## Code Generation Models and the Enigmatic Code

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## Abstract

Models are presented that describe the process of creating the Gray code and other similar codes without any calculation. These models are based on simple and rigid rules. Of the codes obtained in this way, only one does not correspond to a sequence of OEIS, and it remains unknown whether it can be used somewhere.

Below we see the model for creating the Gray code (in decimal) for all integers that have a length of four and five digits in their binary expressions.
Gray code (binary integers 4-bit)


The numbers arranged in the last column are rearranged in gray code in the first column by transferring groups of numbers from column to column, in the way the arrows show. There are only two types of paired arrows for carrying numbers, crosswise $(\times)$ and parallel $(=)$. The number of columns is always equal to the length of the binary expressions of the numbers to be encoded. The patterns formed in the columns of the arrows are the same in both tables.

The rules for the arrangement of arrows are defined by the Thue Morse sequence:
$01101001100101101001011001101001, \ldots$
We omit the first digit and divide the sequence into groups of $1,2,4,8, \ldots$ digits:
$1,10,1001,10010110,1001011001101001, \ldots$
Each term in this sequence is a double copy of the previous term, but the digits of the copy placed on the right side are reversed. We now set " $=$ " = $0, " \times "=1$ and replace with these symbols the digits of the last sequence. So we will have
$\times, \times=, \times==\times, \times==\times=\times \times=, \ldots$
Then we place in this order the symbols in the successive columns of the arrows of the above table so that they are read from top to bottom.

A similar procedure applies to the following models, with the difference that in place of the Thue Morse sequence the binary sequence 101010... is also used.

| Inv. Gray code |  | inary integers 5-bit) | n |
| :---: | :---: | :---: | :---: |
| 31 | 16 | $16-16$ | 16 |
| 30 | 17 | 17 - 17 | 17 |
| 28 | 19 | $19 \longrightarrow 19$ | 18 |
| 29 | 18 | $18-18$ | 19 |
| 24 | 23 | 23 20 | 20 |
| 25 | 22 | $22 \times 21$ | 21 |
| 27 | 20 | $20<23$ | 22 |
| 26 | 21 | 21 22 | 23 |
| 16 | 31 | $24>24$ | 24 |
| 17 | 30 | $25 \longleftarrow 25$ | 25 |
| 19 - | 28 | / $27 \rightarrow 27$ | 26 |
| 18. | 29 | $\checkmark 26<26$ | 27 |
| 23 | 24 | 人 $31 \rightarrow 28$ | 28 |
| 22 | 25 | - $30 \times 29$ | 29 |
| 20 | 27 | $28 \times 31$ | 30 |
| 21 | 26 | $29 \quad 30$ | 31 |




We observe the following.

1. Each column after the first contains the same number of pairs of arrows of the type " $=$ " and " $\times$ ".
2. The number of these pairs is doubled in each subsequent column.
3. The rules for arranging pairs in each column are determined by the arrangement of the pairs in the previous column.
4. In each diagram, all the vertices or all the bottoms of the columns are occupied by pairs of " $\times$ " type arrows.

Violation of these rules leads to insignificant sequences.

We also notice that each code sequence is created in two ways, except for the Gray code and the unknown $x$ code. These two sequences do not coincide with each other as would be expected based on the above.


## The relevant sequences and their definitions, by OEIS

A003188
Decimal equivalent of Gray code for n .
$0,1,3,2,6,7,5,4,12,13,15,14,10,11,9,8,24,25,27,26,30,31,29$, $28,20,21,23,22,18,19,17,16, \ldots$
A006068
$\mathrm{a}(\mathrm{n})$ is Gray-coded into n (inverse Gray code).
$0,1,3,2,7,6,4,5,15,14,12,13,8,9,11,10,31,30,28,29,24,25,27$, $26,16,17,19,18,23,22,20,21, \ldots$

## A154435

Permutation of nonnegative integers induced by Lamplighter group generating wreath recursion, variant $3: a=s(b, a), b=(a, b)$, starting from the state a.
$0,1,3,2,6,7,5,4,13,12,14,15,10,11,9,8,26,27,25,24,29,28,30$, $31,21,20,22,23,18,19,17,16, \ldots$

A154436
Permutation of nonnegative integers induced by Lamplighter group generating wreath recursion, variant $1: \mathrm{a}=\mathrm{s}(\mathrm{a}, \mathrm{b}), \mathrm{b}=(\mathrm{a}, \mathrm{b})$, starting from the
state a.
$0,1,3,2,7,6,4,5,15,14,12,13,9,8,10,11,31,30,28,29,25,24,26$, $27,19,18,16,17,21,20,22,23, \ldots$

A010060
Thue-Morse sequence: let A_k denote the first $2^{\wedge} \mathrm{k}$ terms; then $\mathrm{A}_{-} 0=0$ and for $\mathrm{k}>=0, \mathrm{~A}_{-}\{\mathrm{k}+1\}=\mathrm{A}_{-} \mathrm{k} \mathrm{B}_{-} \mathrm{k}$, where $\mathrm{B}_{-} \mathrm{k}$ is obtained from $\mathrm{A}_{-} \mathrm{k}$ by interchanging 0 's and 1 's.
$0,1,1,0,1,0,0,1,1,0,0,1,0,1,1,0, \ldots$
A133468
A080814 complemented, then interpreted as binary and then re-expressed in decimal form (e.g., " $1221 "=" 0110 "$ ). Alternately, view as A080814 with " 1 " mapped to " 1 " and " 2 " mapped to " 0 ".
$1,2,9,150,38505,2523490710, \ldots$
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