On the mathematical connections between some formulas concerning Ramanujan Modular Forms, ϕ, ζ(2) and various topics and parameters of String Theory and Particle Physics.

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Abstract

In this paper we describe and analyze the mathematical connections between some formulas concerning Ramanujan Modular Forms, ϕ, ζ(2) and various topics and parameters of String Theory and Particle Physics.

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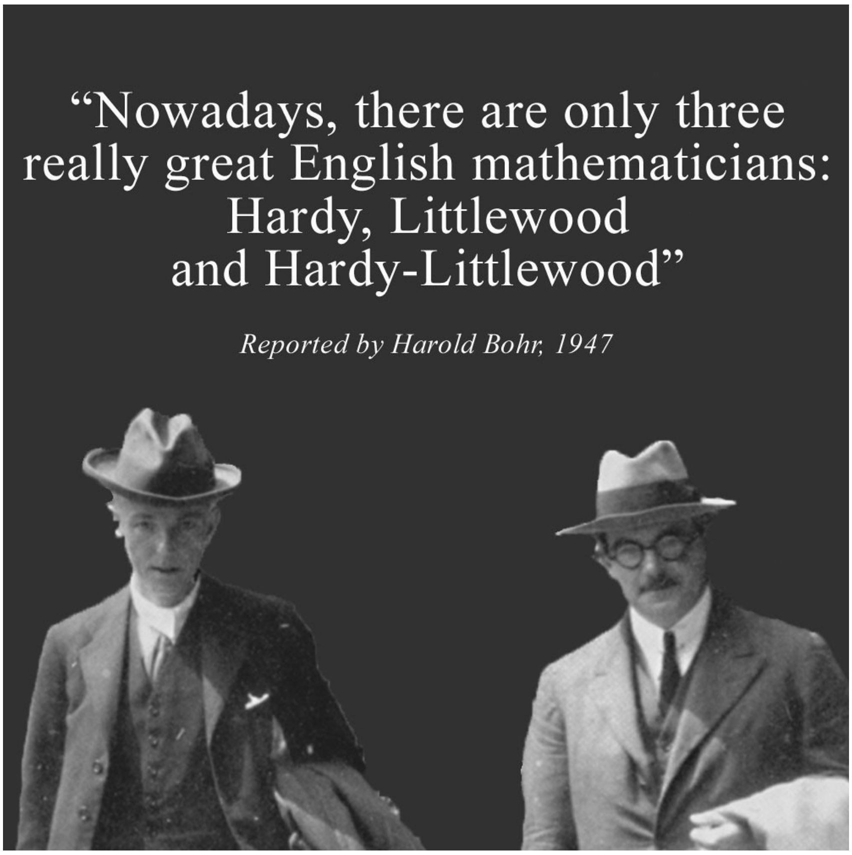

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We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

For more information on the data entered for the development of the various equations, see the "Observations" section.

From

Partitions, *q*-Series, and Modular Forms
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Now, we have that:

As an example of an approximation with more than one term, Ramanujan gave $(q)_\infty^{-120}$. A simpler example is provided by $(q)_\infty^{-48}$. By the functional equation of the Dedekind η-function (see for example [Ap, 48]), we have

$$q^{\frac{1}{24}}(q)_\infty = \sqrt{\frac{2\pi}{\alpha}}\, q_1^{\frac{1}{6}}(q_1^4;q_1^4)_\infty ,$$

where $q = \mathrm{e}^{-\alpha}$ and $q_1 = \mathrm{e}^{-\frac{\pi^2}{\alpha}}$. Hence as $\alpha \to 0^+$,

$$\begin{aligned}
(q)_\infty^{-48} &= q^2[q^{\frac{1}{24}}(q)_\infty]^{-48} \\
&= q^2\left[\sqrt{\frac{2\pi}{\alpha}}\, q_1^{\frac{1}{6}}(q_1^4;q_1^4)_\infty\right]^{-48} \\
&= \frac{\alpha^{24}}{(2\pi)^{24}} q^2 q_1^{-8}[1 - q_1^4 + O(q_1^8)]^{-48} \\
&- \frac{\alpha^{24}}{(2\pi)^{24}} q^2 q_1^{-8}[1 + 48q_1^4 + O(q_1^8)] \\
&= \frac{\alpha^{24}}{(2\pi)^{24}} q^2 q_1^{-8} + \frac{48\alpha^{24}}{(2\pi)^{24}} q^2 q_1^{-4} + o(1) \\
&= \frac{\alpha^{24}}{(2\pi)^{24}} \exp\left(\frac{8\pi^2}{\alpha} - 2\alpha\right) + \frac{48\alpha^{24}}{(2\pi)^{24}} \exp\left(\frac{4\pi^2}{\alpha} - 2\alpha\right) + o(1).
\end{aligned}$$

For:

$$q = e^{-\alpha}$$

we put α = 1/12 and obtain:

e^(-1/12)

Input:

$e^{-1/12}$

Exact result:

$$\frac{1}{\sqrt[12]{e}}$$

Decimal approximation:

0.920044414629323247893155324053717231673187534959974201701...

q = 0.9200444146293….

Property:

$\frac{1}{\sqrt[12]{e}}$ is a transcendental number

All values of 1/e^(1/12):

$\frac{e^0}{\sqrt[12]{e}} \approx 0.920044$ (real root)

$\frac{e^{-(i\pi)/6}}{\sqrt[12]{e}} \approx 0.7968 - 0.46002\, i$

$\frac{e^{-(i\pi)/3}}{\sqrt[12]{e}} \approx 0.46002 - 0.7968\, i$

$\frac{e^{-(i\pi)/2}}{\sqrt[12]{e}} \approx -0.92004\, i$

$\frac{e^{-(2i\pi)/3}}{\sqrt[12]{e}} \approx -0.46002 - 0.7968\, i$

Alternative representation:

$$e^{-1/12} = \exp^{-\frac{1}{12}}(z) \quad \text{for } z = 1$$

Series representations:

$$e^{-1/12} = \frac{1}{\sqrt[12]{\sum_{k=0}^{\infty} \frac{1}{k!}}}$$

$$e^{-1/12} = \frac{1}{\sqrt[12]{\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}}}$$

$$e^{-1/12} = \frac{\sqrt[12]{2}}{\sqrt[12]{\sum_{k=0}^{\infty} \frac{1+k}{k!}}}$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,d\,s}{(2\,\pi\,i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\operatorname{Re}(a) \text{ and } |\arg(z)| < \pi)$$

From:

$$\frac{\alpha^{24}}{(2\pi)^{24}} \exp\left(\frac{8\pi^2}{\alpha} - 2\alpha\right) + \frac{48\alpha^{24}}{(2\pi)^{24}} \exp\left(\frac{4\pi^2}{\alpha} - 2\alpha\right) + o(1).$$

For α = 1/12, we obtain:

(((1/12)^24))/((2Pi)^24) exp((((((8Pi^2)/(1/12))-2*1/12))))+48*(((1/12)^24))/((2Pi)^24) exp((((((4Pi^2)/(1/12))-2*1/12))))

Input:

$$\frac{\left(\frac{1}{12}\right)^{24}}{(2\,\pi)^{24}} \exp\left(\frac{8\,\pi^2}{\frac{1}{12}} - 2\times\frac{1}{12}\right) + 48\times\frac{\left(\frac{1}{12}\right)^{24}}{(2\,\pi)^{24}} \exp\left(\frac{4\,\pi^2}{\frac{1}{12}} - 2\times\frac{1}{12}\right)$$

Exact result:

$$\frac{e^{48\pi^2-1/6}}{27\,786\,162\,017\,714\,252\,592\,689\,197\,350\,912\,\pi^{24}}+\frac{e^{96\pi^2-1/6}}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}$$

Decimal approximation:

$2.2759615293034692593068303077991630295927502015295737\ldots\times10^{366}$

$2.27596152\ldots*10^{366}$

Alternate forms:

$$\frac{e^{48\pi^2-1/6}\left(48+e^{48\pi^2}\right)}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}$$

$$\frac{48\,e^{48\pi^2-1/6}+e^{96\pi^2-1/6}}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}$$

