STRUCTURES AND PROPERTIES OF INTEGER SEQUENCES GENERATED

FROM

PRIME NUMBERS SEEDS

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Abstract:

A previous paper submitted to viXra by the author on 2020-06-07 *(1)* and more specifically its paragraph 3-5 was related to the generation of primefree integer sequences using, as a seed, a prime numbers subset, a recursive algorithm and a specific formula described again in the next background section of this paper extending the initial work.
Indeed, whereas two prime numbers seeds containing respectively the first 10^3 and $2*10^4$ prime numbers were used in the initial study, the seed size range has been enlarged from 10^2 to 10^7 prime number terms. This allowed to confirm previous results and reinforced the so called primefree conjecture referenced CD-3 established from them.

(1) This paper entitled "Structures and Properties of Integer Sequences generated from prime and nonprime number seeds"can be downloaded at: http://viXra.org/abs/2005.0056 under the viXra subject category: number theory and citation number: 2006.0056.

Key Words: prime numbers seeds, stepwise-algorithm and formula, primefree integer sequences conjecture.

1- IT Tools and VBA program:

- PC: AMD (tm) XP 2800+
  2.08 GHz. RAM: 1.00Go.
- softwares: R x64 4.0.1 Ink, RStudio, Windows and Excel 2010.
- a R program has been developed for sequence calculation.

2- Background:

2-1 Recursive algorithm:

The recursive algorithm starts with a subset of the prime numbers set *(2, 3, 5 , 7, 11,13, 17, 19...)* used as a seed to produce with the formula below a first sequence S_1 which is then used as a new seed to produce with the same formula the next sequence S_2 and so one…

2-2 Formula:

The formula leading with the above described algorithm to primefree sequences and referenced in the previous mentioned paper formula n° 3 is:

$$t_{(i+2+2*j,\, j+2)} = t_{(i+2+2*j,\, j+1)} + t_{(i+3+2*j,\, j+1)} - t_{(i+1+2*j,\, j+1)} - t_{(i+2*j,\, j+1)}$$

with i = 1 to n-3-3*j and j = 0 to m-2
and where (n) is the number of terms of the prime numbers seed equal to the number of rows (n) of a (n*m) matrix where (m) is the number of columns. The prime numbers seed is filed in the first column of the matrix and the sequences produced in the columns 2 to m.

2-3 Main results from the initial work:

- Whereas prime numbers are present in the first few sequences produced by the algorithm and the formula, they are totally absent from the subsequent ones, including from long sequences composed of more than several thousand terms.

- The number of prime numbers in the first sequences deacreases along the sequence.

3- Methodology of the extended study:

It is based on these observations that the work as been exetended to the use of longer prime numbers seeds.
So, whereas only two prime numbers seeds containing respectively the first 10^3 and $2*10^4$ terms of the prime numbers set *(2, 3, 5 , 7, 11,13, 17, 19...)* have been used in the initial work, 6 seeds containing respectively the first 10^2, 10^3, 10^4, 10^5, 10^6 and 10^7 prime numbers have been selected to extend the search for longer primefree sequences.

4- Results:

4-1: Sign and primality of the sequence terms:

For the 6 seeds, whereas, the first sequence contains positive integers only, both positive and negative ones are present in the next sequences. primefree sequences excepted, each sequence is composed of prime and nonprime numbers.

4-2: Number and percentage of primes in sequences:

Tables n° 1 to n° 6 give the number and the percentage of prime numbers in sequences for the 6 prime numbers seeds containing themselves: 10^2, 10^3, 10^4, 10^5, 10^6 and 10^7 prime number terms.

Figure n°1 gives the total number of terms in the first 15 sequences for the 6 prime numbers seeds, a total of 166664340 terms.

Figure n° 2 gives the total number of prime numbers in the first 15 sequences for the 6 prime numbers seeds, a total of 202316, thus representing 0,12 % of the total number of terms.

Figure n° 3 gives the overall percentage of prime numbers in the first 15 sequences for the 6 prime numbers seeds.

Figure n° 4 and n° 5 give respectively the number of prime numbers and their percentage in the S_2 sequences for the 6 prime numbers seeds.

Figure n° 6 and n° 7 give respectively the number of prime numbers and their percentage in the S_3 sequences for the 6 prime numbers seeds.

Figures n° 8 to n° 19 give respectively the number of prime numbers and their percentage in each sequence for each prime numbers seed.

Main observations:

- The number of prime numbers in the first sequences increases with the size of the prime numbers seed, but the percentage of these decreases.

- Sequences which follow S_2 sequences overall contain fewer and fewer prime numbers.

- After a few sequences, the algorithm and the formula generate long primefree sequences. With the first 10^7 prime numbers seed the first primefree sequence contains 9999961 terms. By comparison in the initial work using the first 20000 prime numbers seed, the longest primefree sequence contained only 19973 terms.

4-3: Occurrence of prime numbers in sequences:

Table n° 7 to n° 10 show the occurrence of prime numbers in the sequences produced from the first $10^{2,}$ $10^{3,}$, 10^4, and 10^5 prime numbers seeds.

Main observations:

- 2, 7, 19 and 1831 are the only four prime numbers found in all the sequences produced from the 6 prime numbers seeds.

- For all prime numbers seeds, the first sequence S_1 contains only one prime: *(7)* as the first term of the sequence.

