STRUCTURES AND PROPERTIES OF INTEGER SEQUENCES GENERATED

FROM

PRIME AND NONPRIME NUMBERS SEEDS

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Abstract:

A specific recursive algorithm and three fomulas have been used to generate integer sequences from prime and nonprime numbers seeds. After a few generations, some growing structures have been identified in these integer sequences, whereas such structures were absent when a subset of natural numbers was used as an alternative seed.
The sum of the reciprocals of primes of these integer sequences, well fitted by models of the form a*ln(ln(n)) + b, were calculated. Their distances to that of the harmonic series summed only over the primes were estimated and compared to the Meissel-Mertens constant.
Finally, the algorithm used with one of the three formulas led after a few iterations to the production of long primefree sequences containing large numbers and allowed to establish a so called primefree sequences conjecture.

Key Words: prime and nonprime numbers seeds, stepwise-algorithm and formulas, integer sequences structures, sum of the reciprocals of primes, a*ln(ln(n)) + b models, primefree sequences conjecture.

1- IT Tools and VBA program:
- PC: AMD (tm) XP 2800+
  2.08 GHz. RAM: 1.00Go.
- software: Windows and Excel 2010.
- a VBA program has been developed for sequence calculation.

2- Recursive algorithm and formulas:

2-1 Recursive algorithm:

<u>Note</u>: *a complete list of abbreviations is given in annex n° 1.*

The algorithm starts with a subset of the prime numbers set containing the first 1000 primes *(2, 3, 5, 7, 11, 13,17…7919)* used as a seed referenced S_{0P} to produce with a formula a first sequence S_1 which is then used as a new seed to produce with the same formula the next sequence S_2 and so one…A larger subset of the prime numbers set containing the 20000 first primes *(2, 3, 5, 7, 11, 13, 17…224737)* has also been used as a seed referenced S_{0PL}.
A subset of the first 1000 nonprime positive integers *(4, 6, 8, 9, 10... 1197)* and a subset of the first 1000 natural numbers *(1 to 1000)* have been used too as alternative seeds to the seed S_{0P}. They are repectively named S_{0NP} and S_{0N}. Each sequence is composed of a prime numbers subsequence containing all primes of the sequence and a nonprime numbers subsequence containing all nonprime numbers of the sequence. (e.g: the sequence S_1 is composed of the prime numbers subsequence referenced S_{1p} and the nonprime numbers subsequence referenced S_{1np}.

2-2 Formulas:

Formula n° 1

The seed S_{0P} or S_{0NP} or S_{0N} has been filed in the first column of a 1000 rows and 21 columns matrix. Then, 20 sequences S_1 to S_{20}, filed in columns 2 to 21 of the matix, were calculated with the formula:

$$t_{i+1+j,\, j+2} = t_{i+1+j,\, j+1} + t_{i+2+j,\, j+1} - t_{i+j,\, j+1} \quad \textit{(formula n° 1)}$$

i varying from 1 to 998-2*j and j from 0 to 19.

The term on the left side of the equal sign of the formula belongs to the sequence S_1 produced from the seed S_{0P} or to one of the sequence S_2 to S_{20} produced from the one which precedes it, this last being used as a seed.
The terms on the right side belong to the seed S_{0P} or to a sequence S_1 to S_{19} used as a seed to produce the next sequence.

<u>Note</u>: as formula n° 1 leads to a loss of two terms between the seed and the first sequence and between a given sequence and the next one, the i and j ranges and the term indexes in the formula have to be set accordingly.

Formula n° 2

The same recursive algorithm was used with the seed S_{0P} and the formula n° 2 below to produce a total of 13 sequences referenced S_1 to S_{13}.

$t_{i+2+2*j,\, j+2} = t_{(i+2+*2j,\, j+1)} + t_{(i+3+2*j,\, j+1)} + t_{(i+4+2*,j+1)} - t_{(i+1+2*j,\, j+1)} - t_{(i+2*j,\, j+1)}$ *(formula n°2)*

with i = 1 to 996-4*j and j = 0 to 12

Again, the first sequence S_1 has been produced from the seed S_{0P} and the S_2 sequence from S_1 and so on…

Formula n° 3

The same recursive algorithm, again with the seed S_{0P} , was used with the formula n° 3 below to produce a total of 17 sequences referenced S_1 to S_{17}.
As before, S_{0P} has been used to produce S_1 and S_1 to produce S_2 and so on…

$t_{(i+2+2*j,\, j+2)} = t_{(i+2+2*j,\, j+1)} + t_{(i+3+2*j,\, j+1)} - t_{(i+1+2*j,\, j+1)} - t_{(i+2*j,\, j+1)}$
(formula n°:3)

with i = 1 to 997-3*j and j = 0 to 16

3- Results :

3-1 Sequences produced from the prime numbers subset S_{0P} and formula n° 1:

3-1-1 Basic statistics:

The first 10 terms of the seed S_{0P} and those of the sequences S_1 to S_{10} are given in table n° 1.

- Whereas the sequences S_1 to S_5 contain only positive integers, sequences S_6 to S_{20} contain both positive and negative integers.

So, the nonprime subsequences S_{6np} to S_{20np} contain both positive and negative integers.

- Figure n° 1 shows the number of terms of the seed S_{0P} and those of the sequences S_1 to S_{20}, a loss of two terms between the seed and S_1 and between a sequence and the next one being induced by the formula.

- Figure n° 2 shows that the number of primes in the sequences decreases from S_1 to S_{20}.

- Figure n° 3 shows that the percentage of primes in the sequences decreases from S_1 to S_{20}.

- Figure n° 4 shows that the median of primes is rougthly constant from prime numbers subsequences S_{1p} to S_{8p} but increases from S_{9p} to S_{20p} .

- Figure n° 5 shows that the minimum prime number value is roughtly constant from S_{1p} to S_{11p} but increases from S_{12p} to S_{20p} . and that the maximum prime number value is roughtly constant from S_{1p} to S_{8p} but increases from S_{9p} to S_{20p}.

- Figure n° 6 shows that the range of prime number values is roughtly constant from S_{1p} to S_{8p} but increases from S_{9p} to S_{20p}.

<u>Note</u>: *independently of this work, it has been found that the prime subsequence S_{1p} of the sequence S_1 was filled in The On-Line Encyclopedia of Integer Sequences, published electronically at: https://oeis.org/A175873 (1). The first 20 terms of this sequence are: 13, 17, 19, 23, 37, 47, 67, 89, 103, 107, 109, 113, 131, 151, 173, 193, 199, 233, 239, 269…*

3-1-2 Sequence structures:

Figures n° 7 to n° 27 show the seed S_{0p} and the 20 prime numbers subsequences S_{1p} to S_{20p}.
Whereas figure n° 7 does not reveal any particular structure in the prime numbers seed S_{0p}, a more and more visible one appears in the prime numbers subsequences S_{1p} to S_{6p} and two distinct prime number size distributions are clearly present in S_{7p} to S_{20p} prime numbers subsequences.
Then, it has been checked if these indentified patterns were specific or not of the generated prime subsequences. Figures n° 28 to n° 35 show

that these structures also exist in the nonprime subsequences S_{6np},S_{13np} and S_{20np} and in the corresponding sequences S_6, S_{13} and S_{20}.
So, the prime numbers subset *(2, 3,5, 7, 11, 13 ,17…7919)* generates through formula n° 1 structures in integer sequences and also in both prime and nonprime subsequences of them.

3-2 Sequences produced from the nonprime numbers subset S_{0NP} and formula n° 1:

3-2-1 Basic statistics :

The first 10 terms of the seed S_{0NP} and those of the sequences S_1 to S_{10} are given in table n° 2.

- It can be noticed that some duplicates are produced such as 11 and 17 in sequence S_1.

- Whereas the sequences S_1 to S_5 contain only positive integers, sequences S_6 to S_{20} contain both positive and negative integers. So, the nonprime subsequences S_{6np} to S_{20np} contain both positive and negative integers.

- Figure n° 36 shows that the number of primes in the sequences decreases from S_1 to S_{20}.

- Figure n° 37 shows that the percentage of primes in the sequences decreases from S_1 to S_{20}.

- Figure n° 38 shows that the median of primes is rougthly constant from prime numbers subsequences S_{1p} to S_{9p} but increases from S_{10p} to S_{20p} .

- Figure n° 39 shows that the minimum prime number value is roughtly constant from S_{1p} to S_{11p} but increases from S_{12p} to S_{20p} and that the maximum prime number value is roughtly constant from S_{1p} to S_{8p} but increases from S_{9p} to S_{20p}.

- Figure n° 40 shows that the range of prime number values is roughtly constant from S_{1p} to S_{8p} but increases from S_{9p} to S_{20p}.

Overall, these observations are very similar to those seen in the case of the seed S_{0P}.

3-2-2: Sequence structures:

Figures n° 41 to n° 61 show the seed S_{0NP} and the 20 prime number subsequences S_{1p} to S_{20p}.
Whereas figure n°41 does not reveal any particular structure in the prime numbers seed S_{0NP}, a more and more visible one appears in the prime numbers subsequences S_{1p} to S_{6p} and two distinct prime number size distributions are clearly present in S_{7p} to S_{20p} subsequences.
Then, it has been checked if these indentified patterns were specific or not of the generated prime subsequences. Figures n° 62 to figure n° 69 show that the structures also exist in the nonprime subsequences S_{6np},S_{13np} and S_{20np} and in the corresponding sequences S_6, S_{13} and S_{20}. So, the nonprime numbers subset *(4, 6, 8, 9, 10... 1197)* similarly to the prime numbers subset *(2, 3, 5, 7, 11, 13,17...7919)* generates through formula n° 1 structures in integer sequences and also in both prime and nonprime subsequences of them.

3-3 Sequences produced from the natural numbers set S_{0N} and formula n° 1:

3-3-1 Basic statistics:

The first 10 terms of the seed S_{0N} and those of the sequences S_1 to S_{10} are given in table n° 3.

- To the opposite to the two previous cases for which the prime number seed S_{0P} and the nonprime number seed S_{0NP} were used, here none of the sequences S_1 to S_{20} contain negative intergers.

- Figure n° 70 shows that the number of primes in the sequences decreases from S_1 to S_{20}.

- Figure n° 71 shows that the percentage of primes in the sequences decreases from S_1 to S_{20}.

- Figure n° 72 shows that the median of primes regularly increases from prime subsequence S_{1p} to S_{20p} and that is a difference compared to the two previous cases where the prime number seed S_{0P} and the nonprime numbers seed S_{0NP} were used.

- Figure n° 73 shows that the minimum prime number value regularly increases from S_{1p} to S_{20p} and that the maximum prime number value is roughtly constant from S_{1p} to S_{20p}.

- Figure n° 74 shows that the range of prime number values regularly decreases from S_{1p} to S_{20p}.

Overall, these observations show that the prime numbers statistics in this case are quite different from the ones observed when the prime number seed $_{S0P}$ and the nonprime numbers seed S_{0NP} were used.

3-3-2: Sequence structures:

Figures n° 75 to n° 95 show the seed S_{0N} and the 20 prime numbers subsequences S_{1p} to S_{20p}.
Figures n° 96 to n° 103 show the nonprime numbers subsequences S_{6np},S_{13np} and S_{20np} and the S_6, S_{13} and S_{20} sequences.
No stuructures have been identified in either the 20 prime numbers subsequences or in the nonprime numbers subsequences S_{6np},S_{13np} and S_{20np} as in the corresponding sequences S_6, S_{13} and S_{20}.
Therefore, this absence of pattern in the sequence and prime and nonprime numbers subsequences produced from the natural numbers seed S_{0N} and formula n°1 is another major difference compared to the two cases for which the prime numbers seed S_{0P} and the nonprime numbers seed S_{0NP} are used together with formula n° 1.

3-4 Sequences produced from formula n° 2:

3-4-1 Basic statistics:

Note: *identifying large prime numbers requires long computation times and for that reason calculations with formula n° 2 were not performed beyond sequence S_{13}. However, 13 sequences was enough to make a meaningful comparison of results given by formulas n° 1 and n° 2.*

The first 10 terms of the seed S_{0p} and the sequences S_1 to S_{10} are given in table n° 4.

- Whereas the sequences S_1 to S_3 contain only positive integers, sequences S_4 to S_{13} contain both positive and negative integers. So, the nonprime subsequences S_{4np} to S_{13np} contain both positive and negative integers.

- Figure n° 104 shows the number of terms of the seed S_{0P} and those of the sequences S_1 to S_{13}, a loss of four terms between the seed

S_{0P} and $S_1$and between a sequence and the next one being induced by formula n° 2.

- Figure n° 105 shows that the number of primes in the sequences decreases from S_1 to S_{13}.

- Figure n° 106 shows that the percentage of primes in the sequences decreases from S_1 to S_{13}.

- Figures n° 107 to n° 110 show the histograms of primes for the sequences S_{1p}, S_{2p}, S_{3p} and S_{4p} and a slight decrease of the density of primes along the sequences.

- Figure n° 111 shows that the median of primes is rougthly constant from S_{1p} to S_{5p} and increases from S_{6p} to S_{13p}.

- Figure n° 112 shows that the minimum prime number value is roughtly constant from S_{1p} to S_{7p} but increases from S_{8p} to S_{13p} and that the maximum prime number value is roughtly constant from S_{1p} to S_{2p}, then increasing from S_{3p} to S_{13p} .

- Figure n° 113 shows that the range of prime number values increases from S_{1p} to S_{13p}.

3-4-2 Comparison of the basic statistics on data given by formulas n° 1 and n° 2:

- Figure n° 114 shows that formulas n° 1 and n° 2 give a similar percentage of primes for sequences S_1 to S_5 but that formula n°1 leads to a significantly higher percentage of primes in sequences S_6 to S_{13}.

- Figure n° 115 shows that formulas n° 1 and n° 2 give a similar median of primes for subsequences S_{1p} to S_{4p} but that formula n° 2 leads to a higher median of primes in subsequences S_{5p} to S_{13p}.

- Figure n° 116 shows that formulas n° 1 and n° 2 give a similar range of prime number values for sequences S_{1p} to S_{3p} but that formula n° 2 leads to a higher range of primes in subsequences S_{6p} to S_{13p}.

3-4-3 Sequence structures:

Figures n°117 to n°130 show the seed S_{0p} and the 13 prime numbers subsequences S_{1p} to S_{13p}.
Whereas figure n°117 does not reveal any particular structure in the prime numbers seed S_{0p}, a more and more visible one appears in the prime numbers subsequences S_{1p} to S_{3p} and two distinct prime numbers size distributions are clearly present in S_{4p} to S_{13p} prime numbers subsequences.
Then, it has been checked if these indentified patterns were specific or not of the generated prime subsequences. Figures n°131 to figure n°136 show that the structures also exist in the nonprime numbers subsequences S_{1np}, S_{6np},S_{13np} as in the corresponding sequences S_1, S_6 and S_{13}.
So, the prime numbers subset (2, 3,5, 7, 11, 13 ,17…7919) generates through formula n° 2 structures in sequences and also in both prime and nonprime numbers subsequences of them.

3-5 Sequences produced from formula n° 3:

3-5-1 Basic statistics:

<u>Note</u>: here and in a first step, the number of sequences has been restricted to 17 to limit the computing time for searching large prime numbers.

The first 10 terms of the seed S_{0p} and the sequences S_1 to S_{17} are given in table n° 5.

Figures n° 137 and n° 138 show that formula n° 3 leads unlike formulas n° 1 and n° 2 to much less prime numbers, thus lower corresponding percentages.

The occurence of primes in each sequence is given in table n° 6.

It can be noted that:

- there is 49 occurences of the prime number 2 on a total of 53 primes.
- Sequences S_6 to S_9 and s_{11} to S_{17} do not contain any prime number.

On the basis of this last observation, it was worth to investigate further the absence of primes in sequences. For that purpose, a prime number seed S_{0PL} containing 20000 terms from 2 to 224737 *(the first 20000 primes)* was used and 28 sequences have been calculated. The S_{0PL} seed was filed in the first column of a 20000 rows and 29 columns matrix and 28 sequences in the columns 2 to 29.

Figures n° 139 show the number of terms of this large seed S_{0PL} and those of the 28 sequences with a loss of three terms between the seed S_{0PL} and the first sequence and between a sequence and the next one; this loss being induced by formula n° 3.

Figures n° 140 and n° 141 respectvely show the number of primes and the percentage of them in each sequence.

Table n° 7 gives the occurence of primes in the 28 sequences. Compared to the set of sequences produced from formula n° 3 and a seed S_{0P} containing 1000 terms *(the first 1000 primes)* the number of primes in the sequences is now higher due to the higher length of sequences and whereas sequences S_6, S_7 and S_8 were prime free in the case of the 1000 terms S_{0P} prime numbers seed, they do contain 9, 2 and 1 primes in the case of the 20000 terms S_{0PL} prime seed. S_9 contains no prime in both cases, S_{10} still contains one prime *(the same as in the 1000 terms S_{OP} seed (1831)).*

2, 7, 19 and 1831 are the four primes found. With their repeats they count for 617 primes over the 558782 terms of the 28 sequences *(0,11 %)* and they are concentrated in the first 10 sequences. On these 617 primes, 613 are the prime number "2" *(99,35%).* On 28 sequences 19 *(S_9 and S_{11} to S_{28})* of them containing in total 378920 terms are primefree. S_9 is the first and the longest primefree sequence *(19973 terms).* The first and smallest term of sequence n° 28 is: - 429306405948700 *(15 digits)* and the last and largest one is: 449225647786036 *(15 digits)*, so a range of 878532053734736.

<u>Note:</u> *other formulas giving primefree sequences have been found and reported in the litterature (see references (2) to (6)).*

3-5-2 A prime free conjecture *(Ref: conjecture CD-3):*

Considering that:

- the number of primes decreases from S_1 to S_{28}.
- the number of prime decreases along a sequence.
- Sequences S_9 and S_{11} to S_{28} containing a total of 378920 numbers are primefree sequences.

The following conjecture (Ref: conjecture CD3 can be established:

Conjecture CD-3:
The formula below:

$$t_{(i+2+2*j,\, j+2)} = t_{(i+2+2*j,\, j+1)} + t_{(i+3+2*j,\, j+1)} - t_{(i+1+2*j,\, j+1)} - t_{(i+2*j,\, j+1)}$$

applied to the prime numbers set *(2, 3, 5 , 7, 11,13, 17, 19...)* used as a seed, generates a sequence which is then used as a new seed to produce the next sequence and so one. When the number of terms of the prime number set tends to + ∞ and after a certain number of iterations, this recursive process leads to an infinite number of long primefree sequences containing increasingly large composite numbers.

3-6 Sum of the reciprocals of primes:

Figure n° 142 shows the continuous deacease of the reciprocals of primes.

Figure n° 143 shows the asymptotic increase of the sum of the reciprocals of primes.

Figure n° 144: shows the sum of the reciprocals of primes and the function LN(LN(n))
The Meissel-Mertens constant often reffered to as the Mertens constant is defined as the limiting difference between the sum of the reciprocals of primes $p \leq n$ and the function LN(LN(n)) named Model-1.

Figure n° 145 shows, with the S_{0N} natural or S_{0NP} nonprime numbers seeds and formula n° 1, the sum of the reciprocal of prime numbers of the subsequence S_{1p} as a function of p and the Model-2 predicted values.

Figure n° 146 shows with the S_{0P} prime numbers seed and formula n° 1 the sum of the reciprocals of the S_{1p} subsequence prime numbers as a function of prime numbers and the Model-3 predicted values.

Figure n° 147 shows with the S_{0P} prime numbers seed and formula n° 2 the sum of the of the reciprocals of the S_{1p} subsequence prime numbers as a function of prime numbers and the Model-4 predicted values.

These last three figures show the quite good fit of Model-2, Model-3 and Model-4. The three models are of the form a*LN(LN(p)) + b, so the same form as the LN(LN(n)) with the "a" coefficient different from 1 and the " b" coefficient different from 0. The"a" and "b" coefficients of the three models are given in table n° 8-a. The correlation and determination coefficients are also given in this table and the absolute and relative errors reported in table n° 8-b confim the good quality of the three models.

Table n° 8-b also give the D distances *(see definition in the legend of table 8-b or in annex n° 1).*

Finally, figures n° 148 and n° 149 respectively give the sum (1/p) curves calculated from the s_{1p} subsequence and the sum (1/p) predicted by the models, the sum of the reciprocals of primes and the LN(LN(n) function being shown as reference.

4- Conclusions:

- When the number of iterations of the recursive algorithm increases the number of primes in a sequence decreases and the size of its terms *(number of digits)* increases. 15 digits terms are found in the 28^{th} sequence produced when the S_{0PL} seed is used.

- The number of primes slightly decreases along sequences.

- When the algorithm is used with a prime or nonprime numbers seed some structures are already visible in the first sequences and their prime and nonprime numbers subsequences and their development in the next subsequences leads to two distinct number size distributions. Such patterns do not exist when the natural numbers subset is used as seed.

- When the algorithm is used with the prime numbers subset and formula n° 3, only four primes are procuced in the first sequences and rapidely long primefree sequences containing large size numbers are produced. This led to establish a conjecture stating that an infinite number of primefree sequences can be produced using the standard prime numbers set and formula n° 3.

- Acurate models all of the form of sums (1/p) = a*LN(LN(p)) + b have been developed for the various type of seed/formula combinations.
  Finally, D distances equivalent to the Meissel-Mertens constant have been calculated for the different models.

Refrences:

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Annex n° :1 definitions and abbreviations *(specific to this paper)* :

- a sequence : list of positive or negative, prime or composite integers. A sequence "j" is produced from an seed or the " j-1" sequence and a formula. Sequences are noted: S_j, *(j = 1, 2, 3 ,4….).*

- a prime numbers subsequence: part of a S_j sequence composed of all prime numbers of this sequence. Prime numbers subsequences are noted S_{jp}., *(j = 1, 2, 3 ,4…).*

- a nonprime numbers subsequence: part of a S_j sequence composed of all numbers of this sequence except the primes ones. nonprimes subsequences are noted S_{jnp}, *(j = 1, 2, 3 ,4…).*

- a term ($t_{\text{index-1, index-2}}$): an element of a sequence or a subsequence. Terms are indexed to indicate the sequence to which they belong *(index-1)* and their position in the sequence *(index-2).*

- The prime numbers seed S_{0P} : a subset of the standard set of prime numbers *(2, 3, 5, 7, 11 , 13, 17, 19, 23…7919)* used as seed in the algorithm.

- The nonprime numbers seed S_{0NP}: a subset of the standard set of nonprime positive numbers *(2, 4, 6, 8, 9 ,10 …1197)* used in the algorithm.

- The natural numbers seed S_{0N} : a subset of the standard set of natural numbers *(1, 2, 3, 4, 5…1000)* used as seed in the algorithm.

- the recursive algorithm : starts from the seed *(S_{0P} or S_{0N} or S_{0NP})*, to produce with a formula a first sequence S_1 which is then used as a new seed to produce with the same formula the next sequence S_2 and so one, each sequence S_{j+1} being produced from the previous one $_{Sj}$.

- a formula : it is the equation used to produce a sequence from an seed or the previous sequence.

- p: prime number.

- prime (n): the n^{th} prime number *(e.g : 97 is the 25^{th} prime number)*

- sum (1/p), $p \le n$ = sum of the reciprocals of prime numbers at n.

- Meissel-Mertens constant : is defined as the difference between the sum of the reciprocals of prime numbers $\le$ n and the series of general term Ln(Ln(n)) when n tends to $+\infty$. Approximate value : 0,261497.

- Distance D : is defined as the difference between the sum of the reciprocals of prime numbers $\le$ n and the sum of the reciprocal of primes numbers $\le$ n of a sequence generated by the algorithm and one formula as described in this paper. The "distance D" can be seen as the equivalent of the Meissel-Mertens constant.

- Absolute error : the difference, at a given (n), between the sum of the reciprocals of primes $p \le n$ of a sequence and the corresponding model predicted value.

- Relative error : relative difference at a given (n) between the sum of the reciprocals of primes $p \le$ n of a sequence and the corresponding model predicted value.

- R : coefficient of correlation between the sum of the reciprocals of primes of a sequence and the corresponding sum calculated from a model.

- R^2 : coefficent of determination between the sum of the reciprocals of primes of a sequence and the corresponding sum calculated from a model.

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Annex n° 2: tables

Term n°	S_{0P}	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
1	2	6	16	28	30	72	64	8	218	154	-196
2	3	9	21	27	43	81	13	143	39	555	-2041
3	5	13	23	31	59	55	59	83	333	-597	951
4	7	17	25	39	65	39	97	99	261	-889	3251
5	11	19	29	51	49	75	45	317	-525	1243	-1065
6	13	23	35	53	55	61	151	43	-103	1119	-2165
7	17	25	45	47	69	59	211	-251	821	-941	2015
8	19	33	43	61	47	153	-17	191	195	-105	1099
9	23	37	49	55	81	117	-23	379	-315	1179	-2253
10	29	39	55	53	119	19	197	7	405	-185	-579

<u>Note</u>: prime numbers are in red.

Table n° 1: first 10 terms of the prime numbers seed S_{0P} *(1000 terms)* and those of sequences S_1 to $S_{10,}$ formula n° 1.

Term n°	S_{0P}	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
1	4	10	12	19	26	8	90	-104	275	-283	270
2	6	11	13	25	12	49	11	15	180	-422	1310
3	8	11	18	20	22	49	-25	156	-188	409	-163
4	9	13	20	17	39	11	51	39	-54	479	-1071
5	10	16	18	25	32	13	80	-71	275	-233	-94
6	12	17	19	31	18	49	10	56	150	-359	706
7	14	17	24	26	27	44	-1	148	-108	32	653
8	15	19	26	23	40	15	67	58	-101	315	55
9	16	22	24	30	31	28	80	-18	25	370	-686
10	18	23	25	33	24	54	45	-25	189	0	-343

<u>Note</u>: prime numbers are in red.

Table n° 2: first 10 terms of the nonprime numbers seed S_{0NP} *(1000 terms)* and those of sequences S_1 to $S_{10,}$ formula n° 1.

Term n°	S_{0P}	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
1	1	4	7	10	13	16	19	22	25	28	31
2	2	5	8	11	14	17	20	23	26	29	32
3	3	6	9	12	15	18	21	24	27	30	33
4	4	7	10	13	16	19	22	25	28	31	34
5	5	8	11	14	17	20	23	26	29	32	35
6	6	9	12	15	18	21	24	27	30	33	36
7	7	10	13	16	19	22	25	28	31	34	37
8	8	11	14	17	20	23	26	29	32	35	38
9	9	12	15	18	21	24	27	30	33	36	39
10	10	13	16	19	22	25	28	31	34	37	40

<u>Note</u>: prime numbers are in red.

Table n° 3: first 10 terms of the natural numbers seed S_{0N} *(1000 terms)* and those of sequences S_1 to $S_{10,}$ formula n° 1.

Term n°	S_{0P}	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
1	2	18	54	114	20	652	-1872	9202	-37558	161484	-666012
2	3	23	55	127	17	615	-1429	6985	-30623	155727	-753875
3	5	29	63	107	127	151	615	-1215	1259	33455	-289393
4	7	31	77	81	259	-351	2567	-9199	37869	-138959	480339
5	11	35	83	73	303	-405	2719	-10957	54175	-243297	999517
6	13	41	85	97	197	93	885	-4697	34319	-182437	804527
7	17	47	79	145	37	727	-1637	6499	-15911	30837	-32559
8	19	55	77	171	-23	991	-3161	15911	-65323	249683	-869459
9	23	57	85	157	81	689	-2555	16111	-73569	293263	-1014897
10	29	61	99	111	269	25	267	4099	-25137	109981	-365913

<u>Note</u> : primes numbers are in red.

Table n° 4: first 10 terms of the prime numbers seed S_{0P} *(1000 terms)* and those of sequences S_1 to S_{10}, formula n° 2.

Term n°	S_{0P}	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
1	2	7	7	-5	-3	19	93	-777	2469	-4159	1831
2	3	10	2	12	-48	106	-266	1160	-4814	14540	-28928
3	5	12	0	12	-32	128	-628	2326	-6266	12318	-14154
4	7	12	4	-8	0	90	-322	526	-238	-106	-6160
5	11	12	10	-16	26	-122	588	-1854	3698	-1964	-23154
6	13	12	6	-12	70	-288	788	-1560	2116	22	-15836
7	17	16	0	-12	46	-66	4	-6	1238	-8252	34752
8	19	18	0	10	-72	244	-482	290	2612	-16844	72518
9	23	16	-6	36	-100	190	-286	260	764	-7222	39902
10	29	18	-6	8	8	-8	-198	1262	-5166	16878	-49908

<u>Note</u> : primes are in red.

Table n° 5: first 10 terms of the prime numbers seed S_{0P} *(1000 terms)* and those of sequences S_1 to S_{10}, formula n° 3.

Sequence	Prime numbers	Occurence
S_1	7	1
S_2	2 7	32 1
S_3	2	10
S_4	2	3
S_5	2 19	4 1
S_6	----	no prime
S_7	----	no prime
S_8	----	no prime
S_9	----	no prime
S_{10}	1831	1
S_{11} to S_{28}	----	no prime

Table n° 6: occurrence of primes in sequences, *prime numbers seed S_{0P} (1000 terms) and formula n° 3.*

Sequence	Prime numbers	Occurence
S_1	7	1
S_2	2 7	411 1
S_3	2	119
S_4	2	48
S_5	2 19	23 1
S_6	2	9
S_7	2	2
S_8	2	1
S_9	----	no prime
S_{10}	1831	1
S_{11} to S_{28}	----	no prime

Table n° 7: occurrence of primes in the sequences, *prime numbers seed S_{0P} ($2x10^4$ terms) and formula n° 3.*

Model inputs	Model	Correlation coefficient (R)	Determination coefficient (R^2)
Prime numbers	Model-1 : Sum (1/p)= LN(LN(n))	0,999	0,997
Natural or nonprime numbers seed S_{0N} or S_{0NP} Prime numbers subsequence S_{1p} formula n° 1	Model-2 : Sum (1/p)= 0,985*LN(LN(P) - 0,535	0,999	0,998
Prime numbers seed S_{0P} Prime numbers subsequence S_{1p}, formula n° 1	Model-3 : Sum (1/p)= 0,233*LN(LN(P) + 0,038	0,978	0,957
Prime numbers seed S_{0P} Prime numbers subsequence S_{1p} formula n° 2	Model-4 : Sum (1/p)= 0,229*LN(LN(P) - 0,114	0,997	0,994

Table n° 8-a: models of sums of the reciprocals of prime numbers.

Model	Model inputs	Absolute error	Relative error	Distance D *
Model-1	Prime numbers	not appropriate	not appropriate	0,26
Model-2	Natural or nonprime numbers seed S_{0N} or S_{0NP} prime subsequence S_{1p} formula n° 1	0,00115	0,06	0,83
Model-3	Prime numbers S_{0p} subsequence S_{1p}, formula n° 1	-0,0053	0,86	2,16
Model-4	Prime numbers seed S_{0P} Prime numbers subsequence S_{1p} formula n° 2	-0,0037	0,80	2,32

Table n° 8-b: absolute and relative errors of the models and distance *D,*
(at the prime number 224729 of the subsequence S_{1p} for models n° 1, n° 2, n° 3 and at the prime number 224611 of the subsequence S_{1p} for model n° 4; the closest prime number from the prime number 224729.)