Series representations:

$$\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}=\frac{e^{-1/6+288\times\sum_{k=1}^{\infty}1/k^2}}{27\,786\,162\,017\,714\,252\,592\,689\,197\,350\,912\,\pi^{24}}+\frac{e^{-1/6+576\times\sum_{k=1}^{\infty}1/k^2}}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}$$

$$\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}=\frac{\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-1/6+48\pi^2}\left(48+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{48\pi^2}\right)}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}$$

$$\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}=\frac{\left(48+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{48\pi^2}\right)\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-1/6+48\pi^2}}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}$$

Integral representations:

$$\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}=$$

$$\left(\exp\left(-\frac{1}{6}+48\left(\frac{3\sqrt{3}}{4}+24\int_0^{\frac{1}{4}}\sqrt{-(-1+t)\,t}\;dt\right)^2\right)\left(48+\exp\left(48\left(\frac{3\sqrt{3}}{4}+24\int_0^{\frac{1}{4}}\sqrt{-(-1+t)\,t}\;dt\right)^2\right)\right)\right)\Bigg/$$
$$\left(1\,338\,258\,845\,052\,394\,702\,439\,737\,982\,976\left(\sqrt{3}+32\int_0^{\frac{1}{4}}\sqrt{-(-1+t)\,t}\;dt\right)^{24}\right)$$

$$\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}=$$
$$\frac{e^{-1/6+192\left(\int_0^\infty 1/(1+t^2)\,dt\right)^2}}{466\,174\,441\,982\,187\,842\,026\,106\,684\,822\,878\,420\,992\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^{24}}+$$
$$\frac{e^{-1/6+384\left(\int_0^\infty 1/(1+t^2)\,dt\right)^2}}{22\,376\,373\,215\,145\,016\,417\,253\,120\,871\,498\,164\,207\,616\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^{24}}$$

$$\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}=$$
$$\frac{e^{-1/6+192\left(\int_0^\infty \sin(t)/t\,dt\right)^2}}{466\,174\,441\,982\,187\,842\,026\,106\,684\,822\,878\,420\,992\left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^{24}}+$$
$$\frac{e^{-1/6+384\left(\int_0^\infty \sin(t)/t\,dt\right)^2}}{22\,376\,373\,215\,145\,016\,417\,253\,120\,871\,498\,164\,207\,616\left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^{24}}$$

Thence, we obtain:

$$q^{\frac{1}{24}}(q)_\infty=\sqrt{\frac{2\pi}{\alpha}}\,q_1^{\frac{1}{6}}(q_1^4;q_1^4)_\infty$$

$$(q)_\infty^{-48} = q^2[q^{\frac{1}{24}}(q)_\infty]^{-48}$$

$$= q^2\left[\sqrt{\frac{2\pi}{\alpha}}\, q_1^{\frac{1}{6}}(q_1^4;q_1^4)_\infty\right]^{-48}$$

$$= \frac{\alpha^{24}}{(2\pi)^{24}}\exp\left(\frac{8\pi^2}{\alpha}-2\alpha\right)+\frac{48\alpha^{24}}{(2\pi)^{24}}\exp\left(\frac{4\pi^2}{\alpha}-2\alpha\right)+o(1)$$

$$= 2.2759615293034692593068303077991630295927502015295737\ldots \times 10^{366}$$

$2.2759615293\ldots*10^{366}$

Now, we observe that performing the 1752th root (where 1752 is divisible by 8, and is equal to 1729 + 23, where 23 is a Eisenstein prime number), we obtain:

[(((1/12)^24))/((2Pi)^24) exp((((((8Pi^2)/(1/12))-2*1/12))))+48*(((1/12)^24))/((2Pi)^24) exp((((((4Pi^2)/(1/12))-2*1/12))))]^1/1752

Input:

$$\sqrt[1752]{\frac{\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\exp\left(\frac{8\pi^2}{\frac{1}{12}}-2\times\frac{1}{12}\right)+48\times\frac{\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\exp\left(\frac{4\pi^2}{\frac{1}{12}}-2\times\frac{1}{12}\right)}$$

Exact result:

$$\left(\frac{e^{48\pi^2-1/6}}{27\,786\,162\,017\,714\,252\,592\,689\,197\,350\,912\,\pi^{24}}+\frac{e^{96\pi^2-1/6}}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}\right)\hat{}\,(1/1752)$$

Decimal approximation:

1.6184823618532031954271325093077399511254707612788916269 51...

1.61848236…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms:

$$\frac{e^{(2\pi^2)/73-1/10512}\ \sqrt[1752]{48+e^{48\pi^2}}}{2^{3/73}\ \sqrt[73]{3\pi}}$$

$$\frac{\sqrt[1752]{48\,e^{48\pi^2-1/6}+e^{96\pi^2-1/6}}}{2^{3/73}\ \sqrt[73]{3\pi}}$$

All 1752nd roots of e^(48 π^2 - 1/6)/(27786162017714252592689197350912 π^24) + e^(96 π^2 - 1/6)/(1333735776850284124449081472843776 π^24):

$$\left(\frac{e^{48\pi^2-1/6}}{27786162017714252592689197350912\,\pi^{24}}+\frac{e^{96\pi^2-1/6}}{1333735776850284124449081472843776\,\pi^{24}}\right)^\wedge (1/1752)\, e^0 \approx 1.6185 \text{ (real, principal root)}$$

$$\left(\frac{e^{48\pi^2-1/6}}{27786162017714252592689197350912\,\pi^{24}}+\frac{e^{96\pi^2-1/6}}{1333735776850284124449081472843776\,\pi^{24}}\right)^\wedge (1/1752)\, e^{(i\pi)/876} \approx 1.6185+0.005804\,i$$

$$\left(\frac{e^{48\pi^2-1/6}}{27786162017714252592689197350912\,\pi^{24}}+\frac{e^{96\pi^2-1/6}}{1333735776850284124449081472843776\,\pi^{24}}\right)^\wedge (1/1752)\, e^{(i\pi)/438} \approx 1.6184+0.011609\,i$$

$$\left(\frac{e^{48\pi^2-1/6}}{27786162017714252592689197350912\,\pi^{24}}+\frac{e^{96\pi^2-1/6}}{1333735776850284124449081472843776\,\pi^{24}}\right)^\wedge (1/1752)\, e^{(i\pi)/292} \approx 1.6184+0.017413\,i$$

$$\left(\frac{e^{48\pi^2-1/6}}{27786162017714252592689197350912\,\pi^{24}}+\frac{e^{96\pi^2-1/6}}{1333735776850284124449081472843776\,\pi^{24}}\right)^\wedge (1/1752)\, e^{(i\pi)/219} \approx 1.6183+0.023217\,i$$

Series representations:

$$\sqrt[1752]{\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}} = \frac{\sqrt[1752]{e^{-1/6+288\times\sum_{k=1}^{\infty}1/k^2}\left(48+e^{288\times\sum_{k=1}^{\infty}1/k^2}\right)}}{2^{3/73}\sqrt[73]{3\pi}}$$

$$\sqrt[1752]{\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}} = \frac{\sqrt[1752]{\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-1/6+48\pi^2}\left(48+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{48\pi^2}\right)}}{2^{3/73}\sqrt[73]{3\pi}}$$

$$\sqrt[1752]{\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}} = \frac{\sqrt[1752]{\left(48+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{48\pi^2}\right)\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-1/6+48\pi^2}}}{2^{3/73}\sqrt[73]{3\pi}}$$

Integral representations:

$$\sqrt[1752]{\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}} = \frac{\sqrt[1752]{\frac{e^{-1/6+192\left(\int_0^{\infty}1/(1+t^2)\,dt\right)^2}\left(48+e^{192\left(\int_0^{\infty}1/(1+t^2)\,dt\right)^2}\right)}{\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{24}}}}{2^{4/73}\sqrt[73]{3}}$$

$$\sqrt[1752]{\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}} = \frac{\sqrt[1752]{\frac{e^{-1/6+192\left(\int_0^\infty \sin(t)/t\,dt\right)^2}\left(48+e^{192\left(\int_0^\infty \sin(t)/t\,dt\right)^2}\right)}{\left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^{24}}}}{2^{4/73}\sqrt[73]{3}}$$

$$\sqrt[1752]{\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}} = \frac{\sqrt[1752]{\frac{e^{-1/6+768\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^2}\left(48+e^{768\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^2}\right)}{\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^{24}}}}{2^{5/73}\sqrt[73]{3}}$$

From which, we obtain, subtracting 23, that is a Eisenstein prime number:

log base 1.6184823618532((([(((1/12)^24))/((2Pi)^24) exp((((((8Pi^2)/(1/12))-2*1/12))))+48*(((1/12)^24))/((2Pi)^24) exp((((((4Pi^2)/(1/12))-2*1/12))))]))) -23

Input interpretation:

$$\log_{1.6184823618532}\left(\frac{\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\exp\left(\frac{8\pi^2}{\frac{1}{12}}-2\times\frac{1}{12}\right)+48\times\frac{\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\exp\left(\frac{4\pi^2}{\frac{1}{12}}-2\times\frac{1}{12}\right)\right)-23$$

$\log_b(x)$ is the base- b logarithm

Result:

1729.0000000000...

