

# Further mathematical connections between some Number Theory formulas, $\phi$ , $\zeta(2)$ and various topics and parameters of D-branes and Particle Physics. VII

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## Abstract

*In this paper we describe and analyze some Number Theory expressions. Furthermore, we have obtained several mathematical connections with  $\phi$ ,  $\zeta(2)$  and various topics and parameters of D-branes and Particle Physics.*

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“Nowadays, there are only three  
really great English mathematicians:  
Hardy, Littlewood  
and Hardy-Littlewood”

*Reported by Harold Bohr, 1947*



<https://www.flickr.com/photos/greshamcollege/26156541272>

**We want to highlight that the development of the various equations was carried out according to our possible logical and original interpretation**

**For more information on the data entered for the development of the various equations, see the "Observations" section.**

From:

### **D-brane Disformal Coupling and Thermal Dark Matter**

*Bhaskar Dutta, Esteban Jimenez, Ivonne Zavala* - arXiv:1708.07153v2 [hep-ph] 3

Nov 2017

Now, we have:

$$L = 2 + 2\varphi'^2 - \frac{7}{3}\varphi'^4 + \frac{2}{27}\varphi'^6 - 3\varphi'^4 R, \quad (3.10)$$

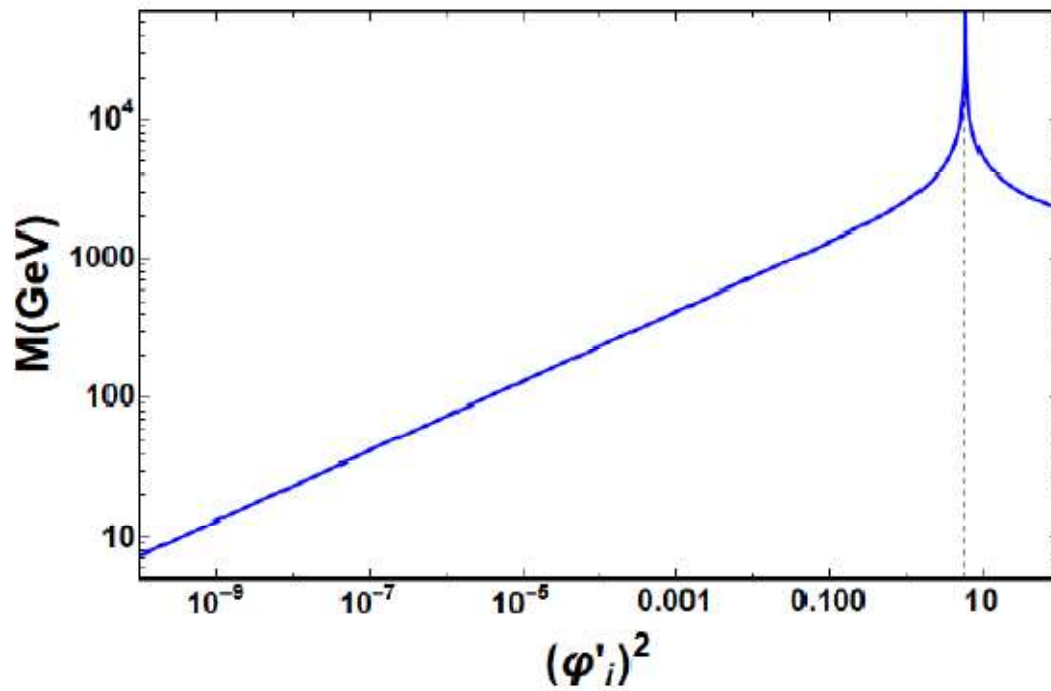
$$\mathbb{L} = 4\ell^3 - L^2 = -\frac{\varphi'^4}{9}(1+R) [81\varphi'^4 R - (3+4\varphi'^2)(\varphi'^2-6)^2], \quad (3.11)$$

$$\ell = \left(1 + \frac{\varphi'^2}{3}\right)^2, \quad (3.12)$$

$$R = \frac{\tilde{\rho}}{M^4} \left(\frac{\tilde{\rho}}{M^4} + 2\right). \quad (3.13)$$

$$R \leq \frac{(3+4\varphi_i'^2)(\varphi_i'^2-6)^2}{81\varphi_i'^4}. \quad (3.14)$$

$$\left[ \left( \frac{3\pi g_{eff}(\tilde{T}_i) \varphi_i'^2}{-90\varphi_i'^2 + 20\sqrt{(3+\varphi_i'^2)^3}} \right)^{1/4} \tilde{T}_i, +\infty \right]. \quad (3.15)$$



From

$$R \leq \frac{(3 + 4\varphi_i'^2)(\varphi_i'^2 - 6)^2}{81\varphi_i'^4}.$$

we obtain, for  $\varphi_i'^2 = 8$ :

$$\frac{((3+4*8)(8-6)^2)}{(81*8^2)}$$

**Input:**

$$\frac{(3 + 4 \times 8)(8 - 6)^2}{81 \times 8^2}$$

**Exact result:**

$$\frac{35}{1296}$$

**Decimal approximation:**

0.027006172839506172839506172839506172839506172839...