** Distance D : difference between the sum of the reciprocals of prime numbers and the sum of the reciprocals of prime numbers of the S_{1p} subsequence.*

Annex n° 3 : figures

Figures from the prime numbers seed S_{0P} and formula n° 1

Figure n° 1: numbers of terms in the seed S_{0P} and the sequences
prime numbers seeds and formula n° 1

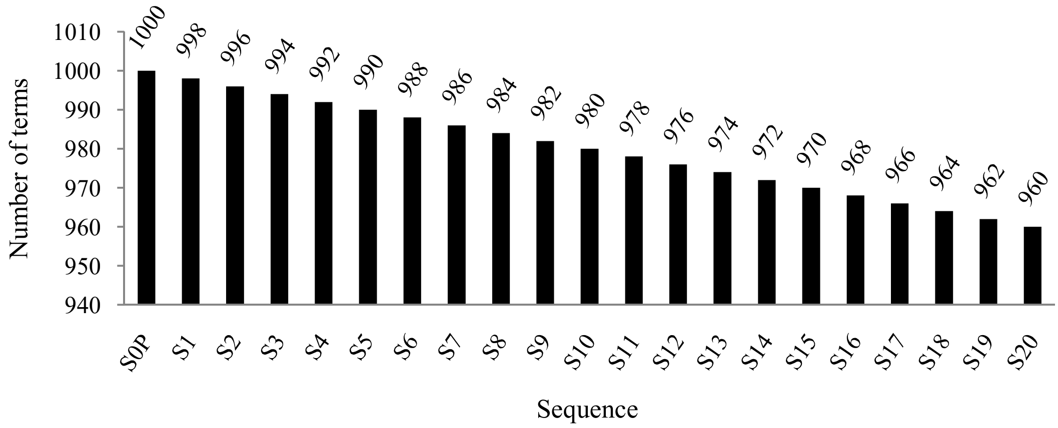


Figure n° 2: number of primes in each sequence
prime numbers seed S_{0P} and formuma n° 1

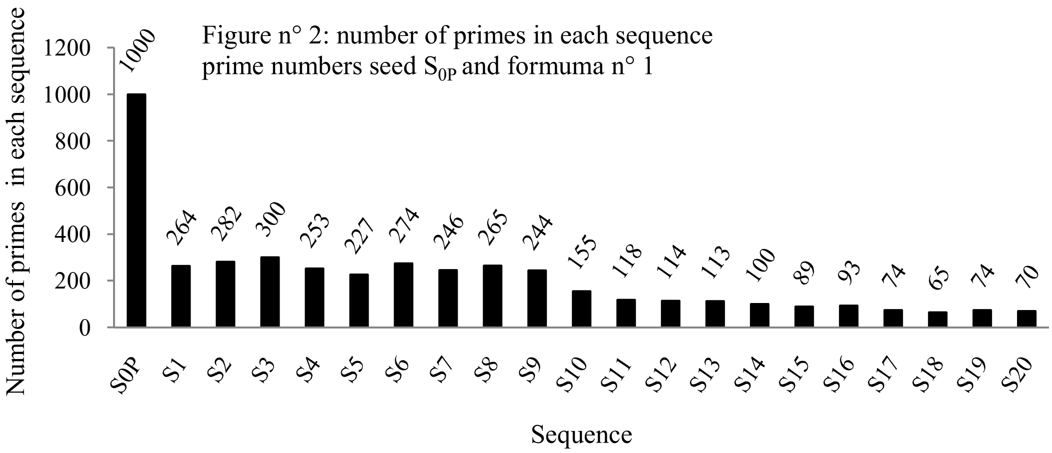


Figure n° 3: percentage of prime numbers in each sequence
prime numbers seed S_{0P} and formula n° 1

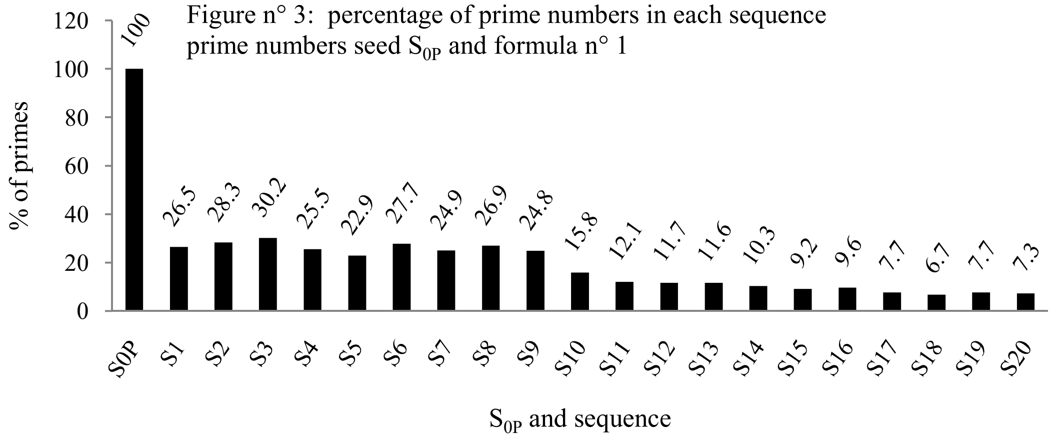


Figure n° 4: median of primes in S_{0P} and each prime numbers subsequence
prime numbers seed S_{0P} and formula n° 1

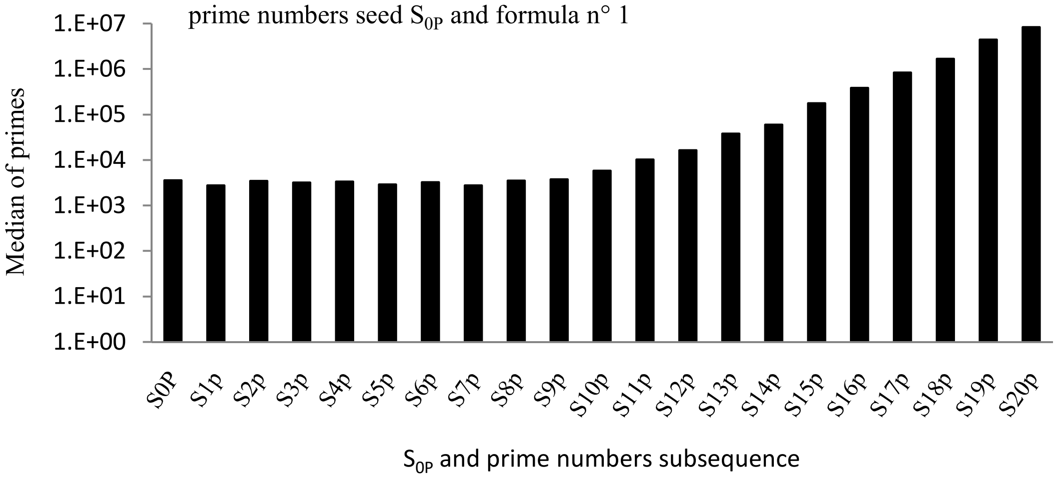


Figure n° 5: prime number minimum and maximum values in S_{0P} and each prime numbers subsequence
prime numbers seed S_{0P} and formula n° 1

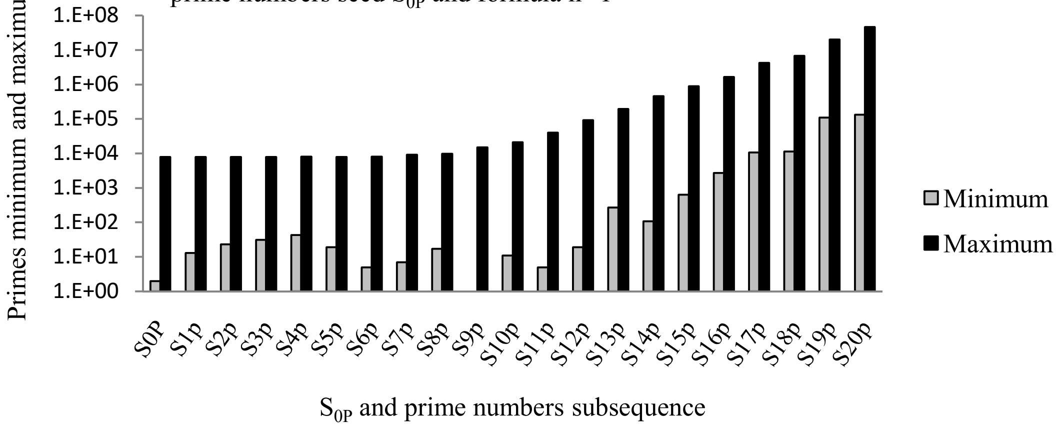
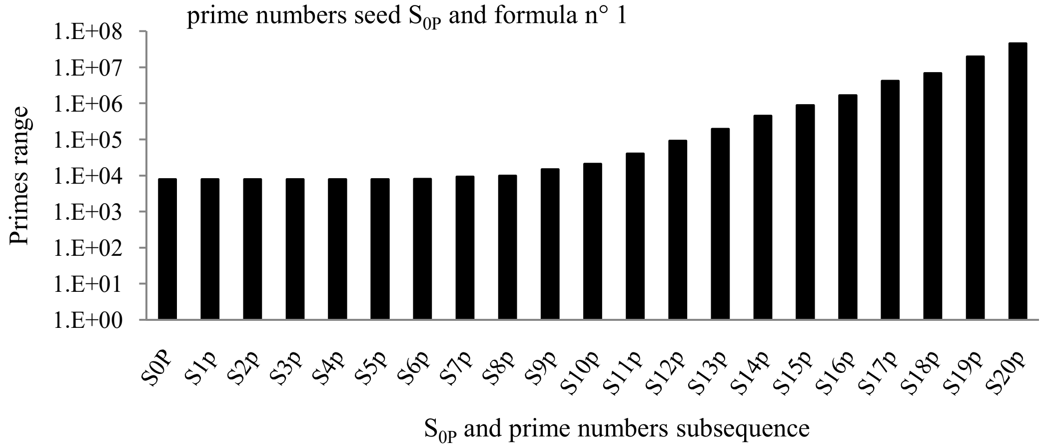


Figure n° 6: prime number range values for S_{0P} and each prime numbers subsequence.
prime numbers seed S_{0P} and formula n° 1



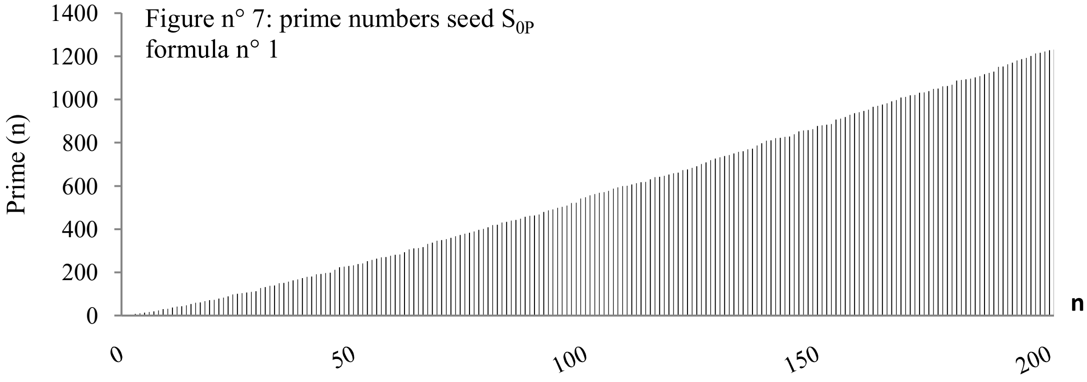


Figure n° 7: prime numbers seed S_{0P} formula n° 1

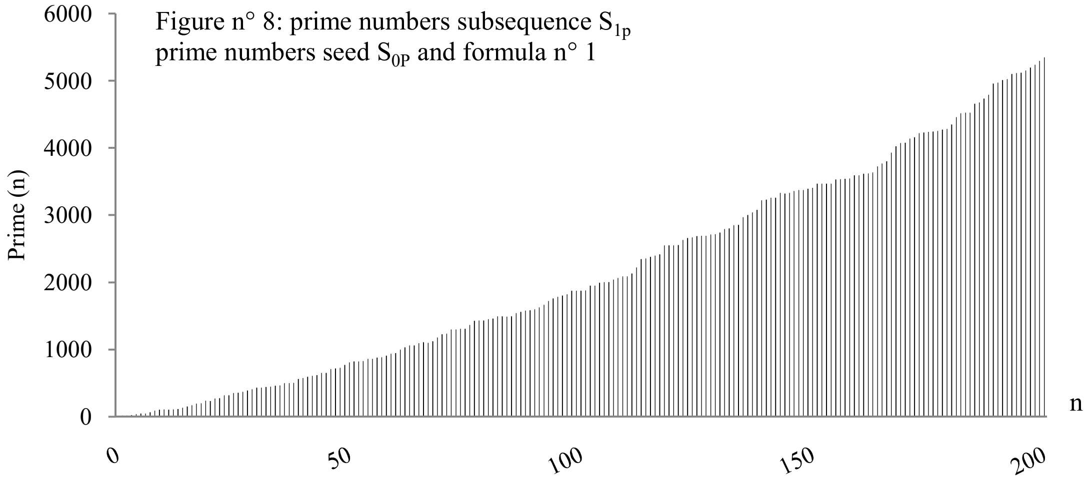


Figure n° 8: prime numbers subsequence S_{1p} prime numbers seed S_{0P} and formula n° 1

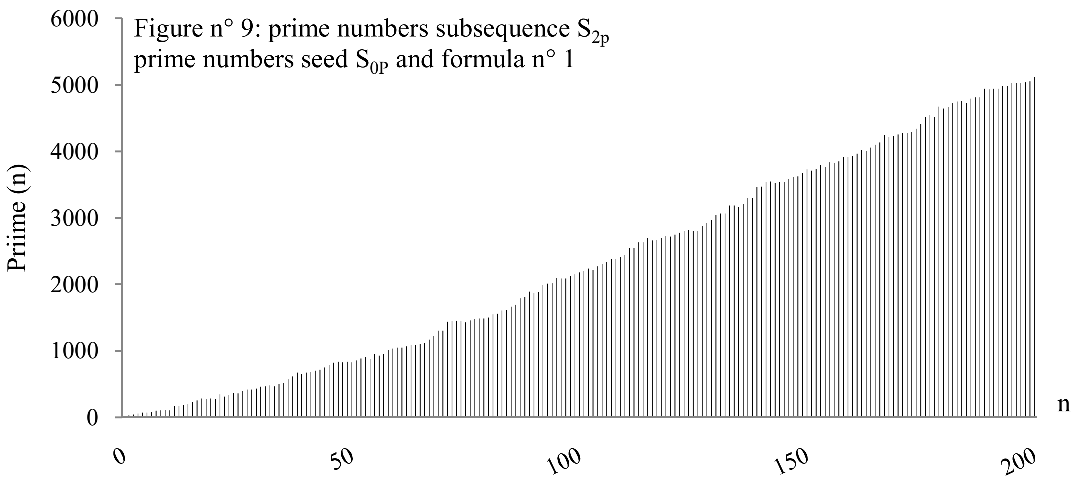


Figure n° 9: prime numbers subsequence S_{2p} prime numbers seed S_{0P} and formula n° 1

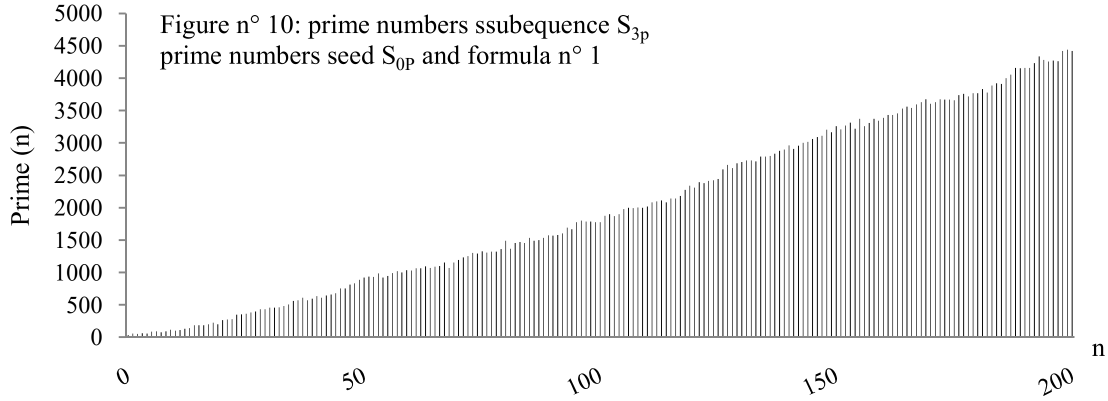


Figure n° 10: prime numbers ssubequence S_{3p} prime numbers seed S_{0P} and formula n° 1

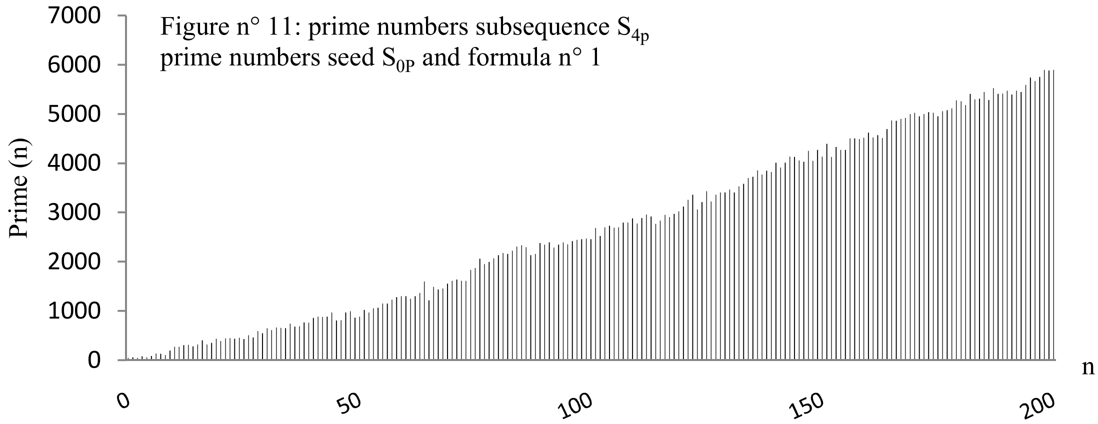


Figure n° 11: prime numbers subsequence S_{4p} prime numbers seed S_{0P} and formula n° 1

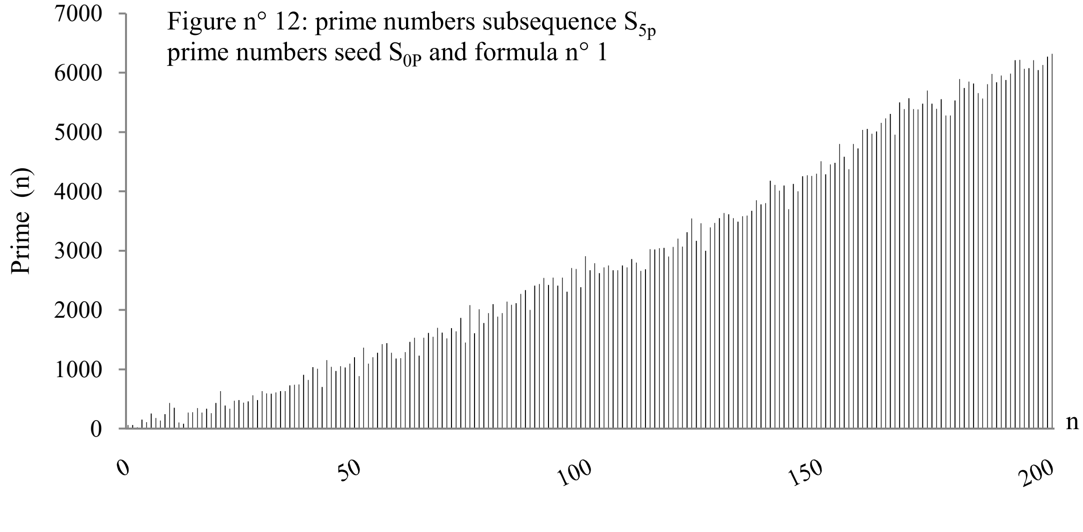


Figure n° 12: prime numbers subsequence S_{5p} prime numbers seed S_{0P} and formula n° 1

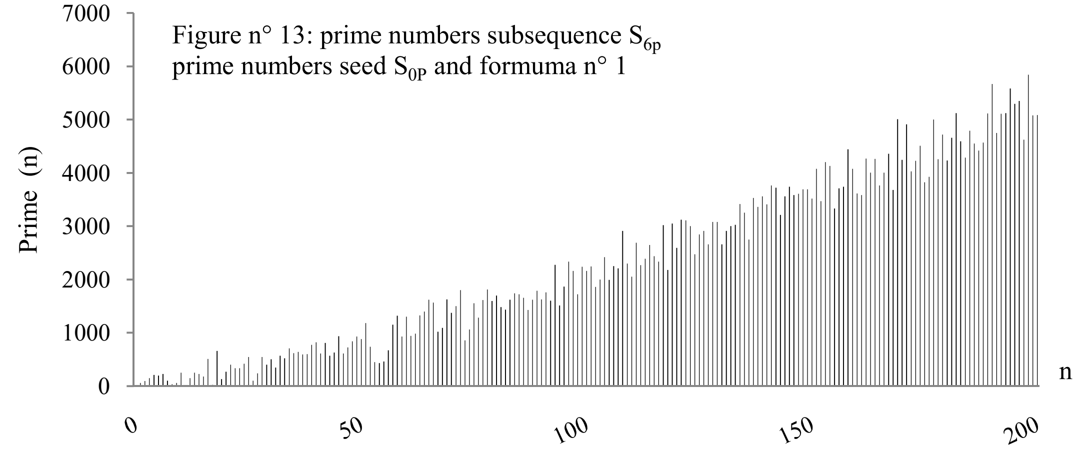


Figure n° 13: prime numbers subsequence S6p
prime numbers seed S0P and formuma n° 1
Prime (n)
n
0
1000
2000
3000
4000
5000
6000
7000
0
50
100
150
200

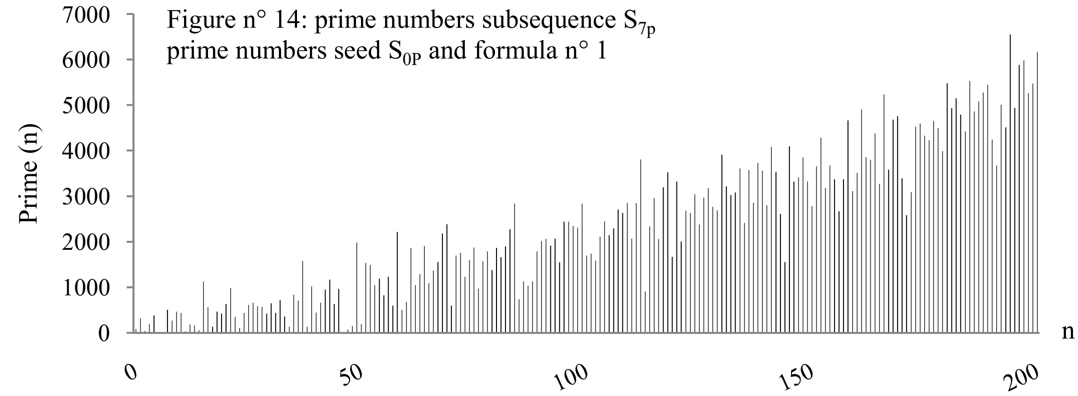


Figure n° 14: prime numbers subsequence S7p
prime numbers seed S0P and formula n° 1
Prime (n)
n
0
1000
2000
3000
4000
5000
6000
7000
0
50
100
150
200

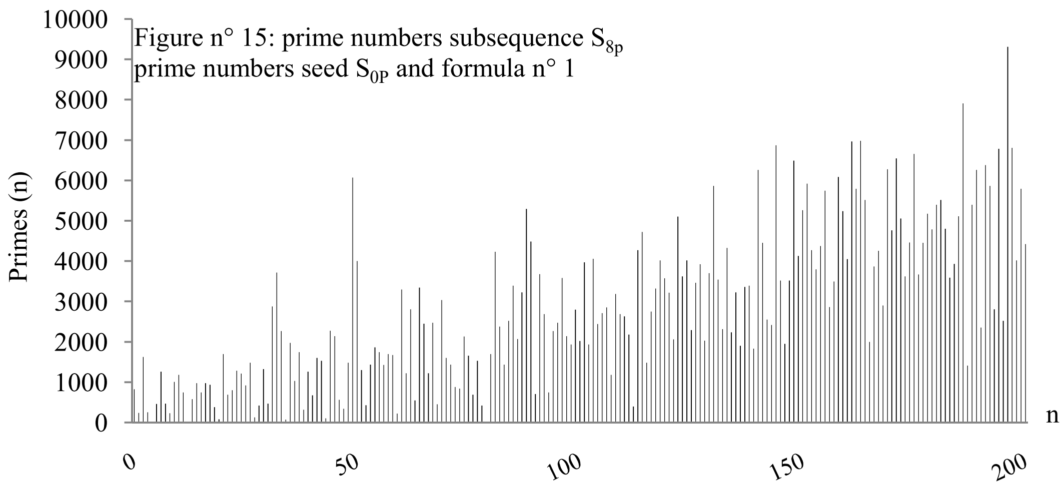


Figure n° 15: prime numbers subsequence S8p
prime numbers seed S0P and formula n° 1
Primes (n)
n
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
0
50
100
150
200

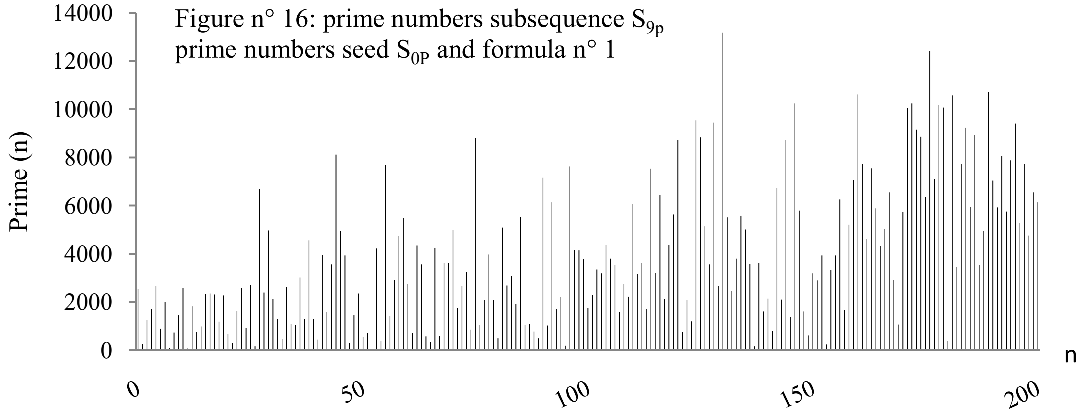


Figure n° 16: prime numbers subsequence S9p
prime numbers seed S0P and formula n° 1
Prime (n)
n

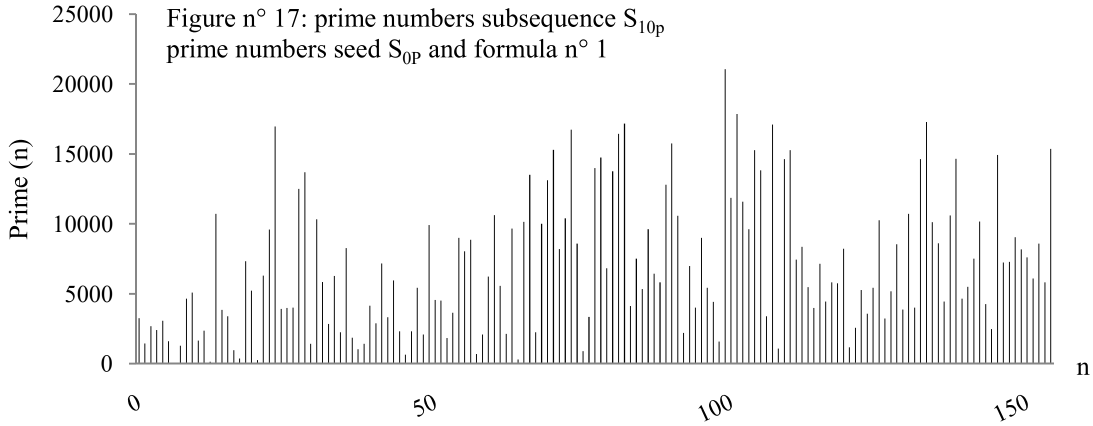


Figure n° 17: prime numbers subsequence S10p
prime numbers seed S0P and formula n° 1
Prime (n)
n

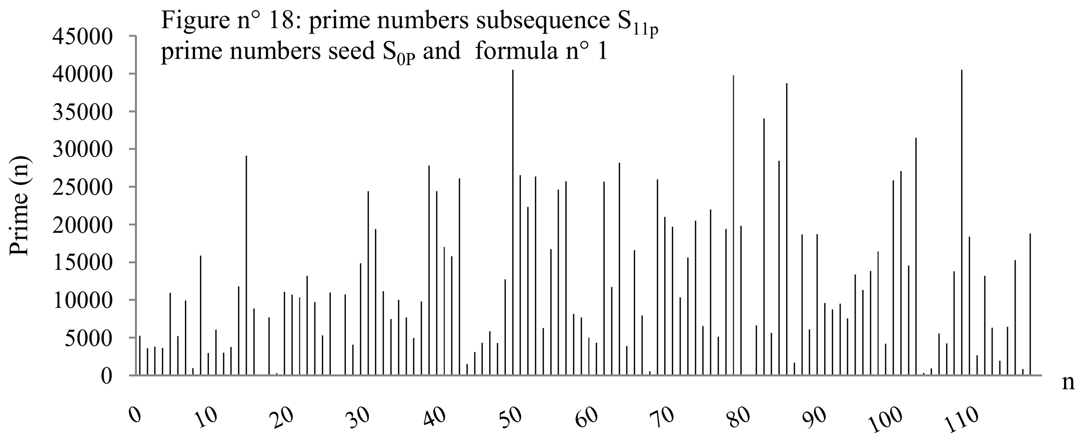


Figure n° 18: prime numbers subsequence S11p
prime numbers seed S0P and formula n° 1
Prime (n)
n

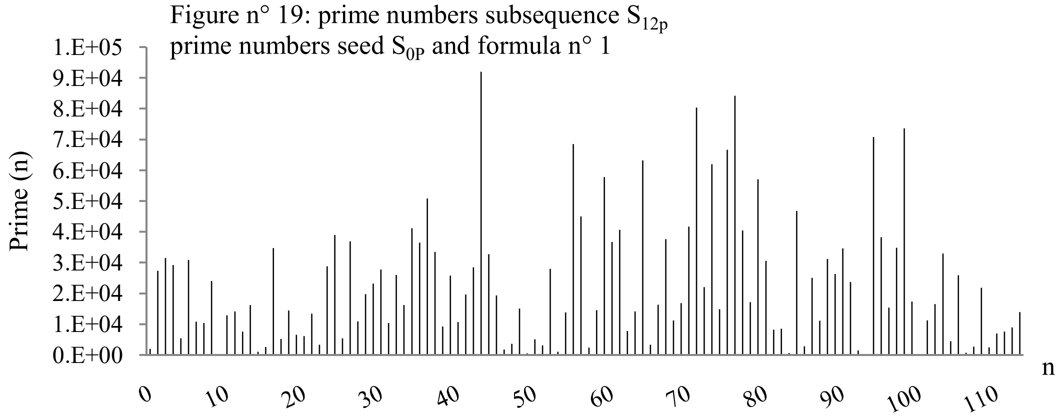


Figure n° 19: prime numbers subsequence S_{12p} prime numbers seed S_{0P} and formula n° 1

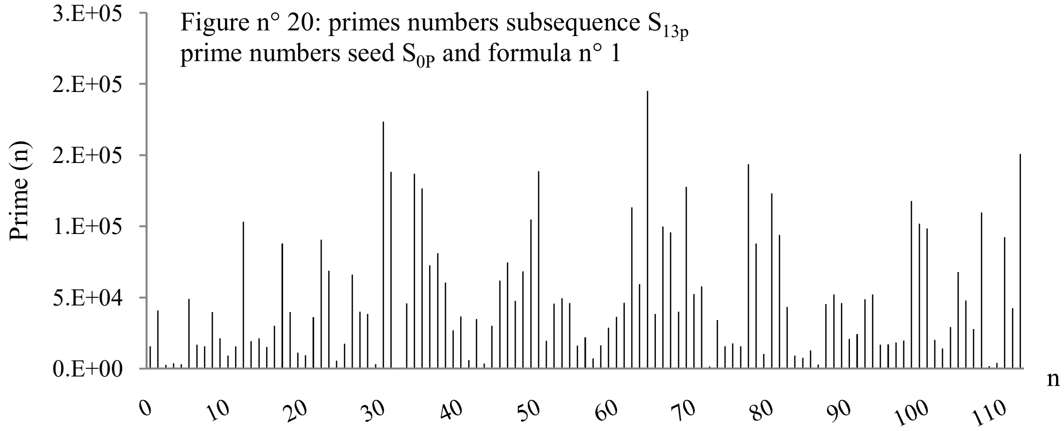


Figure n° 20: primes numbers subsequence S_{13p} prime numbers seed S_{0P} and formula n° 1

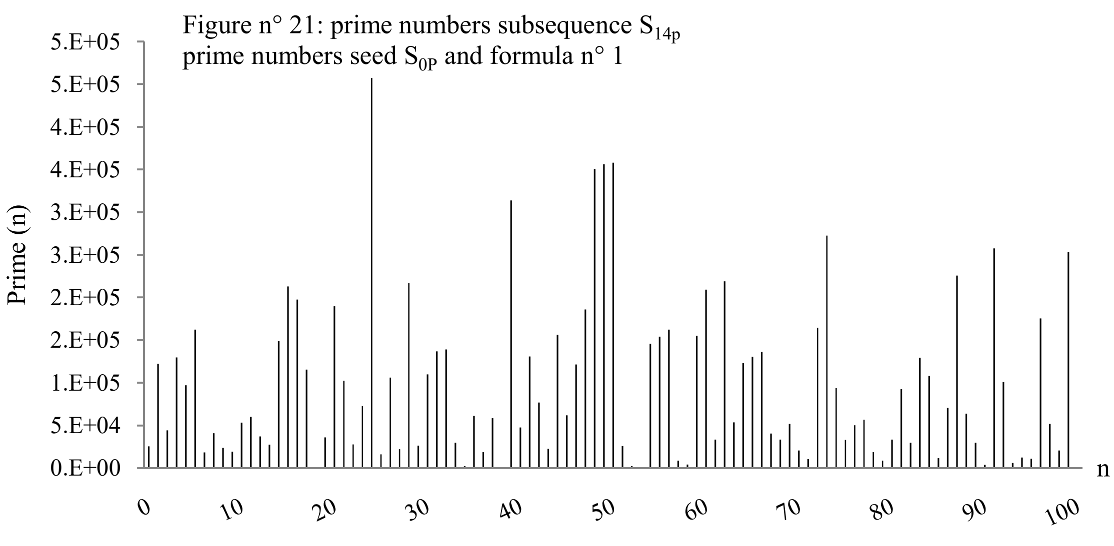


Figure n° 21: prime numbers subsequence S_{14p} prime numbers seed S_{0P} and formula n° 1

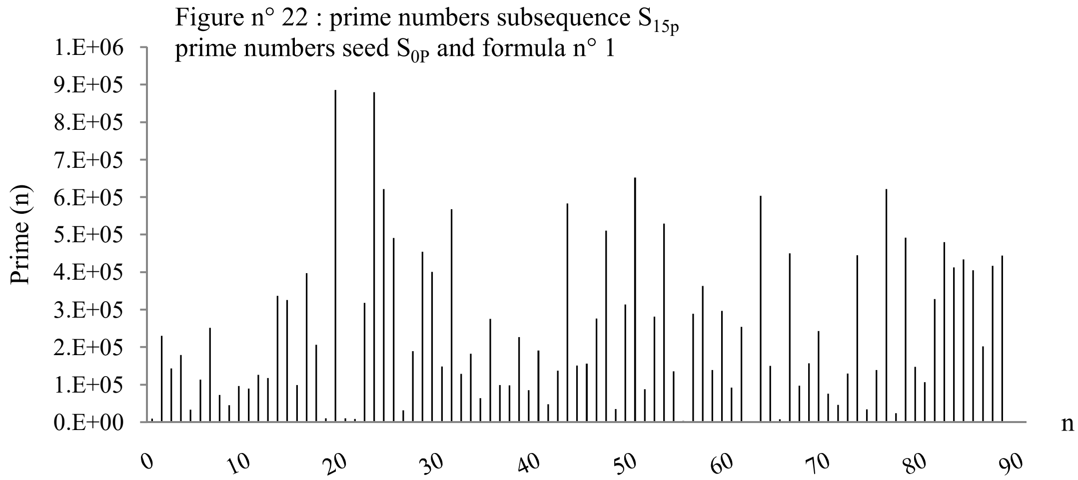


Figure n° 22 : prime numbers subsequence S_{15p} prime numbers seed S_{0P} and formula n° 1

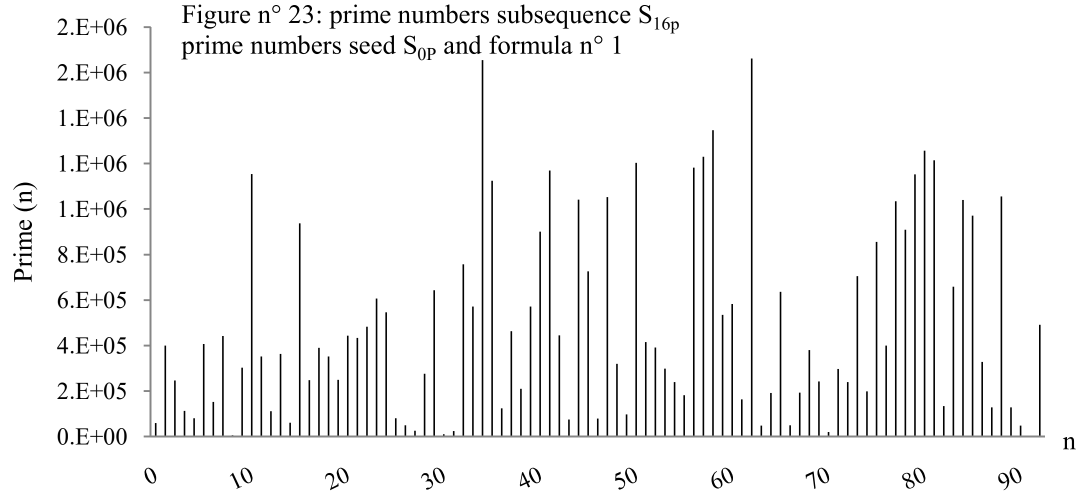


Figure n° 23: prime numbers subsequence S_{16p} prime numbers seed S_{0P} and formula n° 1

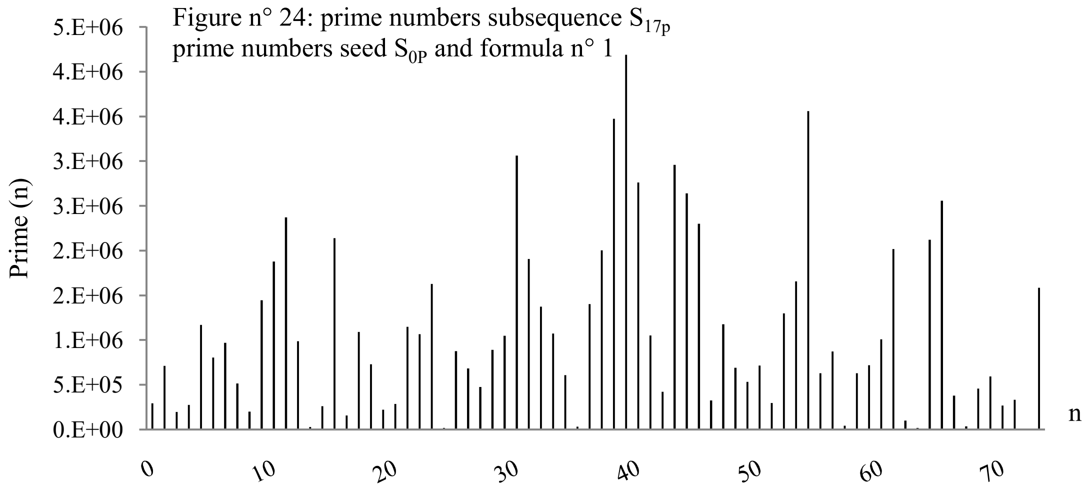


Figure n° 24: prime numbers subsequence S_{17p} prime numbers seed S_{0P} and formula n° 1