- S_2 sequences contain only 2 primes *(7 and 2), (7)* as the first term, a first *(2)* in the second position and many *(2's)* along the sequence.

- S_5 sequences contain only 2 primes *(19 and 2), (19)* as the first term and *(2's)* along the sequence.

- S_{10} sequences contain only one prime *(1831)* as the first term.

- Other sequences than the ones above mentionned contain either no primes or *(2's).*

- Figure n° 20 shows the distribution of prime numbers in the S_6 sequence for the first 10^4 prime numbers seed.

5- Conclusions:

This extended work well confirm the initial findings of the first paper, filed on 2020-06-07 under the viXra citation number: 2006.0056 in the subject category: number theory, and consolidate the referenced CD- 3 conjecture which can be slightly reformulated as follows:
The formula below:

$$t_{(i+2+2*j,\, j+2)} = t_{(i+2+2*j,\, j+1)} + t_{(i+3+2*j,\, j+1)} - t_{(i+1+2*j,\, j+1)} - t_{(i+2*j,\, j+1)}$$

applied to the prime numbers set *(2, 3, 5 , 7, 11,13, 17, 19...)* used as a seed, generates a sequence which is then used as a new seed to produce the next sequence and so one. When the number of terms of the prime number set tends to + ∞ and after a certain number of iterations, this recursive process leads to an infinite number of primefree sequences containing an infinity of increasingly large composite numbers.

Annex n° 1: tables

Seed/sequence	Seed	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
Number of terms in seed and sequences	10^2	97	94	91	88	85	82	79	76	73
Number of primes in seed and sequences	10^2	1	7	0	0	1	0	0	0	0
% of primes in seed and sequences	100	1,03	7,45	0	0	1,18	0	0	0	0

Sequence	S_{10}	S_{11}	S_{12}	S_{33}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}
Number of terms in sequences	70	67	64	61	58	55	52	49	46
Number of primes in sequences	1	0	0	0	0	0	0	0	0
% of primes in sequences	1,43	0	0	0	0	0	0	0	0

Table n° 1: seed and sequence length and prime numbers.
seed: first 10^2 prime numbers from 2 to 541.

Seed/sequence	Seed	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
Number of terms in seed and sequences	10^3	997	994	991	988	985	982	979	976	973
Number of primes in seed and sequences	10^3	1	33	10	3	5	0	0	0	0
% of primes in seed and sequences	100	0,10	3,32	1,01	0,30	0,51	0	0	0	0

Sequence	S_{10}	S_{11}	S_{12}	S_{33}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}
Number of terms in sequences	970	967	964	961	958	955	952	949	946
Number of primes in sequences	1	0	0	0	0	0	0	0	0
% of primes in sequences	0,10	0	0	0	0	0	0	0	0

Table n° 2: seed and sequence length and prime numbers.
seed: first 10^3 prime numbers from 2 to 7919.

Seed/sequence	Seed	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
Number of terms in seed and sequences	10^4	9997	9994	9991	9988	9985	9982	9979	9976	9973
Number of primes in seed and sequences	10^4	1	222	68	27	14	6	1	0	0
% of primes in seed and sequences	100	0,01	2,22	0,68	0,27	0,14	0,06	0,01	0	0

Sequence	S_{10}	S_{11}	S_{12}	S_{33}	S_{14}	S_{15}	S_{16}	S_{17}
Number of terms in sequences	9970	9967	9964	9961	9958	9955	9952	9949
Number of primes in sequences	1	0	0	0	0	0	0	0
% of primes in sequences	0,01	0	0	0	0	0	0	0

Table n° 3: seed and sequence length and prime numbers.
seed: first 10^4 prime numbers from 2 to 104729.

Seed/sequence	Seed	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
Number of terms in seed and sequences	10^5	99997	99994	99991	99988	99985	99982	99979	99976
Number of primes in seed and sequences	10^5	1	1685	505	228	86	32	7	1
% of primes in seed and sequences	100	0,001	1,69	0,51	0,23	0,086	0,032	0,007	0,001

Sequence	S_9	S_{10}	S_{11}	S_{12}	S_{33}	S_{14}	S_{15}	S_{16}	S_{17}
Number of terms in sequences	99973	99970	99967	99964	99961	99958	99955	99952	99949
Number of primes in sequences	1	2	0	0	0	0	0	0	0
% of primes in sequences	0,001	0,002	0	0	0	0	0	0	0

Table n° 4: seed and sequence length and prime numbers.
seed: first 10^5 prime numbers from 2 to 1299709.

Seed/sequence	Seed	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
Number of terms in seed and sequences	10^6	999997	999994	999991	999988	999985	999982	999979	999976
Number of primes in seed and sequences	10^6	1	13786	4338	1783	587	199	72	28
% of primes in seed and sequences	100	0,0001	1,38	0,43	0,18	0,059	0,02	0,0072	0,0028

Sequence	S_9	S_{10}	S_{11}	S_{12}	S_{33}	S_{14}	S_{15}	S_{16}	S_{17}
Number of terms in sequences	999973	999970	999967	999964	999961	999958	999955	999952	999949
Number of primes in sequences	8	3	0	0	0	0	0	0	0
% of primes in sequences	0,0008	0,0003	0	0	0	0	0	0	0

Table n° 5: seed and sequence length and prime numbers.
seed: first 10^6 prime numbers from 2 to 15485863.