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representation:

$$\log_{1.61848236185320000}\left(\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\right)-23=$$

$$-23+\frac{\log\left(\frac{48\exp\left(-\frac{2}{12}+\frac{4\pi^2}{\frac{1}{12}}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\exp\left(-\frac{2}{12}+\frac{8\pi^2}{\frac{1}{12}}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\right)}{\log(1.61848236185320000)}$$

Series representations:

$$\log_{1.61848236185320000}\left(\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\right)-23=-23-$$

$$1.0000000000000000\log\left(\frac{48\exp\left(-\frac{1}{6}+48\pi^2\right)+\exp\left(-\frac{1}{6}+96\pi^2\right)}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}\right)$$

$$\left(-2.1168609837209152+\sum_{k=0}^{\infty}0.61848236185320000^k\,G(k)\right)$$

$$\text{for}\left(G(0)=0\text{ and }\frac{(-1)^k\,k}{2(1+k)(2+k)}+G(k)=\sum_{j=1}^{k}\frac{(-1)^{1+j}\,G(-j+k)}{1+j}\right)$$

$$\log_{1.61848236185320000}\left(\frac{\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\left(48\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\right)-23=-23-$$

$$1.0000000000000000\log\left(\frac{48\exp\left(-\frac{1}{6}+48\pi^2\right)+\exp\left(-\frac{1}{6}+96\pi^2\right)}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}\right)$$

$$\left(-2.1168609837209152+\sum_{k=0}^{\infty}0.61848236185320000^k\,G(k)\right)$$

$$\text{for}\left(G(0)=0\text{ and }G(k)=\frac{(-1)^{1+k}\,k}{2(1+k)(2+k)}+\sum_{j=1}^{k}\frac{(-1)^{1+j}\,G(-j+k)}{1+j}\right)$$

Multiplying by 1/10 and subtracting 34, that is a Fibonacci number, and the golden ratio value, we obtain from the previous expression:

1/10 log base 1.6184823618532((([(((1/12)^24))/((2Pi)^24) exp((((((8Pi^2)/(1/12))-2*1/12))))+48*(((1/12)^24))/((2Pi)^24) exp((((((4Pi^2)/(1/12))-2*1/12))))]))) -34 - golden ratio

Input interpretation:

$$\frac{1}{10}\log_{1.6184823618532}\left(\frac{\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\exp\left(\frac{8\pi^2}{\frac{1}{12}}-2\times\frac{1}{12}\right)+48\times\frac{\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\exp\left(\frac{4\pi^2}{\frac{1}{12}}-2\times\frac{1}{12}\right)\right)-34-\phi$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

139.58196601125...

139.58196601125... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representation:

$$\frac{1}{10}\log_{1.61848236185320000}\left(\frac{\left(\frac{1}{12}\right)^{24}\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)}{(2\pi)^{24}}+\frac{48\left(\frac{1}{12}\right)^{24}\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)}{(2\pi)^{24}}\right)-34-\phi=$$

$$-34-\phi+\frac{\log\left(\frac{48\exp\left(-\frac{2}{12}+\frac{4\pi^2}{\frac{1}{12}}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\exp\left(-\frac{2}{12}+\frac{8\pi^2}{\frac{1}{12}}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\right)}{10\log(1.61848236185320000)}$$

Series representations:

$$\frac{1}{10}\log_{1.61848236185320000}\left(\frac{\left(\frac{1}{12}\right)^{24}\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)}{(2\pi)^{24}}+\frac{48\left(\frac{1}{12}\right)^{24}\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)}{(2\pi)^{24}}\right)-34-\phi=$$

$$-34.0000000000000000-1.00000000000000000\,\phi+\log\left(\frac{48\exp\left(-\frac{1}{6}+48\pi^2\right)+\exp\left(-\frac{1}{6}+96\pi^2\right)}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}\right)\left(0.21168609837209152-0.100000000000000000\times\sum_{k=0}^{\infty}0.61848236185320000^k\,G(k)\right)$$

$$\text{for }\left(G(0)=0\text{ and }\frac{(-1)^k\,k}{2(1+k)(2+k)}+G(k)=\sum_{j=1}^{k}\frac{(-1)^{1+j}\,G(-j+k)}{1+j}\right)$$

$$\frac{1}{10}\log_{1.61848236185320000}\left(\frac{\left(\frac{1}{12}\right)^{24}\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)}{(2\pi)^{24}}+\frac{48\left(\frac{1}{12}\right)^{24}\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)}{(2\pi)^{24}}\right)-34-\phi=$$

$$-34.0000000000000000-1.00000000000000000\,\phi+\log\left(\frac{48\exp\left(-\frac{1}{6}+48\pi^2\right)+\exp\left(-\frac{1}{6}+96\pi^2\right)}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}\right)\left(0.21168609837209152-0.100000000000000000\times\sum_{k=0}^{\infty}0.61848236185320000^k\,G(k)\right)$$

$$\text{for }\left(G(0)=0\text{ and }G(k)=\frac{(-1)^{1+k}\,k}{2(1+k)(2+k)}+\sum_{j=1}^{k}\frac{(-1)^{1+j}\,G(-j+k)}{1+j}\right)$$

and, multiplying by 1/10, subtracting 55, adding 8 (55 and 8 are Fibonacci numbers) and again subtracting the square of the golden ratio value, we obtain:

1/10 log base 1.6184823618532((([(((1/12)^24))/((2Pi)^24) exp((((((8Pi^2)/(1/12))-2*1/12))))+48*(((1/12)^24))/((2Pi)^24) exp((((((4Pi^2)/(1/12))-2*1/12))))]))) -55+8 -golden ratio^2

Input interpretation:

$$\frac{1}{10}\log_{1.6184823618532}\left(\frac{\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\exp\left(\frac{8\pi^2}{\frac{1}{12}}-2\times\frac{1}{12}\right)+48\times\frac{\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\exp\left(\frac{4\pi^2}{\frac{1}{12}}-2\times\frac{1}{12}\right)\right)-55+8-\phi^2$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

125.58196601125...

125.58196601125... result very near to the Higgs boson mass 125.18 GeV

Alternative representation:

$$\frac{1}{10}\log_{1.61848236185320000}\left(\frac{\left(\frac{1}{12}\right)^{24}\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)}{(2\pi)^{24}}+\frac{48\left(\frac{1}{12}\right)^{24}\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)}{(2\pi)^{24}}\right)-55+8-\phi^2=-47-\phi^2+\frac{\log\left(\frac{48\exp\left(-\frac{2}{12}+\frac{4\pi^2}{\frac{1}{12}}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}+\frac{\exp\left(-\frac{2}{12}+\frac{8\pi^2}{\frac{1}{12}}\right)\left(\frac{1}{12}\right)^{24}}{(2\pi)^{24}}\right)}{10\log(1.61848236185320000)}$$

Series representations:

$$\frac{1}{10}\log_{1.61848236185320000}\left(\frac{\left(\frac{1}{12}\right)^{24}\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)}{(2\pi)^{24}}+\frac{48\left(\frac{1}{12}\right)^{24}\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)}{(2\pi)^{24}}\right)-55+8-\phi^2 = -47.0000000000000000-1.00000000000000000\,\phi^2+\log\left(\frac{48\exp\left(-\frac{1}{6}+48\pi^2\right)+\exp\left(-\frac{1}{6}+96\pi^2\right)}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}\right)\left(0.21168609837209152-0.100000000000000000\times\sum_{k=0}^{\infty}0.61848236185320000^k\,G(k)\right)$$

$$\text{for}\left(G(0)=0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)}+G(k)=\sum_{j=1}^{k}\frac{(-1)^{1+j}\,G(-j+k)}{1+j}\right)$$

$$\frac{1}{10}\log_{1.61848236185320000}\left(\frac{\left(\frac{1}{12}\right)^{24}\exp\left(\frac{8\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)}{(2\pi)^{24}}+\frac{48\left(\frac{1}{12}\right)^{24}\exp\left(\frac{4\pi^2}{\frac{1}{12}}-\frac{2}{12}\right)}{(2\pi)^{24}}\right)-55+8-\phi^2 = -47.0000000000000000-1.00000000000000000\,\phi^2+\log\left(\frac{48\exp\left(-\frac{1}{6}+48\pi^2\right)+\exp\left(-\frac{1}{6}+96\pi^2\right)}{1\,333\,735\,776\,850\,284\,124\,449\,081\,472\,843\,776\,\pi^{24}}\right)\left(0.21168609837209152-0.100000000000000000\times\sum_{k=0}^{\infty}0.61848236185320000^k\,G(k)\right)$$

$$\text{for}\left(G(0)=0 \text{ and } G(k)=\frac{(-1)^{1+k} k}{2(1+k)(2+k)}+\sum_{j=1}^{k}\frac{(-1)^{1+j}\,G(-j+k)}{1+j}\right)$$