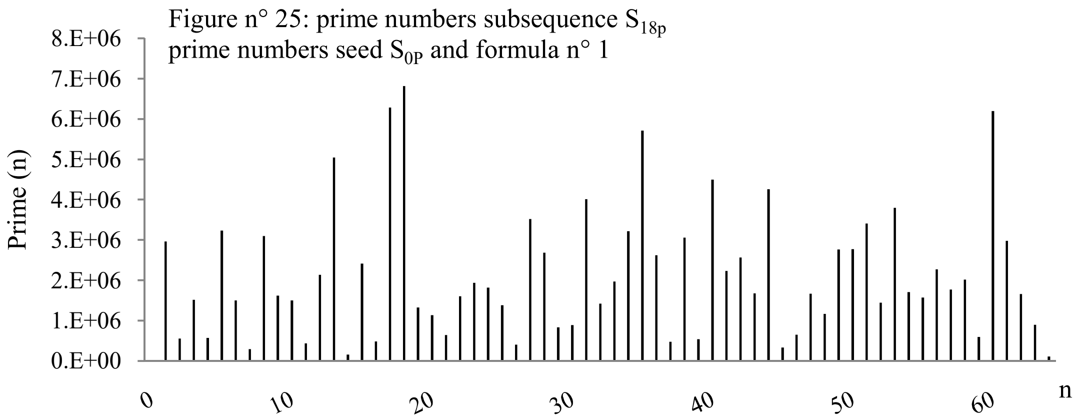


Figure n° 25: prime numbers subsequence S_{18p}
prime numbers seed S_{0P} and formula n° 1
Prime (n)
8.E+06
7.E+06
6.E+06
5.E+06
4.E+06
3.E+06
2.E+06
1.E+06
0.E+00
0
10
20
30
40
50
60
n

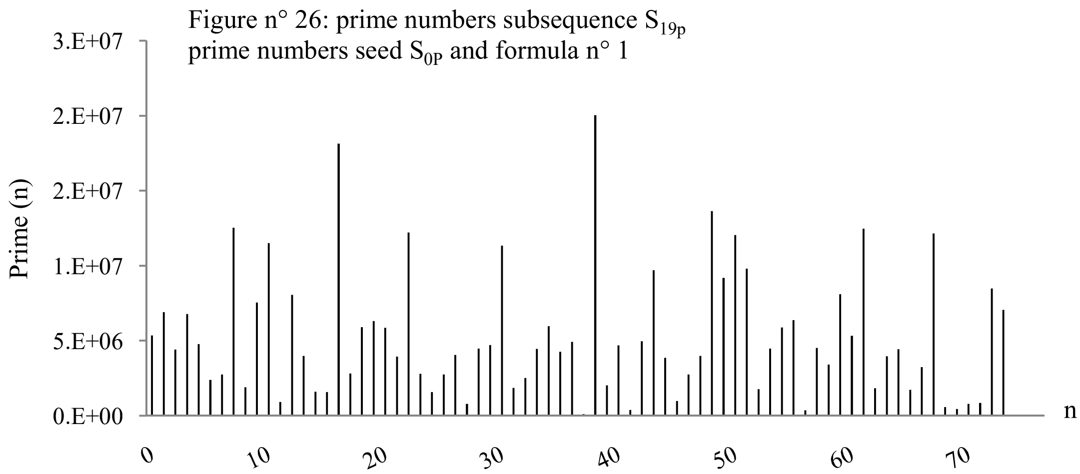


Figure n° 26: prime numbers subsequence S_{19p}
prime numbers seed S_{0P} and formula n° 1
Prime (n)
3.E+07
2.E+07
2.E+07
1.E+07
5.E+06
0.E+00
0
10
20
30
40
50
60
70
n

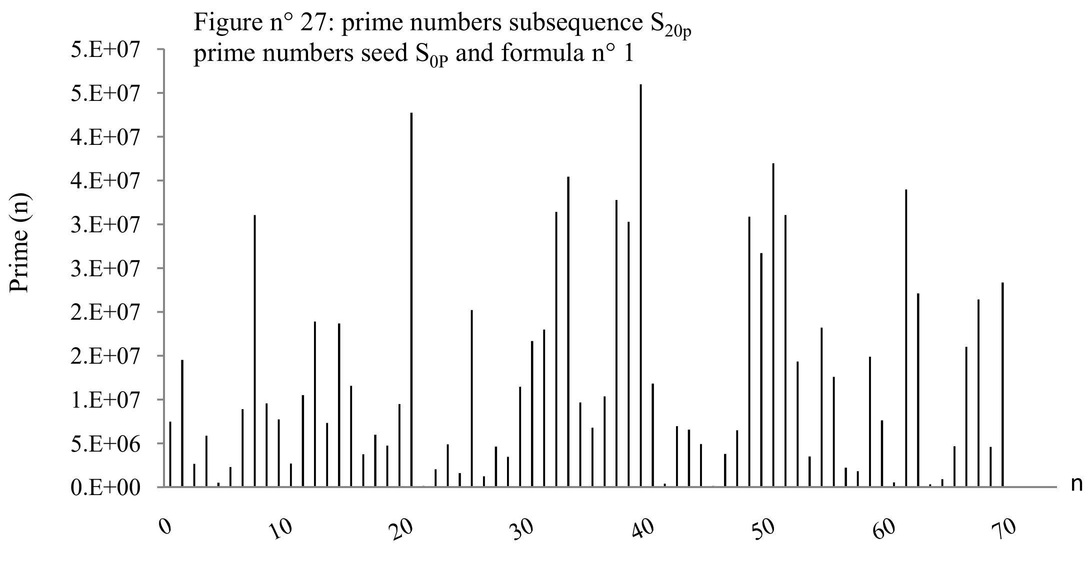


Figure n° 27: prime numbers subsequence S_{20p}
prime numbers seed S_{0P} and formula n° 1
Prime (n)
5.E+07
5.E+07
4.E+07
4.E+07
3.E+07
3.E+07
2.E+07
2.E+07
1.E+07
5.E+06
0.E+00
0
10
20
30
40
50
60
70
n

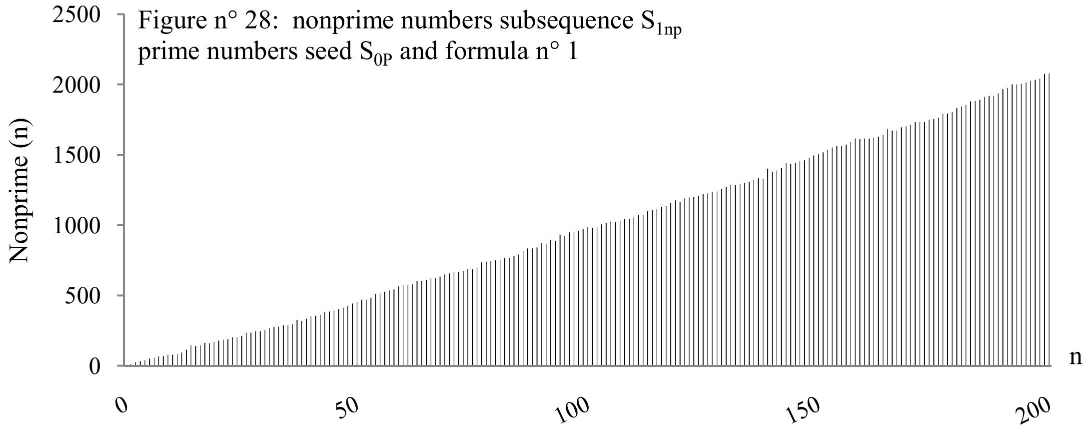


Figure n° 28: nonprime numbers subsequence S_{1np}
prime numbers seed S_{0P} and formula n° 1
Nonprime (n)
2500
2000
1500
1000
500
0
n
0
50
100
150
200

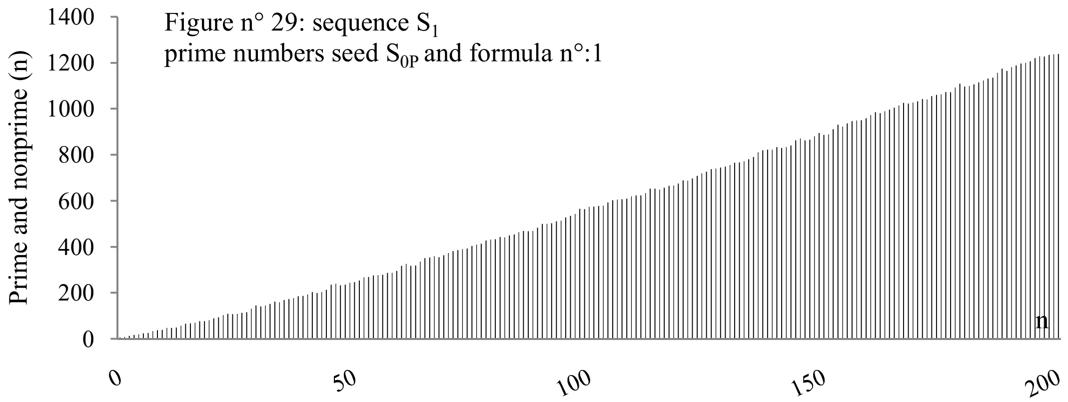


Figure n° 29: sequence S_1
prime numbers seed S_{0P} and formula n°:1
Prime and nonprime (n)
1400
1200
1000
800
600
400
200
0
n
0
50
100
150
200

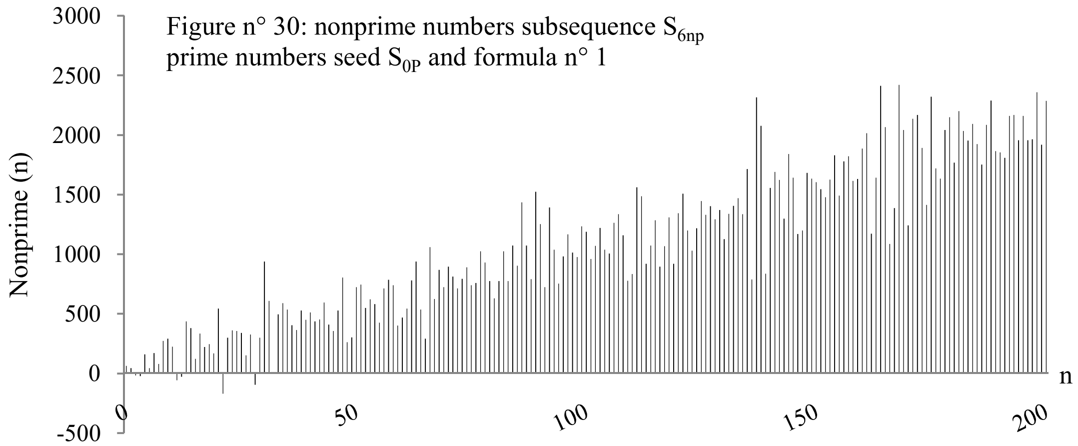


Figure n° 30: nonprime numbers subsequence S_{6np}
prime numbers seed S_{0P} and formula n° 1
Nonprime (n)
3000
2500
2000
1500
1000
500
0
-500
n
0
50
100
150
200

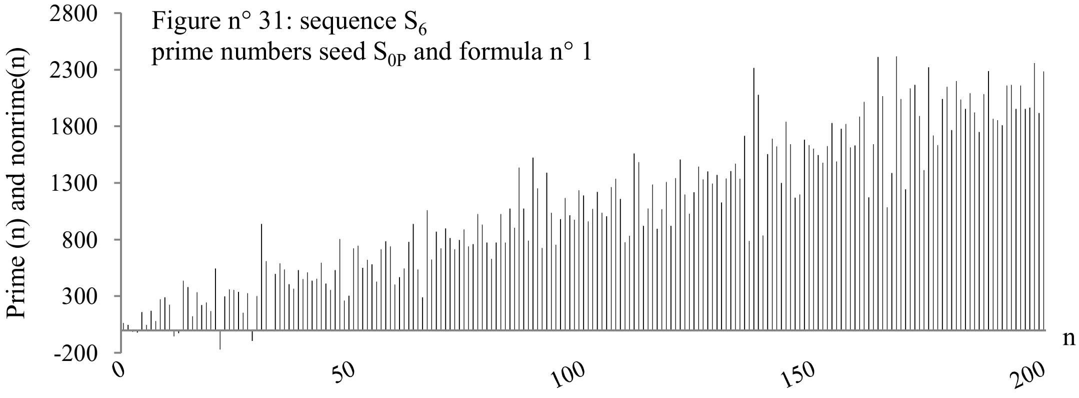


Figure n° 31: sequence S_6
prime numbers seed S_{0P} and formula n° 1

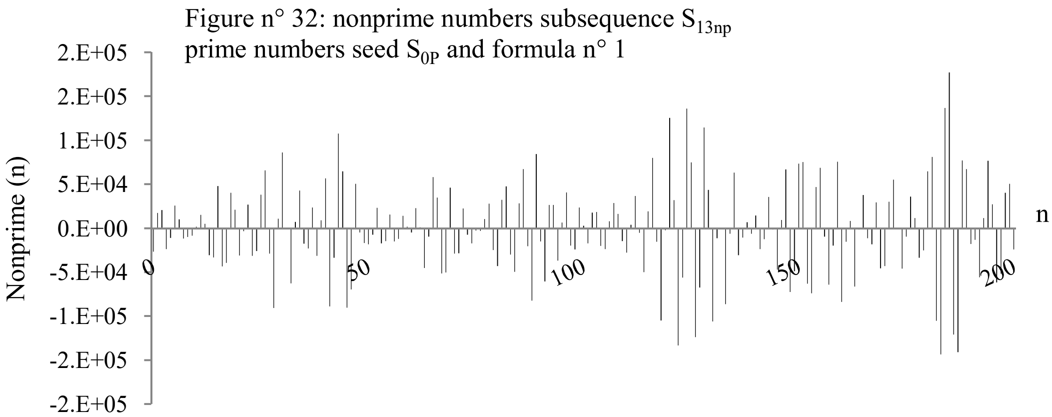


Figure n° 32: nonprime numbers subsequence S_{13np}
prime numbers seed S_{0P} and formula n° 1

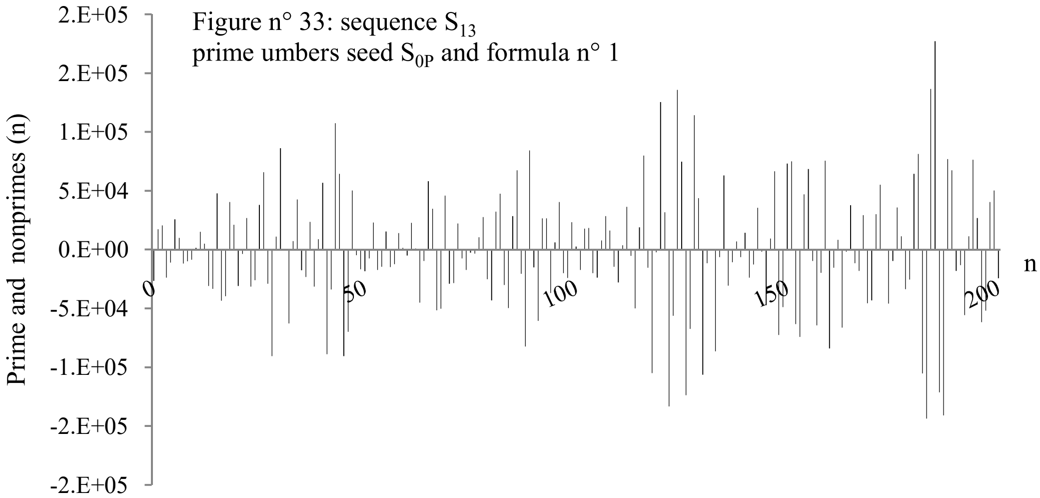


Figure n° 33: sequence S_{13}
prime umbers seed S_{0P} and formula n° 1

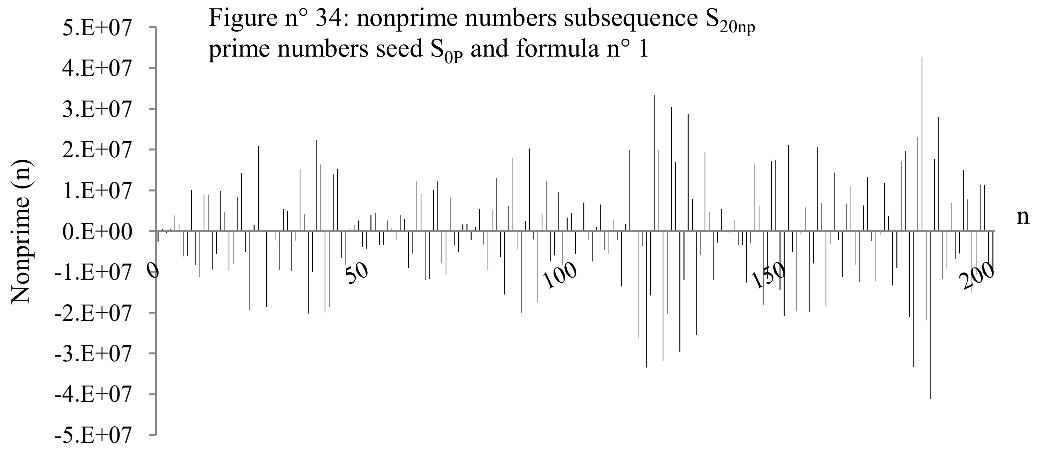


Figure n° 34: nonprime numbers subsequence S_{20np}
prime numbers seed S_{0P} and formula n° 1
5.E+07
4.E+07
3.E+07
2.E+07
1.E+07
0.E+00
-1.E+07
-2.E+07
-3.E+07
-4.E+07
-5.E+07
Nonprime (n)
n
0
50
100
150
200

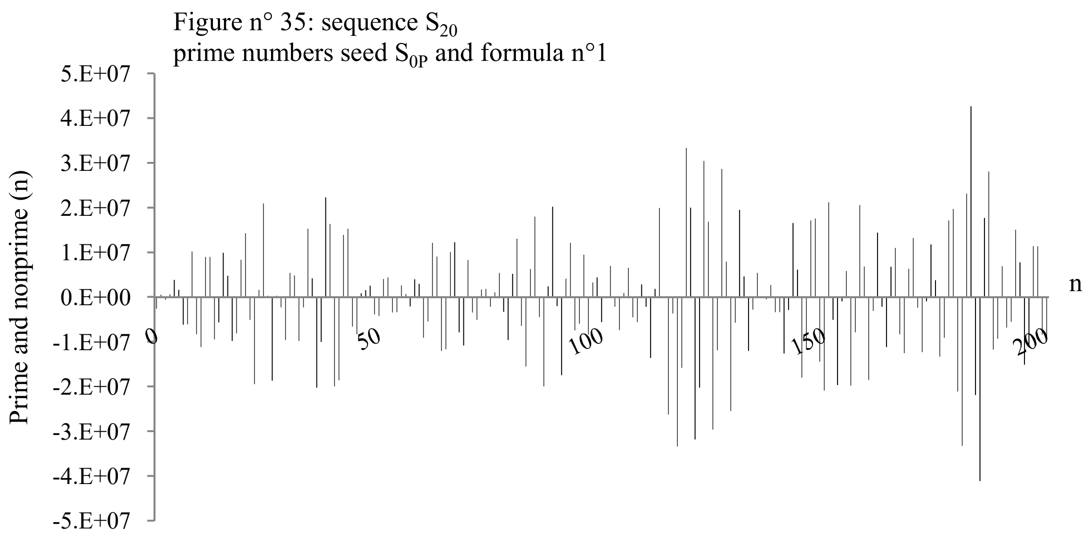


Figure n° 35: sequence S_{20}
prime numbers seed S_{0P} and formula n°1
5.E+07
4.E+07
3.E+07
2.E+07
1.E+07
0.E+00
-1.E+07
-2.E+07
-3.E+07
-4.E+07
-5.E+07
Prime and nonprime (n)
n
0
50
100
150
200

Figures from the nonprime numbers seed S_{0NP} and formula n°:1

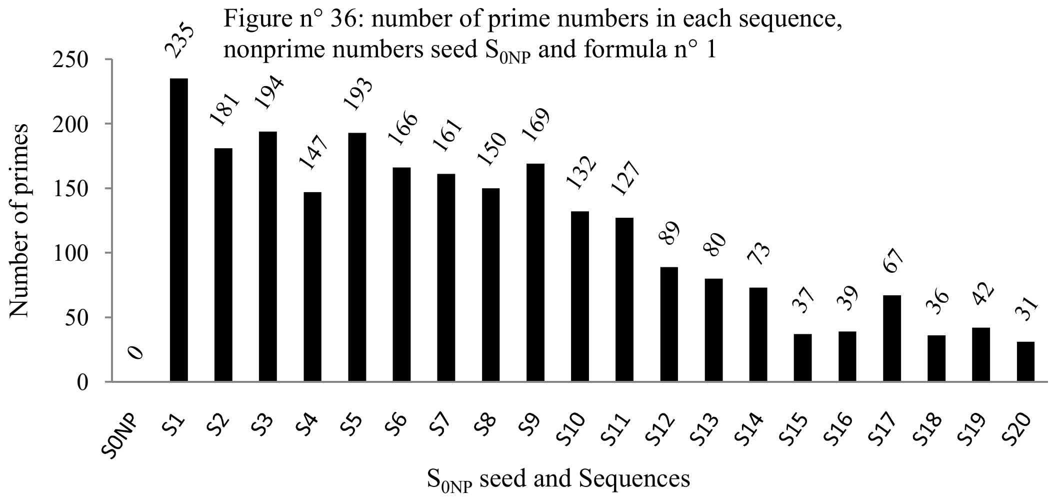


Figure n° 36: number of prime numbers in each sequence, nonprime numbers seed S_{0NP} and formula n° 1

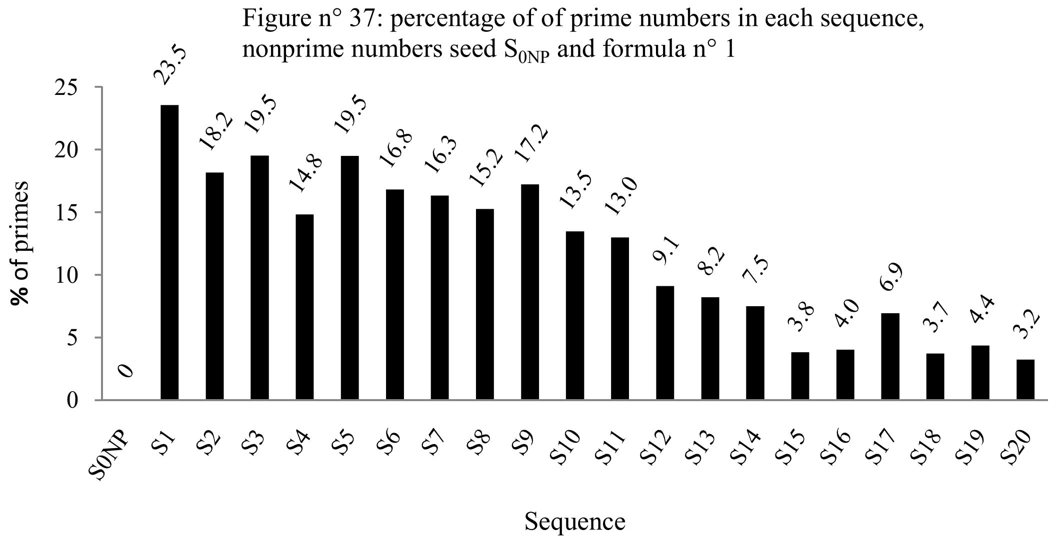


Figure n° 37: percentage of of prime numbers in each sequence, nonprime numbers seed S_{0NP} and formula n° 1

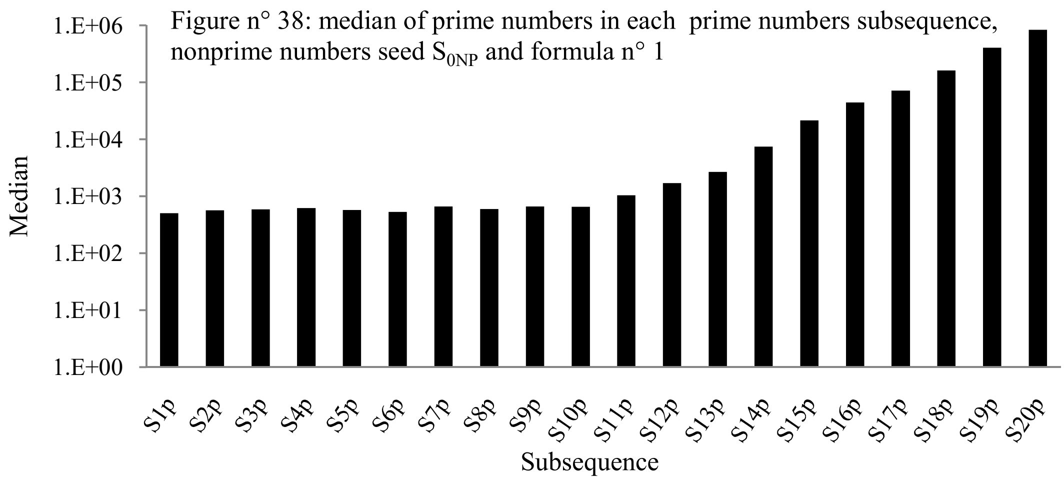


Figure n° 38: median of prime numbers in each prime numbers subsequence, nonprime numbers seed S_{0NP} and formula n° 1

Figure n° 39: prime number minimum and maximum values in each prime numbers subsequence,
nonprime numbers seed S_{0NP} and formula n° 1

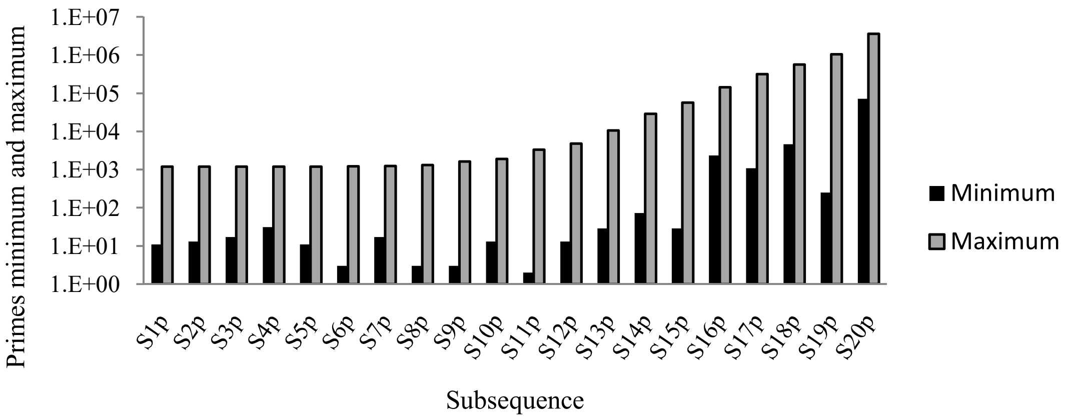


Figure n° 40: prime number range values for each prime numbers subsequence
nonprime numbers seed S_{0NP} and formula n° 1

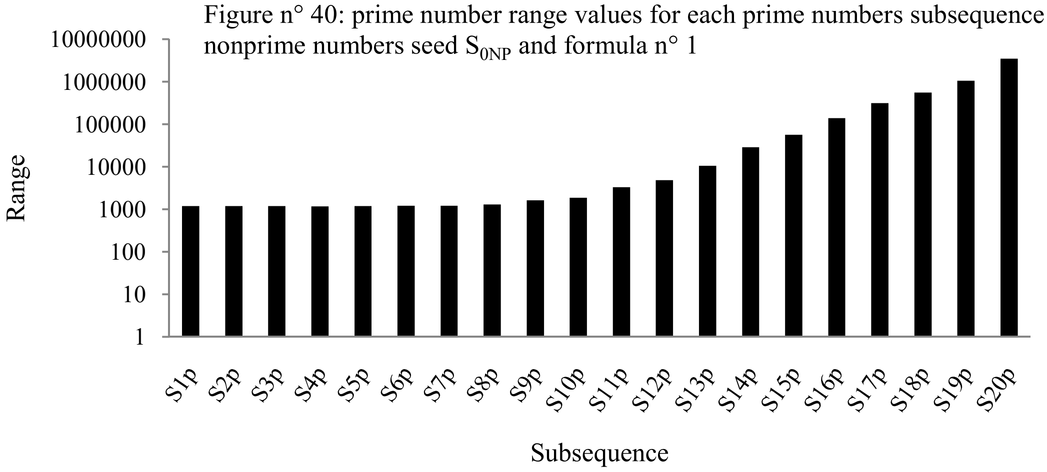
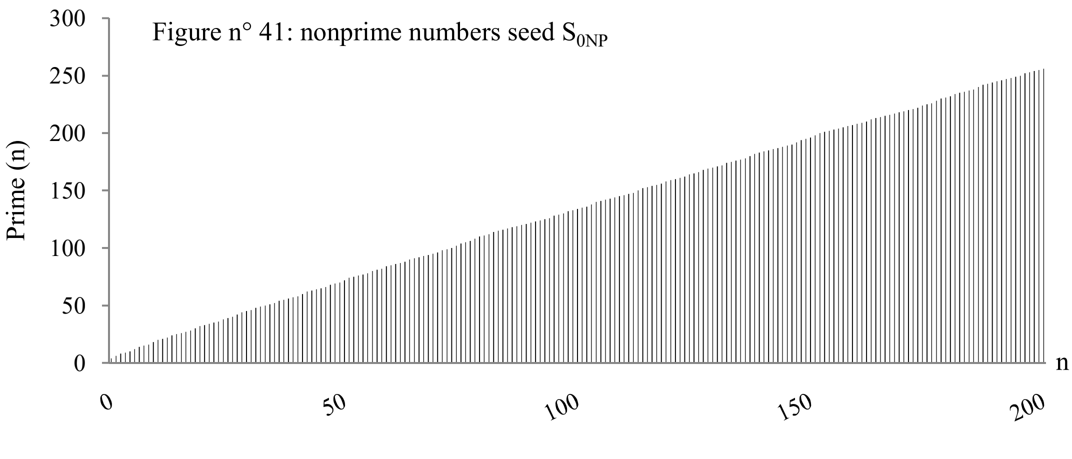


Figure n° 41: nonprime numbers seed S_{0NP}



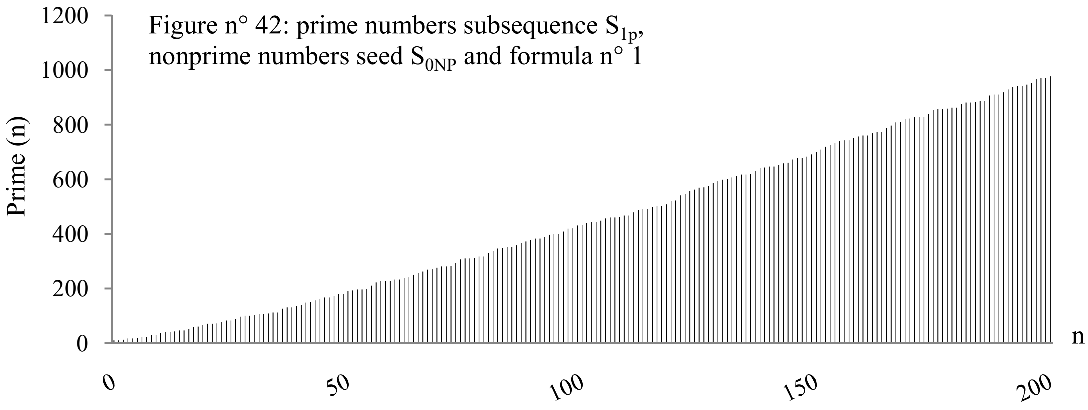


Figure n° 42: prime numbers subsequence S_{1p}, nonprime numbers seed S_{0NP} and formula n° 1

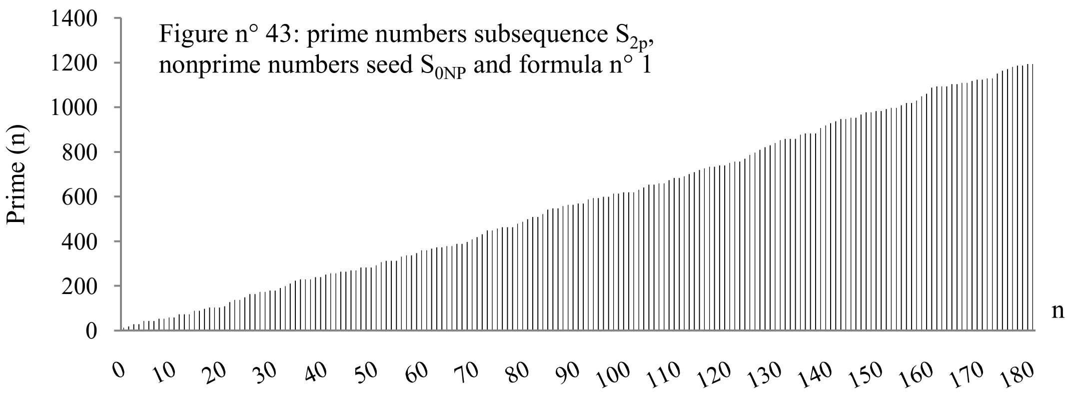


Figure n° 43: prime numbers subsequence S_{2p}, nonprime numbers seed S_{0NP} and formula n° 1

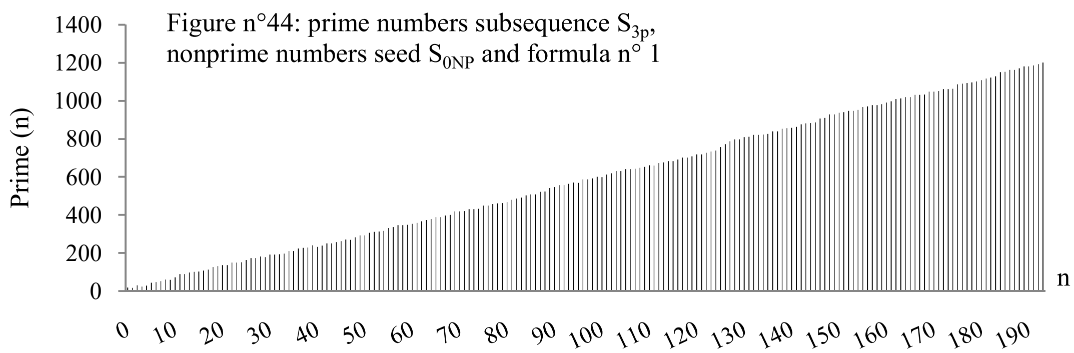


Figure n°44: prime numbers subsequence S_{3p}, nonprime numbers seed S_{0NP} and formula n° 1

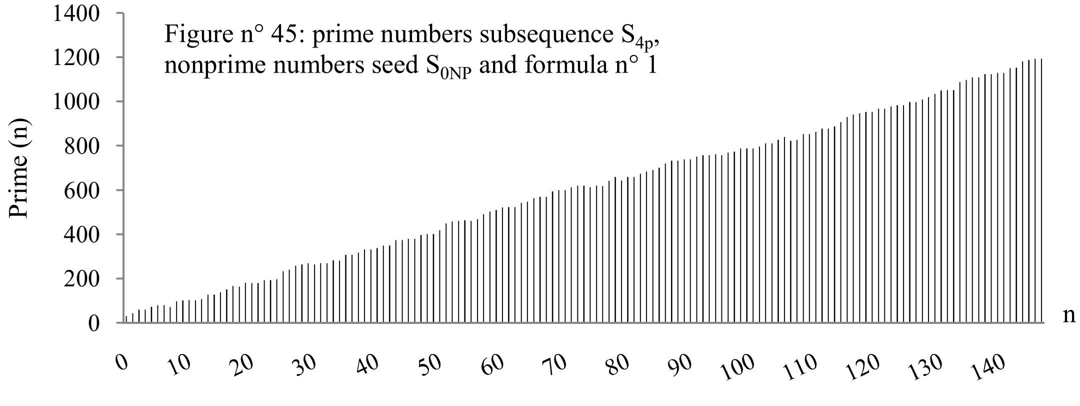


Figure n° 45: prime numbers subsequence S_{4p}, nonprime numbers seed S_{0NP} and formula n° 1

