"NEW TYPES OF PYTHAGORIAN EQUATIONS AND THE RESULTING OUTCONSE" (ELEMENTARY ASPECT)

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Keywords: Pythagorean equations, new types, irrational, complex, whole numbers.

Abstract

Annotation. In the present paper, new types of Pythagorean equations are derived (which are countless) based on the irrational ones that generate them, complex and integer numbers, those proven the fact of the rehabilitation of the system of solutions of the Pythagorean Euclidean equations based on the identities I received.

It is believed, starting with Euclid (the third century BC) to the present, that these (2) and (2a) - and only these formulas given in [1], [2], [3] cover all integer solutions of the Pythagorean equation (1).

It will be proved below that this is not so: the system of solutions given in [1], [2], [3] is not complete, which is a breakthrough in number theory.

§1

1.1. All, as it is considered, integer mutually simple solutions of the equation $x^2+y^2=z^2$ (1) are defined by the following formulas: x= m^2-n^2 y=2mn z= m^2+n^2 (2) , where m and n are arbitrary coprime numbers [1], [2], [3].

1.2. It is believed that integer non-mutually simple solutions (1) are obtained from (2) by the formulas: x=($m^2 - n^2$)s y=2mns z=(m^2+ n^2)s (2a), [1] , [2] , [3] , where s is an arbitrary integer.

§2

Statement: “The classical system for obtaining integer solutions Pythagorean equations (1) in accordance with formulas (2) and (2a) is not complete and should be supplemented by integer solutions of the new types of Pythagorean equations (which are countless) based on the irrational ones that generate them, complex and integer numbers. " (A)

Evidence

2.1. Find integer solutions of the following system of equations:

$\begin{cases} uv = t^2 \\ u + v = c^3 \end{cases}$ (3). Let be $x^2 + y^2 = z^2$,где x= m^2- n^2 y=2mn

z= m^2+ n^2 , (m,n)=1.

Then, if u=(m^2+ n^2)(m^2- n^2)^2=zx^2 v=(m^2+ n^2)(2mn)^2=zy^2

$$\begin{cases} \mathrm{uv} = (m^2 + n^2)^2(m^2 - n^2)^2(2\mathrm{mn})^2 = (\mathrm{xyz})^2 = t^2 \\ \mathrm{u} + \mathrm{v} = (m^2 + n^2)[(m^2 - n^2)^2 + (2\mathrm{mn})^2] = (m^2 + n^2)^3 = z^3 \end{cases} \quad (4).$$

2.2. If u=(m^2- n^2)(m^2+ n^2)^2=xz^2 v=(m-n)(2mn)^2=xy^2 , then

$$\begin{cases} uv = (m^2 - n^2)^2(m^2 + n^2)^2(2mn)^2 = (xyz)^2 = t^2 \\ \mathrm{u} - \mathrm{v} = (m^2 - n^2)^3 = x^3 \end{cases} \quad (5).$$

under identical with p.2.1. m and n.

2.3. If u=(2mn)(m^2+ n^2)^2=yz^2 v=(2mn)(m^2- n^2)^2=yx^2 ,то

$$\begin{cases} \mathrm{uv} = (2\mathrm{mn})^2(m^2 + n^2)^2(m^2 - n^2)^2 = (\mathrm{xyz})^2 = t^2 \\ \mathrm{u} - \mathrm{v} = (2\mathrm{mn})^3 = y^3 \end{cases} \quad (6).$$

under identical with p.2.1. m and n.

2.4. Thus, in (3) all cases related to integer coprime values of m and n.

§3

3.1. Continue the proof. From p.2.1. uv= t^2 u+v= z^3 Then, v= $z^3 - u$

$(z^3 - \text{u})\text{u} = t^2$ $\quad u^2 - z^3 u + t^2 = 0$ $\quad \text{u} = \frac{1}{2}[z^3 \pm \sqrt{(z^3)^2 - (2t)^2}]$.

The root of the root expression will be integer if the equation $(z^3)^2$-(2t)^2 $=\gamma^2$- Pythagoras (7). From here, $z^3 = m_z^2 + n_z^2$ $\quad$ 2t=2 $m_z n_z$ $\quad \gamma = m_z^2 - n_z^2$.

Let us prove that the γ-integer for integers z and t. The signs of absolute values are omitted.