Seed/sequence	Seed	S_1	S_2	S_3	S_4	S_5	S_6	S_7
Number of terms in seed and sequences	10^7	9999997	9999994	9999991	9999988	9999985	9999982	9999979
Number of primes in seed and sequences	10^7	1	117710	38267	14900	5052	1738	591
% of primes in seed and sequences	100	0,00001	1,18	0,38	0,15	0,051	0,017	0,0059

Sequence	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{33}	S_{14}	S_{15}
Number of terms in sequences	9999976	9999973	9999970	9999967	9999964	9999961	9999958	9999955
Number of primes in sequences	198	75	20	7	1	0	0	0
% of primes in sequences	0,002	0,0008	0,0002	0,0001	0,00001	0	0	0

Table n° 6: seed and sequence length and prime numbers.
seed: first 10^7 prime numbers from 2 to 179424673.

Sequence	Prime numbers	Occurence
S_1	7	1
S_2	7 2	1 6
S_3	-----	0 prime
S_4	-----	0 prime
S_5	19	1
S_6	-----	0 prime
S_7	-----	0 prime
S_8	-----	0 prime
S_9	-----	0 prime
S_{10}	1831	1
S_{11} to S_{18}	-----	0 prime

Table n° 7: occurrence of prime numbers in sequences.
seed: first 10^2 prime numbers from 2 to 541

Sequence	Prime numbers	Occurence
S_1	7	1
S_2	7 2	1 32
S_3	2	10
S_4	2	3
S_5	19 2	1 4
S_6	-----	0 prime
S_7	-----	0 prime
S_8	-----	0 prime
S_9	-----	0 prime
S_{10}	1831	1
S_{11} to S_{18}	----	no prime

Table n° 8: occurrence of prime numbers in sequences.
seed: first 10^3 prime numbers from 2 to 7919

Sequence	Prime numbers	Occurence
S_1	7	1
S_2	7 2	1 221
S_3	2	68
S_4	2	27
S_5	19 2	1 13
S_6	2	6
S_7	2	1
S_8	----	no prime
S_9	----	no prime
S_{10}	1831	1
S_{11} to S_{17}	----	no prime

Table n° 9: occurrence of prime numbers in sequences.
seed: first 10^4 prime numbers from 2 to 104729

Sequence	Prime numbers	Occurence
S_1	7	1
S_2	2 7	1684 1
S_3	2	505
S_4	2	228
S_5	19 2	1 85
S_6	2	32
S_7	2	7
S_8	2	1
S_9	2	1
S_{10}	1831 2	1 1
S_{11} to S_{16}	----	no prime

Table n° 10: occurrence of prime numbers in sequences.
seed: first 10^5 prime numbers from 2 to 1299709

Annex n° 2 : figures

Figure n° 1: total number of terms in the first 15 sequences = f (seed size)

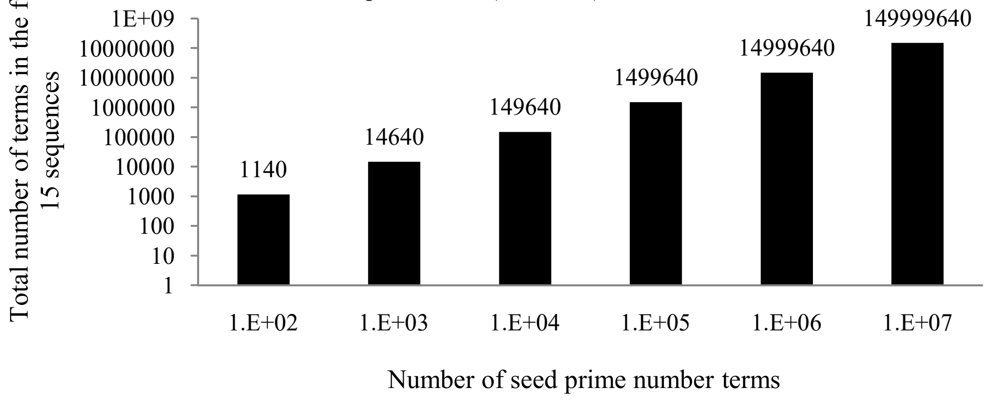

Figure n° 2 : total number of prime numbers in the first 15 sequences = f (seed size)

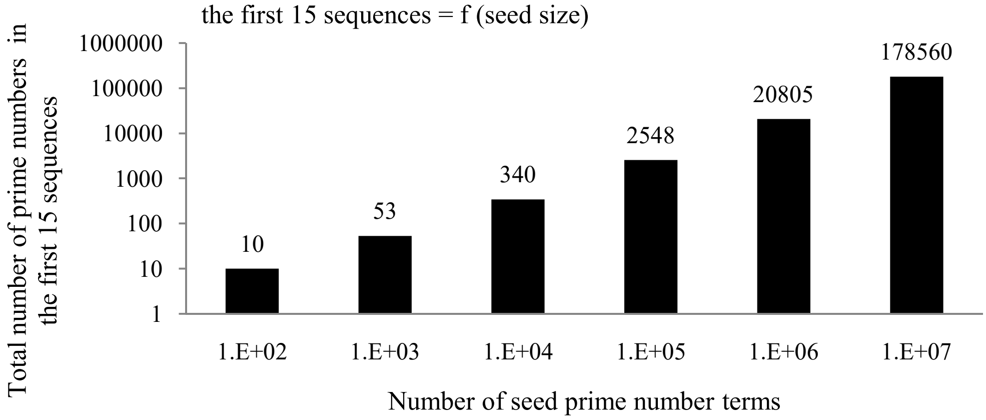

Figure n° 3 : overall percentage of prime numbers in the first 15 sequences = f(seed size)