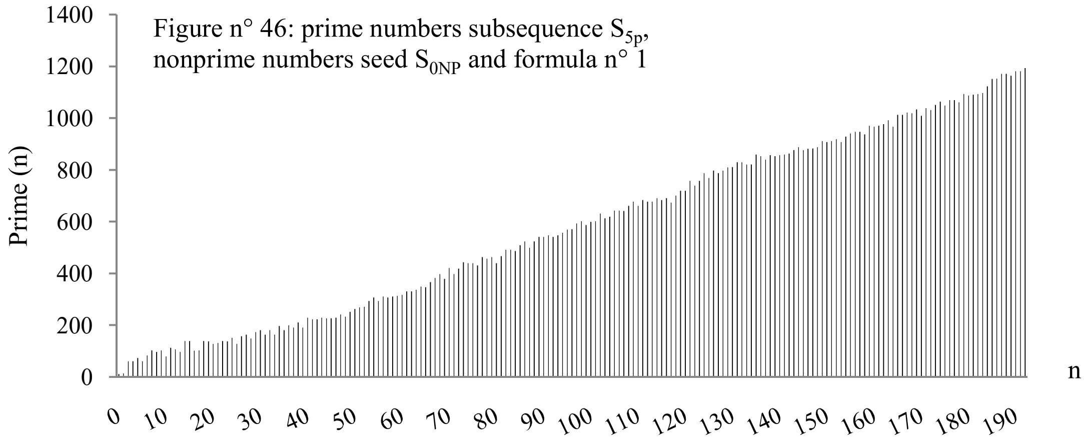


Figure n° 46: prime numbers subsequence S_{5p}, nonprime numbers seed S_{0NP} and formula n° 1

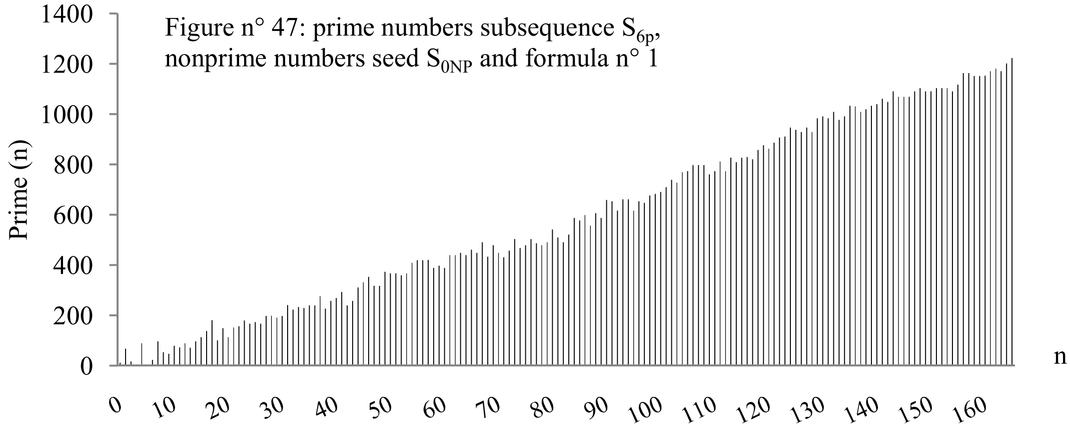


Figure n° 47: prime numbers subsequence S_{6p}, nonprime numbers seed S_{0NP} and formula n° 1

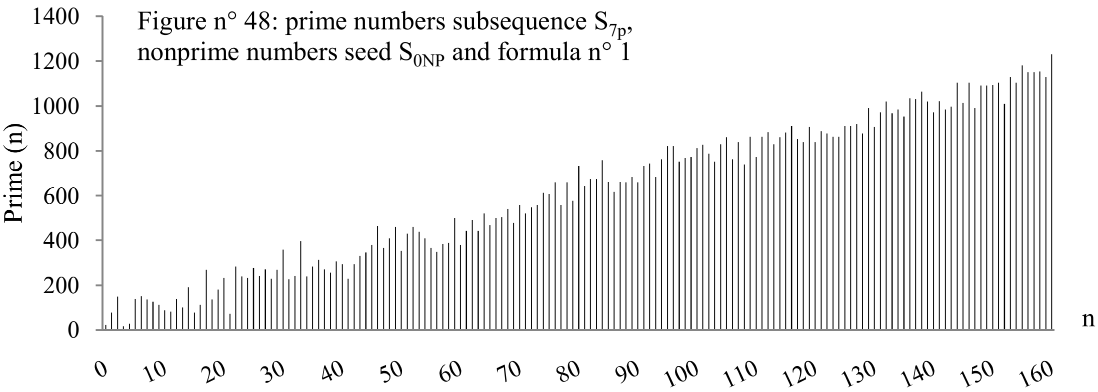


Figure n° 48: prime numbers subsequence S_{7p}, nonprime numbers seed S_{0NP} and formula n° 1

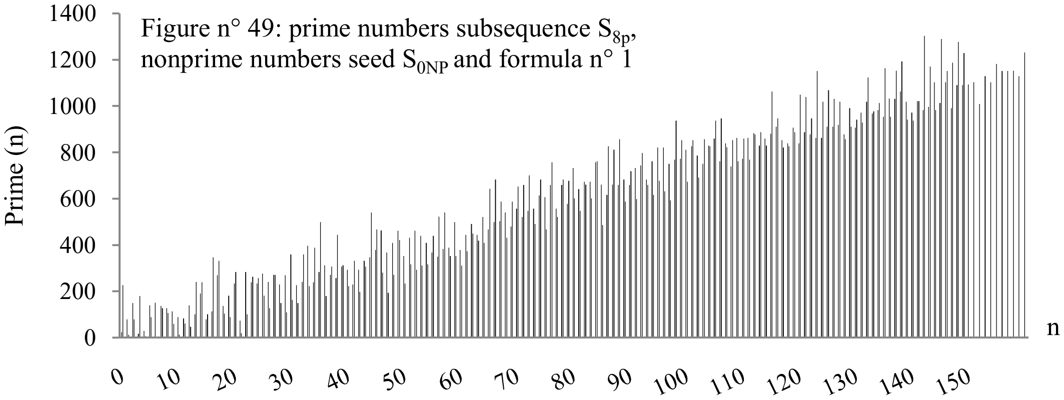


Figure n° 49: prime numbers subsequence S_{8p}, nonprime numbers seed S_{0NP} and formula n° 1

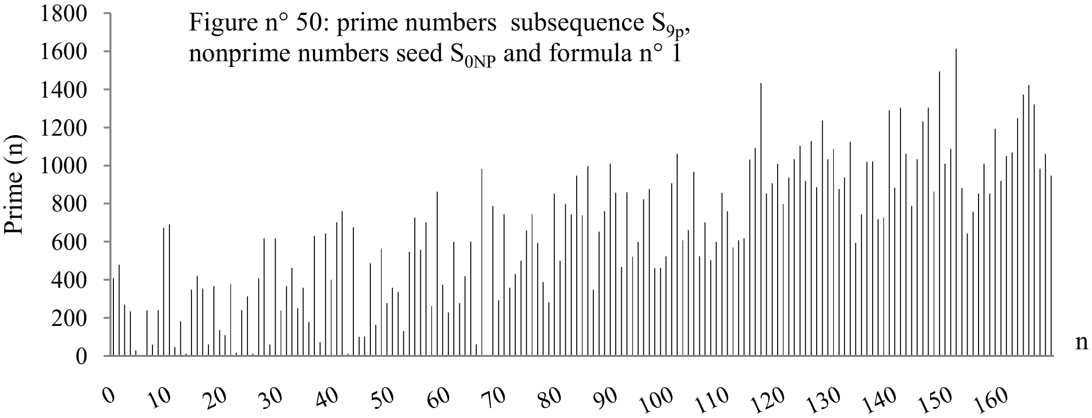


Figure n° 50: prime numbers subsequence S_{9p}, nonprime numbers seed S_{0NP} and formula n° 1

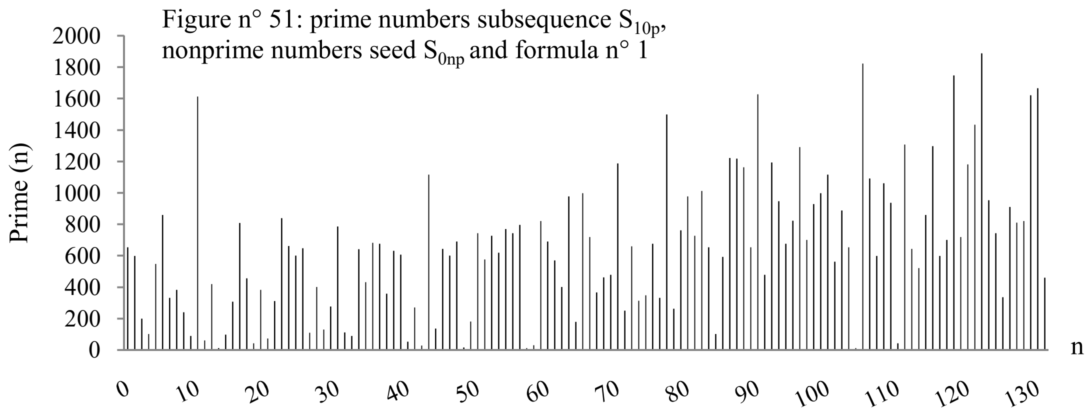


Figure n° 51: prime numbers subsequence S_{10p}, nonprime numbers seed S_{0np} and formula n° 1
Prime (n)
n
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0
10
20
30
40
50
60
70
80
90
100
110
120
130

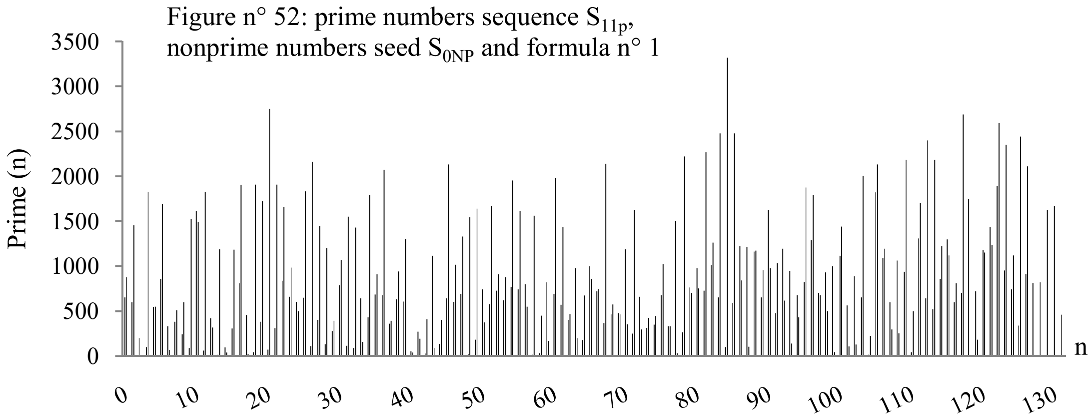


Figure n° 52: prime numbers sequence S_{11p}, nonprime numbers seed S_{0NP} and formula n° 1
Prime (n)
n
3500
3000
2500
2000
1500
1000
500
0
0
10
20
30
40
50
60
70
80
90
100
110
120
130

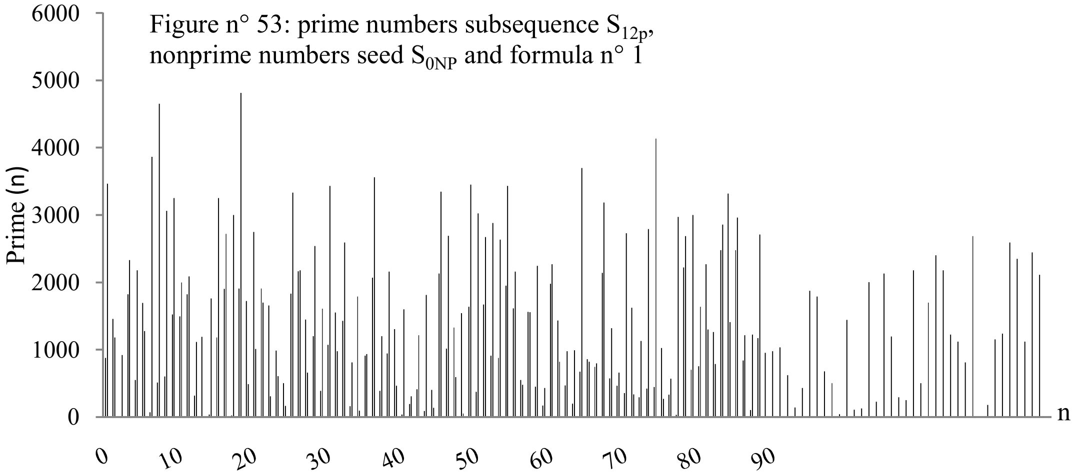


Figure n° 53: prime numbers subsequence S_{12p}, nonprime numbers seed S_{0NP} and formula n° 1
Prime (n)
n
6000
5000
4000
3000
2000
1000
0
0
10
20
30
40
50
60
70
80
90

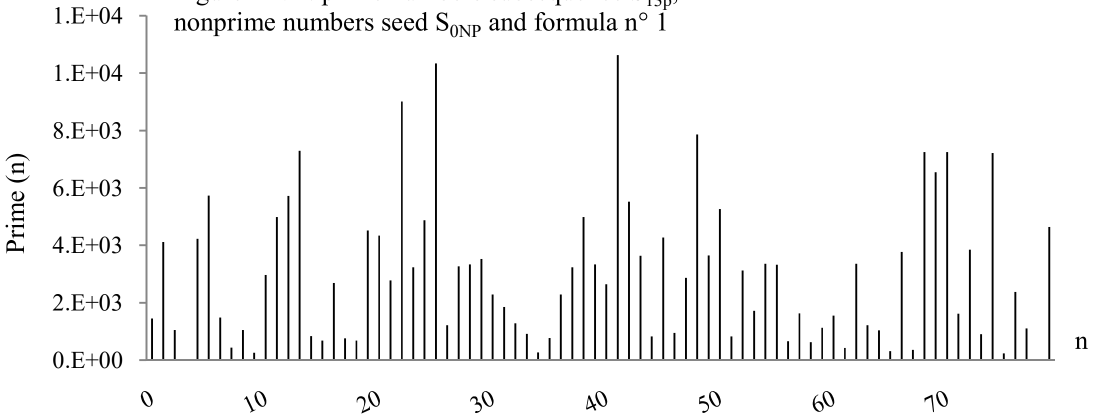


Figure n° 54: prime numbers subsequence S_{13p}, nonprime numbers seed S_{0NP} and formula n° 1

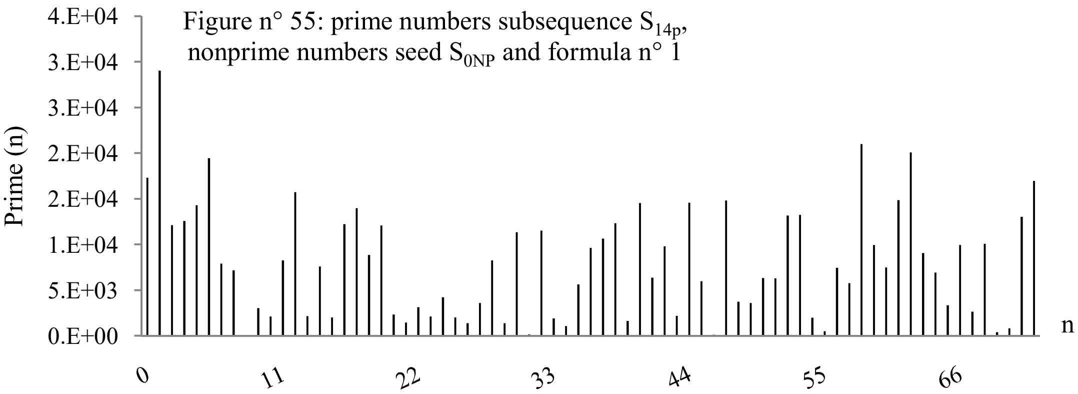


Figure n° 55: prime numbers subsequence S_{14p}, nonprime numbers seed S_{0NP} and formula n° 1

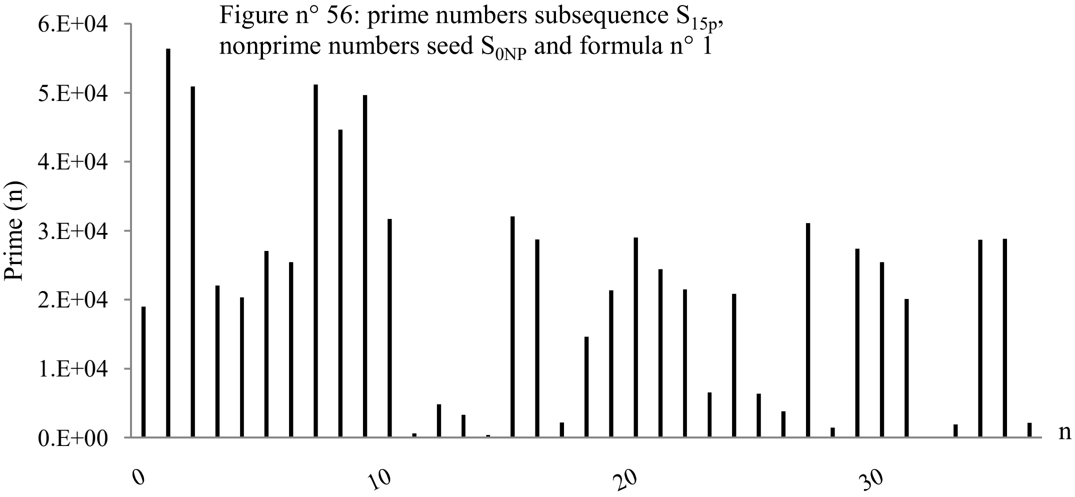


Figure n° 56: prime numbers subsequence S_{15p}, nonprime numbers seed S_{0NP} and formula n° 1

Figure n° 57: prime numbers subsequence S_{16p}, nonprime numbers seed S_{0NP} and formula n° 1

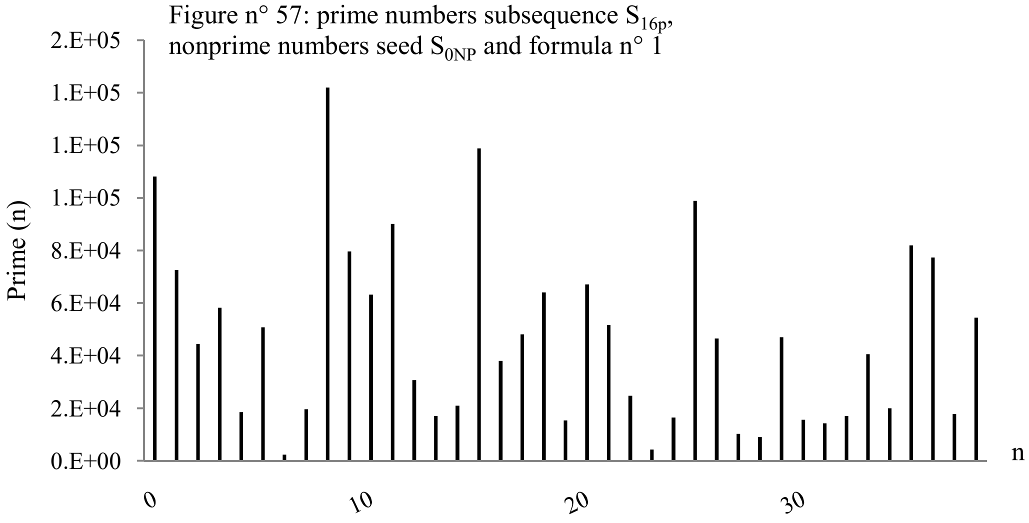


Figure n° 58: prime numbers subsequence S_{17p}, nonprime numbers seed S_{0NP} and formula n° 1

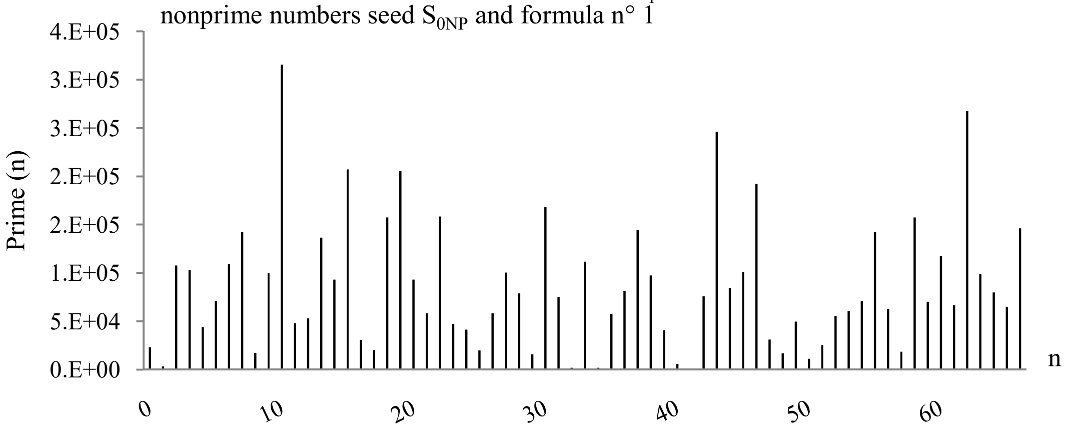
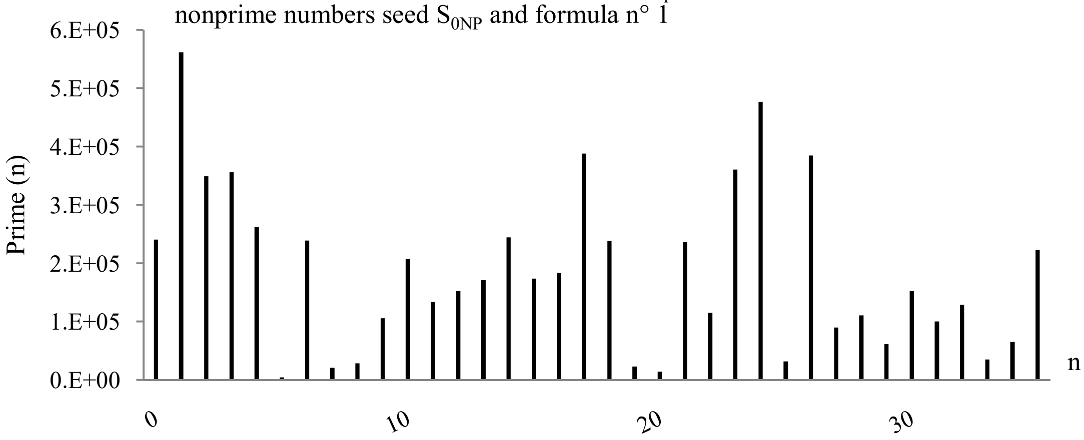


Figure n° 59: prime numbers subsequence S_{18p}, nonprime numbers seed S_{0NP} and formula n° 1



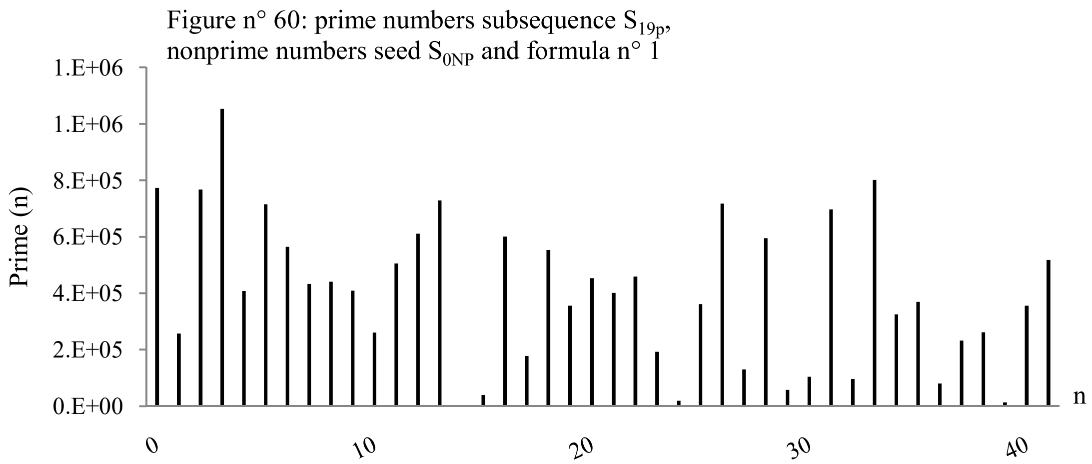


Figure n° 60: prime numbers subsequence S_{19p}, nonprime numbers seed S_{0NP} and formula n° 1

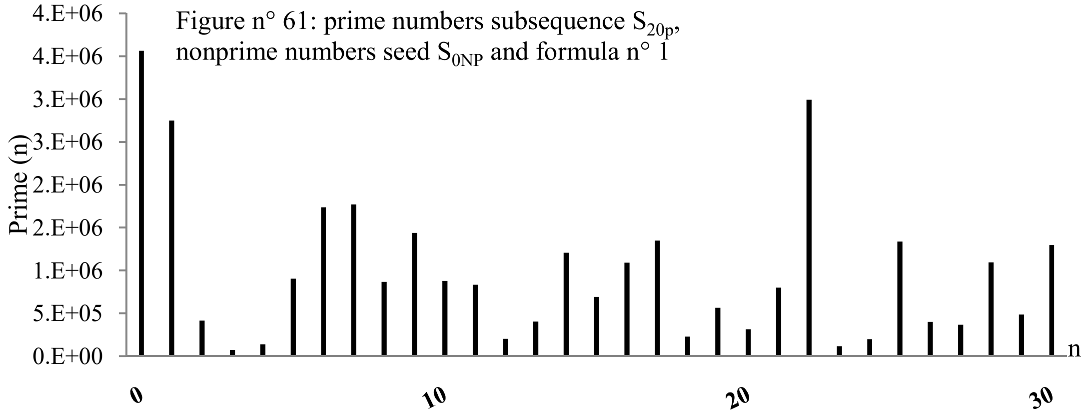


Figure n° 61: prime numbers subsequence S_{20p}, nonprime numbers seed S_{0NP} and formula n° 1

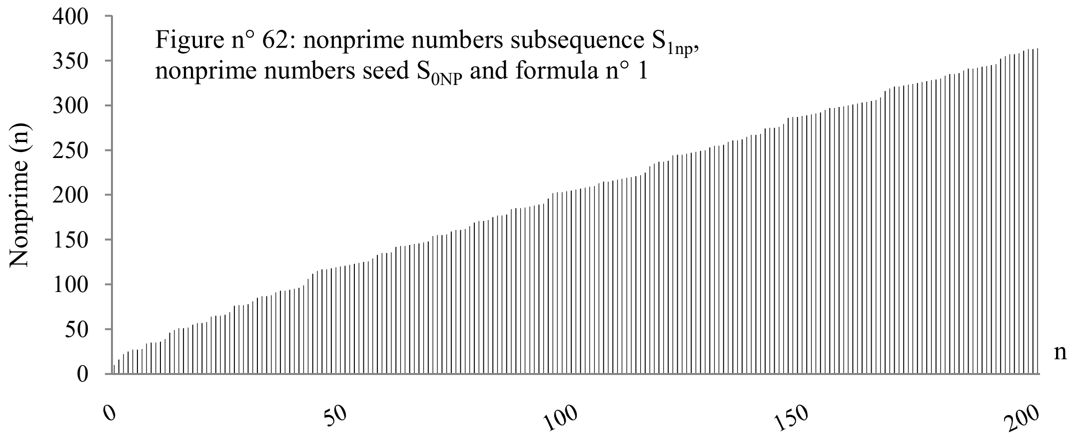


Figure n° 62: nonprime numbers subsequence S_{1np}, nonprime numbers seed S_{0NP} and formula n° 1

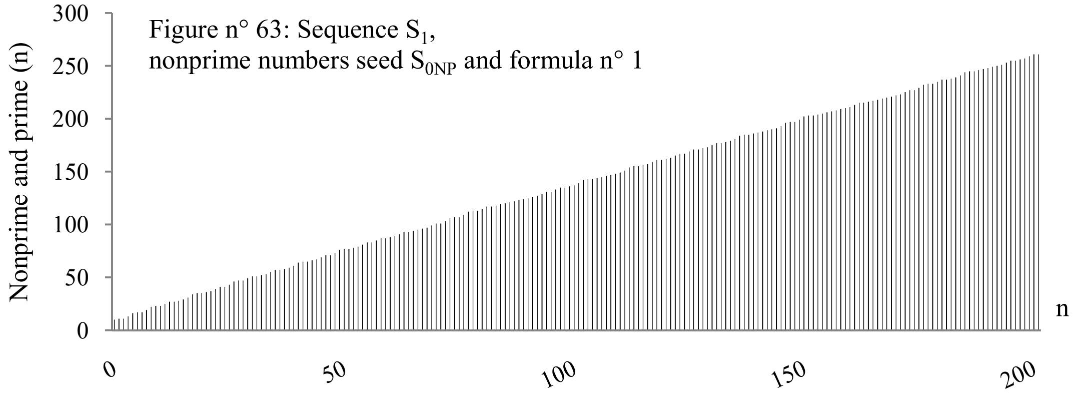


Figure n° 63: Sequence S_1,
nonprime numbers seed S_{0NP} and formula n° 1
Nonprime and prime (n)
300
250
200
150
100
50
0
n
0
50
100
150
200

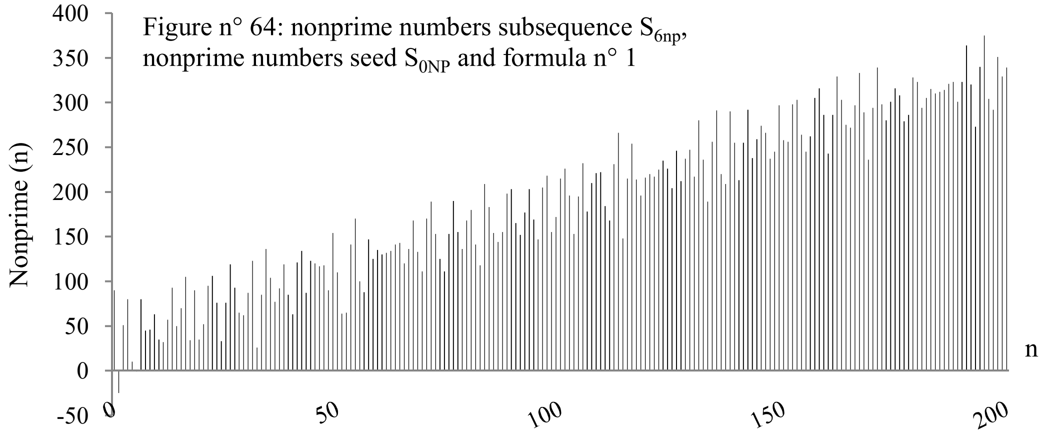


Figure n° 64: nonprime numbers subsequence S_{6np},
nonprime numbers seed S_{0NP} and formula n° 1
Nonprime (n)
400
350
300
250
200
150
100
50
0
-50
n
0
50
100
150
200

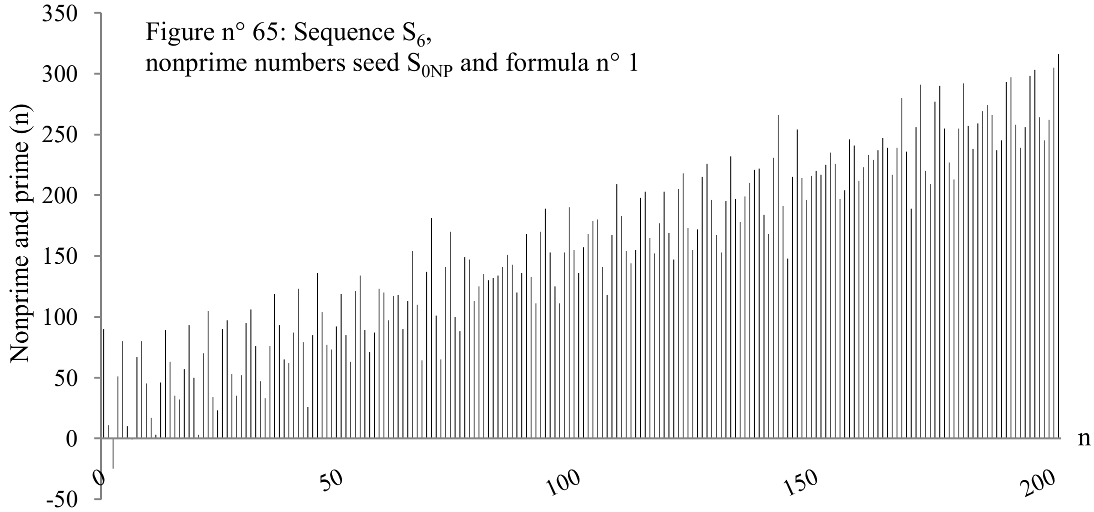


Figure n° 65: Sequence S_6,
nonprime numbers seed S_{0NP} and formula n° 1
Nonprime and prime (n)
350
300
250
200
150
100
50
0
-50
n
0
50
100
150
200

Figure n° 66: nonprime numbers subsequence S_{13np},
nonprime numbers seed S_{0NP} and formula n° 1