$z^6 - 4x^2y^2z^2 = z^2(z^4 - 4x^2y^2) = z^2[z^4 - 4(z^2 - x^2)\,x^2] = z^2(z^4 - 4z^2x^2 + 4x^4) =$

$z^2(z^2 - 2x^2)^2 = \gamma^2$ $\quad \gamma = \text{z}(z^2 - 2x^2) = \text{z}(y^2 - x^2) = \text{z}y^2 - \text{z}x^2 = m_z^2 - n_z^2$

$$m_z^2 = \text{z}y^2 \quad m_z = \text{y}\sqrt{z} \qquad n_z^2 = \text{z}x^2 \quad n_z = \text{x}\sqrt{z} \qquad (8).$$

The conclusions are confirmed by the following: $m_z = \frac{1}{2}\sqrt{z^3 + \gamma}$ $\quad n_z = \frac{1}{2}\sqrt{z^3 - \gamma}$,

as required to prove.

Thus irrational m_zи n_z generate the Pythagorean equation (7).

Dividing each term of equation (7) by $(\sqrt{z})^4$, we obtain a new classical Pythagorean equation:

$(x^2 + y^2)^2$-(2xy)^2=$(y^2 - x^2)^2$ with the corresponding m and n.

Examples: m=2 n=1 $5^2 - 4^2 = 3^2$ $\quad m_5 = 4\sqrt{5}$ $\quad n_5 = 3\sqrt{5}$ $\quad (5^3)^2$-(2.3.4.5)^2=(16-9)^2

$125^2 - 120^2 = 35^2$. This type of equation is countless.

Dividing each term by 25, we get: $7^2 + 24^2 = 25^2 = 5^4$ m=4 n=3.

(or divide each term without squares by 5).

$5^2 + 12^2 = 13^2$ m=3 n=2 $2197^2 = 1547^2 + 1560^2$. Dividing by 13, we get: $119^2 + 120^2 = 13^4$.

$21^2 + 20^2 = 29^2$ $\quad 1189^2 + 24360^2 = 24389^2$ $\quad 41^2 + 840^2 = 29^4 = 841^2$.

§4

4.1. From p.2.2. by analogy with p.3.1. $\quad \gamma^2 = (x^3)^2 + (2\text{xyz})^2 = \ldots = x^2(x^4 - 4x^2z^2 + 4z^4) =$

$x^2(x^2 - 2z^2)^2$ и $\gamma = \text{x}(x^2 - 2z^2)$ $\quad m_x^2 = \frac{1}{2}(x^3 + x^3 - 2xz^2) = -\text{x}y^2$ $\quad m_x = \text{y}\sqrt{-x}$

$n_x^2 = \frac{1}{2}(x^3 - 2\text{x}z^2 - x^3) = -\text{x}z^2$ $\quad n_x = z\sqrt{-x}$.

Example: $3^2 + 4^2 = 5^2$ $m_x = 5\sqrt{-3}$ $n_x = 4\sqrt{-3}$ $(3^3)^2 + 120^2 = (-123)^2$.
Dividing by 3, we get $3^4 + 40^2 = 41^2$.

§5

5.1. From p.2.3. by analogy with p.4.1. $3^2 + 4^2 = 5^2$ $m_y = 5\sqrt{4}$ $n_y = 3\sqrt{4}$
$(4^3)^2 + 120^2 = 136^2$ $(2^3)^4 + 120^2 = 136^2$ $8^2 + 15^2 = 17^2$.
Thus, the above statement (A) is completely proved. [4]

§6

Another way to obtain new types of Pythagorean equations (from sections 3-5), or, in other words, the fact of rehabilitation of the decision system Euclid based on the identities I obtained (proof):

6.1. If $x^2 + y^2 = z^2$, then $x^4 + (2yz)^2 = (x^2 - 2z^2)^2$ (1) ;
$y^4 + (2xz)^2 = (y^2 - 2z^2)^2$ (2) ;
$z^4 - (2xy)^2 = (z^2 - 2x^2)^2 = (z^2 - 2y^2)^2$ (3).

6.2. Multiplying (1)- on x^2, (2)-on y^2, (3)-on z^2,get:

$(x^3)^2 + (2xyz)^2 = (x^3 - 2z^2x)^2$,
$(y^3)^2 + (2xyz)^2 = (y^3 - 2z^2y)^2$,
$(z^3)^2 - (2xyz)^2 = (z^3 - 2x^2z)^2 = (z^3 - 2y^2z)^2$.

P.S. 1. Frey's elliptic curve: $y^2 = (A^N - x)(B^N + x)$.
Let be $y^2 = t^2$, $u = A^N - x$ $v = B^N + x$. Then, $u + v = A^N + B^N = z^3$.
1.1. By $u_1 = au$ $v_1 = av$ and $a = z^{N-3}$ $A^N + B^N = z^N$ (in particular),
a- arbitrary natural number, including unit.
2. Instructive isomorphism: human immune system and Pythagoras the equation that the new solutions satisfy is completely different, but already known nature.

Literature.

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