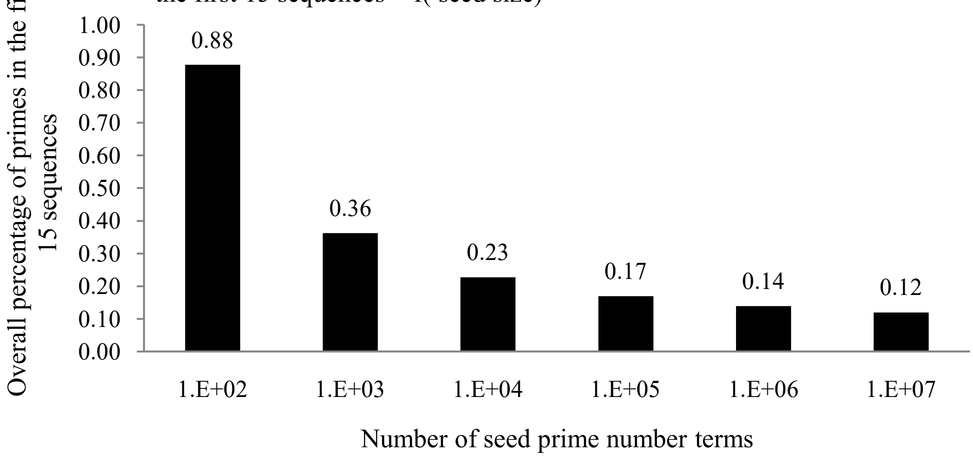

Figure n° 4: number of prime numbers in the S_2 sequence = f (seed size)

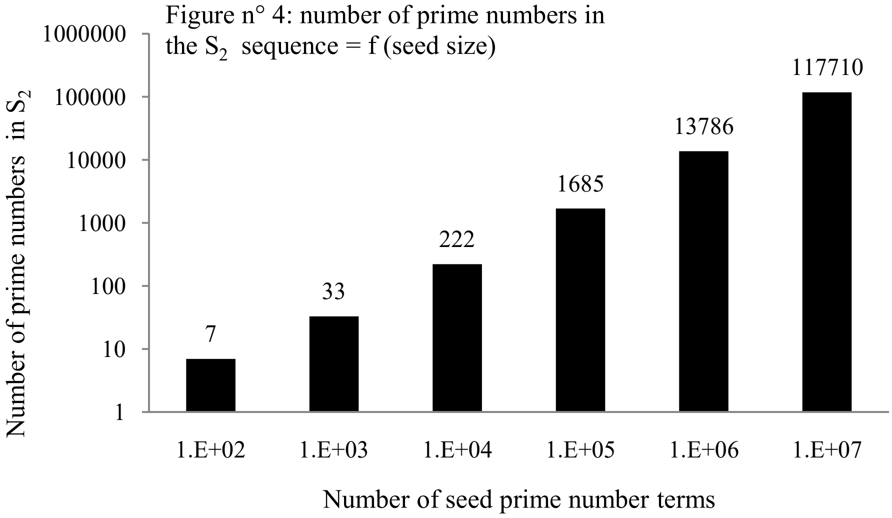

Figure n° 5: percentage of prime numbers in the S_2 sequence = f (seed size)

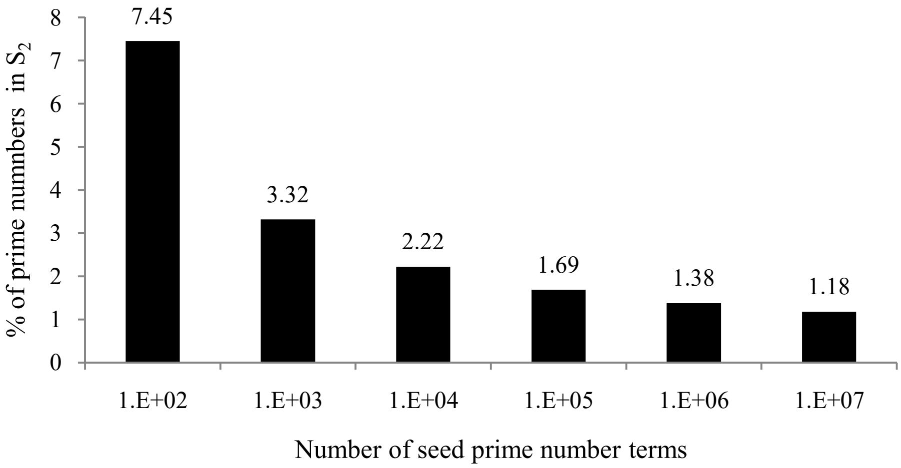

Figure n° 6: number of prime numbers in the S_3 sequence = f (seed size)