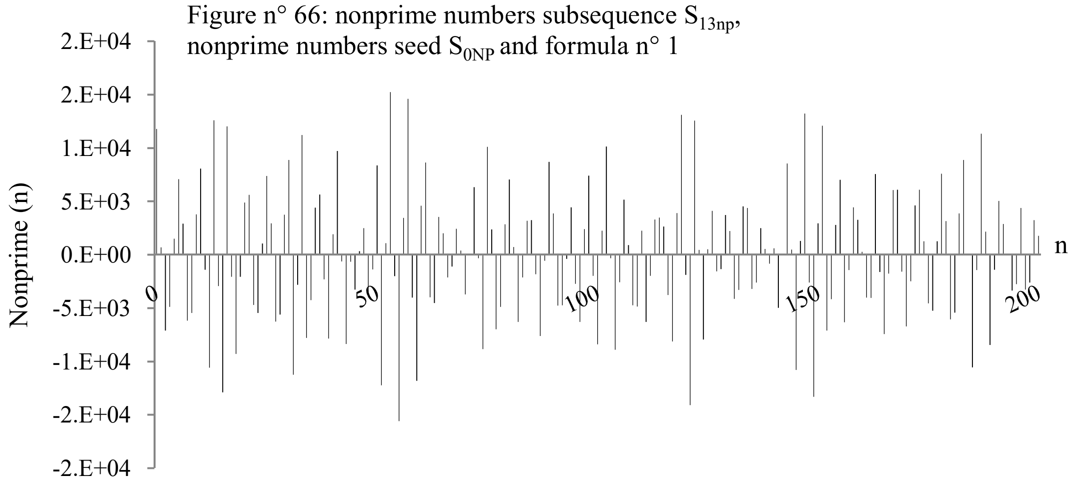


Figure n° 67: Sequence S_{13},
nonprime numbers seed S_{0NP} and formula n° 1

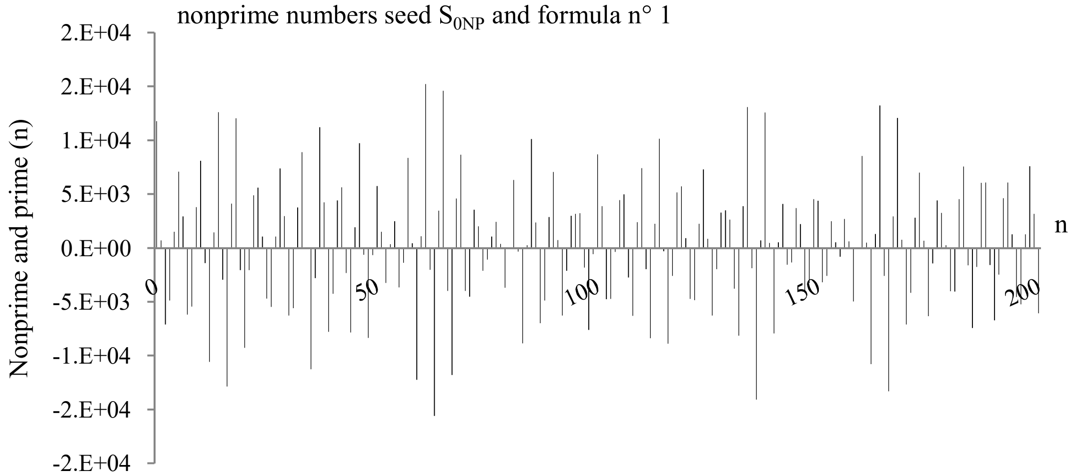
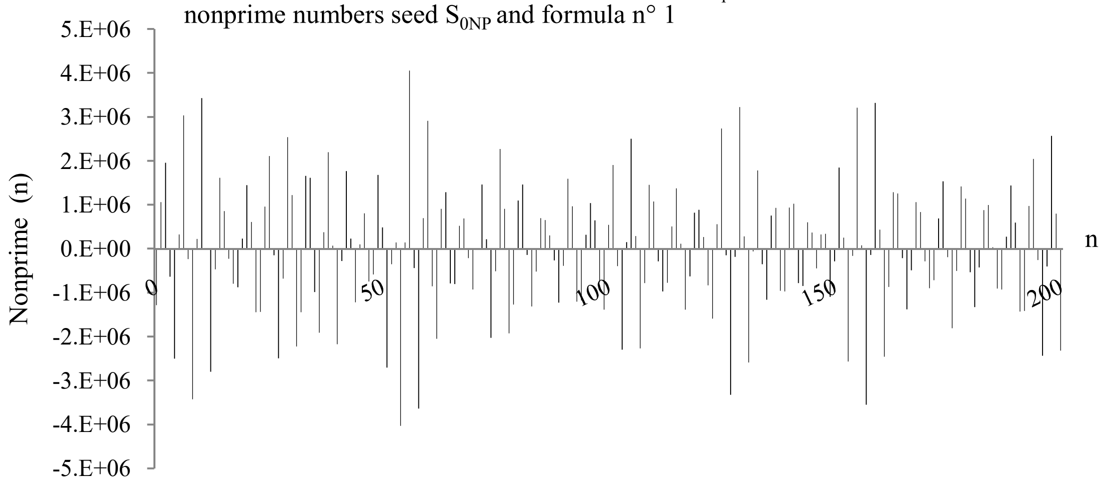


Figure n° 68: nonprime numbers subsequence S_{20np},
nonprime numbers seed S_{0NP} and formula n° 1



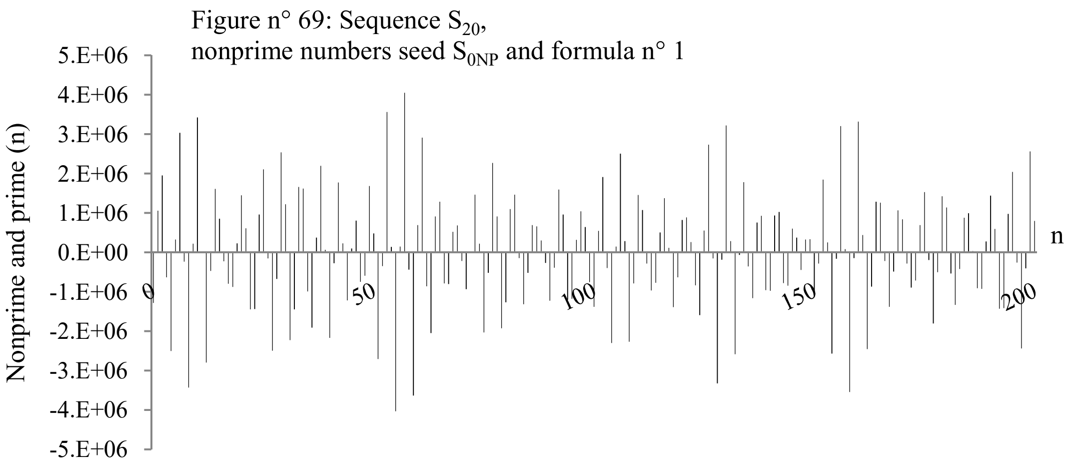


Figure n° 69: Sequence S_{20},
nonprime numbers seed S_{0NP} and formula n° 1
5.E+06
4.E+06
3.E+06
2.E+06
1.E+06
0.E+00
-1.E+06
-2.E+06
-3.E+06
-4.E+06
-5.E+06
Nonprime and prime (n)
n
0
50
100
150
200

Figures from the natural numbers seed and formula n° 1

Figure n° 70 : number of primes in the seed S_{0N} and in each sequence, natural numbers sedd S_{0N} and formula n° 1

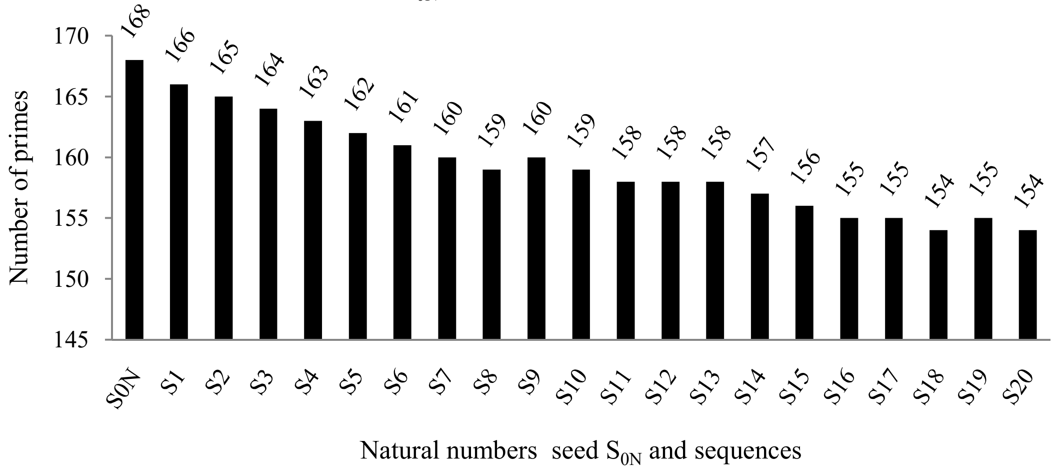


Figure n° 71: percentage of primes in each sequence natural numbers seed S_{0N} and formula n° 1

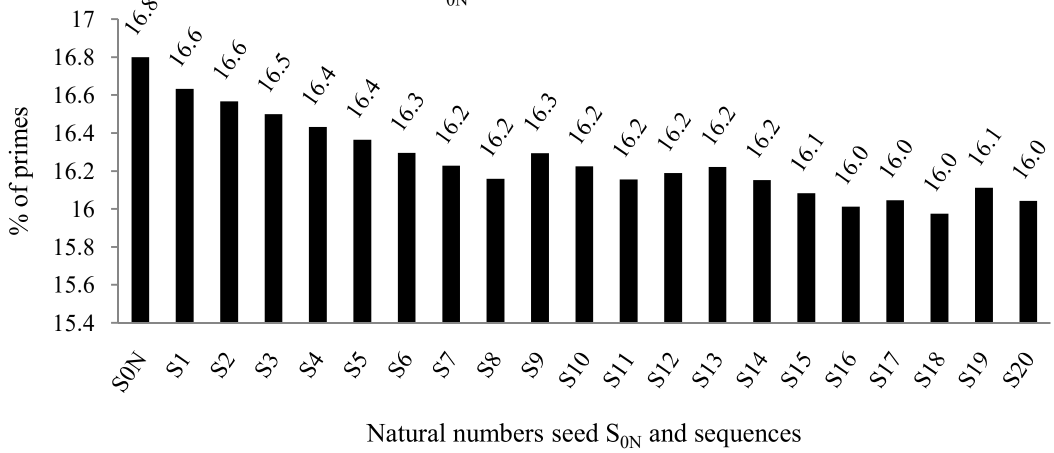


Figure n° 72 : median of primes in the seed S_{0N} and in each prime numbers subsequence natural numbers seed S_{0N} and formula n° 1

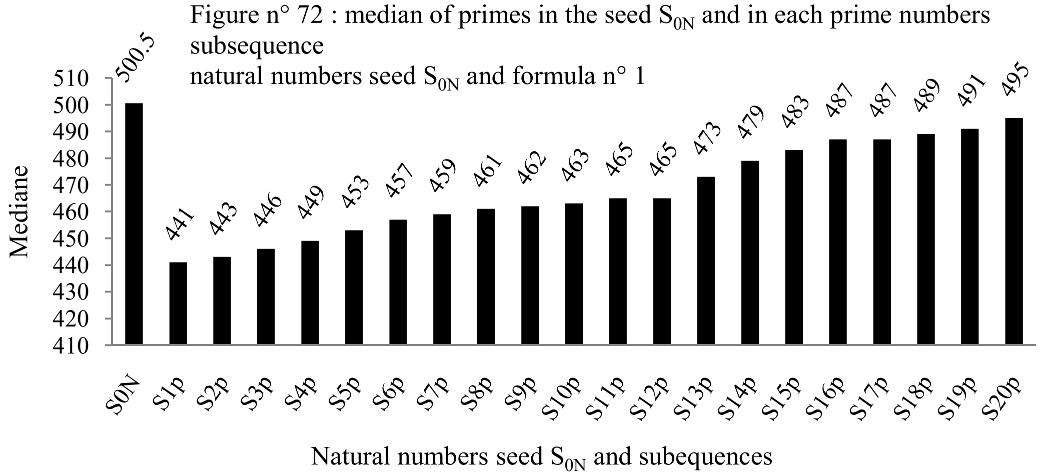


Figure n° 73 : prime number minimum and maximum values in each prime numbers subsequence
natural numbers seed S_{0N} and formula n° 1

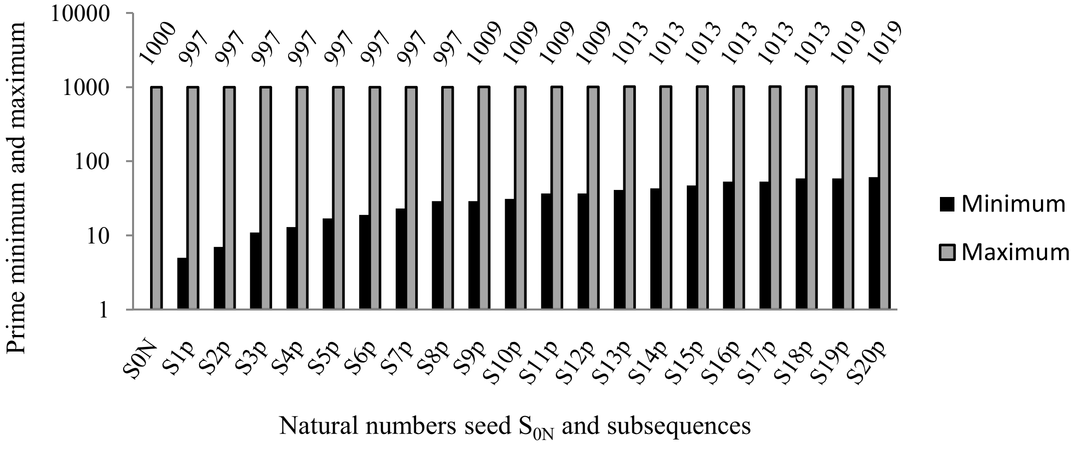


Figure n° 74: prime number range values for the natural numbers seed S_{0N} and each prime numbers subsequence,
natural numbers seed S_{0N} and formula n° 1

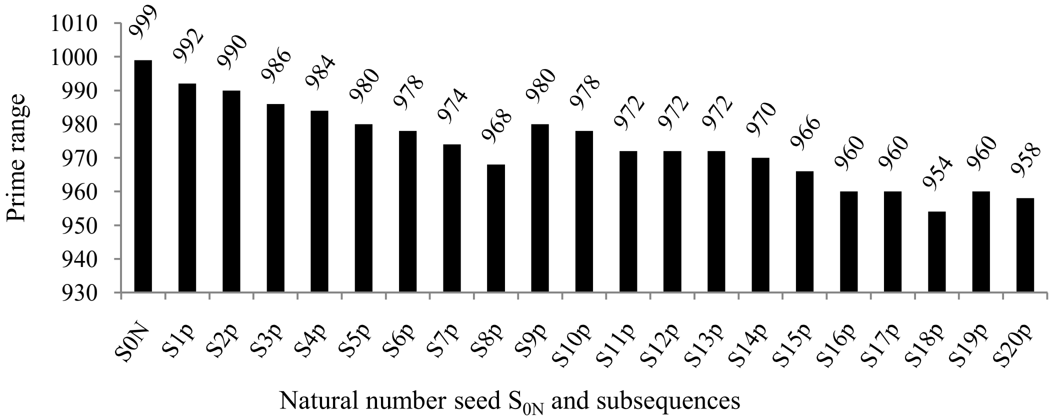
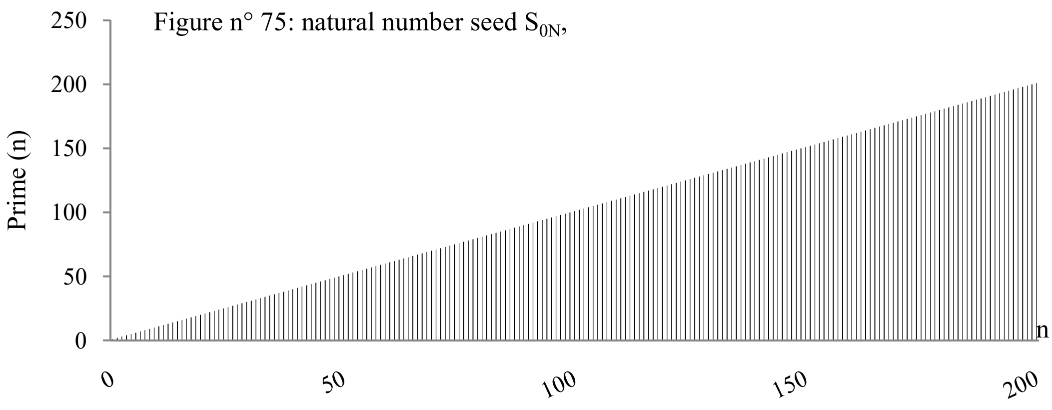


Figure n° 75: natural number seed S_{0N},



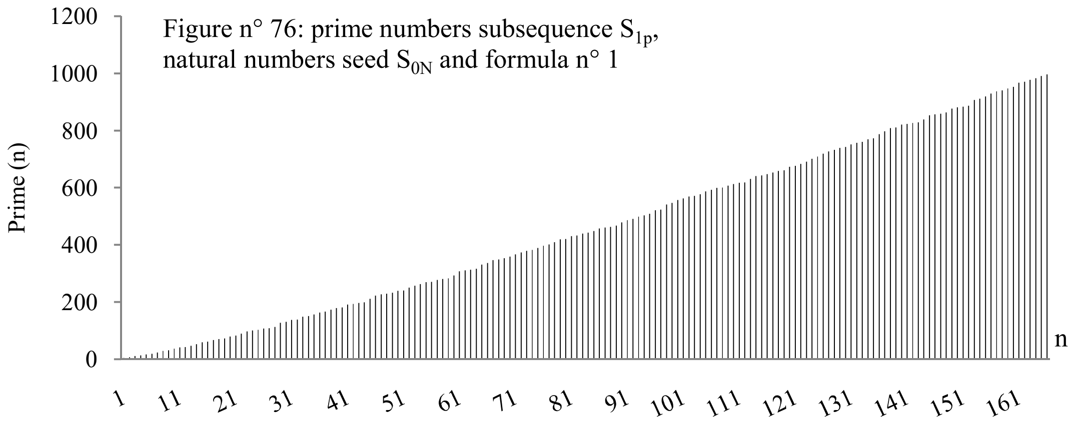


Figure n° 76: prime numbers subsequence S_{1p}, natural numbers seed S_{0N} and formula n° 1
Prime (n)
1200
1000
800
600
400
200
0
n
1
11
21
31
41
51
61
71
81
91
101
111
121
131
141
151
161

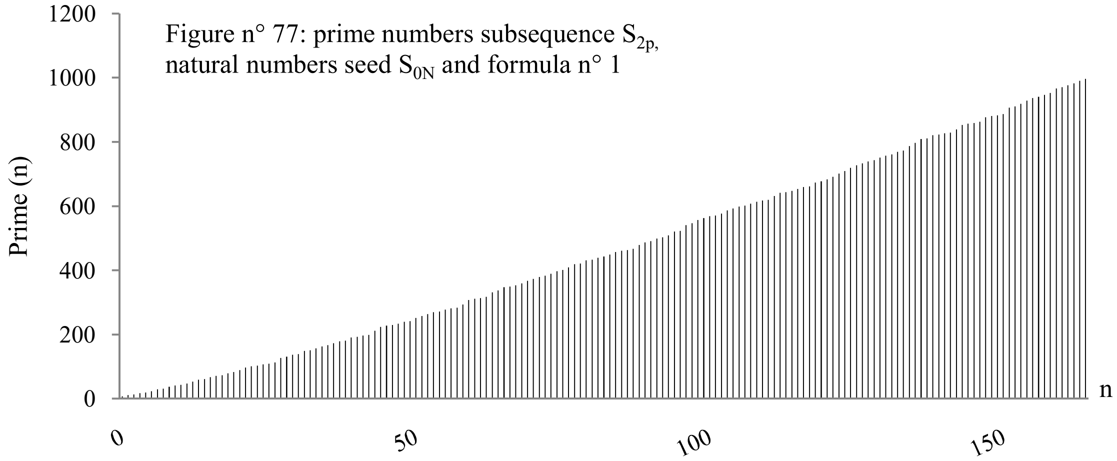


Figure n° 77: prime numbers subsequence S_{2p}, natural numbers seed S_{0N} and formula n° 1
Prime (n)
1200
1000
800
600
400
200
0
n
0
50
100
150

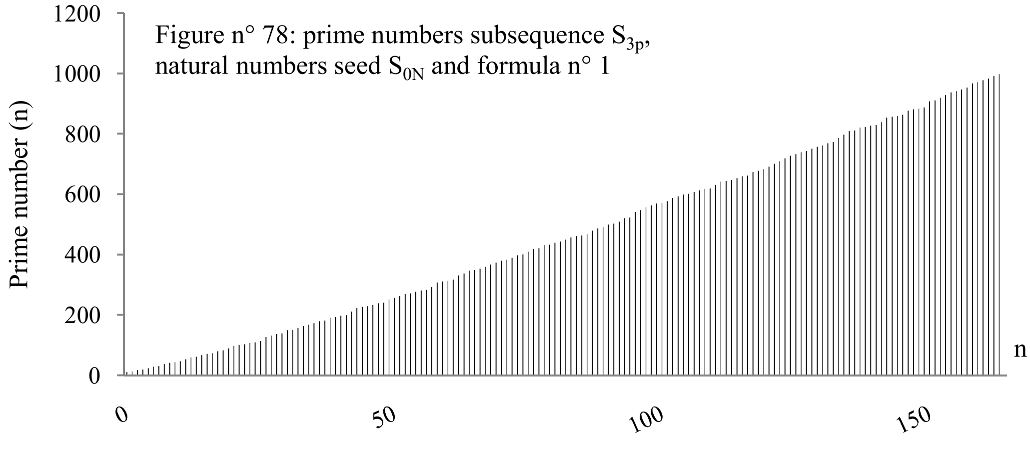


Figure n° 78: prime numbers subsequence S_{3p}, natural numbers seed S_{0N} and formula n° 1
Prime number (n)
1200
1000
800
600
400
200
0
n
0
50
100
150

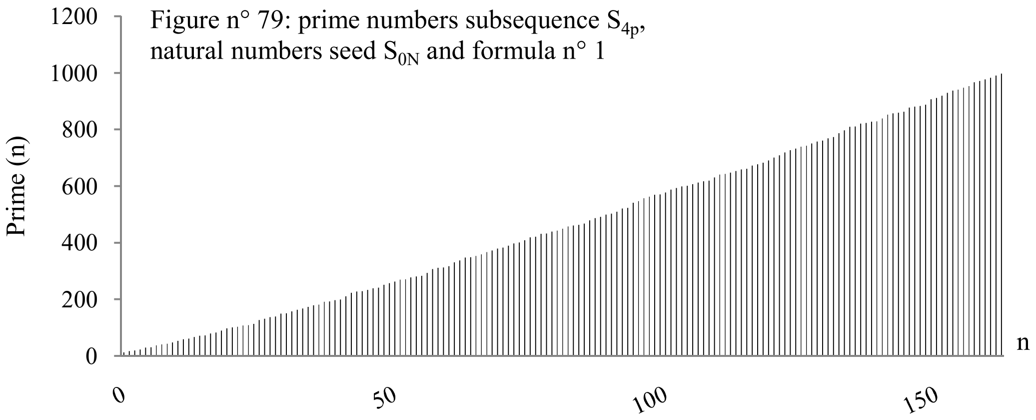


Figure n° 79: prime numbers subsequence S_{4p}, natural numbers seed S_{0N} and formula n° 1

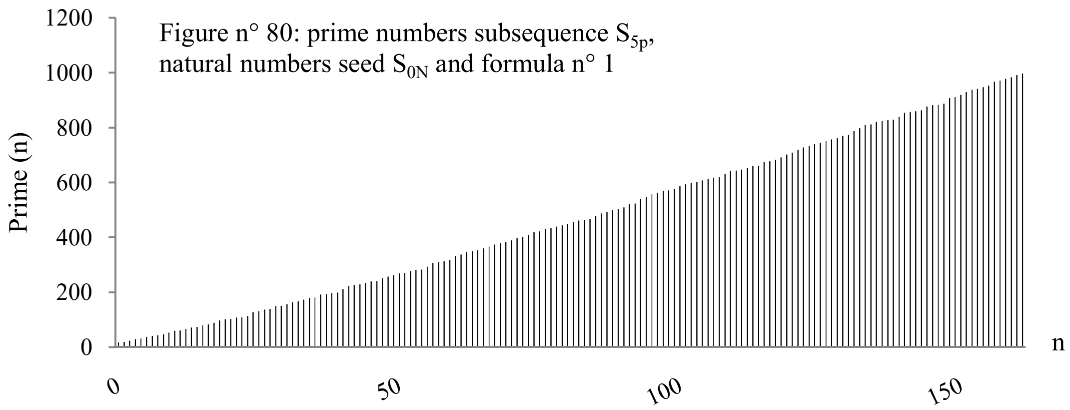


Figure n° 80: prime numbers subsequence S_{5p}, natural numbers seed S_{0N} and formula n° 1

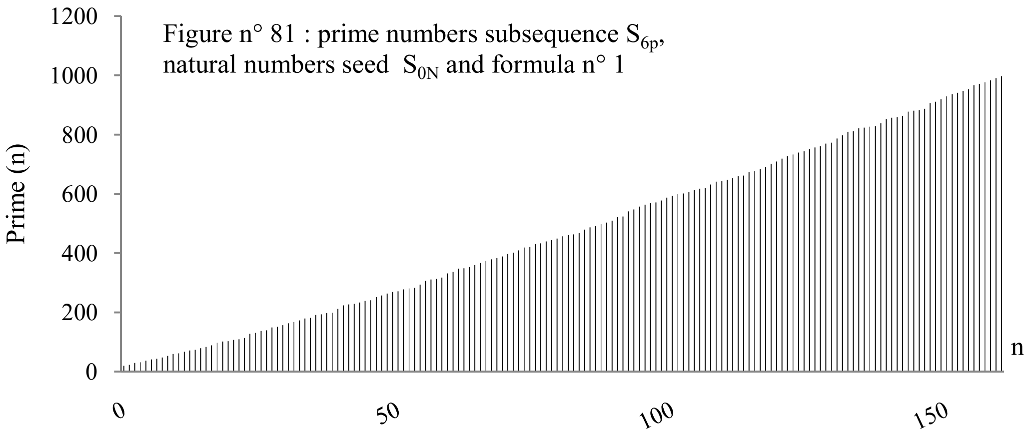


Figure n° 81 : prime numbers subsequence S_{6p}, natural numbers seed S_{0N} and formula n° 1

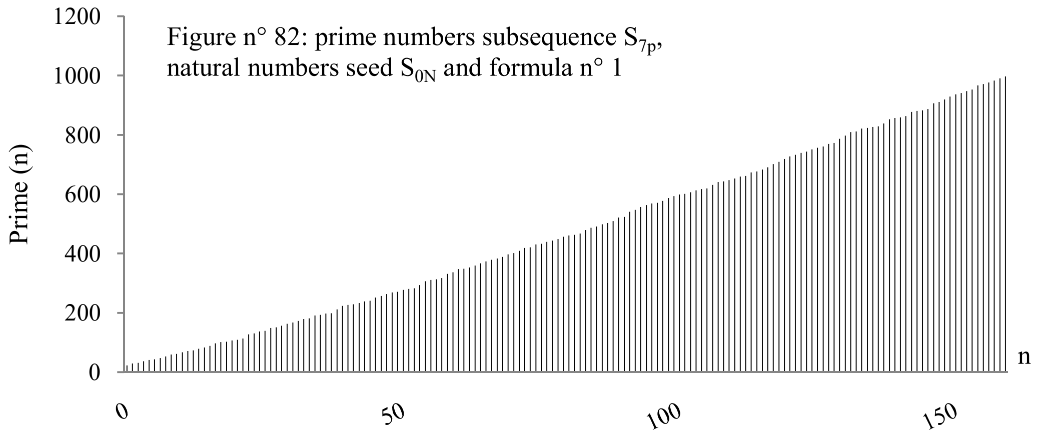


Figure n° 82: prime numbers subsequence S_{7p}, natural numbers seed S_{0N} and formula n° 1
Prime (n)
n
1200
1000
800
600
400
200
0
0
50
100
150

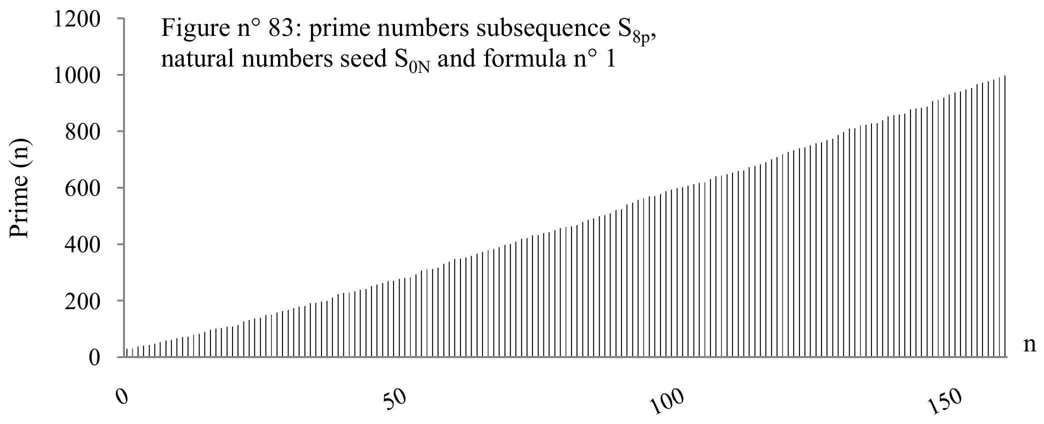


Figure n° 83: prime numbers subsequence S_{8p}, natural numbers seed S_{0N} and formula n° 1
Prime (n)
n
1200
1000
800
600
400
200
0
0
50
100
150

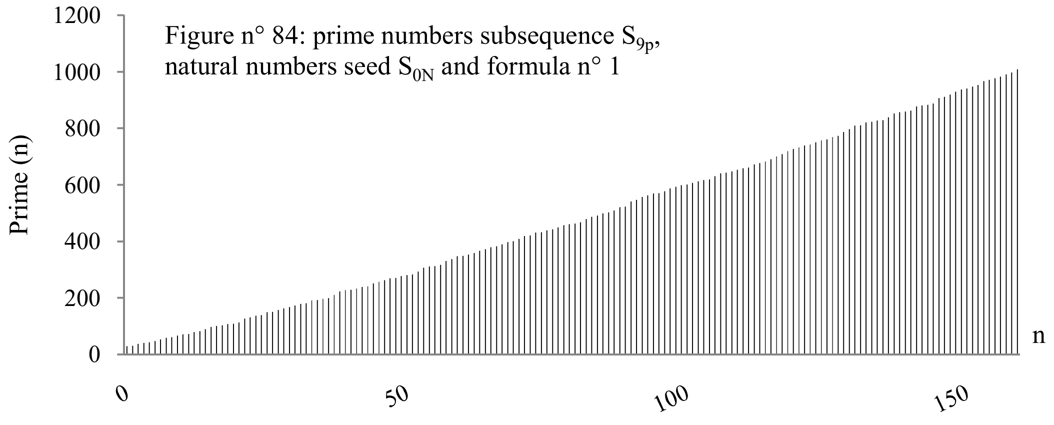


Figure n° 84: prime numbers subsequence S_{9p}, natural numbers seed S_{0N} and formula n° 1
Prime (n)
n
1200
1000
800
600
400
200
0
0
50
100
150

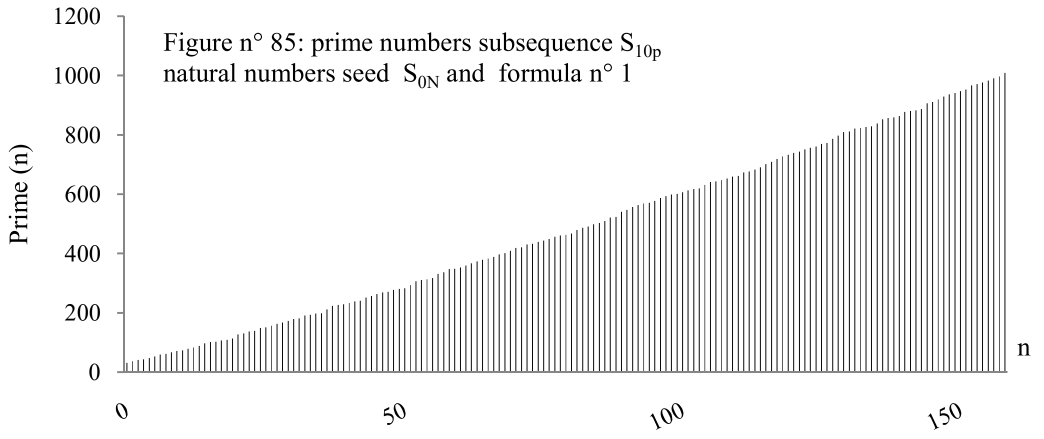


Figure n° 85: prime numbers subsequence S_{10p}
natural numbers seed S_{0N} and formula n° 1

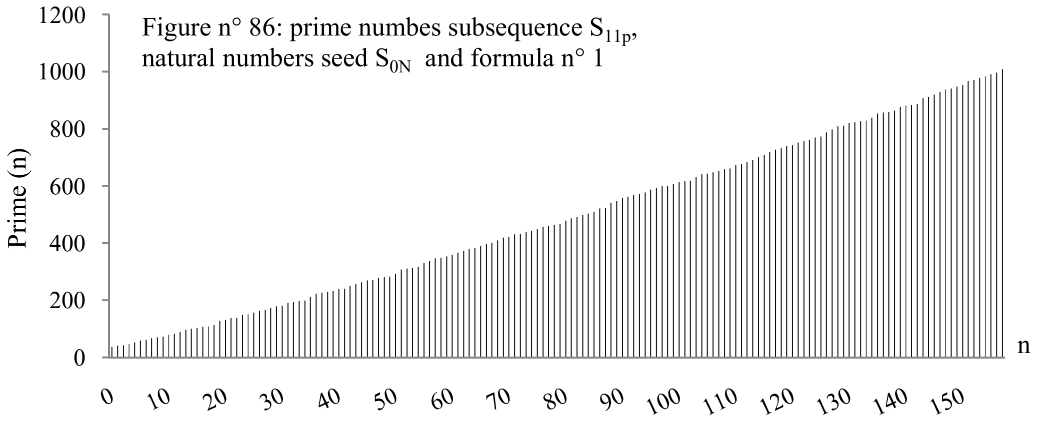


Figure n° 86: prime numbes subsequence S_{11p},
natural numbers seed S_{0N} and formula n° 1

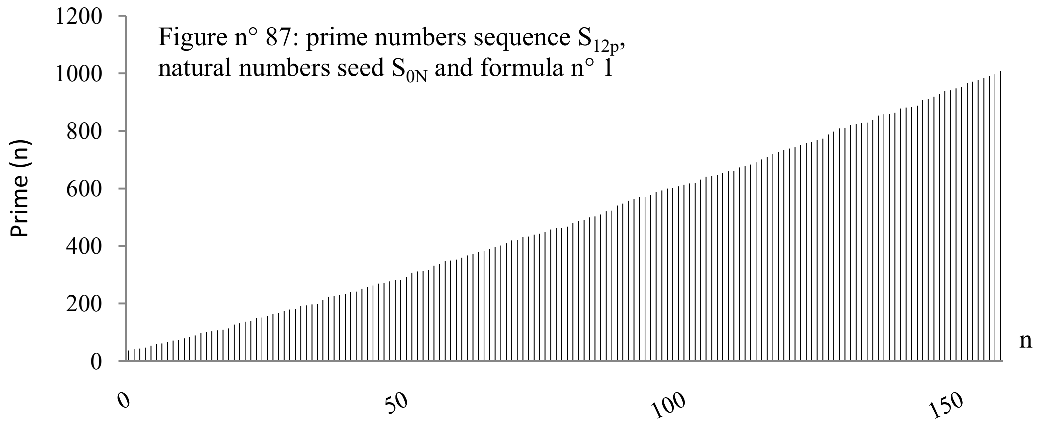


Figure n° 87: prime numbers sequence S_{12p},
natural numbers seed S_{0N} and formula n° 1

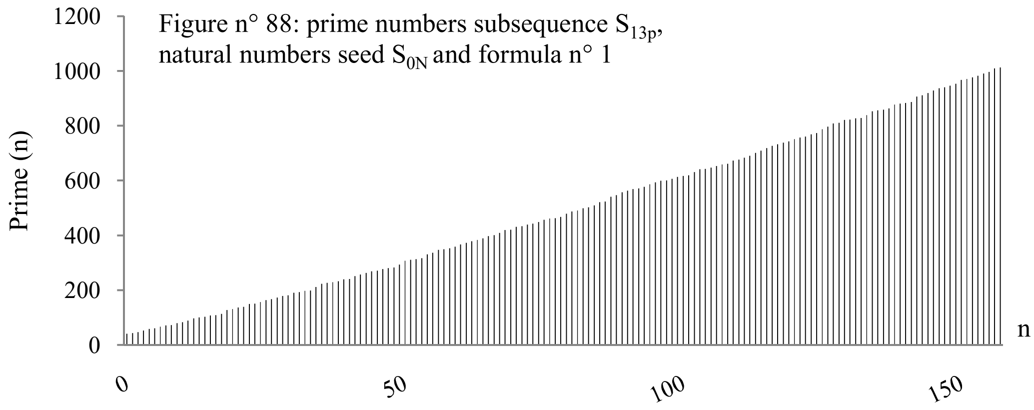


Figure n° 88: prime numbers subsequence S13p,
natural numbers seed S0N and formula n° 1
1200
1000
800
600
400
200
0
Prime (n)
n
0
50
100
150