**R = 0.0270061728395....**

From which:

$$2\left(\frac{1}{((3+4 \times 8)(8-6)^2) \times \frac{1}{81 \times 8^2}} - 5\right)$$

**Input:**

$$2\left(\frac{1}{((3+4 \times 8)(8-6)^2) \times \frac{1}{81 \times 8^2}} - 5\right)$$

**Exact result:**

$$\frac{2242}{35}$$

**Decimal approximation:**

64.05714285714285714285714285714285714285714285714285...

64.057142857...  $\approx$  64

and:

$$27 \times 2\left(\frac{1}{((3+4 \times 8)(8-6)^2) \times \frac{1}{81 \times 8^2}} - 5\right)$$

**Input:**

$$27 \times 2\left(\frac{1}{((3+4 \times 8)(8-6)^2) \times \frac{1}{81 \times 8^2}} - 5\right)$$

**Exact result:**

$$\frac{60534}{35}$$

**Decimal approximation:**

1729.542857142857142857142857142857142857142857142857142857...

**Repeating decimal:**

1729.5428571 (period 6)

1729.5428571

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*“The fundamental group of the complex form, compact real form, or any algebraic version of  $E_6$  is the cyclic group  $\mathbf{Z}/3\mathbf{Z}$ , and its outer automorphism group is the cyclic group  $\mathbf{Z}/2\mathbf{Z}$ . Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics,  $E_6$  plays a role in some grand unified theories”.*

$$[27*2((1/((((3+4*8)(8-6)^2))1/(81*8^2)))-5)]^{1/15}$$

**Input:**

$$\sqrt[15]{27 \times 2 \left( \frac{1}{((3 + 4 \times 8)(8 - 6)^2) \times \frac{1}{81 \times 8^2}} - 5 \right)}$$

**Result:**

$$\sqrt[5]{3} \sqrt[15]{\frac{2242}{35}}$$

**Decimal approximation:**

1.643849631143818087794971508682869397989364722745499772781...

$$1.6438496311\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

$$[(((\pi)/((((3+4*8)(8-6)^2))1/(81*8^2)))]+23$$

Where 23 is an Eisenstein prime number

**Input:**

$$\frac{\pi}{((3 + 4 \times 8)(8 - 6)^2) \times \frac{1}{81 \times 8^2}} + 23$$

**Result:**

$$23 + \frac{1296 \pi}{35}$$

### Decimal approximation:

139.3286879729249153442167378494353067977009011882896326852...

139.32868797... result practically equal to the rest mass of Pion meson 139.57 MeV

### Property:

$23 + \frac{1296\pi}{35}$  is a transcendental number

### Alternate form:

$$\frac{1}{35} (1296\pi + 805)$$

### Alternative representations:

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 23 = 23 + \frac{180^\circ}{\frac{140}{81 \times 8^2}}$$

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 23 = 23 - \frac{i \log(-1)}{\frac{140}{81 \times 8^2}}$$

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 23 = 23 + \frac{\cos^{-1}(-1)}{\frac{140}{81 \times 8^2}}$$

### Series representations:

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 23 = 23 + \frac{5184}{35} \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 23 = 23 + \sum_{k=0}^{\infty} -\frac{5184 (-1)^k 5^{-2(1+k)} \times 239^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{7(1+2k)}$$

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 23 = 23 + \frac{1296}{35} \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

### Integral representations:

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 23 = 23 + \frac{5184}{35} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 23 = 23 + \frac{2592}{35} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 23 = 23 + \frac{2592}{35} \int_0^\infty \frac{1}{1+t^2} dt$$

$$[\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}}] + 11 - 2$$

Where 11 and 2 are two Eisenstein prime numbers

**Input:**

$$\frac{\pi}{(3+4 \times 8)(8-6)^2 \times \frac{1}{81 \times 8^2}} + 11 - 2$$

**Result:**

$$9 + \frac{1296 \pi}{35}$$

**Decimal approximation:**

125.3286879729249153442167378494353067977009011882896326852...