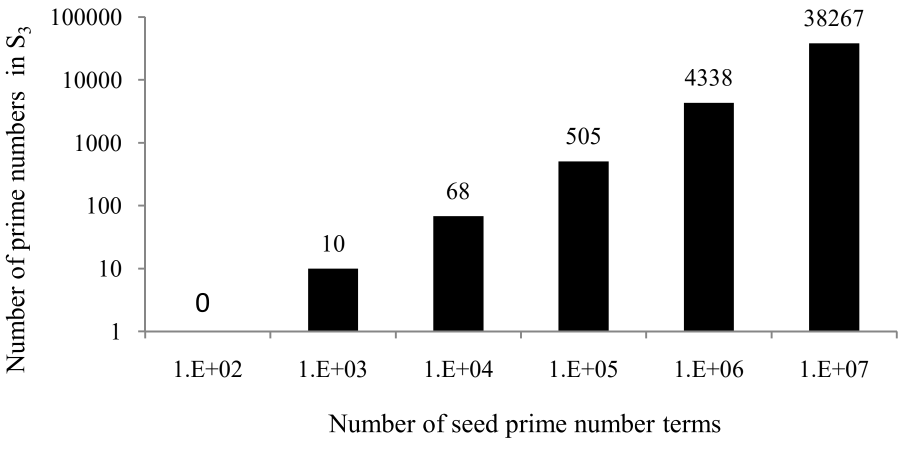

Figure n° 7: percentage of prime numbers in the S_3 sequence = f (seed size)

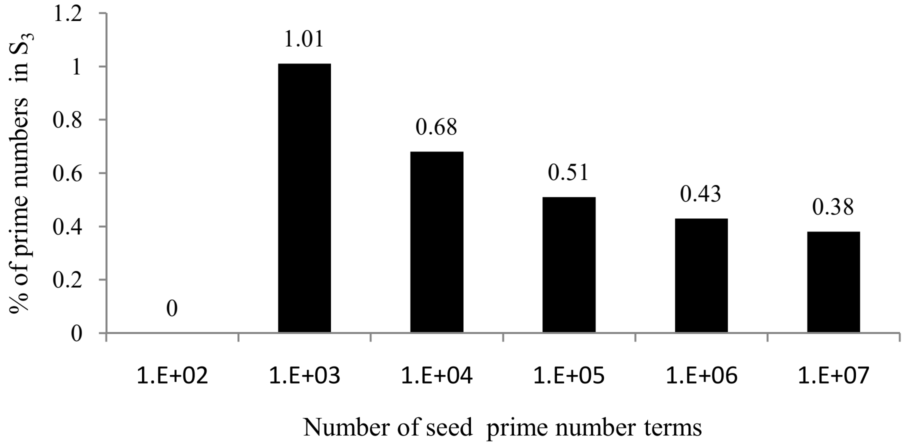

Figure n° 8: number of prime numbers in each sequence. seed of 10^2 prime number terms.

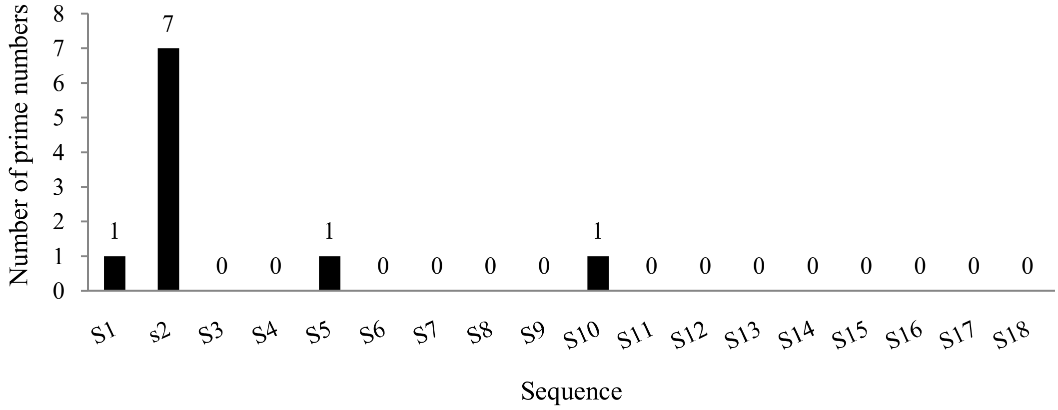

Figure n° 9: percentage of prime numbers in each sequence. seed of 10^2 prime number terms.