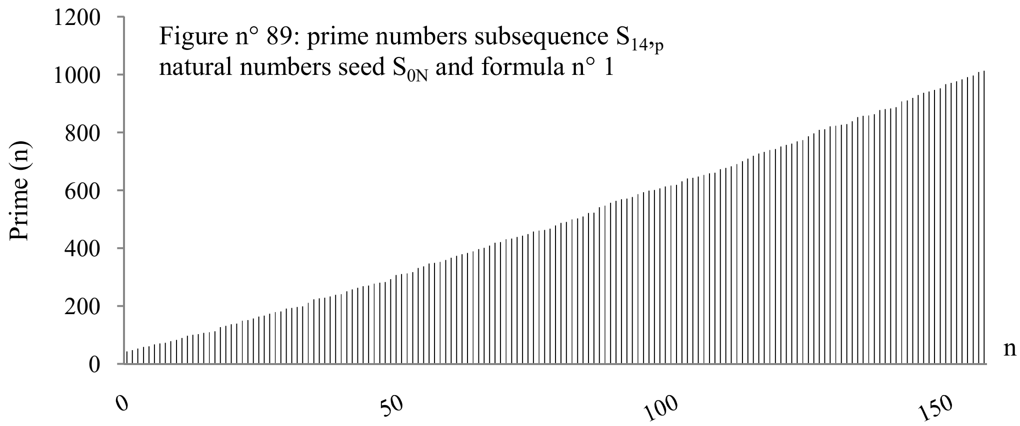


Figure n° 89: prime numbers subsequence S14,p
natural numbers seed S0N and formula n° 1
1200
1000
800
600
400
200
0
Prime (n)
n
0
50
100
150

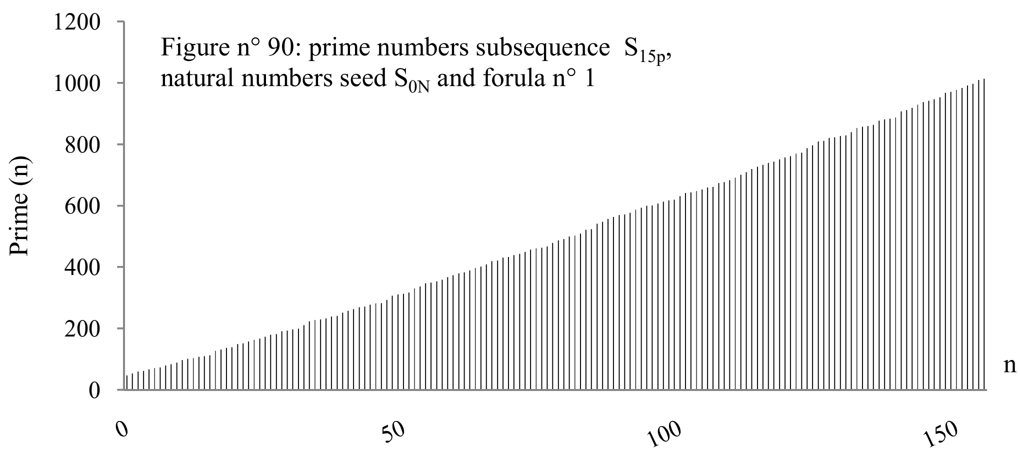


Figure n° 90: prime numbers subsequence S15p,
natural numbers seed S0N and forula n° 1
1200
1000
800
600
400
200
0
Prime (n)
n
0
50
100
150

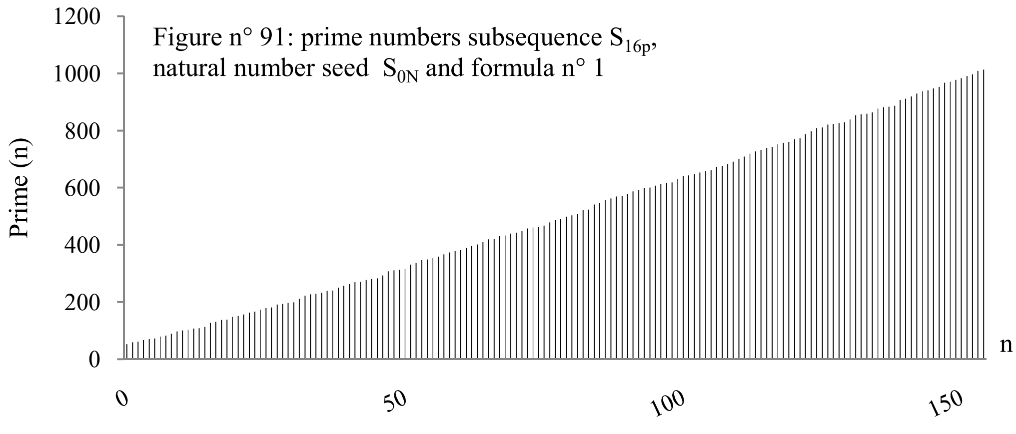


Figure n° 91: prime numbers subsequence S_{16p}, natural number seed S_{0N} and formula n° 1

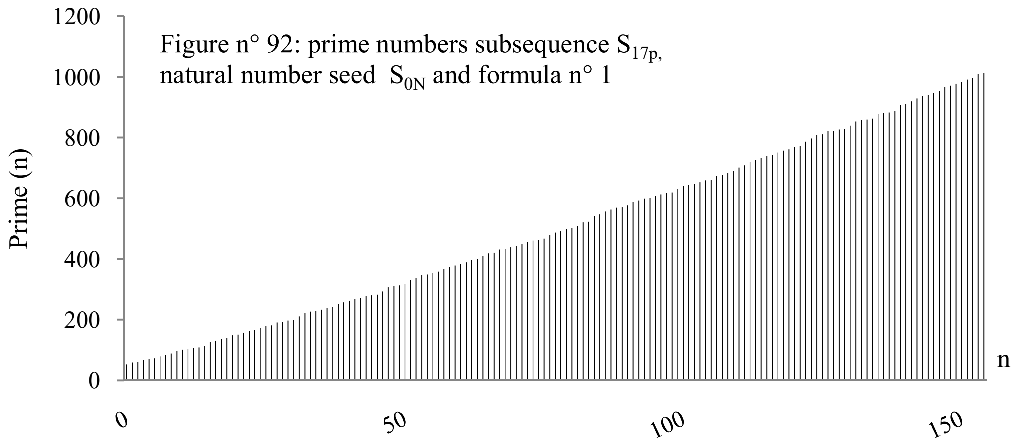


Figure n° 92: prime numbers subsequence $S_{17p,}$ natural number seed S_{0N} and formula n° 1

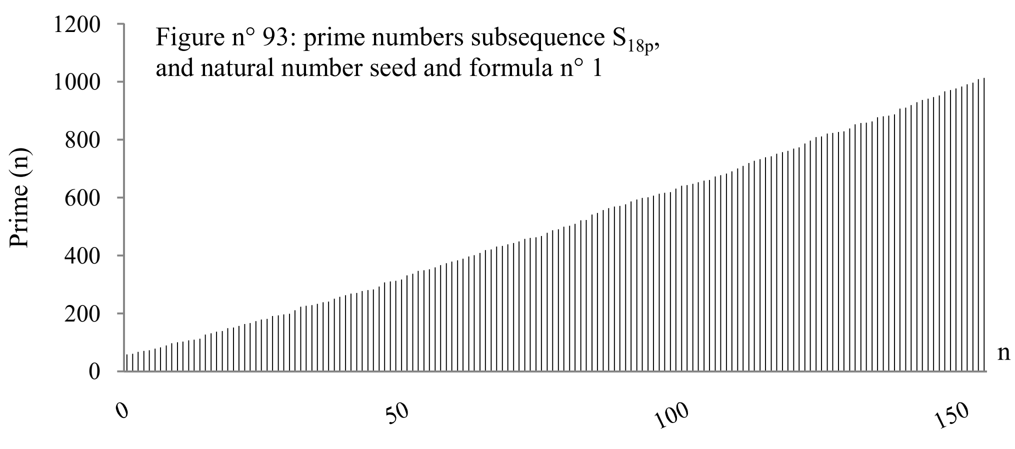


Figure n° 93: prime numbers subsequence S_{18p}, and natural number seed and formula n° 1

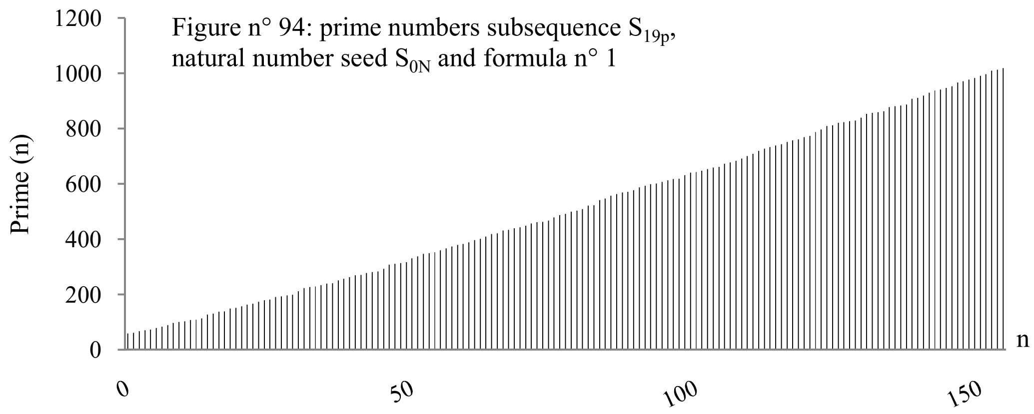


Figure n° 94: prime numbers subsequence S_{19p}, natural number seed S_{0N} and formula n° 1

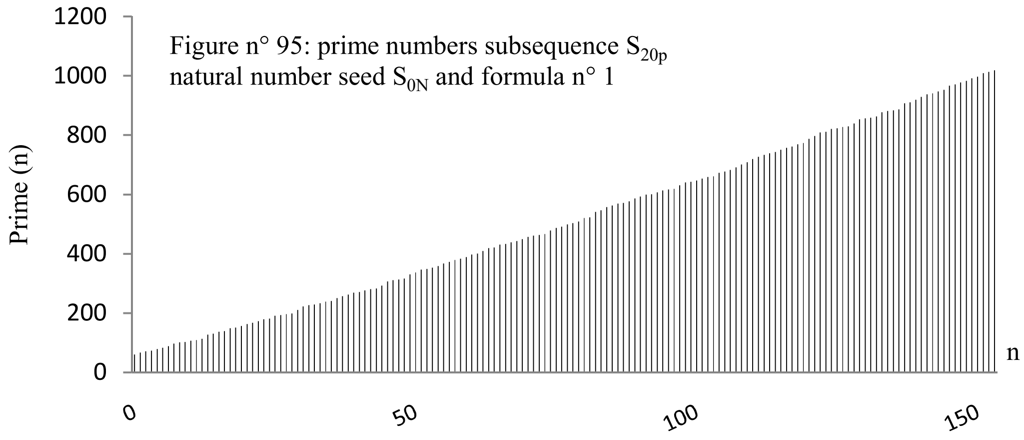


Figure n° 95: prime numbers subsequence S_{20p} natural number seed S_{0N} and formula n° 1

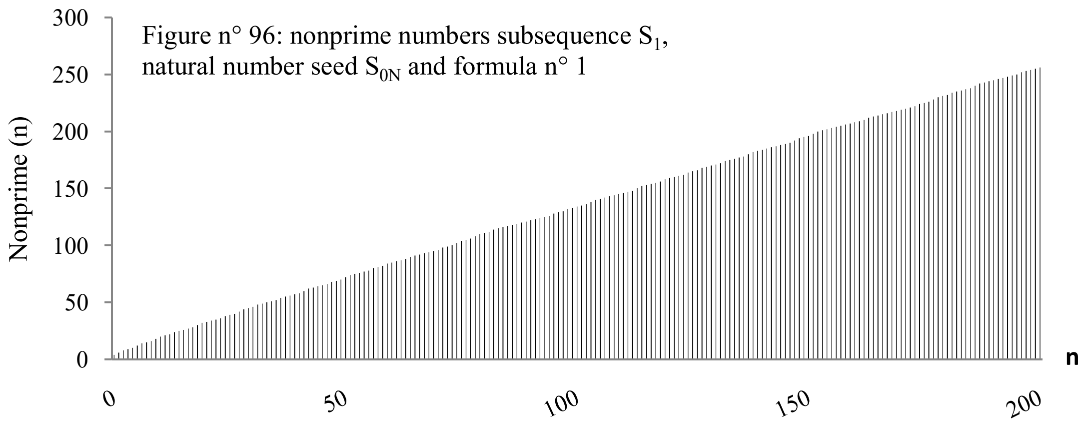


Figure n° 96: nonprime numbers subsequence S_1, natural number seed S_{0N} and formula n° 1

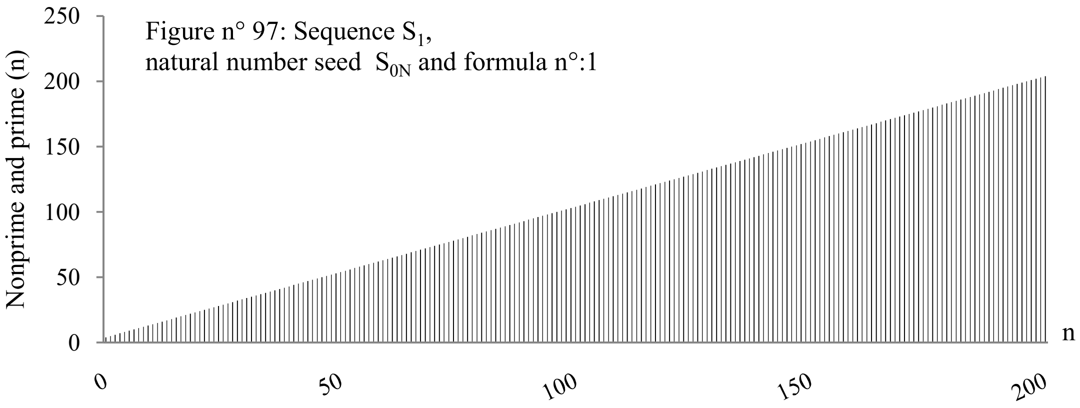


Figure n° 97: Sequence S_1, natural number seed S_{0N} and formula n°:1

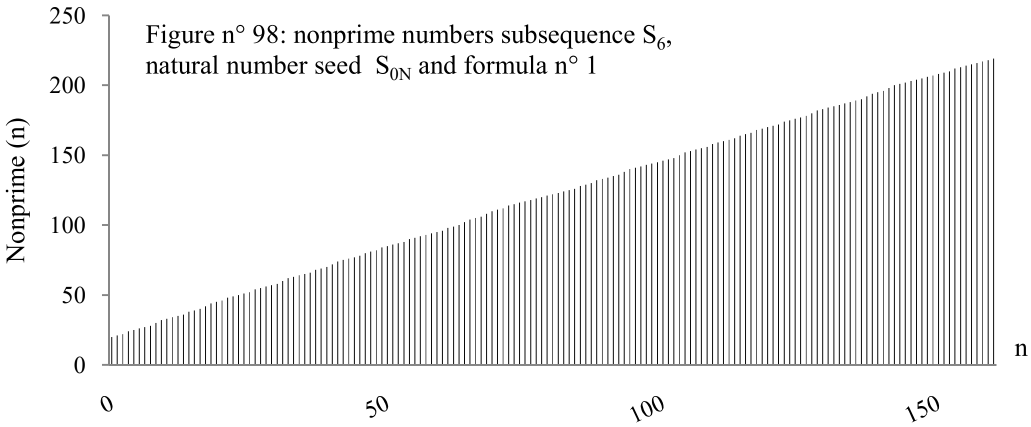


Figure n° 98: nonprime numbers subsequence S_6, natural number seed S_{0N} and formula n° 1

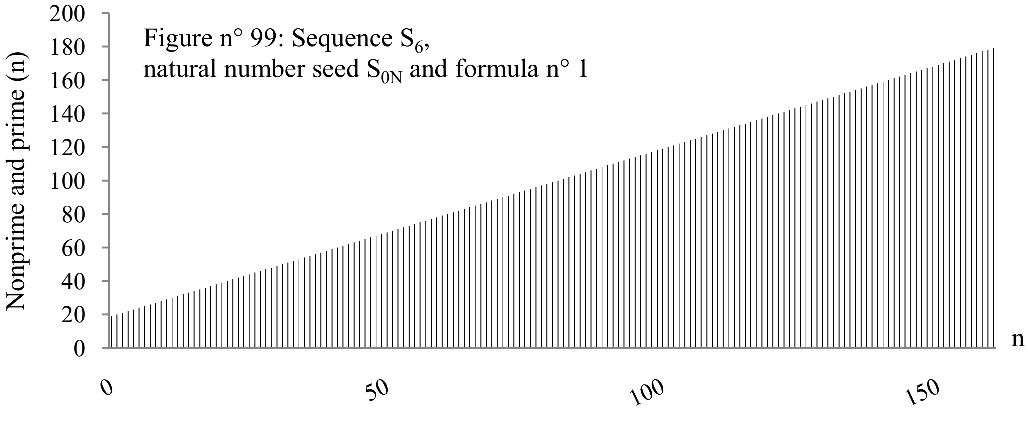


Figure n° 99: Sequence S_6, natural number seed S_{0N} and formula n° 1

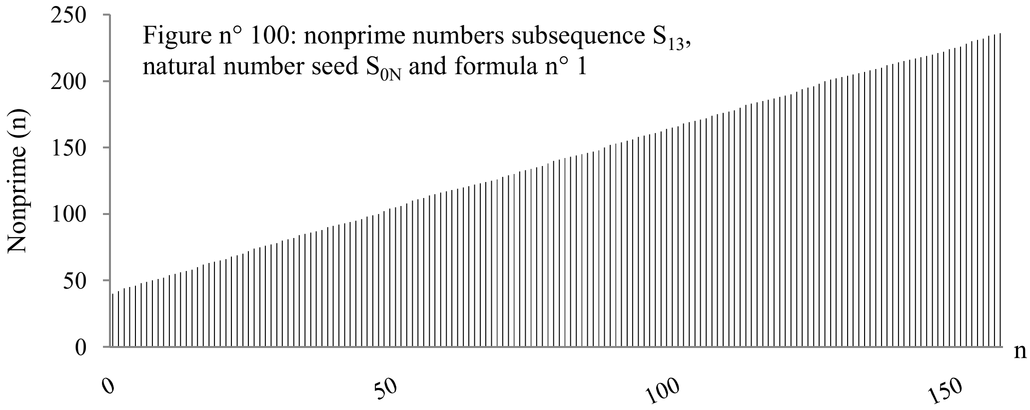


Figure n° 100: nonprime numbers subsequence S_{13}, natural number seed S_{0N} and formula n° 1

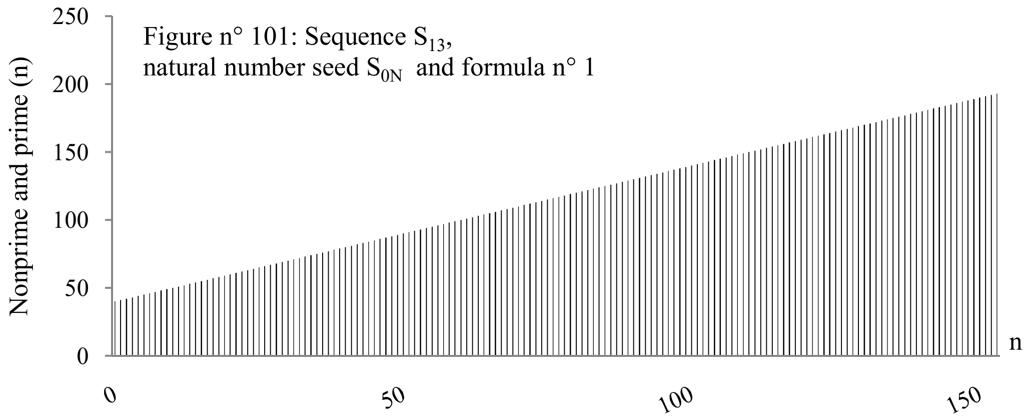


Figure n° 101: Sequence S_{13}, natural number seed S_{0N} and formula n° 1

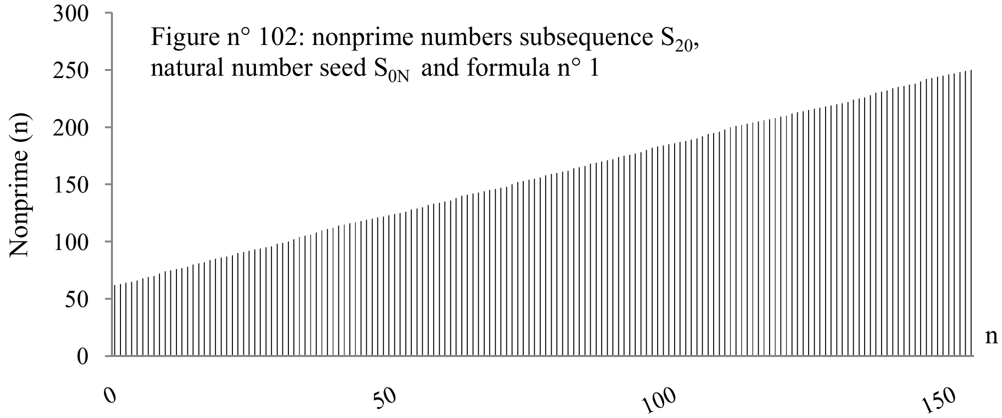


Figure n° 102: nonprime numbers subsequence S_{20}, natural number seed S_{0N} and formula n° 1

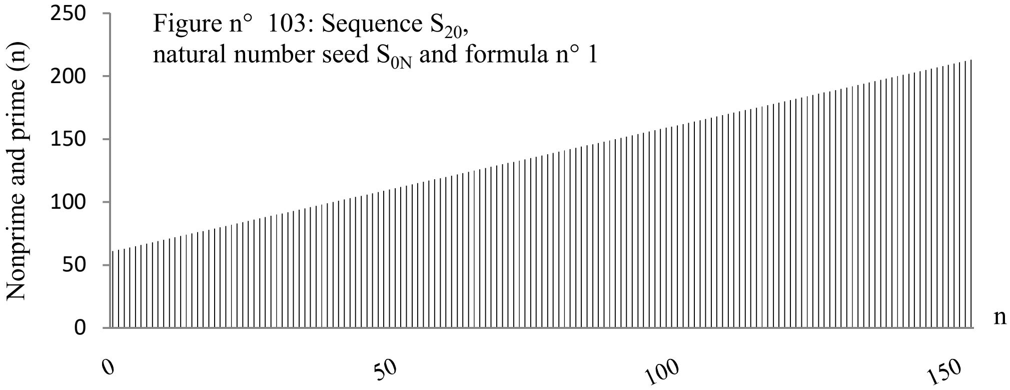


Figure n° 103: Sequence S_{20},
natural number seed S_{0N} and formula n° 1
Nonprime and prime (n)
250
200
150
100
50
0
n
0
50
100
150

Figures from the prime numbers seed S_{0P} and formula n° 2

Figure n° 104: number of terms in the seed S_{0P} and sequences prime numbers seed S_{0P} and formula n° 2

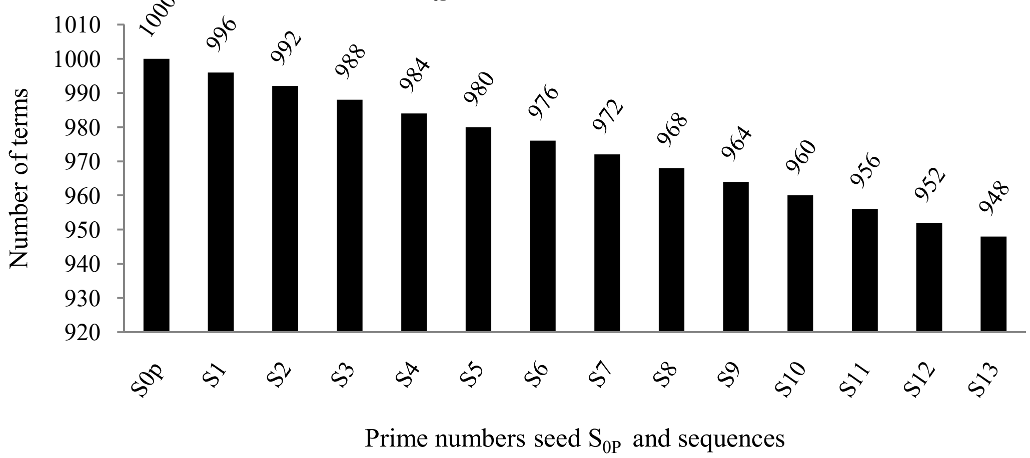


Figure n° 105: number of prime numbers in S_{0p} and each sequence, prime numbers seed S_{0P} and formula n° 2

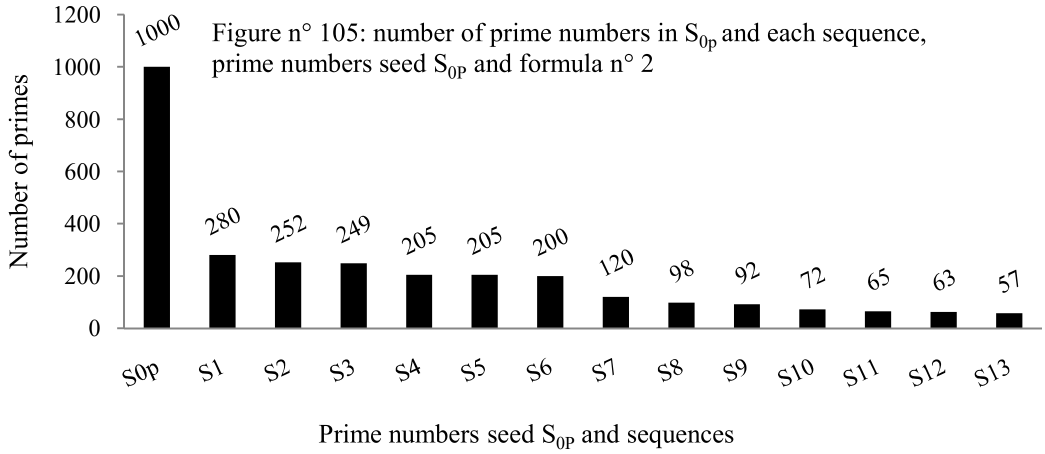


Figure n° 106: percentage of prime numbers in S_{0p} and each sequence, prime numbers seed S_{0P} and formula n° 2

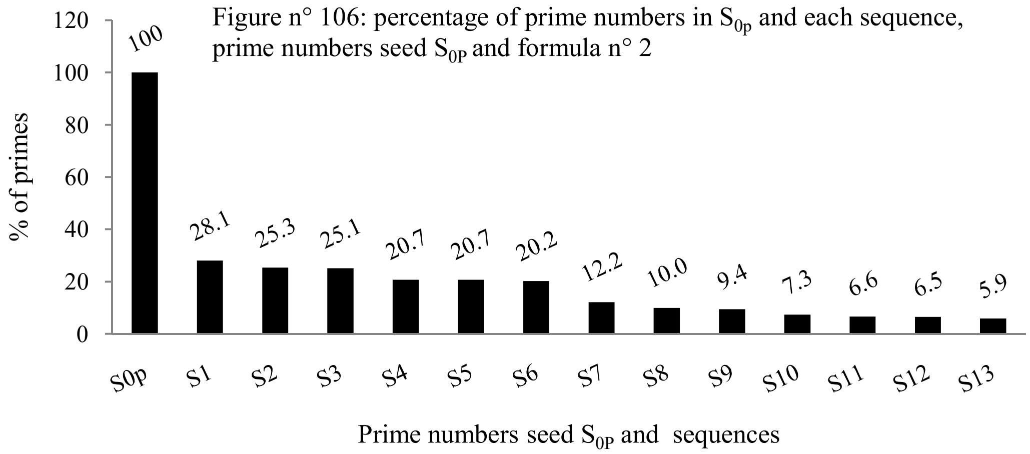


Figure n° 107: histogram of prime numbers in the S_{1p} subsequence, prime numbers seed S_{0P} and formula n° 2

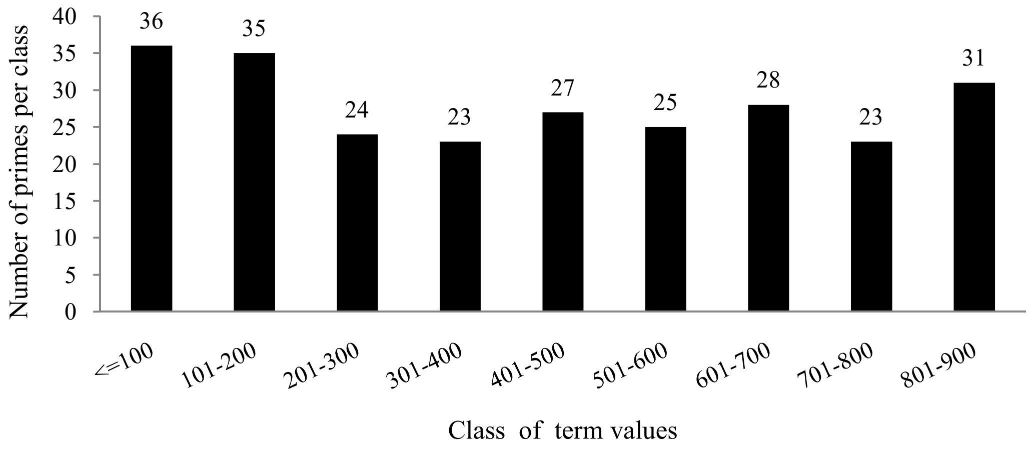


Figure n°108: histogram of prime numbers in the S_{2p} subsequence, prime numbers seed S_{0P} and formula n° 2

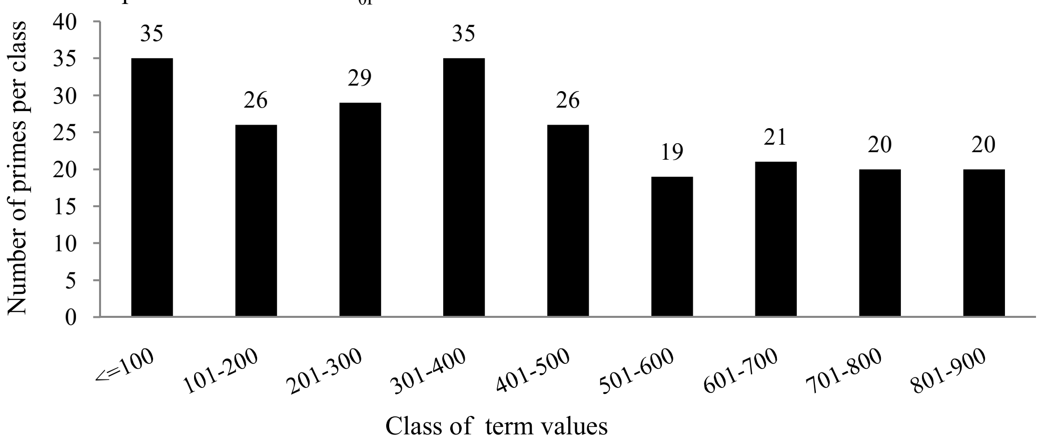


Figure n°109: histogram of prime numbers in the S_{3p} subsequence, prime numbers seed S_{0P} and formula n° 2

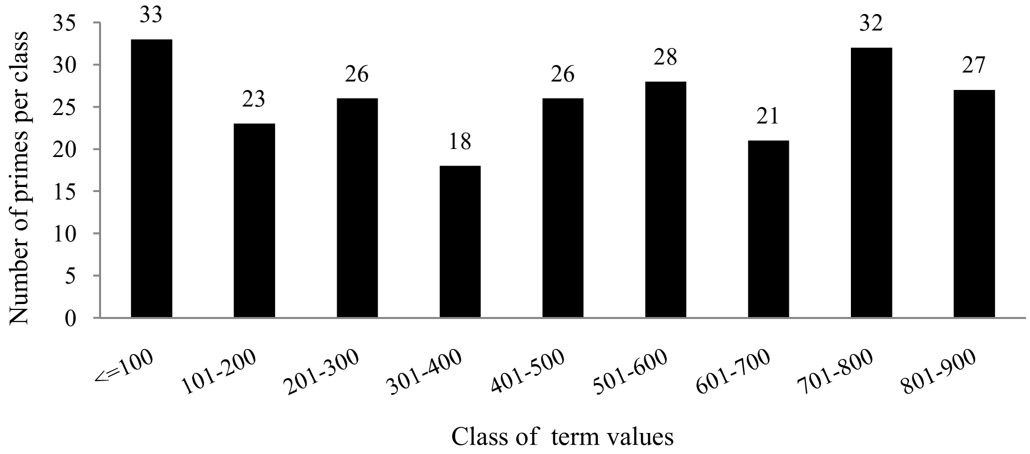


Figure n° 110: histogram of prime numbers in the S_{4p} subsequence, prime numbers seed S_{0P} and formula n° 2

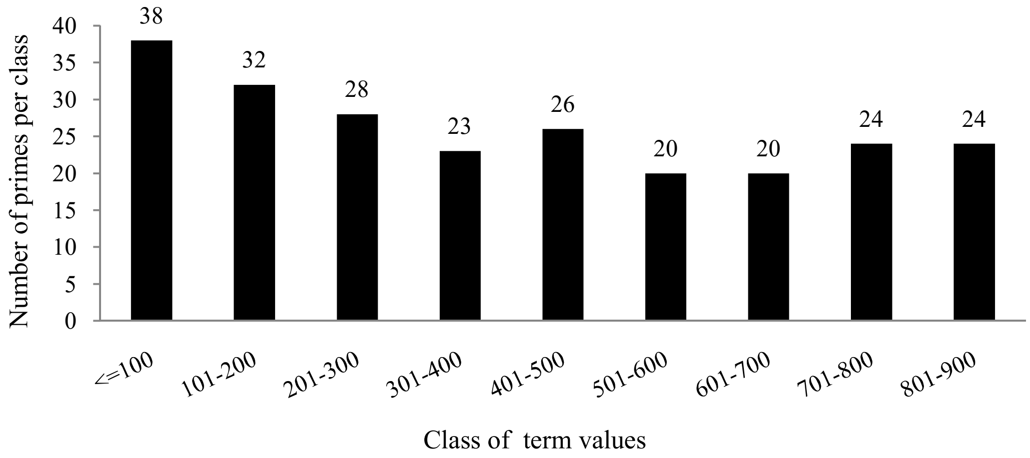


Figure n° 111: median of prime numbers in S_{0p} and each prime numbers subsequence, prime numbers seed S_{0P} and formula n° 2