Now, we have that:

We begin with the partition generating function $P(q) = (q)_\infty^{-1}$, where as usual

$$(q)_0 = 1, \quad (q)_n = \prod_{m=1}^{n}(1-q^m), \quad \text{and} \quad (q)_\infty = \prod_{m=1}^{\infty}(1-q^m), \quad |q|<1.$$

More generally, we put

$$(a;q^k)_0 = 1, \quad (a;q^k)_n = \prod_{m=0}^{n-1}(1-aq^{mk}), \quad \text{and} \quad (a;q^k)_\infty = \prod_{m=0}^{\infty}(1-aq^{mk}),$$

so that $(q)_n = (q;q)_n$ and $(q)_\infty = (q;q)_\infty$. We have

$$(a;q^k)_n = \frac{(a;q^k)_\infty}{(aq^{nk};q^k)_\infty}$$

for $n \geq 0$, and for other real n, we take this as the definition of $(a;q^k)_n$.

$P(q)$ satisfies the Euler and Durfee identities

$$P(q) = \sum_{n=0}^{\infty}\frac{q^n}{(q)_n} = \sum_{n=0}^{\infty}\frac{q^{n^2}}{(q)_n^2}. \tag{1.1}$$

These express $P(q)$ in what S. Ramanujan, in his last letter to Hardy [R1, 354–355], [R2, 127–131], [W1, 56–61], called *transformed Eulerian form*. Other examples are provided by the Rogers-Ramanujan identities

$$\left.\begin{aligned} G(q) &= \prod_{m=1}^{\infty}\frac{1}{(1-q^{5m-4})(1-q^{5m-1})} = \sum_{n=0}^{\infty}\frac{q^{n^2}}{(q)_n}, \\ H(q) &= \prod_{m=1}^{\infty}\frac{1}{(1-q^{5m-3})(1-q^{5m-2})} = \sum_{n=0}^{\infty}\frac{q^{n(n+1)}}{(q)_n}. \end{aligned}\right\} \tag{1.2}$$

In his letter, Ramanujan remarked that as q tends radially to exponential singularities at roots of unity, the functions $P(q)$ and $G(q)$ have asymptotic approximations involving "closed exponential factors." To describe these approximations, he introduced a complex variable α with $\mathrm{Re}(\alpha) > 0$ and put $q = \mathrm{e}^{-\alpha}$. Then, for example, if α is real and $\alpha \to 0^+$, we have

$$\left.\begin{aligned} P(q) &= \sqrt{\frac{\alpha}{2\pi}}\exp\left(\frac{\pi^2}{6\alpha} - \frac{\alpha}{24}\right) + o(1), \\ G(q) &= \sqrt{\frac{2}{5-\sqrt{5}}}\exp\left(\frac{\pi^2}{15\alpha} - \frac{\alpha}{60}\right) + o(1), \end{aligned}\right\} \tag{1.3}$$

As noted by Watson [W1], it is desirable to supplement the transformation laws by rules governing the behavior of Mordell integrals such as

$$W_c(r,\alpha) = \int_0^\infty e^{-\alpha x^2} \frac{\cosh r\alpha x}{\cosh \alpha x}\,dx \quad \text{and} \quad W_s(r,\alpha) = \int_0^\infty e^{-\alpha x^2} \frac{\sinh r\alpha x}{\sinh \alpha x}\,dx$$

under the map $\alpha \mapsto \beta = \pi^2/\alpha$ (and thus $q \mapsto q_1$). These laws are [M7]

$$\left.\begin{aligned} \sqrt{\frac{\alpha^3}{\pi^3}} W_c(r,\alpha) &= 2\cos\left(\frac{\pi r}{2}\right) \int_0^\infty e^{-\beta x^2} \frac{\cosh \beta x}{\cos \pi r + \cosh 2\beta x}\,dx, \\ \sqrt{\frac{\alpha^3}{\pi^3}} W_s(r,\alpha) &= \sin(\pi r) \int_0^\infty \frac{e^{-\beta x^2}}{\cos \pi r + \cosh 2\beta x}\,dx \end{aligned}\right\} \tag{4.4}$$

for $|r| < 1$.

We now outline a proof of (4.2). It is more convenient to work with the functions

$$g_3(q^r, q) = \frac{1}{(q)_\infty} \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{\frac{3}{2}n(n+1)}}{1-q^{n+r}} \tag{4.5}$$

and

$$h_3(e^{2\pi i r}, q) = \frac{4\sin^2 \pi r}{(q)_\infty} \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{\frac{1}{2}n(3n+1)}}{(1-e^{2\pi i r} q^n)(1-e^{-2\pi i r} q^n)}, \tag{4.6}$$

where $r = a/b$. The series here are called *generalized Lambert series*. As in [GM2,

From

$$(q)_\infty = \prod_{m=1}^{\infty} (1-q^m), \quad |q| < 1.$$

for q = 0.9200444146293, we obtain:

Product (1-0.9200444146293^m), m = 1..infinity

Input interpretation:

$$\prod_{m=1}^{\infty} (1 - 0.9200444146293^m)$$

Infinite product:

$$\prod_{m=1}^{\infty}(1-0.9200444146293^m) = 2.331090122972\times10^{-8}$$

2.331090122972e-8 = $(q)_\infty$

Now, for q = 0.9200444146293…. ; r = 0.5; a = 1; b = 2   (where r = a/b)

$$h_3(e^{2\pi ir},q) = \frac{4\sin^2\pi r}{(q)_\infty}\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{\frac{1}{2}n(3n+1)}}{(1-e^{2\pi ir}q^n)(1-e^{-2\pi ir}q^n)}, \tag{4.6}$$

we obtain:

(4*sin^2(Pi*0.5)) / (2.331090122972e-8) * sum ((((-1)^n * 0.9200444146293^(1/2*n(3n+1))))) / ((((1-exp(2Pi*i*0.5)*0.9200444146293^n) (1-exp(-2Pi*i*0.5)*0.9200444146293^n)))), n=-2..2*10^4

Input interpretation:

$$\frac{4\sin^2(\pi\times0.5)}{2.331090122972\times10^{-8}}\sum_{n=-2}^{2\times10^4}\left((-1)^n\times0.9200444146293^{1/2\,n\,(3\,n+1)}\right)/\left((1-\exp(2\,\pi\,i\times0.5)\times0.9200444146293^n)(1-\exp(-2\,\pi\,i\times0.5)\times0.9200444146293^n)\right)$$

i is the imaginary unit

Result:

$8.49326\times10^6+0\,i$

Alternate form:

8.49326×10^6

$8.49326*10^6$

Now, performing the 32th root of the previous expression, we obtain:

(((((4*sin^2(Pi*0.5)) / (2.331090122972e-8) * sum ((((-1)^n * 0.9200444146293^(1/2*n(3n+1))))) / ((((1-exp(2Pi*i*0.5)*0.9200444146293^n) (1-exp(-2Pi*i*0.5)*0.9200444146293^n)))), n=-2..2*10^4))))^1/32

Input interpretation:

$$\left(\frac{4 \sin^2(\pi \times 0.5)}{2.331090122972 \times 10^{-8}} \sum_{n=-2}^{2 \times 10^4} \left((-1)^n \times 0.9200444146293^{1/2\, n(3n+1)}\right) / \left(\left(1 - \exp(2\pi i \times 0.5) \times 0.9200444146293^n\right) \left(1 - \exp(-2\pi i \times 0.5) \times 0.9200444146293^n\right)\right)\right) \text{^} (1/32)$$

i is the imaginary unit

Result:

$1.64639 + 0\, i$

Alternate form:

1.64639

$1.64639 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

We note that from the following formula concerning the number of planar partitions (or plane partitions) of n (see A000219 - OEIS):

a(n) ~ (c_2 / n^(25/36)) * exp(c_1 * n^(2/3)), where c_1 = 2.00945... and c_2 = 0.23151...

a(n) ~ (0.23151 / n^(25/36)) * exp(2.00945 * n^(2/3))   for n = 30.932775 ,

we obtain:

(0.23151 / 30.932775^(25/36)) * exp(2.00945 * 30.932775^(2/3))

Input interpretation:

$$\frac{0.23151}{30.932775^{25/36}} \exp\left(2.00945 \times 30.932775^{2/3}\right)$$

Result:

$8.49326... \times 10^6$

8.49326…*10^6

or, for

3/19 (26 + sqrt(1322)) π ≈ 30.93277501220

we obtain the same result:

(0.23151 / (3/19 (26 + sqrt(1322)) π)^(25/36)) * exp(2.00945 * (3/19 (26 + sqrt(1322)) π)^(2/3))

Input interpretation:

$$\frac{0.23151}{\left(\frac{3}{19}\left(26+\sqrt{1322}\right)\pi\right)^{25/36}} \exp\left(2.00945\left(\frac{3}{19}\left(26+\sqrt{1322}\right)\pi\right)^{2/3}\right)$$

Result:

$8.49326... \times 10^6$

8.49326…*10^6

Series representations:

$$\frac{\exp\left(2.00945\left(\frac{3}{19}\left(26+\sqrt{1322}\right)\pi\right)^{2/3}\right)0.23151}{\left(\frac{3}{19}\left(26+\sqrt{1322}\right)\pi\right)^{25/36}} = \left(0.834178 \exp\left(0.587024\left(\pi\left(26+\sqrt{1321}\sum_{k=0}^{\infty}1321^{-k}\binom{\frac{1}{2}}{k}\right)\right)^{2/3}\right)\left(\pi\left(26+\sqrt{1321}\sum_{k=0}^{\infty}1321^{-k}\binom{\frac{1}{2}}{k}\right)\right)^{11/36}\right) \Big/ \left(\pi\left(26+\sqrt{1321}\sum_{k=0}^{\infty}1321^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$\frac{\exp\left(2.00945\left(\frac{3}{19}\left(26+\sqrt{1322}\right)\pi\right)^{2/3}\right)0.23151}{\left(\frac{3}{19}\left(26+\sqrt{1322}\right)\pi\right)^{25/36}} =$$

$$\left(0.834178\exp\left(0.587024\left(\pi\left(26+\sqrt{1321}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{1321}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)^{2/3}\right)\right.$$
$$\left.\left(\pi\left(26+\sqrt{1321}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{1321}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)^{11/36}\right)/$$
$$\left(\pi\left(26+\sqrt{1321}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{1321}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$\frac{\exp\left(2.00945\left(\frac{3}{19}\left(26+\sqrt{1322}\right)\pi\right)^{2/3}\right)0.23151}{\left(\frac{3}{19}\left(26+\sqrt{1322}\right)\pi\right)^{25/36}} =$$

$$\left(0.834178\exp\left(0.587024\left(\pi\left(26+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1322-z_0)^k z_0^{-k}}{k!}\right)\right)^{2/3}\right)\right.$$
$$\left.\left(\pi\left(26+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1322-z_0)^k z_0^{-k}}{k!}\right)\right)^{11/36}\right)/$$
$$\left(\pi\left(26+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1322-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From

$$g_3(q^r,q)=\frac{1}{(q)_\infty}\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{\frac{3}{2}n(n+1)}}{1-q^{n+r}} \tag{4.5}$$

For q = 0.9200444146293…. ; r = 0.5; a = 1; b = 2 (where r = a/b) and n=-2..2*10^4

we obtain:

1/(2.331090122972e-8) * sum ((((-1)^n * 0.9200444146293^(3/2*n(n+1))))) / ((((1-0.9200444146293^(n+0.5))))), n=-2..2*10^4

Input interpretation:

$$\frac{1}{2.331090122972\times 10^{-8}} \sum_{n=-2}^{2\times 10^4} \frac{(-1)^n \times 0.9200444146293^{3/2\,n\,(n+1)}}{1-0.9200444146293^{n+0.5}}$$

Result:

1.60307×10^9

$1.60307*10^9$

From which, performing the 43th root, where 43 is a prime number:

(((((1/(2.331090122972e-8) * sum ((((-1)^n * 0.9200444146293^(3/2*n(n+1))))) / ((((1-0.9200444146293^(n+0.5))))), n=-2..2*10^4)))))^1/43

Input interpretation:

$$\sqrt[43]{\frac{1}{2.331090122972\times 10^{-8}} \sum_{n=-2}^{2\times 10^4} \frac{(-1)^n \times 0.9200444146293^{3/2\,n\,(n+1)}}{1-0.9200444146293^{n+0.5}}}$$

Result:

1.63708

$1.63708 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

From the ratio between the two results, performing the 11th root, where 11 is an Eisenstein prime number and an Ulam number (note that 11 is also the dimensions number of M-Theory), we obtain:

(1.60307×10^9 / 8.49326 × 10^6)^1/11

Input interpretation:

$$\sqrt[11]{\frac{1.60307\times 10^9}{8.49326\times 10^6}}$$

Result:

1.610267420233879788333178126395966190182800117617438411127...

1.6102674202…. result that is a good approximation to the value of the golden ratio 1.618033988749...

Now

$$\sum_{n=0}^{\infty} \frac{(-1)^n q^{n(n+1)}}{(-q;q)_{2n}} = \sum_{n=0}^{\infty} q^{\frac{1}{2}n(5n+1)}(1-q^{4n+2}) = \left(\sum_{n\geq 0} - \sum_{n<0}\right) q^{\frac{1}{2}n(5n+1)},$$

for q = 0.9200444146293…., from

$$\sum_{n=0}^{\infty} q^{\frac{1}{2}n(5n+1)}(1-q^{4n+2})$$

we obtain:

Sum(((0.9200444146293^(1/2*n(5n+1)) * (1-0.9200444146293^(4n+2)))), n = 0..infinity

Infinite sum:

$$\sum_{n=0}^{\infty} 0.92004441462930000^{1/2\,n\,(5\,n+1)}\left(1-0.92004441462930000^{4\,n+2}\right) = 0.806819$$

0.806819

From which, multiplying by 2, we obtain:

2(((((sum(((0.9200444146293^(1/2*n(5n+1)) * (1-0.9200444146293^(4n+2))))), n = 0..infinity)))))

Input interpretation:

$$2\sum_{n=0}^{\infty} 0.9200444146293^{1/2\,n\,(5\,n+1)}\left(1-0.9200444146293^{4\,n+2}\right)$$

Result:

1.61364

1.61364 result that is a good approximation to the value of the golden ratio 1.618033988749...

Dividing 52 = 26*2 by the entire expression, and adding 1 - √2, we obtain:

(26*2)/(((((sum(((0.9200444146293^(1/2*n(5n+1)) * (1-0.9200444146293^(4n+2))))), n = 0..infinity)))))+1-sqrt2

(note that 26 is the dimensions number of Bosonic String Theory)

Input interpretation:

$$\frac{26\times 2}{\sum_{n=0}^{\infty} 0.9200444146293^{1/2\,n\,(5\,n+1)}\left(1-0.9200444146293^{4\,n+2}\right)}+1-\sqrt{2}$$

Result:

64.0365

64.0365 ≈ 64

Multiplying by 27 the previous expression, we obtain:

27*(((((26*2)/(((((sum(((0.9200444146293^(1/2*n(5n+1)) * (1-0.9200444146293^(4n+2))))), n = 0..infinity)))))+1-sqrt2))))

Input interpretation:

$$27\left(\frac{26\times 2}{\sum_{n=0}^{\infty} 0.9200444146293^{1/2\,n\,(5\,n+1)}\left(1-0.9200444146293^{4\,n+2}\right)}+1-\sqrt{2}\right)$$

Result:

1728.98

1728.98 ≈ 1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

and adding 7, that is a Lucas number and the number of extra dimensions in M-Theory, dividing 96 = 48*2 by the entire expression, and adding 1 - √2, we obtain:

7+(((((48*2)/(((((sum(((0.9200444146293^(1/2*n(5n+1)) * (1-0.9200444146293^(4n+2))))), n = 0..infinity)))))+1-sqrt2))))

Input interpretation:

$$7+\left(\frac{48\times 2}{\sum_{n=0}^{\infty} 0.9200444146293^{1/2\, n\,(5\,n+1)}\left(1-0.9200444146293^{4\,n+2}\right)}+1-\sqrt{2}\right)$$

Result:

125.572

125.572 result very near to the Higgs boson mass 125.18 GeV

and again, instead of 7, we put 21, that is a Fibonacci number, we obtain:

21+(((((48*2)/(((((sum(((0.9200444146293^(1/2*n(5n+1)) * (1-0.9200444146293^(4n+2))))), n = 0..infinity)))))+1-sqrt2))))

Input interpretation:

$$21+\left(\frac{48\times 2}{\sum_{n=0}^{\infty} 0.9200444146293^{1/2\, n\,(5\,n+1)}\left(1-0.9200444146293^{4\,n+2}\right)}+1-\sqrt{2}\right)$$

Result:

139.572

139.572

Now, we have that:

Throughout the rest of this section we will use the modern notation $q = e^{2\pi i\tau}$. For $l > 0$ the Appell function of level l (not to be confused with the level of a modular form) is defined by

$$A_l(u,v;\tau) = a^{\frac{l}{2}} \sum_{n=-\infty}^{\infty} \frac{(-1)^{ln} q^{\frac{1}{2}ln(n+1)} b^n}{1 - aq^n},$$

where $a = e^{2\pi iu}$, $b = e^{2\pi iv}$. These functions are related to Lerch sums [L1], [L2]. Zagier and Zwegers (see [Zw3]) showed that

$$A_l(u,v;\tau) = \sum_{m=0}^{l-1} a^m A_1(lu, v + m\tau + (l-1)/2; l\tau).$$

From

$$A_l(u,v;\tau) = a^{\frac{l}{2}} \sum_{n=-\infty}^{\infty} \frac{(-1)^{ln} q^{\frac{1}{2}ln(n+1)} b^n}{1 - aq^n},$$

for: q = exp(2Pi*1); a = exp(2Pi*0.5); b = exp(2Pi*1.5); 1 = 2 we obtain:

exp(2Pi*0.5) * sum(((((-1)^(2n)*(exp(2Pi))^(n(n+1))*exp(2Pi*1.5)^n))))/(((1-(exp(2Pi*0.5))*(exp(2Pi))^n))), n = -2..2

Input interpretation:

$$\exp(2\pi \times 0.5) \sum_{n=-2}^{2} \frac{(-1)^{2n} \exp^{n(n+1)}(2\pi) \exp^n(2\pi \times 1.5)}{1 - \exp(2\pi \times 0.5) \exp^n(2\pi)}$$

Result:

-1.26261×10^{19}

$-1.26261*10^{19}$

From the Ramanujan's formula for partitions p(n), that is:

An asymptotic expression for $p(n)$ is given by

$$p(n) \sim \frac{1}{4n\sqrt{3}} \exp\left(\pi\sqrt{\frac{2n}{3}}\right) \text{ as } n \to \infty.$$

for n = 410, we obtain:

1/(4*410*sqrt3) * exp(Pi*sqrt((2*410)/3))

Input:

$$\frac{1}{4\times 410\sqrt{3}} \exp\left(\pi\sqrt{\frac{2\times 410}{3}}\right)$$

Exact result:

$$\frac{e^{2\sqrt{205/3}\,\pi}}{1640\sqrt{3}}$$

Decimal approximation:

1.2692612667955803382188732093814590185614963226436226... × 10^{19}

1.26926….*10^{19} result that is very near to the previous solution -1.26261*10^{19} with positive sign

Property:

$$\frac{e^{2\sqrt{205/3}\,\pi}}{1640\sqrt{3}} \text{ is a transcendental number}$$

Series representations:

$$\frac{\exp\left(\pi\sqrt{\frac{2\times 410}{3}}\right)}{4\times 410\sqrt{3}} = \frac{\exp\left(\pi\sqrt{\frac{817}{3}}\sum_{k=0}^{\infty}\left(\frac{817}{3}\right)^{-k}\binom{\frac{1}{2}}{k}\right)}{1640\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(\pi\sqrt{\frac{2\times 410}{3}}\right)}{4\times 410\sqrt{3}} = \frac{\exp\left(\pi\sqrt{\frac{817}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{817}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}{1640\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(\pi\sqrt{\frac{2\times 410}{3}}\right)}{4\times 410\sqrt{3}} = \frac{\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{820}{3}-z_0\right)^k z_0^{-k}}{k!}\right)}{1640\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From the ratio between the two results and performing the 6th root, we obtain:

(-(-12626100000000000000 /12692612667955803382.188732))^1/6

Input interpretation:

$$\sqrt[6]{-\left(-\frac{12\,626\,100\,000\,000\,000\,000}{1.2692612667955803382188732\times 10^{19}}\right)}$$

Result:

0.999124709214663866607676080...

0.9991247092…. result very near to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

= 0.9991104684

Now, we have:

$$\mu(u,v;\tau) = \frac{1}{\vartheta(v;\tau)}A_1(u,v;\tau),$$

where

$$\vartheta(v;\tau) = i\sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{1}{2}(n+\frac{1}{2})^2} b^{n+\frac{1}{2}} = -ib^{-\frac{1}{2}}q^{\frac{1}{8}}j(b,q).$$

From the following expression

$$i \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{1}{2}(n+\frac{1}{2})^2} b^{n+\frac{1}{2}}$$

we obtain:

i * sum(((((-1)^(n)*(exp(2Pi))^(1/2(n+1/2)^2)*exp(2Pi*1.5)^(n+1/2))))), n = -2..2

Input interpretation:

$$i \sum_{n=-2}^{2} (-1)^n \exp^{\frac{1}{2}\left(n+\frac{1}{2}\right)^2}(2\pi) \exp^{n+\frac{1}{2}}(2\pi \times 1.5)$$

i is the imaginary unit

Result:

$5.75672 \times 10^{18} i$

$5.75672*10^{18}\ i$

Thence, from

$$\mu(u,v;\tau) = \frac{1}{\vartheta(v;\tau)} A_1(u,v;\tau)$$

we have:

-1.26261*10^19 / ((((i * sum(((((-1)^(n)*(exp(2Pi))^(1/2(n+1/2)^2)*exp(2Pi*1.5)^(n+1/2))))), n = -2..2))))

Input interpretation:

$$-\frac{1.26261 \times 10^{19}}{i \sum_{n=-2}^{2} (-1)^n \exp^{\frac{1}{2}\left(n+\frac{1}{2}\right)^2}(2\pi) \exp^{n+\frac{1}{2}}(2\pi \times 1.5)}$$

i is the imaginary unit

Result:

$2.19328 i$

$2.19328\ i$

We know that in 1914, Godfrey Harold Hardy proved that $\zeta(1/2 + it)$ has infinitely many real zeros. If we multiply, considering t = 1, this formula by the above expression, we obtain:

zeta(1/2+i) * [-1.26261*10^19 / ((((i * sum(((((-1)^(n)*(exp(2Pi))^(1/2(n+1/2)^2)*exp(2Pi*1.5)^(n+1/2))))), n = -2..2))))]

Input interpretation:

$$\zeta\left(\frac{1}{2}+i\right)\left(-\frac{1.26261\times 10^{19}}{i\sum_{n=-2}^{2}(-1)^n \exp^{\frac{1}{2}\left(n+\frac{1}{2}\right)^2}(2\pi)\exp^{n+\frac{1}{2}}(2\pi\times 1.5)}\right)$$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

Result:

$1.58377 + 0.315693\,i$

That is:

$1.58377 + 0.315693\,i$

i is the imaginary unit

Result:

$1.58377\ldots +$
$0.315693\ldots\, i$

Polar coordinates:

$r = 1.61493$ (radius), $\theta = 11.273°$ (angle)

1.61493 result that is a good approximation to the value of the golden ratio 1.618033988749...