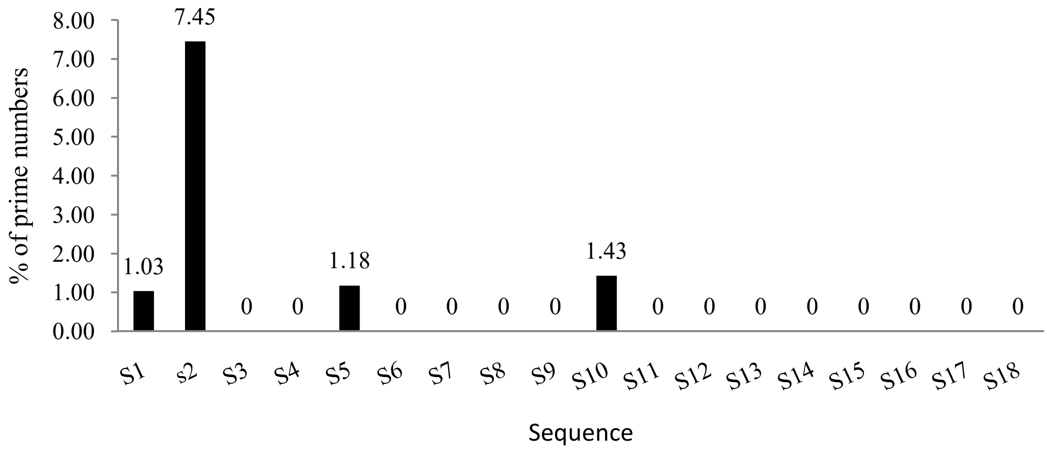

Figure n° 10: number of prime numbers in each sequence.
seed of 10^3 prime number terms.

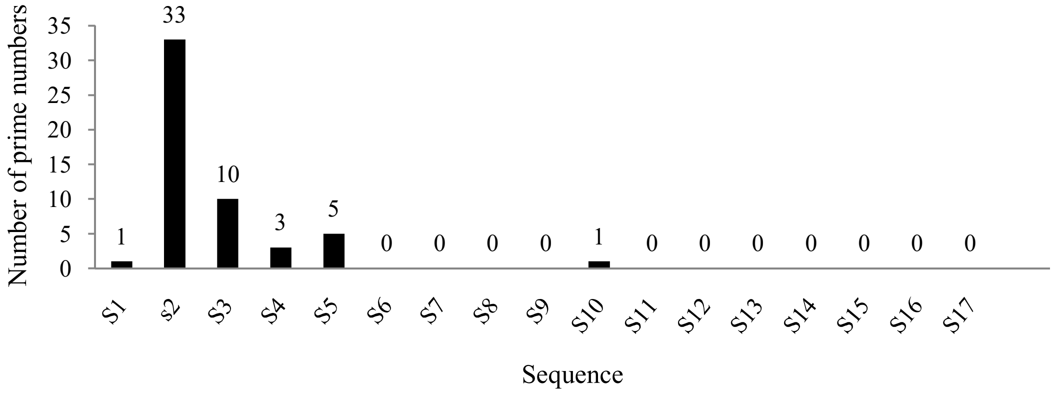

Figure n° 11: percentage of prime numbers in each sequence.
seed of 10^3prime number terms)

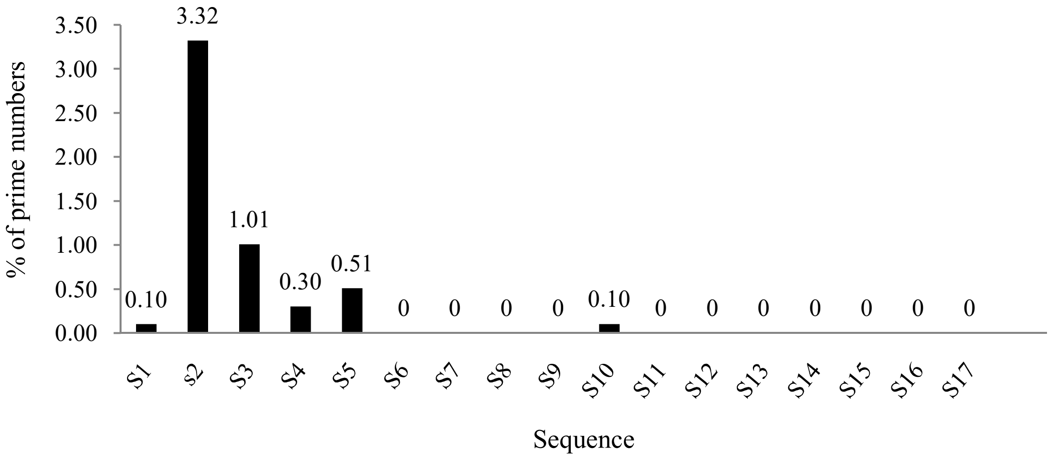

Figure n° 12: number of prime numbers in each sequence.
seed of 10^4 prime number terms.

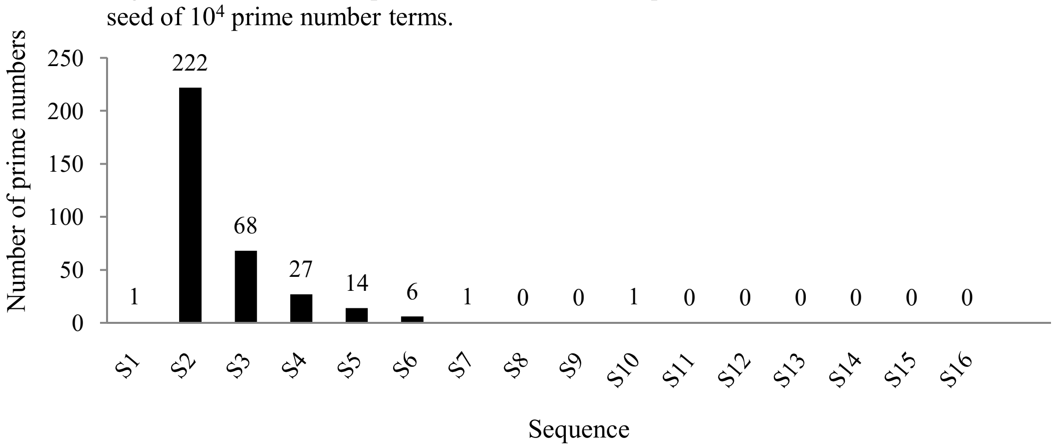

Figure n° 13: percentage of prime numbers in each sequence.
seed of 10^4 prime number terms.