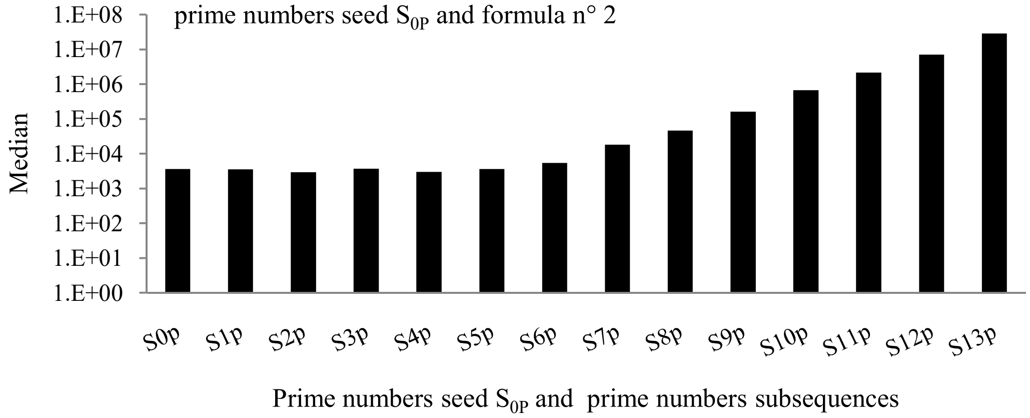
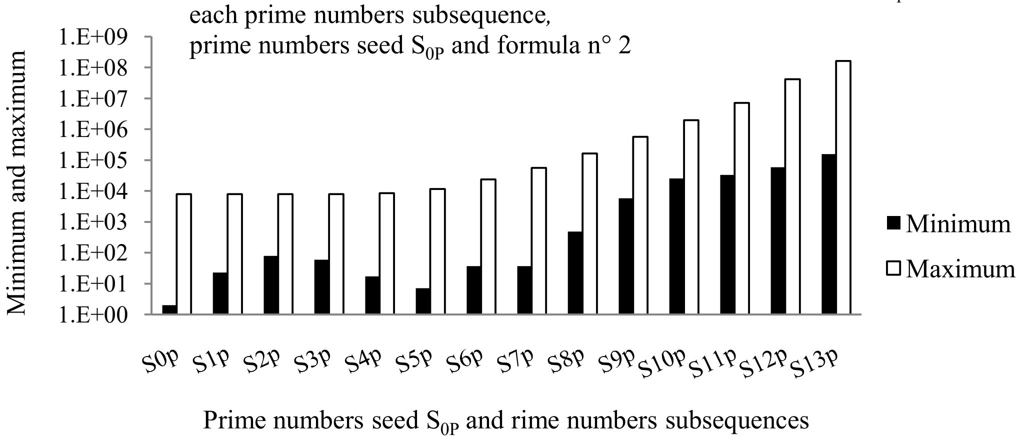


Figure n° 112: prime number minimum and maximum values in S_{0p} and each prime numbers subsequence, prime numbers seed S_{0P} and formula n° 2



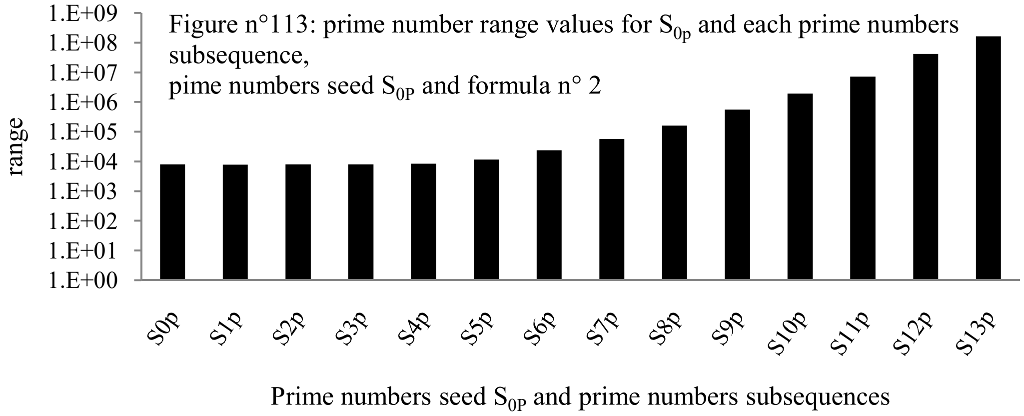


Figure n°113: prime number range values for S_{0p} and each prime numbers subsequence,
pime numbers seed S_{0P} and formula n° 2
range
1.E+09
1.E+08
1.E+07
1.E+06
1.E+05
1.E+04
1.E+03
1.E+02
1.E+01
1.E+00
S0p
S1p
S2p
S3p
S4p
S5p
S6p
S7p
S8p
S9p
S10p
S11p
S12p
S13p
Prime numbers seed S_{0P} and prime numbers subsequences

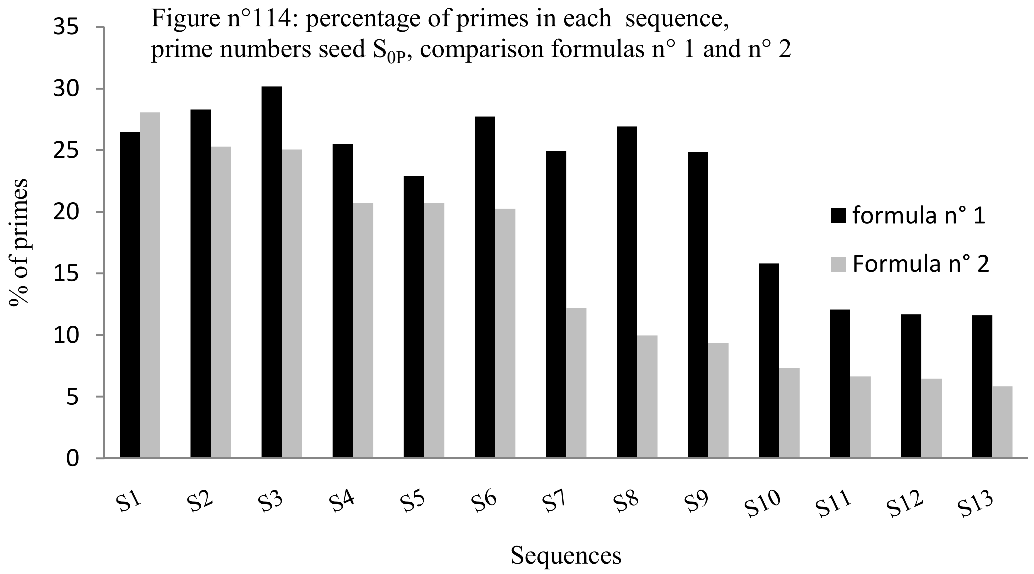


Figure n°114: percentage of primes in each sequence,
prime numbers seed S_{0P}, comparison formulas n° 1 and n° 2
% of primes
35
30
25
20
15
10
5
0
formula n° 1
Formula n° 2
S1
S2
S3
S4
S5
S6
S7
S8
S9
S10
S11
S12
S13
Sequences

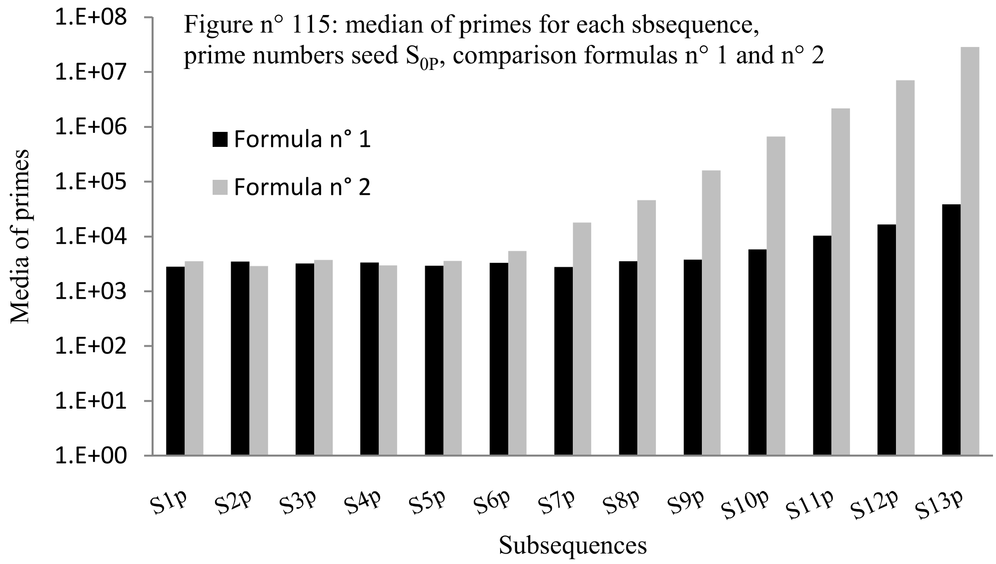


Figure n° 115: median of primes for each sbsequence,
prime numbers seed S_{0P}, comparison formulas n° 1 and n° 2
Media of primes
1.E+08
1.E+07
1.E+06
1.E+05
1.E+04
1.E+03
1.E+02
1.E+01
1.E+00
Formula n° 1
Formula n° 2
S1p
S2p
S3p
S4p
S5p
S6p
S7p
S8p
S9p
S10p
S11p
S12p
S13p
Subsequences

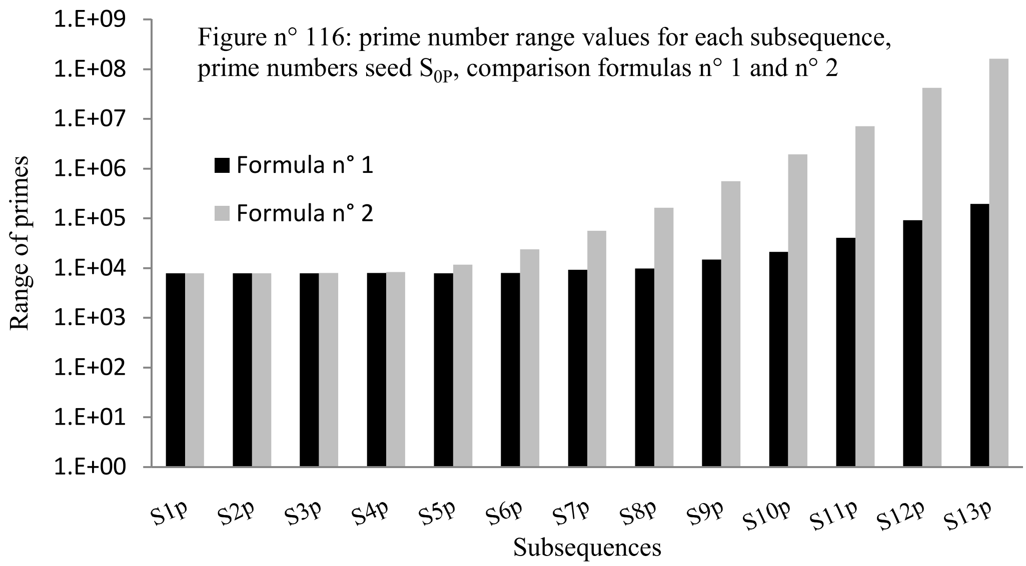


Figure n° 116: prime number range values for each subsequence, prime numbers seed S_{0P}, comparison formulas n° 1 and n° 2
1.E+09
1.E+08
1.E+07
1.E+06
1.E+05
1.E+04
1.E+03
1.E+02
1.E+01
1.E+00
Range of primes
Formula n° 1
Formula n° 2
S1p S2p S3p S4p S5p S6p S7p S8p S9p S10p S11p S12p S13p
Subsequences

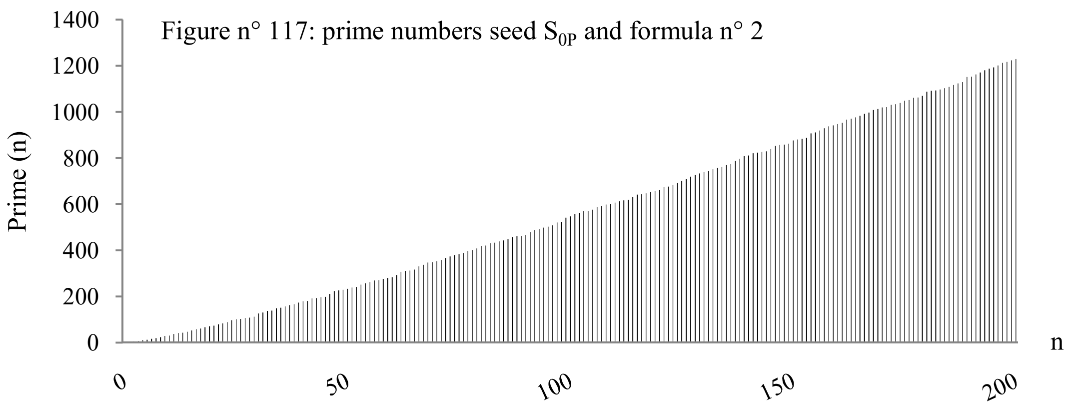


Figure n° 117: prime numbers seed S_{0P} and formula n° 2
1400
1200
1000
800
600
400
200
0
Prime (n)
n
0 50 100 150 200

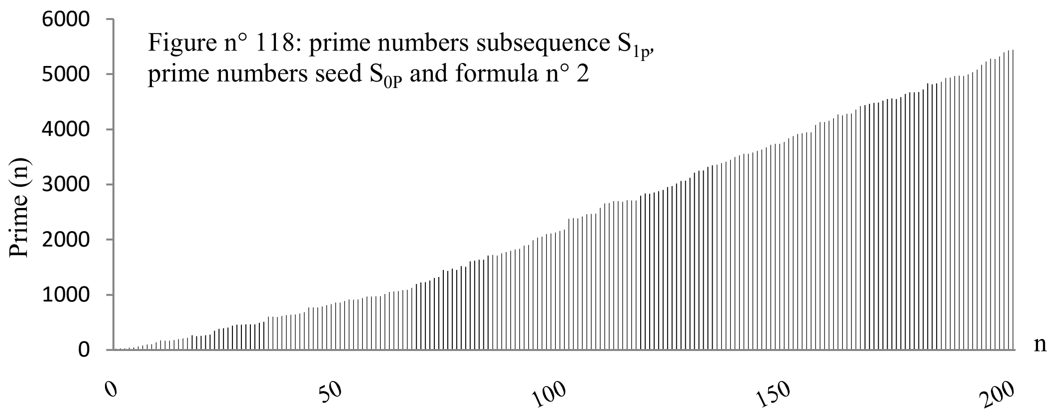


Figure n° 118: prime numbers subsequence S_{1p}, prime numbers seed S_{0P} and formula n° 2
6000
5000
4000
3000
2000
1000
0
Prime (n)
n
0 50 100 150 200

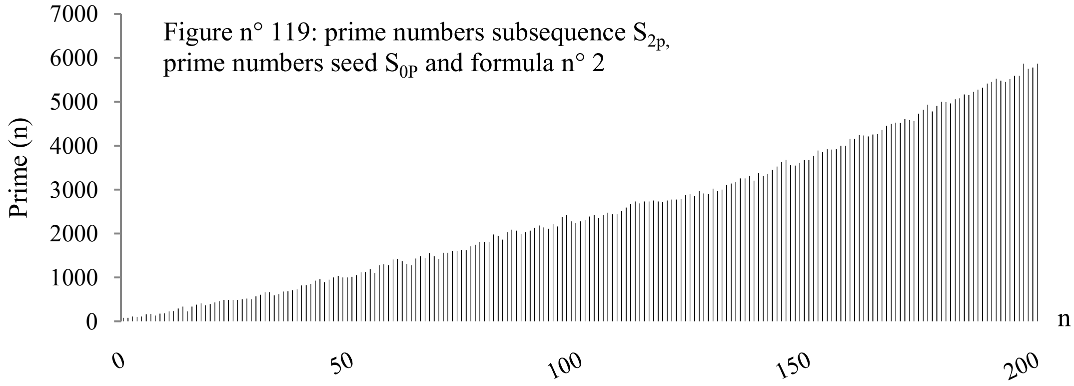


Figure n° 119: prime numbers subsequence $S_{2p,}$ prime numbers seed S_{0P} and formula n° 2

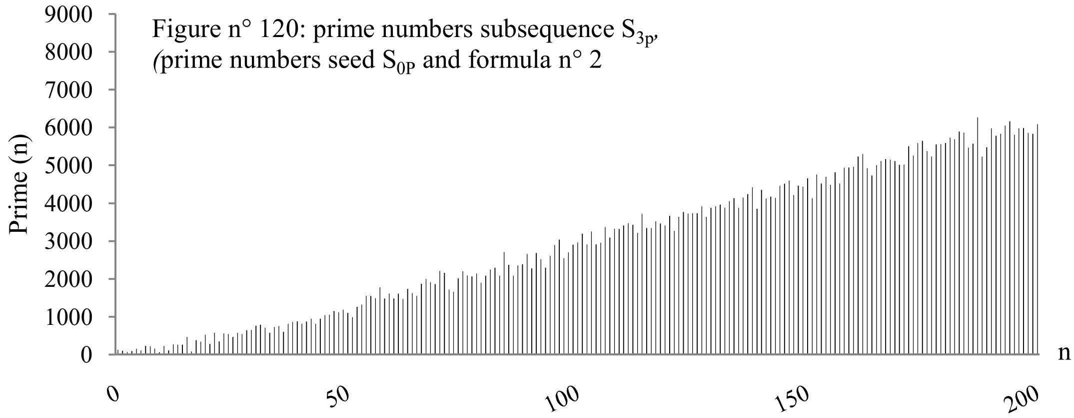


Figure n° 120: prime numbers subsequence S_{3p}, (prime numbers seed S_{0P} and formula n° 2

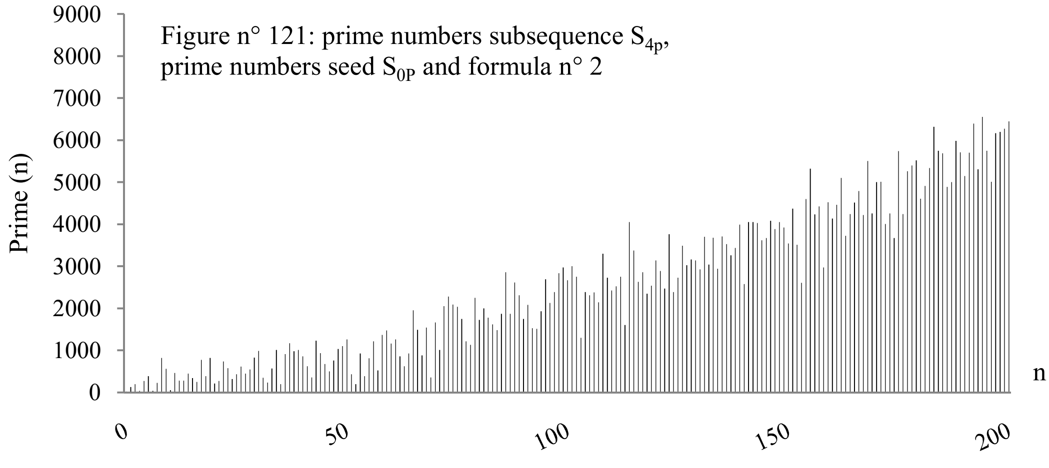


Figure n° 121: prime numbers subsequence S_{4p}, prime numbers seed S_{0P} and formula n° 2

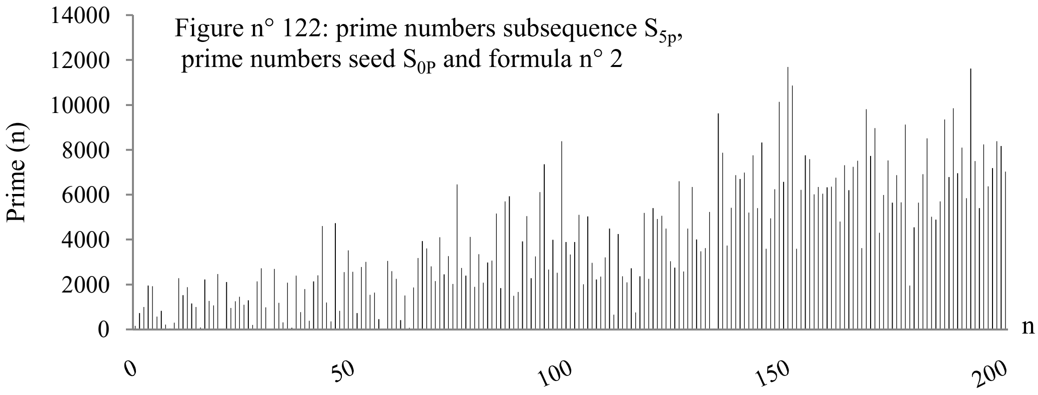


Figure n° 122: prime numbers subsequence S_{5p}, prime numbers seed S_{0P} and formula n° 2

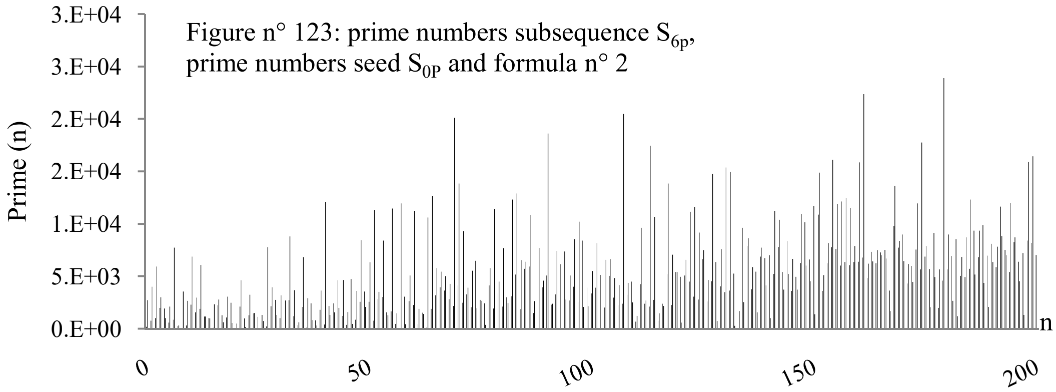


Figure n° 123: prime numbers subsequence S_{6p}, prime numbers seed S_{0P} and formula n° 2

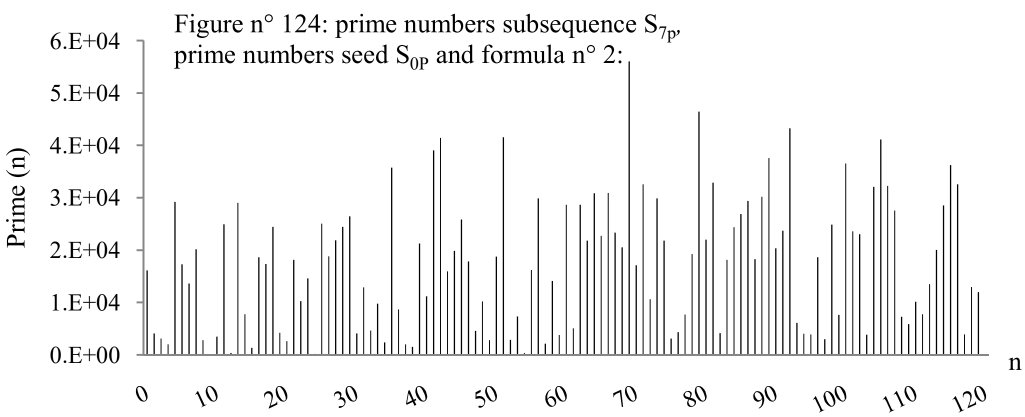


Figure n° 124: prime numbers subsequence S_{7p}, prime numbers seed S_{0P} and formula n° 2:

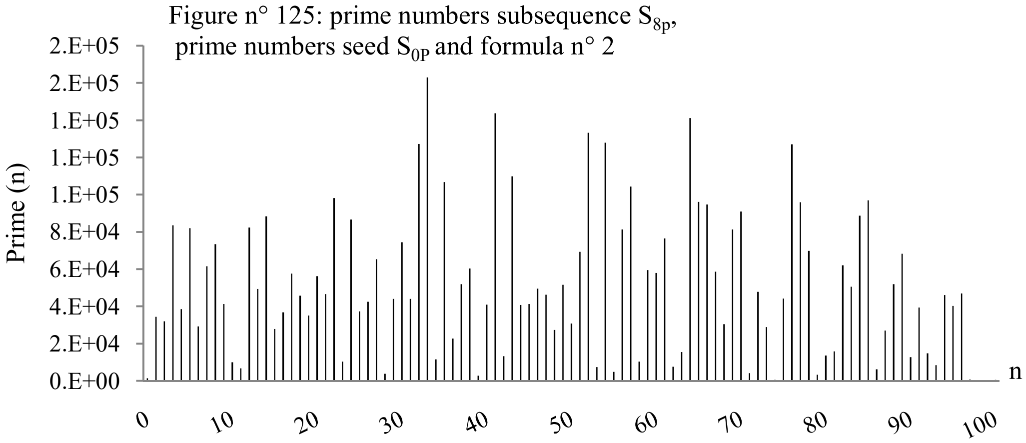


Figure n° 125: prime numbers subsequence S_{8p},
prime numbers seed S_{0P} and formula n° 2
Prime (n)
2.E+05
2.E+05
1.E+05
1.E+05
1.E+05
8.E+04
6.E+04
4.E+04
2.E+04
0.E+00
n
0
10
20
30
40
50
60
70
80
90
100

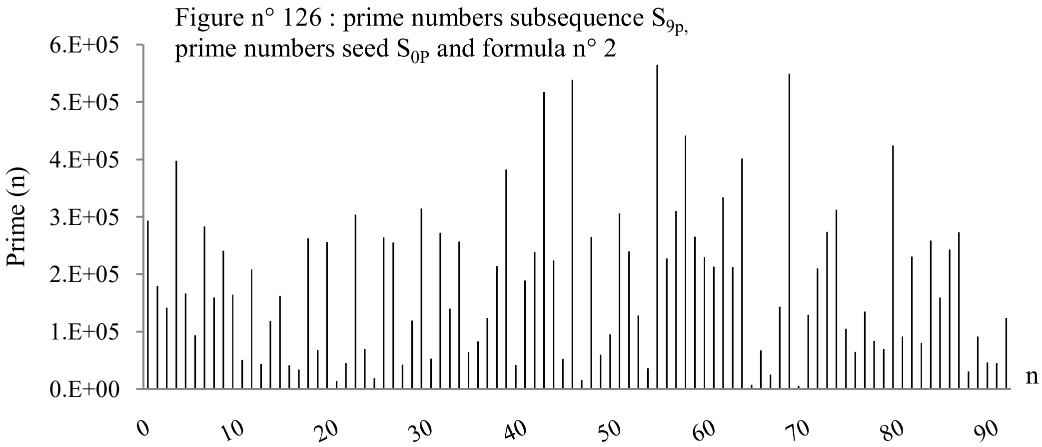


Figure n° 126 : prime numbers subsequence S_{9p},
prime numbers seed S_{0P} and formula n° 2
Prime (n)
6.E+05
5.E+05
4.E+05
3.E+05
2.E+05
1.E+05
0.E+00
n
0
10
20
30
40
50
60
70
80
90

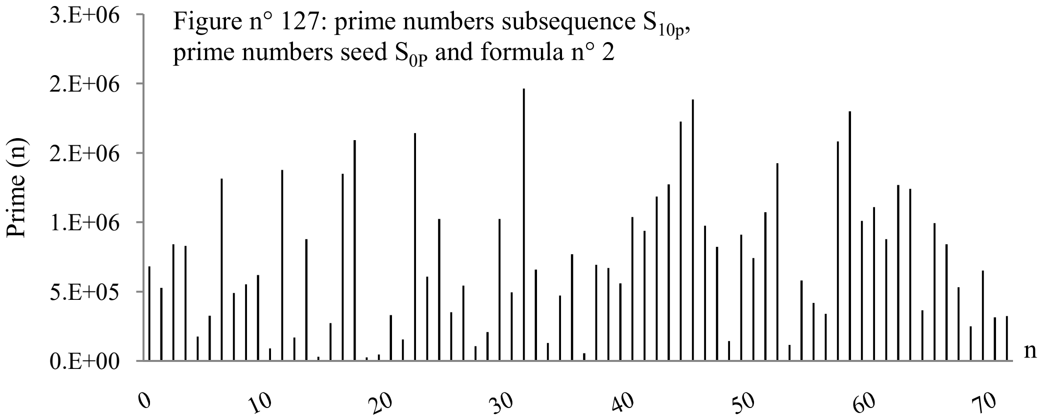


Figure n° 127: prime numbers subsequence S_{10p},
prime numbers seed S_{0P} and formula n° 2
Prime (n)
3.E+06
2.E+06
2.E+06
1.E+06
5.E+05
0.E+00
n
0
10
20
30
40
50
60
70

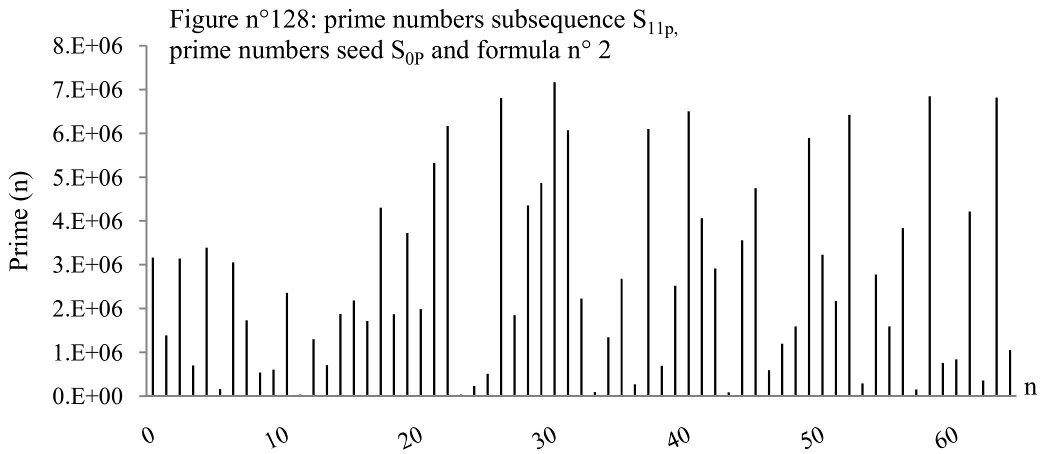


Figure n°128: prime numbers subsequence S_{11p}, prime numbers seed S_{0P} and formula n° 2
8.E+06
7.E+06
6.E+06
5.E+06
4.E+06
3.E+06
2.E+06
1.E+06
0.E+00
Prime (n)
n
0
10
20
30
40
50
60

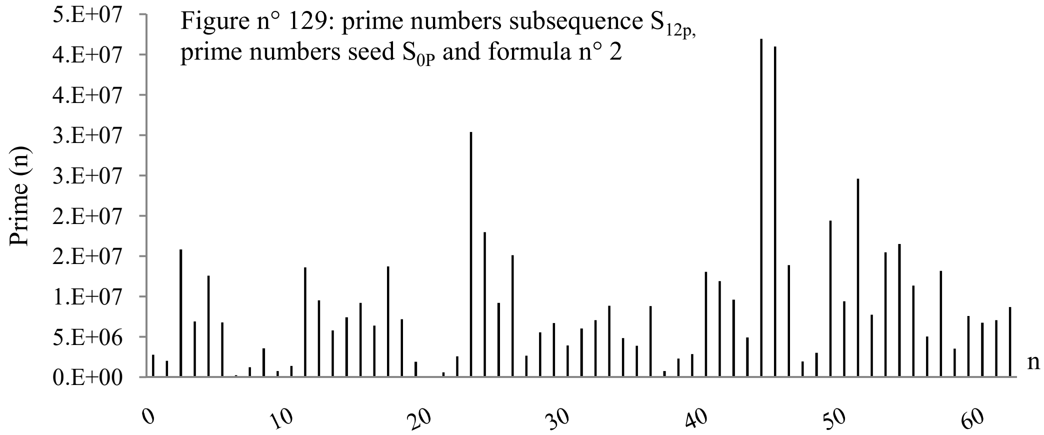


Figure n° 129: prime numbers subsequence S_{12p}, prime numbers seed S_{0P} and formula n° 2
5.E+07
4.E+07
4.E+07
3.E+07
3.E+07
2.E+07
2.E+07
1.E+07
5.E+06
0.E+00
Prime (n)
n
0
10
20
30
40
50
60

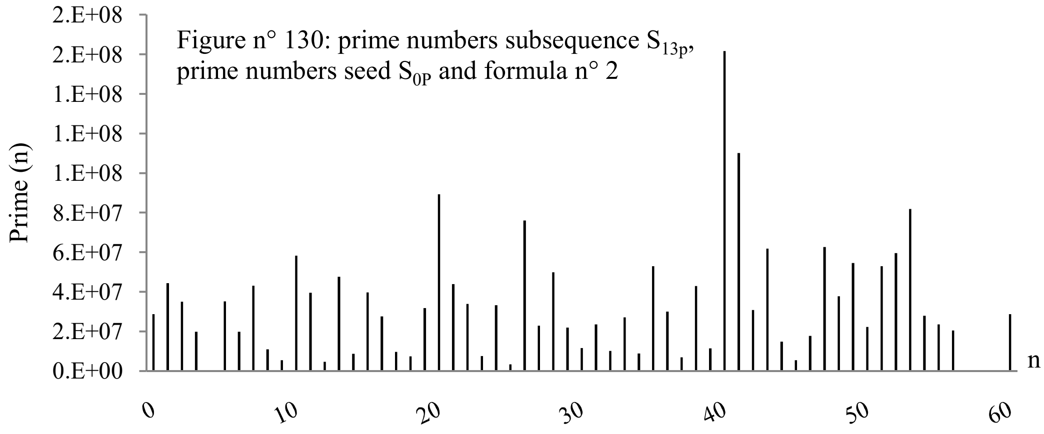


Figure n° 130: prime numbers subsequence S_{13p}, prime numbers seed S_{0P} and formula n° 2
2.E+08
2.E+08
1.E+08
1.E+08
1.E+08
8.E+07
6.E+07
4.E+07
2.E+07
0.E+00
Prime (n)
n
0
10
20
30
40
50
60

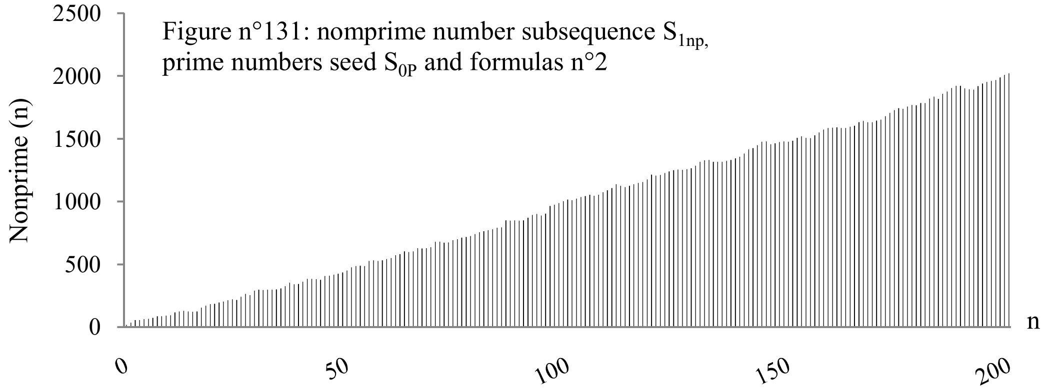


Figure n°131: nomprime number subsequence S_{1np}, prime numbers seed S_{0P} and formulas n°2