Possible closed forms:

$$\frac{1}{2}\sqrt{\frac{3}{2}}\,(e\log(2))^{3/2}-\frac{1}{2}\,i\,e^{2-1/e+e+1/\pi-\pi}\,\pi^{1-e}\,\sin(2\,e\,\pi)\approx 1.5837825305+0.3156922395\,i$$

$$80\,\mathcal{W}_{\text{Wy}}+1-\frac{1}{2}\,i\,e^{2-1/e+e+1/\pi-\pi}\,\pi^{1-e}\,\sin(2\,e\,\pi)\approx 1.5837878504+0.3156922395\,i$$

$$\sqrt{\frac{35}{3}}\,L_2^{3/2}-\frac{1}{2}\,i\,e^{2-1/e+e+1/\pi-\pi}\,\pi^{1-e}\,\sin(2\,e\,\pi)\approx 1.5837618596+0.3156922395\,i$$

Considering the same expression in the quantum form, i.e. multiplying it by $1/10^{35}$, we obtain:

1/10^35*zeta(1/2+i) * [-1.26261*10^19 / ((((i * sum(((((-1)^(n)*(exp(2Pi))^(1/2(n+1/2)^2)*exp(2Pi*1.5)^(n+1/2))))), n = -2..2))))]

Input interpretation:

$$\frac{1}{10^{35}}\,\zeta\left(\frac{1}{2}+i\right)\left(-\frac{1.26261\times 10^{19}}{i\sum_{n=-2}^{2}(-1)^n\exp^{\frac{1}{2}\left(n+\frac{1}{2}\right)^2}(2\,\pi)\exp^{n+\frac{1}{2}}(2\,\pi\times 1.5)}\right)$$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

Result:

$1.58377\times 10^{-35}+3.15693\times 10^{-36}\,i$

that is:

$1.58377\times 10^{-35}+3.15693\times 10^{-36}\,i$

i is the imaginary unit

Result:

$1.58377\ldots\times 10^{-35}+$
$3.15693\ldots\times 10^{-36}\,i$

Polar coordinates:

$r = 1.61493 \times 10^{-35}$ (radius), $\theta = 11.273°$ (angle)

$1.61493*10^{-35}$ result that is very near to the value of Planck length $1.61623*10^{-35}$

Now, we have that:

$$0 < |q| < 1$$

$$(aq)_\infty \sum_{\gamma\in\mathbb{N}^n} \left((-1)^{n+1}b\right)^{|\gamma|} (x_1\cdots x_n)^{-|\gamma|} x_1^{n\gamma_1}\cdots x_n^{n\gamma_n} q^{\frac{1}{2}|\gamma|(|\gamma|+1)+\frac{n}{2}\sum_{i=1}^n \gamma_i(\gamma_i-1)}$$
$$\times \frac{\Delta(xq^\gamma)}{\Delta(x)} \left[(aq)_{|\gamma|}\right]^{-1} \prod_{1\le i,j\le n} \left[(qx_i/x_j)_{\gamma_i}\right]^{-1} = \sum_{k\in\mathbb{N}} (-a)^k q^{\frac{1}{2}k(k-1)} \frac{(b/a)_k}{(q)_k}. \quad (5.5)$$

In the case when $a = 1$, $b = q$, (5.5) reduces to

$$(q)_\infty \sum_{\gamma\subset\mathbb{N}^n} (-1)^{(n+1)|\gamma|} (x_1\cdots x_n)^{\ \ |\gamma|} x_1^{n\gamma_1}\cdots x_n^{n\gamma_n} q^{\frac{1}{2}|\gamma|(|\gamma|-3)+\frac{n}{2}\sum_{i=1}^n \gamma_i(\gamma_i-1)}$$
$$\times \frac{\Delta(xq^\gamma)}{\Delta(x)} \left[(q)_{|\gamma|}\right]^{-1} \prod_{1\le i,j\le n} \left[(qx_i/x_j)_{\gamma_i}\right]^{-1} = \sum_{k\in\mathbb{N}} (-1)^k q^{\frac{1}{2}k(k-1)}. \quad (5.8)$$

Note that, in the case when $n = 1$ and $x_1 = 1$, (5.8) reduces to the following q-series transformation formula:

$$(q)_\infty \sum_{k\subset\mathbb{N}} \frac{q^{k^2+k}}{(q)_k^2} = \sum_{k\subset\mathbb{N}} (-1)^k q^{\frac{1}{2}k(k+1)}, \quad (5.9)$$

For k = 24, that are the "modes" corresponding to the physical vibrations of a bosonic string, we calculate the (5.8) as follows:

(-1)^24 * 0.5^(1/2*24(24-1))

Input:

$(-1)^{24} \times 0.5^{1/2\times 24\,(24-1)}$

Result:

$8.2360921431488462690223234775547023497734073299753965\ldots \times 10^{-84}$

$8.236092143\ldots*10^{-84}$

while, from (5.9), we obtain:

(-1)^24 * 0.5^(1/2*24(24+1))

Input:

$(-1)^{24} \times 0.5^{1/2 \times 24\,(24+1)}$

Result:

$4.9090934652977265530957719549862756429752155124994495\ldots \times 10^{-91}$

$4.90909346\ldots*10^{-91}$

From the ratio between the two expression, we obtain:

((((-1)^24 * 0.5^(1/2*24(24-1))))) / ((((-1)^24 * 0.5^(1/2*24(24+1)))))

Input:

$$\frac{(-1)^{24} \times 0.5^{1/2 \times 24\,(24-1)}}{(-1)^{24} \times 0.5^{1/2 \times 24\,(24+1)}}$$

Result:

16 777 216

16777216

Performing the square root of the above expression, we obtain:

sqrt[((((-1)^24 * 0.5^(1/2*24(24-1))))) / ((((-1)^24 * 0.5^(1/2*24(24+1)))))]

Input:

$$\sqrt{\frac{(-1)^{24} \times 0.5^{1/2 \times 24\,(24-1)}}{(-1)^{24} \times 0.5^{1/2 \times 24\,(24+1)}}}$$

Result:

4096

4096

and performing the 4th root:

[((((-1)^24 * 0.5^(1/2*24(24-1))))) / ((((-1)^24 * 0.5^(1/2*24(24+1)))))]^1/4

Input:

$$\sqrt[4]{\frac{(-1)^{24} \times 0.5^{1/2 \times 24 (24-1)}}{(-1)^{24} \times 0.5^{1/2 \times 24 (24+1)}}}$$

Result:

64

64

and multiplying this last expression by 27, we obtain:

27*[((((-1)^24 * 0.5^(1/2*24(24-1))))) / ((((-1)^24 * 0.5^(1/2*24(24+1)))))]^1/4 + 1

Input:

$$27 \sqrt[4]{\frac{(-1)^{24} \times 0.5^{1/2 \times 24 (24-1)}}{(-1)^{24} \times 0.5^{1/2 \times 24 (24+1)}}} + 1$$

Result:

1729

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Furthermore, multiplying by 2, subtracting 3 and adding 1/golden ratio to the previous expression, we have:

2*[((((-1)^24 * 0.5^(1/2*24(24-1))))) / ((((-1)^24 * 0.5^(1/2*24(24+1)))))]^1/4 - 3+1/golden ratio

Input:

$$2\sqrt[4]{\frac{(-1)^{24}\times 0.5^{1/2\times 24\,(24-1)}}{(-1)^{24}\times 0.5^{1/2\times 24\,(24+1)}}}-3+\frac{1}{\phi}$$

ϕ is the golden ratio

Result:

125.618...

125.618... result very near to the Higgs boson mass 125.18 GeV

Alternative representations:

$$2\sqrt[4]{\frac{(-1)^{24}\,0.5^{(24\,(24-1))/2}}{(-1)^{24}\,0.5^{(24\,(24+1))/2}}}-3+\frac{1}{\phi}=-3+2\sqrt[4]{\frac{(-1)^{24}\,0.5^{276}}{(-1)^{24}\,0.5^{300}}}+\frac{1}{2\sin(54^\circ)}$$

$$2\sqrt[4]{\frac{(-1)^{24}\,0.5^{(24\,(24-1))/2}}{(-1)^{24}\,0.5^{(24\,(24+1))/2}}}-3+\frac{1}{\phi}=-3+-\frac{1}{2\cos(216^\circ)}+2\sqrt[4]{\frac{(-1)^{24}\,0.5^{276}}{(-1)^{24}\,0.5^{300}}}$$

$$2\sqrt[4]{\frac{(-1)^{24}\,0.5^{(24\,(24-1))/2}}{(-1)^{24}\,0.5^{(24\,(24+1))/2}}}-3+\frac{1}{\phi}=-3+2\sqrt[4]{\frac{(-1)^{24}\,0.5^{276}}{(-1)^{24}\,0.5^{300}}}+-\frac{1}{2\sin(666^\circ)}$$

Multiplying by 2 and adding 11, that is a Lucas prime number, and 1/golden ratio to the previous expression, we have:

2*[((((-1)^24 * 0.5^(1/2*24(24-1))))) / ((((-1)^24 * 0.5^(1/2*24(24+1)))))]^1/4 +11+1/golden ratio

Input:

$$2\sqrt[4]{\frac{(-1)^{24}\times 0.5^{1/2\times 24\,(24-1)}}{(-1)^{24}\times 0.5^{1/2\times 24\,(24+1)}}}+11+\frac{1}{\phi}$$

ϕ is the golden ratio

Result:

139.618...