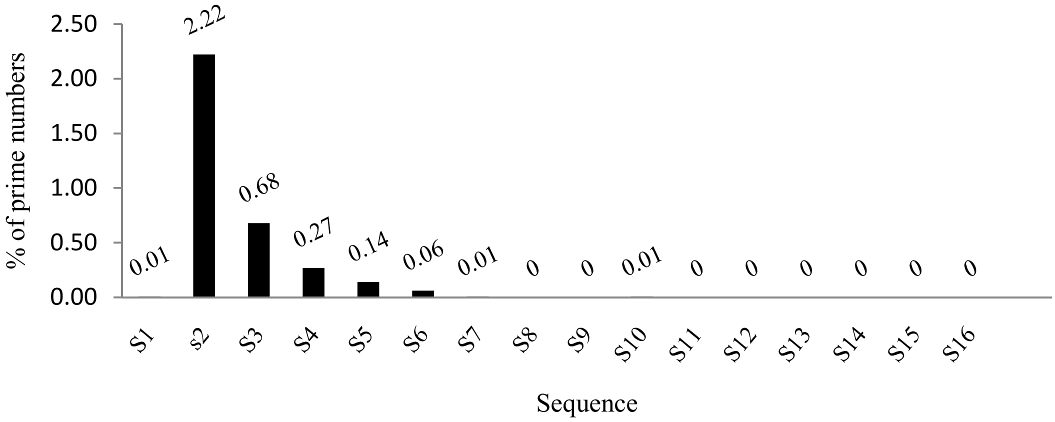

Figure n° 14: number of prime numbers in each sequence.
seed of 10^5 prime number terms.

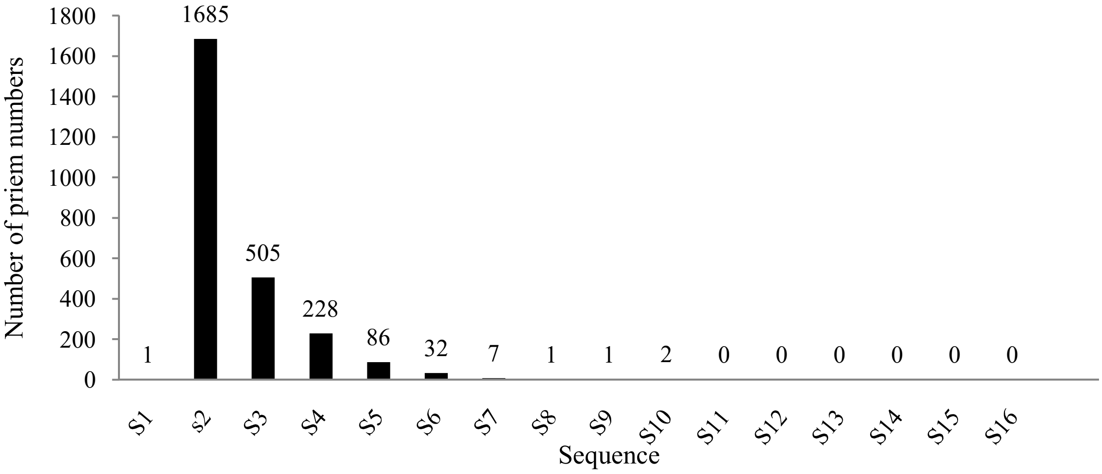

Figure n° 15: percentage of prime numbers in each sequence.
seed of 10^5 prime number terms.

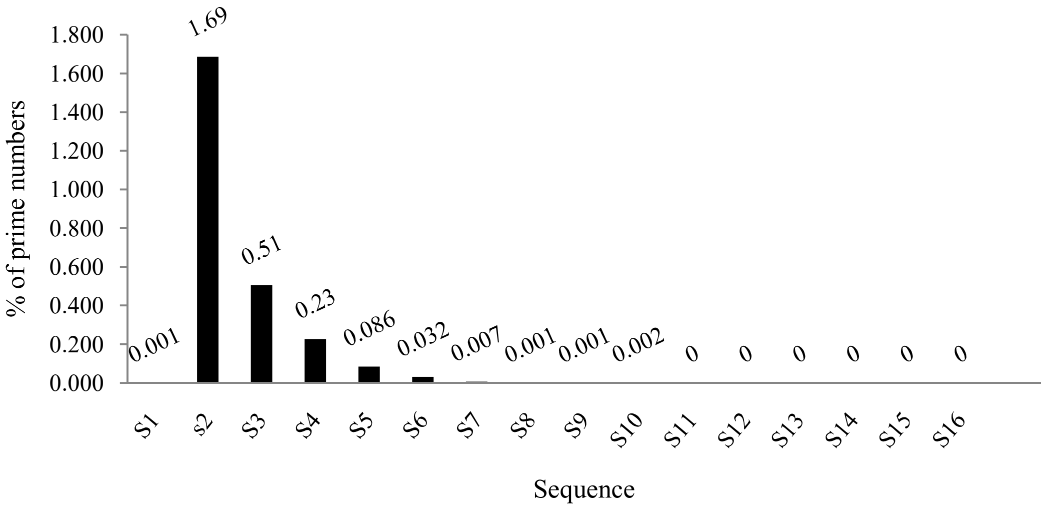

Figure n° 16: number of primes numbers in each sequence.
seed of 10^6 prime number terms.