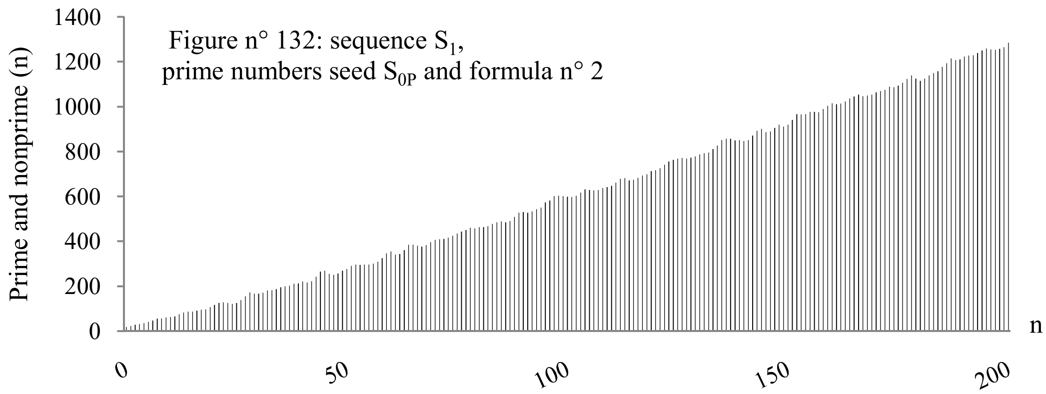


Figure n° 132: sequence S_1, prime numbers seed S_{0P} and formula n° 2

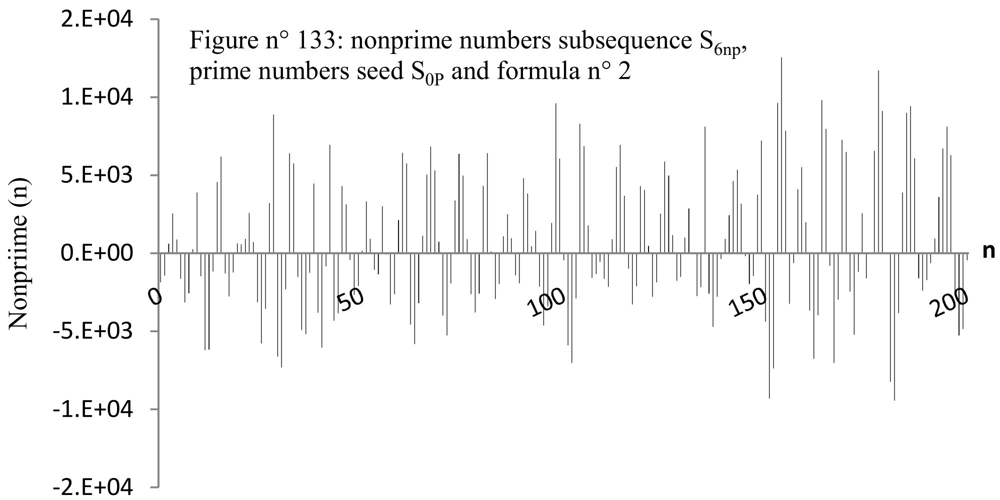


Figure n° 133: nonprime numbers subsequence S_{6np}, prime numbers seed S_{0P} and formula n° 2

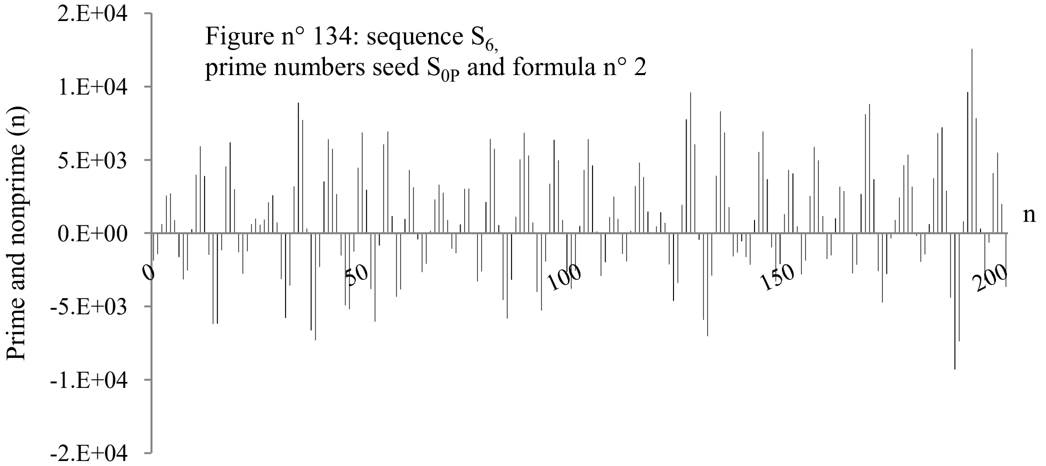


Figure n° 134: sequence S_{6}, prime numbers seed S_{0P} and formula n° 2

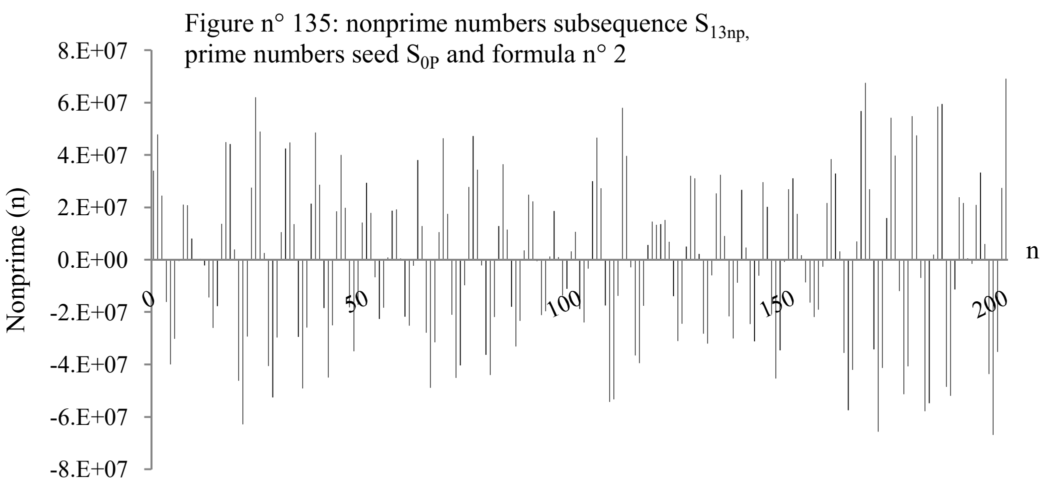


Figure n° 135: nonprime numbers subsequence S_{13np}, prime numbers seed S_{0P} and formula n° 2

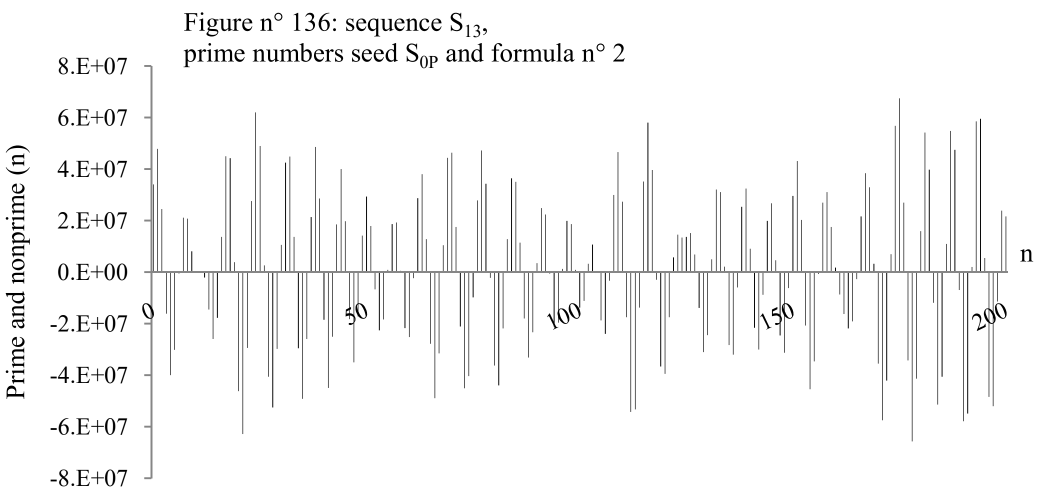


Figure n° 136: sequence S_{13}, prime numbers seed S_{0P} and formula n° 2

Figures from the prime numbers seeds S_{0P} and S_{0PL} and formula n° 3

Figure n° 137: number of pime numbers in each sequence, prime numbers seed S_{0P} (1000 terms) and formula n° 3

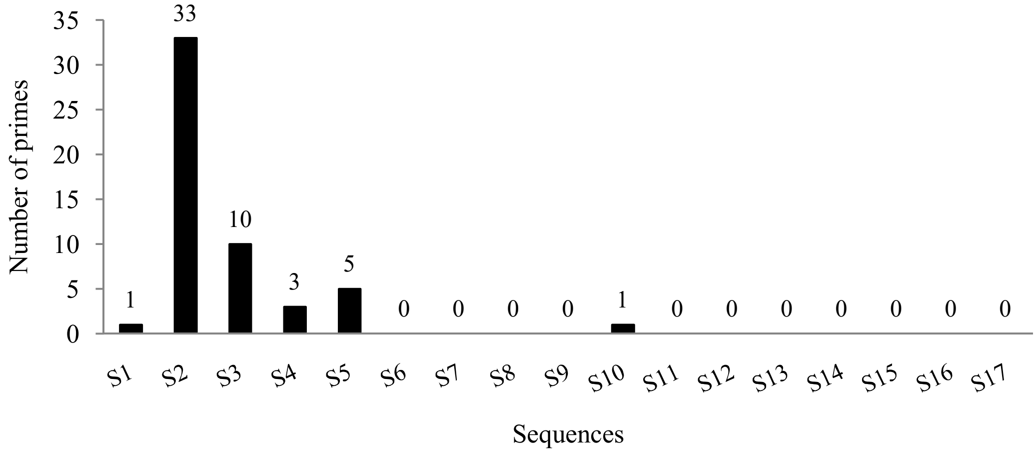


Figure n° 138: percentage of prime numbers in each sequence prime numbers seed S_{0P} (1000 terms) and formula n° 3

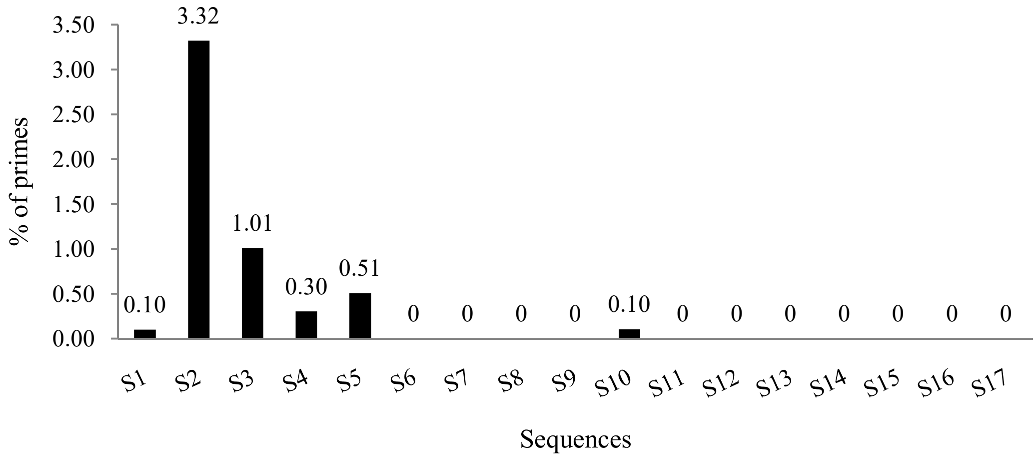


Figure n° 139: number of terms of each sequence prime numbers seed S_{0PL} ($2x10^4$ terms), formula n° 3

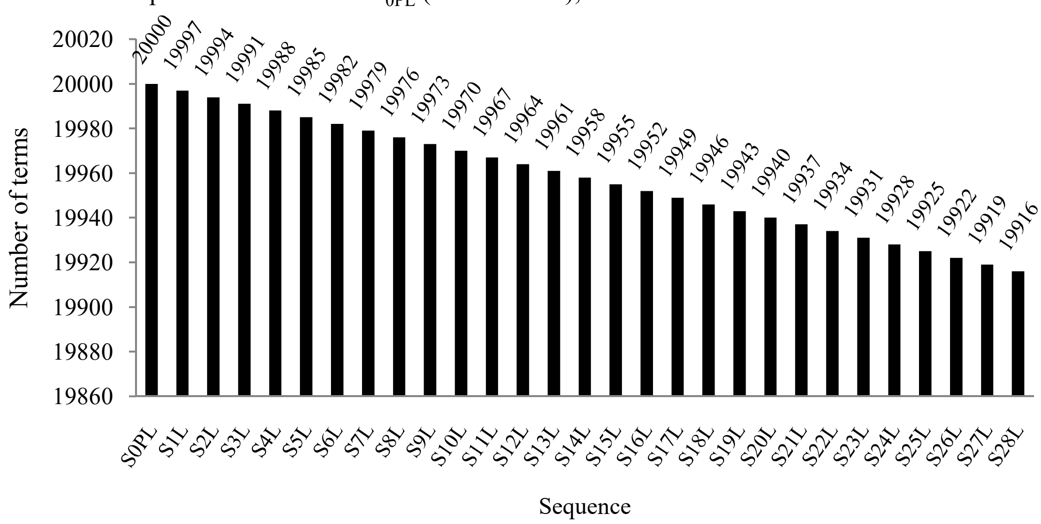


Figure n° 140: number of primes in each sequence,
Prime number seed S_{0PL} ($2x10^4$ terms), formula n° 3

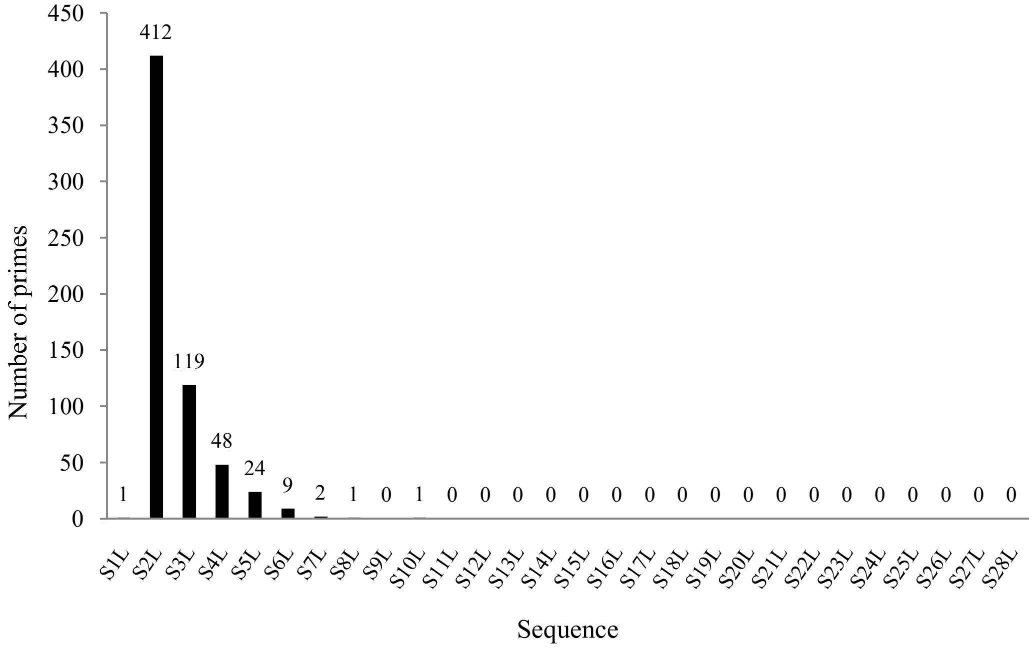
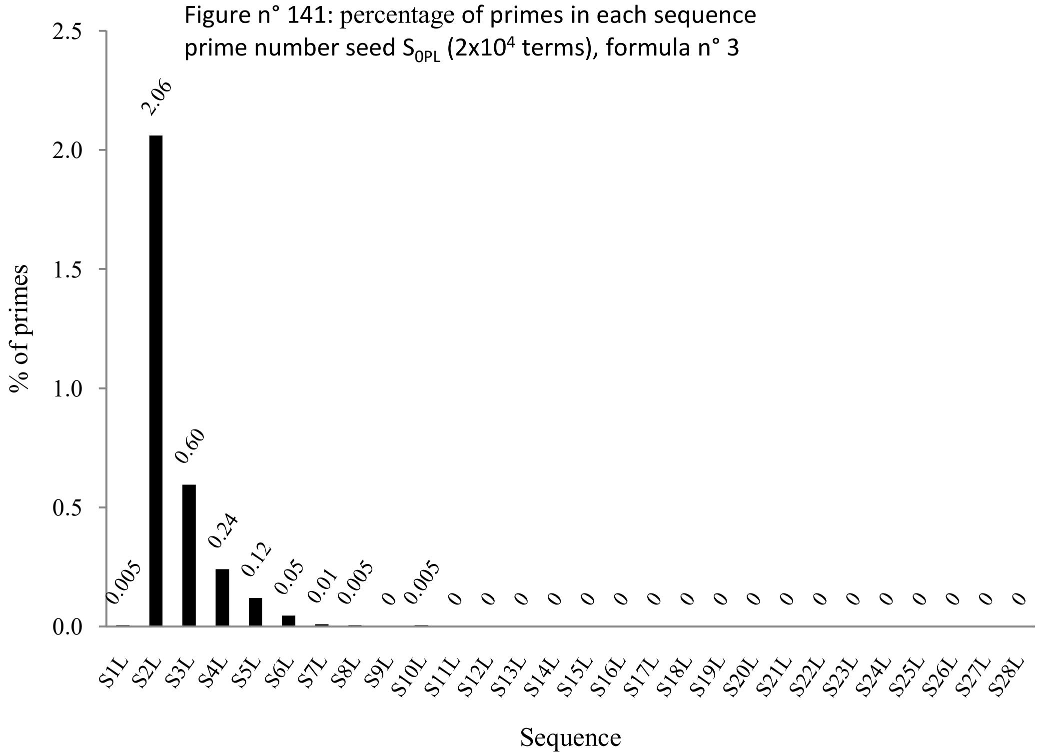


Figure n° 141: percentage of primes in each sequence
prime number seed S_{0PL} ($2x10^4$ terms), formula n° 3



Figures of the sums of the reciprocals of prime numbers :

Figure n° 142: subsequence $S_{1pl,}$ reciprocals of primes (1/p) = f(p), prime numbers seed S_{0PL}, formula n° 1
First 50 terms.

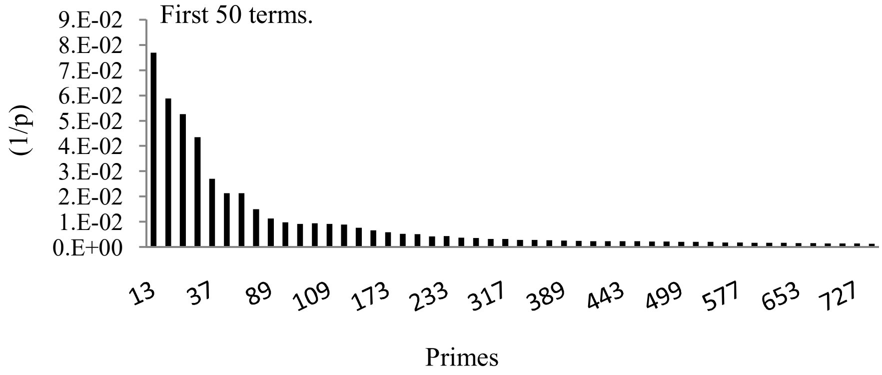


Figure n° 143: subsequence S_{1pl}, sum of the reciprocals of primes; sum of (1/p) = f(p),
prime numbers seed S_{0PL}, formula n° 1
First 50 terms.

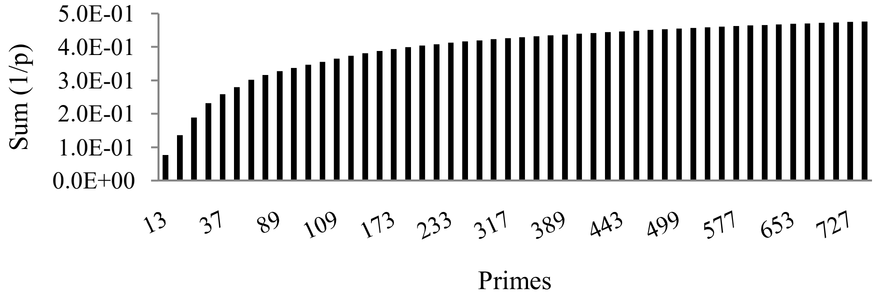
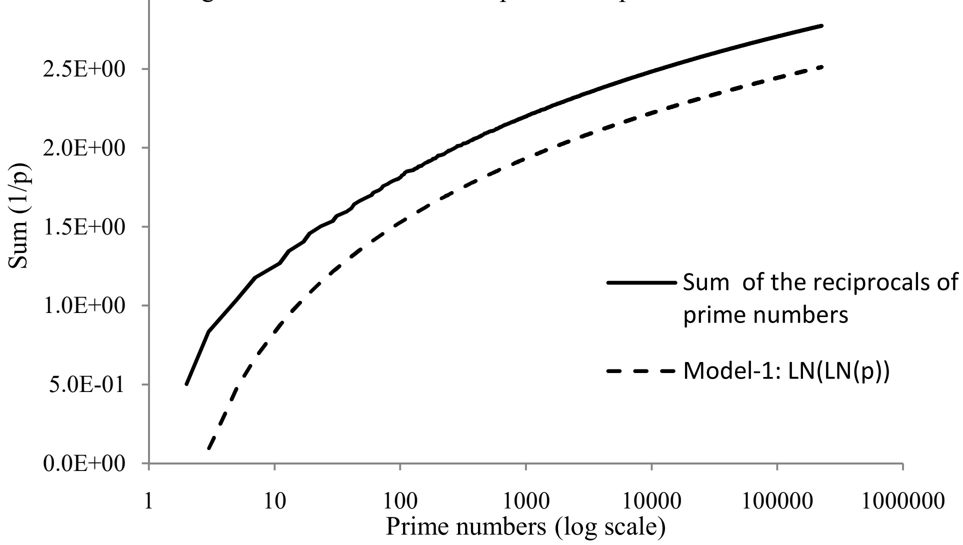


Figure n° 144: sum of the reciprocals of prime numbers and Model-1



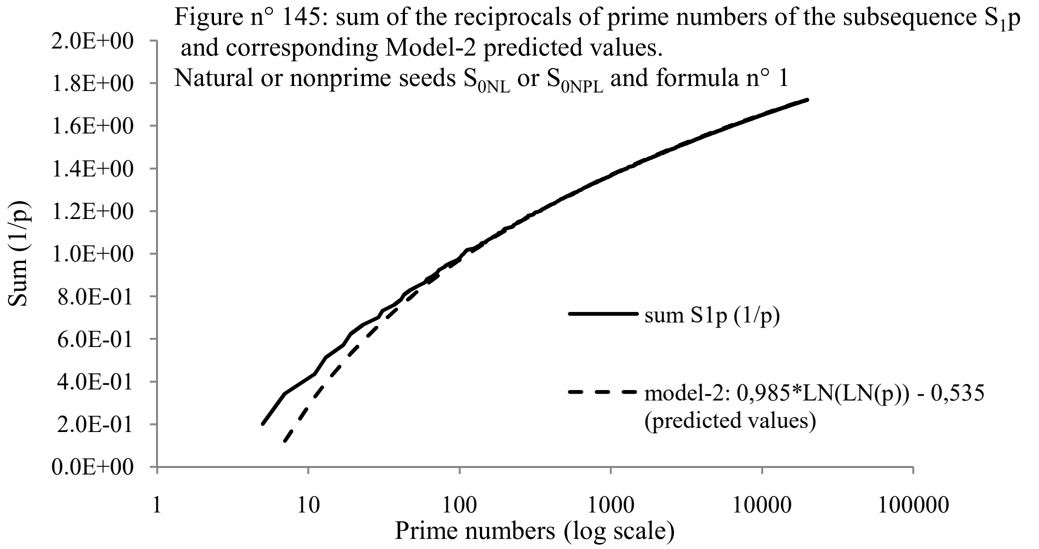


Figure n° 145: sum of the reciprocals of prime numbers of the subsequence S1p
and corresponding Model-2 predicted values.
Natural or nonprime seeds S0NL or S0NPL and formula n° 1
Sum (1/p)
2.0E+00
1.8E+00
1.6E+00
1.4E+00
1.2E+00
1.0E+00
8.0E-01
6.0E-01
4.0E-01
2.0E-01
0.0E+00
1
10
100
1000
10000
100000
Prime numbers (log scale)
sum S1p (1/p)
model-2: 0,985*LN(LN(p)) - 0,535
(predicted values)

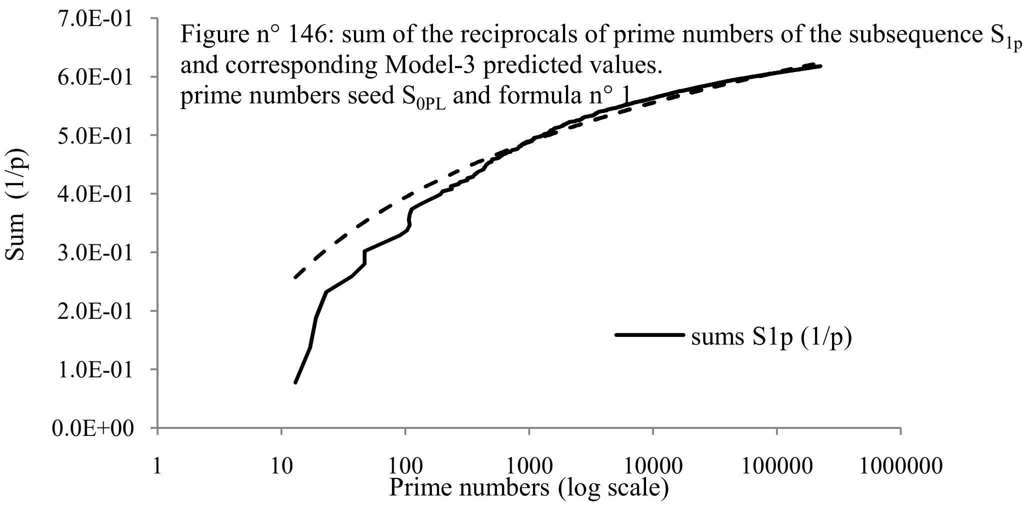


Figure n° 146: sum of the reciprocals of prime numbers of the subsequence S1p
and corresponding Model-3 predicted values.
prime numbers seed S0PL and formula n° 1
Sum (1/p)
7.0E-01
6.0E-01
5.0E-01
4.0E-01
3.0E-01
2.0E-01
1.0E-01
0.0E+00
1
10
100
1000
10000
100000
1000000
Prime numbers (log scale)
sums S1p (1/p)

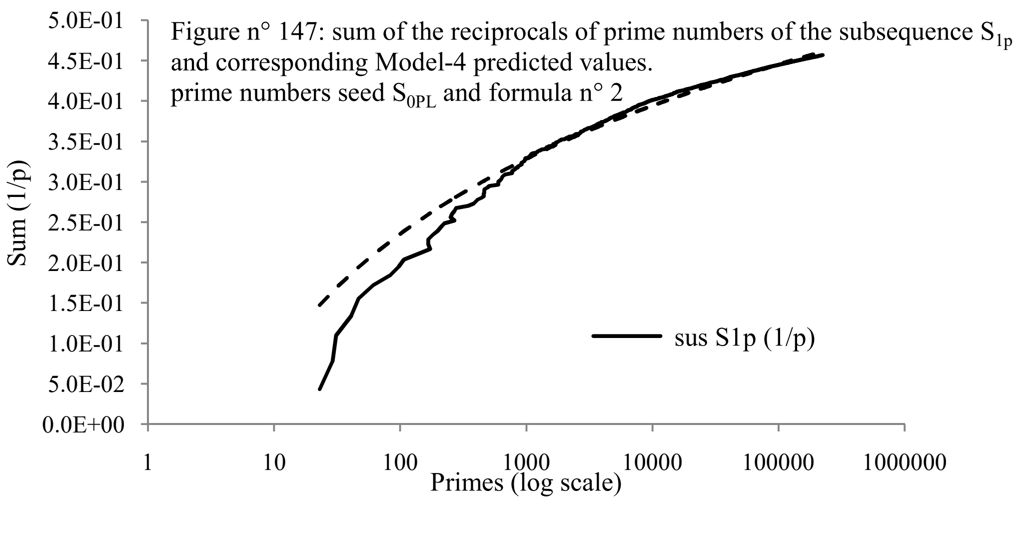
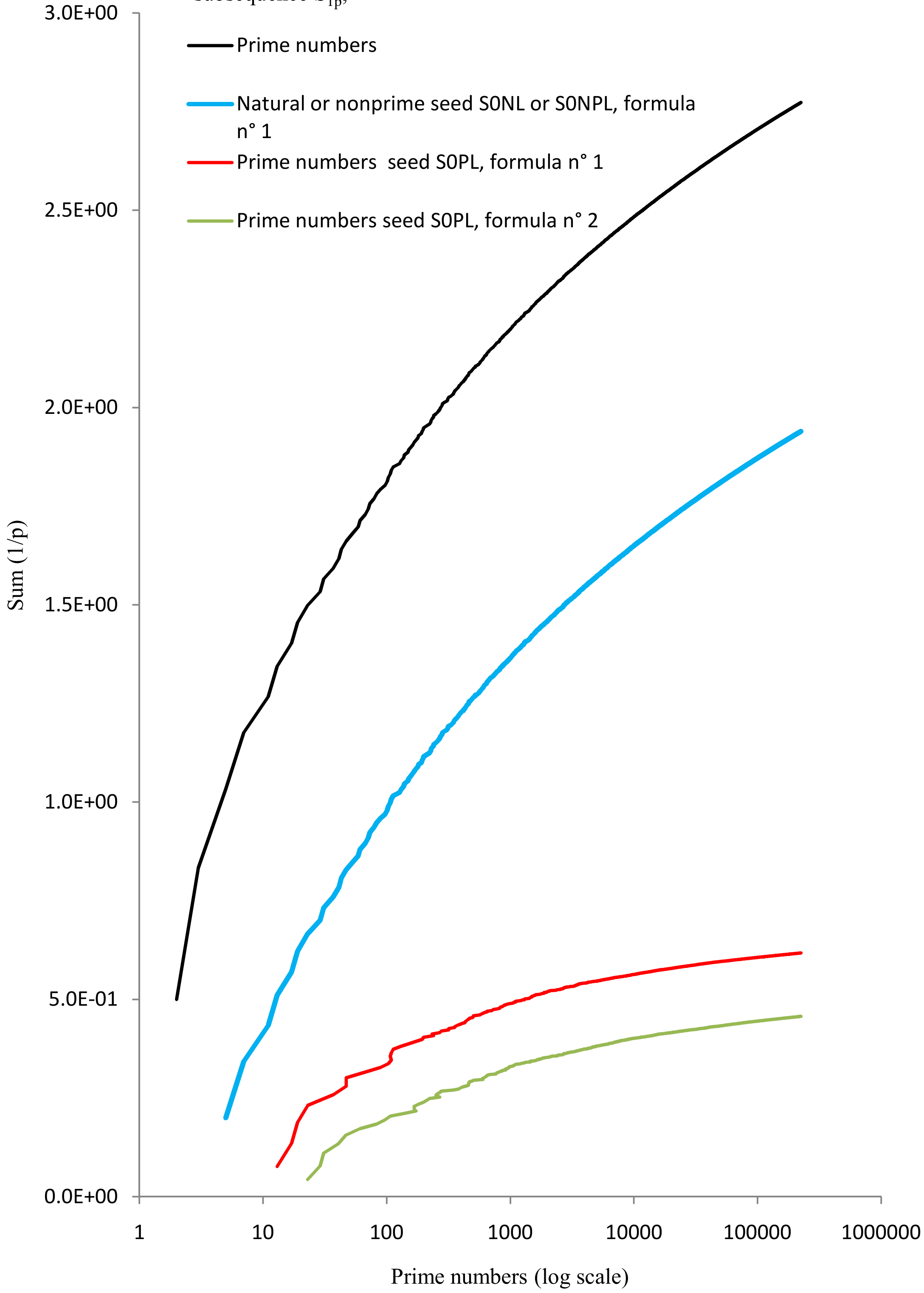


Figure n° 147: sum of the reciprocals of prime numbers of the subsequence S1p
and corresponding Model-4 predicted values.
prime numbers seed S0PL and formula n° 2
Sum (1/p)
5.0E-01
4.5E-01
4.0E-01
3.5E-01
3.0E-01
2.5E-01
2.0E-01
1.5E-01
1.0E-01
5.0E-02
0.0E+00
1
10
100
1000
10000
100000
1000000
Primes (log scale)
sus S1p (1/p)

Figure n° 148: sum of the reciprocals of prime numbers of the subsequence S_{1p},



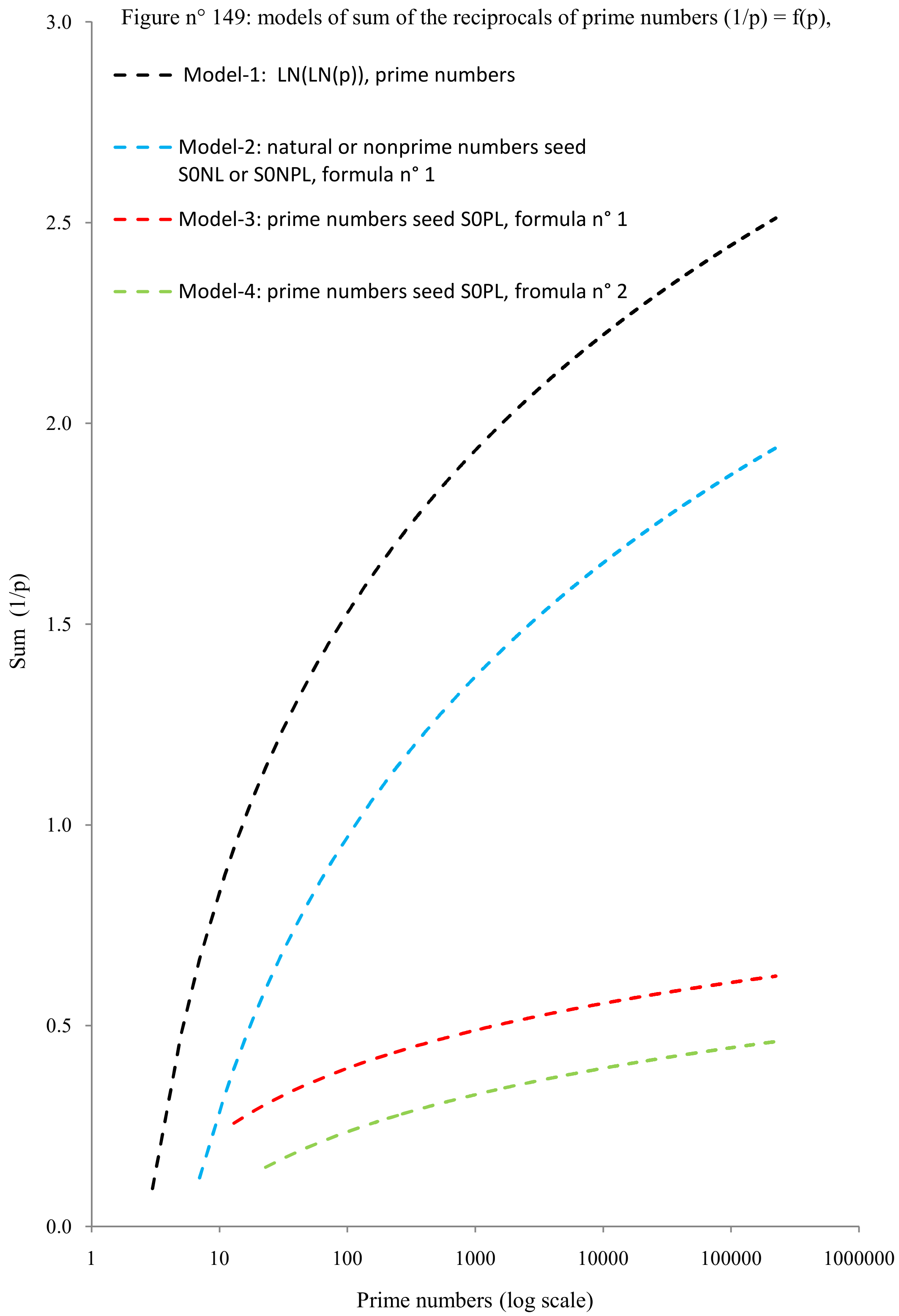


Figure n° 149: models of sum of the reciprocals of prime numbers (1/p) = f(p),
Model-1: LN(LN(p)), prime numbers
Model-2: natural or nonprime numbers seed S0NL or S0NPL, formula n° 1
Model-3: prime numbers seed S0PL, formula n° 1
Model-4: prime numbers seed S0PL, fromula n° 2
Sum (1/p)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
1
10
100
1000
10000
100000
1000000
Prime numbers (log scale)