139.618… result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representations:

$$2\sqrt[4]{\frac{(-1)^{24}\,0.5^{(24\,(24-1))/2}}{(-1)^{24}\,0.5^{(24\,(24+1))/2}}}+11+\frac{1}{\phi}=11+2\sqrt[4]{\frac{(-1)^{24}\,0.5^{276}}{(-1)^{24}\,0.5^{300}}}+\frac{1}{2\sin(54^\circ)}$$

$$2\sqrt[4]{\frac{(-1)^{24}\,0.5^{(24\,(24-1))/2}}{(-1)^{24}\,0.5^{(24\,(24+1))/2}}}+11+\frac{1}{\phi}=11+-\frac{1}{2\cos(216^\circ)}+2\sqrt[4]{\frac{(-1)^{24}\,0.5^{276}}{(-1)^{24}\,0.5^{300}}}$$

$$2\sqrt[4]{\frac{(-1)^{24}\,0.5^{(24\,(24-1))/2}}{(-1)^{24}\,0.5^{(24\,(24+1))/2}}}+11+\frac{1}{\phi}=11+2\sqrt[4]{\frac{(-1)^{24}\,0.5^{276}}{(-1)^{24}\,0.5^{300}}}+-\frac{1}{2\sin(666^\circ)}$$

and again, performing the 15th root of the previous expression

$$27\sqrt[4]{\frac{(-1)^{24}\times 0.5^{1/2\times 24\,(24-1)}}{(-1)^{24}\times 0.5^{1/2\times 24\,(24+1)}}}+1$$

we obtain:

(((((27*[((((-1)^24 * 0.5^(1/2*24(24-1))))) / ((((-1)^24 * 0.5^(1/2*24(24+1)))))]^1/4 + 1)))))^1/15

Input:

$$\sqrt[15]{27\sqrt[4]{\frac{(-1)^{24}\times 0.5^{1/2\times 24\,(24-1)}}{(-1)^{24}\times 0.5^{1/2\times 24\,(24+1)}}}+1}$$

Result:

1.643815228748728130580088031324769514329283143699940172645...

1.6438152287….. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

From:

Stringhe, Brane e (Super)Gravità - *Augusto Sagnotti* - Scuola Normale Superiore e INFN, Piazza dei Cavalieri 7, 56126 Pisa - Ithaca: Viaggio nella Scienza XII, 2018 • Stringhe, Brane e (Super)Gravità

We have the Virasoro-Shapiro Amplitude:

$$\mathcal{A}_{SV} = \frac{\Gamma\left(-1-\frac{\alpha' s}{4}\right)\Gamma\left(-1-\frac{\alpha' t}{4}\right)\Gamma\left(-1-\frac{\alpha' u}{4}\right)}{\Gamma\left(2+\frac{\alpha' s}{4}\right)\Gamma\left(2+\frac{\alpha' t}{4}\right)\Gamma\left(2+\frac{\alpha' u}{4}\right)}$$

$$= \frac{1}{\pi}\int d^2z\ |z|^{-4-\frac{\alpha' s}{2}}\ |1-z|^{-4-\frac{\alpha' t}{2}}\ , \quad (22)$$

or:

$$\frac{1}{\pi}\int d^2z\ |z|^{-4-\frac{\alpha' s}{2}}\ |1-z|^{-4-\frac{\alpha' t}{2}}$$

$$= \frac{\Gamma\left(-1-\frac{\alpha' s}{4}\right)\Gamma\left(-1-\frac{\alpha' t}{4}\right)\Gamma\left(-1-\frac{\alpha' u}{4}\right)}{\Gamma\left(2+\frac{\alpha' s}{4}\right)\Gamma\left(2+\frac{\alpha' t}{4}\right)\Gamma\left(2+\frac{\alpha' u}{4}\right)}$$

We observe that this fundamental equation of the String Theory can be related with the following Ramanujan definite integral (From: **Some definite integrals** – *Srinivasa Ramanujan* - Messenger of Mathematics, XLIV, 1915, 10 – 18) :

$$\int_0^\infty \left(\frac{1+x^2/b^2}{1+x^2/a^2}\right)\left(\frac{1+x^2/(b+1)^2}{1+x^2/(a+1)^2}\right)\left(\frac{1+x^2/(b+2)^2}{1+x^2/(a+2)^2}\right)\cdots\, dx$$
$$= \tfrac{1}{2}\sqrt{\pi}\frac{\Gamma(a+\frac{1}{2})\Gamma(b)\Gamma(b-a-\frac{1}{2})}{\Gamma(a)\Gamma(b-\frac{1}{2})\Gamma(b-a)}, \qquad (3)$$

From which Ramanujan obtain:

$$\int_0^\infty \frac{dx}{(x^2+11^2)(x^2+21^2)(x^2+31^2)(x^2+41^2)(x^2+51^2)}$$
$$= \frac{5\pi}{12\cdot 13\cdot 16\cdot 17\cdot 18\cdot 22\cdot 23\cdot 24\cdot 31\cdot 32\cdot 41}.$$

Thence, we have that:

(5Pi)/(12*13*16*17*18*22*23*24*31*32*41)

Input:

$$\frac{5\pi}{12\times 13\times 16\times 17\times 18\times 22\times 23\times 24\times 31\times 32\times 41}$$

Result:

$$\frac{5\pi}{377244828499968}$$

Decimal approximation:

$4.1638644406096341301956938554345688115959384341554306\ldots \times 10^{-14}$

4.16386444…*10^{-14}

We have previously calculated that

From

$$A_l(u,v;\tau) = a^{\frac{l}{2}} \sum_{n=-\infty}^{\infty} \frac{(-1)^{ln} q^{\frac{1}{2}ln(n+1)} b^n}{1 - aq^n},$$

for: q = exp(2Pi*1); a = exp(2Pi*0.5); b = exp(2Pi*1.5); l = 2 we obtain:

Input interpretation:

$$\exp(2\pi \times 0.5) \sum_{n=-2}^{2} \frac{(-1)^{2n} \exp^{n(n+1)}(2\pi) \exp^n(2\pi \times 1.5)}{1 - \exp(2\pi \times 0.5) \exp^n(2\pi)}$$

Result:

-1.26261×10^{19}

$-1.26261*10^{19}$

From which, inverting the formula and inserting to the numerator -1729 * 728 and to the denominator 1.8928 + 0.5, we obtain:

(-1729*728)/[(1.8928+0.5)((((exp(2Pi*0.5) * sum(((((-1)^(2n)*(exp(2Pi))^(n(n+1))*exp(2Pi*1.5)^n))))/(((1-(exp(2Pi*0.5))*(exp(2Pi))^n))), n = -2..2))))]

where 1729 and 728 are two Ramanujan taxicab numbers, while 1.8928 is a Hausdorff dimension

Input interpretation:

$$\frac{-1729 \times 728}{(1.8928 + 0.5)\left(\exp(2\pi \times 0.5) \sum_{n=-2}^{2} \frac{(-1)^{2n} \exp^{n(n+1)}(2\pi) \exp^n(2\pi \times 1.5)}{1 - \exp(2\pi \times 0.5) \exp^n(2\pi)}\right)}$$

Result:

4.1663×10^{-14}

$4.1663*10^{-14}$ result practically equal to the previous solution $4.16386444…*10^{-14}$, that is related to the Virasoro-Shapiro Amplitude

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} \quad - \quad 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24}+g_{22}^{-24})=e^{\pi\sqrt{22}}-24+4372e^{-\pi\sqrt{22}}+\cdots=64\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of *the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934\ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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