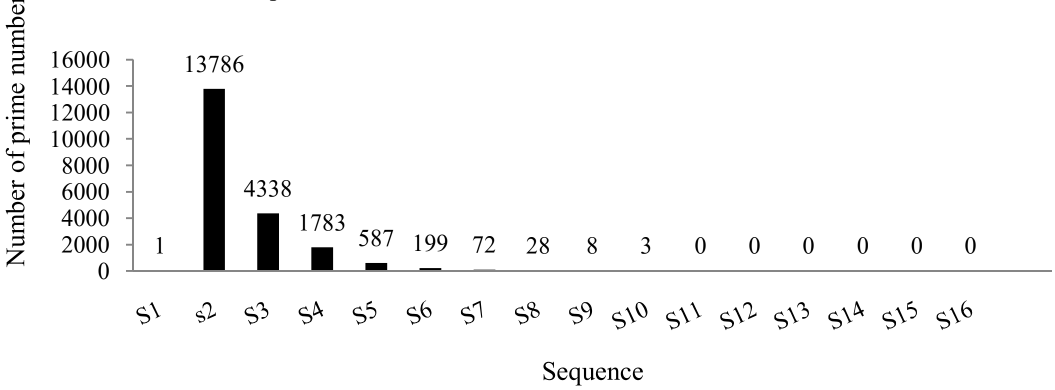

Figure n° 17: percentage of prime numbers in each sequence.
seed of 10^6 prime number terms.

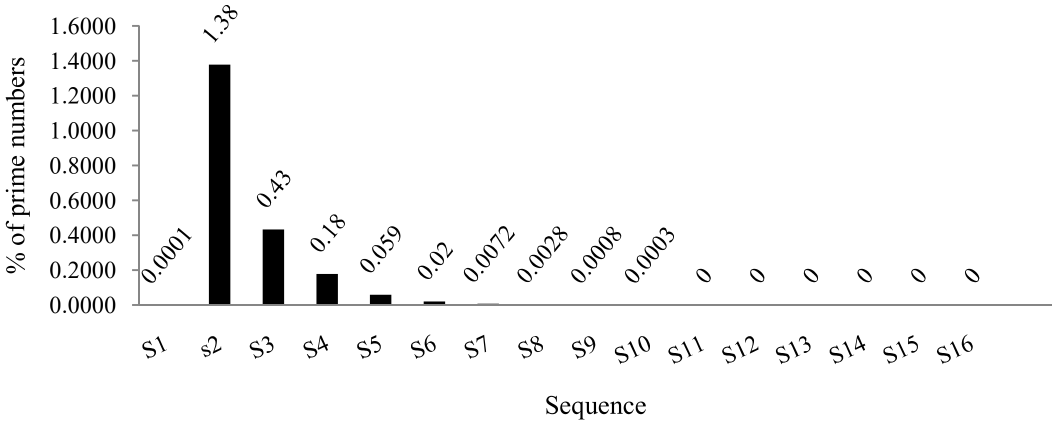

Figure n° 18: number of prime numbers in eacc sequence.
seed 10^7 of prime number terms.

Figure n° 19: percentage of prime numbers in each sequence.
seed of 10^7 prime number terms.

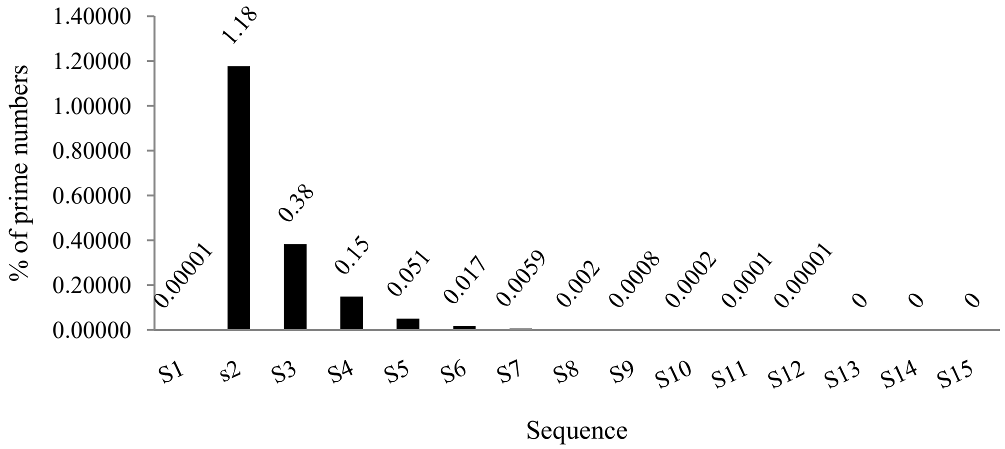

Figure n° 20: distribution of 2's in the S_6 sequence,
seed: first 10^4 prime number terms.