocked sequence;

(0,1,2,3,4,5,6)(7,8,9,10,11,12,)(13,14,15,16,17,18)(19,20,21,22,23,24)(25,26,27,28,29,30),

the new generated remainining numbers are;

(,1,,,,5,)(7,,,,11,,)(13,,,,17,)(19,,,,23,)(25,,,,29,),

there remaining twin pairs of

(5,7), (11,13), (17,19),(23,25), (29,1),

if we treat (29,1) as one twin.

after taking out the two remaining numbers 1 and 5 time 5, it left the following remaining sequence;

(,1,,,,,...,)(7,,,,11,,)(13,,,,17,)(19,,,,23,)(...,,,,,29,),

and the remaining twin pairs are,

(11,13),(17,19),(29,1);

It is clear that all the remining number twins are generated by basic numbers, 1 and 5 too, And all the primes and twin primes are also from the two basic numbers, 1 and 5.

Similarly we have the following for the remaining twins,

Theorem 2;

When sieve upto p_m, the total number of the remaining number twins inside of (0,$p_m\sharp$) is $\prod_{i=2...m}(p_i - 2)$, In general not all the remaining twins are twin primes. We need to sieve more larger primes to get twin primes.

Let p_M be the least prime satisfied the $p_m\sharp < p_M^2$, then we sieve upto p_M for the period (0, $p_m\sharp$), then all those still remaining numbers are primes and remaining twins are twin primes.

Theorem 3;

When sieve upto p_{m+1}, the total number of the remaining numbers inside period ((k-1)x$p_m\sharp$, k x $p_m\sharp$) is equal approximately to $\prod_{i=1\ldots m+1}(p_i-1)$ / p_{m+1} ± 1,

particulary for the period of (0, $p_m\sharp$),

This is equivalent to the following theorem,

Theorem 4;

For any number d with (d,$p_m\sharp$) =1, no common factor with $p_m\sharp$, when sieve upto p_m, the total number of the remaining numbers inside period (0, $p_m\sharp/d$) is equal approximately to $\prod_{i=1\ldots m}(p_i-1)$ / d ± 1,

When sieve upto p_M, the total number of the remaining numbers inside period (0, $p_m\sharp$) are those remaining numbers when sieve upto p_{M-1} in the same period (0, $p_m\sharp$) subtract those remaining numbers when sieve upto p_{M-1} in the period (0, $p_m\sharp/p_M$) multiplied by p_M.

We use $\{(a,b)\}^M$ to denote those remaining numbers in period (a, b) when sieve upto p_M. We have,

$$\{(0,p_m\sharp)\}^M = \{(0,p_m\sharp)\}^{M-1} - \{\{(0,p_m\sharp/p_M)\}^{M-1} \times p_M\}, \qquad (1)$$

and so on, we have,

$$\{(0, p_m\sharp)\}^{M-1} = \{(0, p_m\sharp)\}^{M-2} - \{\{(0, p_m\sharp/p_{M-1})\}^{M-2} \times p_{M-1}\}, \tag{2}$$

and

$$\{(0, p_m\sharp/p_M)\}^{M-1} = \{(0, p_m\sharp/p_M)\}^{M-2} - \{\{(0, p_m\sharp/p_M p_{M-1})\}^{M-2} \times p_{M-1}\}, \tag{3}$$

and so on and on, we will have,

$$\{(0, p_m\sharp)\}^{M} = \sum_{d|P} \mu(d)\{\{(0, p_m\sharp/d)\}^{m} \times d\}, \tag{4}$$

here P $=\prod_{i=m+1\dots M} p_i$.

There are no remaining number in period (0, $p_m\sharp/d$) when $p_m\sharp/d < 1$, and only one remaining number, 1, when $1 < p_m\sharp/d < p_m$,

We have,

$$|\{(0, p_m\sharp)\}^{M}| = \sum_{d|P} \mu(d)|\{(0, p_m\sharp)\}^{m}|/d \pm ER_m \tag{5}$$

we have,

$$|\{(0, p_m\sharp)\}^{M}| = [\prod_{i=1\dots m} (p_i - 1)] \times [\prod_{i=m+1,\dots M} (1 - 1/p_i)] \pm ER_m \tag{6}$$

here, the ER_m is the possible error,

$ER_m = |\{d; d \mid P, p_m < d < p_{m-1}\sharp\}|$,

$$ER_m = |\{(0, p_{m-1}\sharp)\}^{m}| - |\{(0, p_{m-1}\sharp)\}^{M}| \tag{7}$$

$$ER_m = \prod_{i=1\dots m} (p_i - 1)/p_m - [\prod_{i=1\dots m-1} (p_i - 1)] \times [\prod_{i=m,\dots M} (1 - 1/p_i)] + ER_{m-1} \tag{8}$$

We have,

$$ER_m = \sum_{l=1,m} [\prod_{i=1...l} (p_i - 1)/p_l] \times [1 - \prod_{i=l+1,..M} (1 - 1/p_i)], \tag{9}$$

Then we have,

Theorem 5;

when sieve upto p_M for the $(0,\ p_m\sharp)$, the total number of the remaining primes inside $(0,\ p_m\sharp)$is equal approximately to $\prod_{i=1...m}(p_i - 1)\prod_{j=m+1...M}(1 - 1/p_j)\ \pm ER_m$,

here ER_m as above.

Similarly for the twin primes we have,

Theorem 6;

when sieve upto p_M for the $(0,\ p_m\sharp)$, the total number of the remaining twin primes inside $(0,\ p_m\sharp)$is equal approximately to $\prod_{i=2...m}(p_i - 2)\prod_{j=m+1...M}(1 - 2/p_j)\ \pm ER_m$,

and here ER_m is the same as above.

This also proves the twin prime conjecture.