125.3286879... result very near to the Higgs boson mass 125.18 GeV

**Property:**

$9 + \frac{1296 \pi}{35}$  is a transcendental number

**Alternate form:**

$$\frac{9}{35} (35 + 144 \pi)$$

**Alternative representations:**

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 11 - 2 = 9 + \frac{180^\circ}{\frac{140}{81 \times 8^2}}$$

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 11 - 2 = 9 - \frac{i \log(-1)}{\frac{140}{81 \times 8^2}}$$



$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 11 - 2 = 9 + \frac{\cos^{-1}(-1)}{\frac{140}{81 \times 8^2}}$$

**Series representations:**

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 11 - 2 = 9 + \frac{5184}{35} \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 11 - 2 = 9 + \sum_{k=0}^{\infty} -\frac{5184(-1)^k 5^{-2(1+k)} \times 239^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{7(1+2k)}$$

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 11 - 2 = 9 + \frac{1296}{35} \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

**Integral representations:**

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 11 - 2 = 9 + \frac{5184}{35} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 11 - 2 = 9 + \frac{2592}{35} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{\pi}{\frac{(3+4 \times 8)(8-6)^2}{81 \times 8^2}} + 11 - 2 = 9 + \frac{2592}{35} \int_0^{\infty} \frac{1}{1+t^2} dt$$

From

$$\left[ \left( \frac{3 \pi g_{eff}(\tilde{T}_i) \varphi_i'^2}{-90 \varphi_i'^2 + 20 \sqrt{(3 + \varphi_i'^2)^3}} \right)^{1/4} \tilde{T}_i, +\infty \right]$$

we obtain, for  $g_{eff} = 1$  and  $T_i = 1$ :

$$[\frac{3\pi \times 8}{-90 \times 8 + 20\sqrt{(3+8)^3}}]^{0.25}$$

for x = 1

$$[\frac{3\pi \times 8}{-90 \times 8 + 20\sqrt{(3+8)^3}}]^{0.25}$$

**Input:**

$$\left( \frac{3\pi \times 8}{-90 \times 8 + 20\sqrt{(3+8)^3}} \right)^{0.25}$$

**Result:**

1.671570191473248993243402673182071893116337920455733800081...

1.6715701914...

**All 4th roots of (24 π)/(220 sqrt(11) - 720):**

$$2^{3/4} \sqrt[4]{\frac{3\pi}{220\sqrt{11}-720}} e^0 \approx 1.672 \text{ (real, principal root)}$$

$$2^{3/4} \sqrt[4]{\frac{3\pi}{220\sqrt{11}-720}} e^{(i\pi)/2} \approx 1.672 i$$

$$2^{3/4} \sqrt[4]{\frac{3\pi}{220\sqrt{11}-720}} e^{i\pi} \approx -1.672 \text{ (real root)}$$

$$2^{3/4} \sqrt[4]{\frac{3\pi}{220\sqrt{11}-720}} e^{-(i\pi)/2} \approx -1.672 i$$

**Series representations:**

$$\left( \frac{3(\pi 8)}{-90 \times 8 + 20\sqrt{(3+8)^3}} \right)^{0.25} = 2.21336 \left( \frac{\pi}{-720 + 20\sqrt{1330} \sum_{k=0}^{\infty} 1330^{-k} \binom{1/2}{k}} \right)^{0.25}$$

$$\left( \frac{3(\pi 8)}{-90 \times 8 + 20\sqrt{(3+8)^3}} \right)^{0.25} = 2.21336 \left( \frac{\pi}{-720 + 20\sqrt{1330} \sum_{k=0}^{\infty} \frac{(-\frac{1}{1330})^k \binom{-1/2}{k}}{k!}} \right)^{0.25}$$

$$\left( \frac{3 \pi 8}{-90 \times 8 + 20 \sqrt{(3+8)^3}} \right)^{0.25} = 2.21336 \left( \frac{\pi}{-720 + \frac{10 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 1330^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}} \right)^{0.25}$$

### Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \text{ for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

and also, multiplying by  $1/10^{27}$ :

$$1/10^{27} [(((3\pi \cdot 8)/(-90 \cdot 8 + 20 \sqrt{(3+8)^3}))^{0.25})]$$

### Input:

$$\frac{1}{10^{27}} \left( \frac{3 \pi \times 8}{-90 \times 8 + 20 \sqrt{(3+8)^3}} \right)^{0.25}$$

### Result:

$$1.67157... \times 10^{-27}$$

$1.67157... \cdot 10^{-27}$  result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_p = 1.6714213 \times 10^{-27} \text{ kg}$$

that is the holographic proton mass (N. Hamein)

### Series representations:

$$\frac{\left( \frac{3 \pi 8}{-90 \times 8 + 20 \sqrt{(3+8)^3}} \right)^{0.25}}{10^{27}} = 2.21336 \times 10^{-27} \left( \frac{\pi}{-720 + 20 \sqrt{1330} \sum_{k=0}^{\infty} 1330^{-k} \binom{\frac{1}{2}}{k}} \right)^{0.25}$$

$$\frac{\left(\frac{3\pi 8}{-90 \times 8 + 20 \sqrt{(3+8)^3}}\right)^{0.25}}{10^{27}} = 2.21336 \times 10^{-27} \left( \frac{\pi}{-720 + 20 \sqrt{1330} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{1330}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right)^{0.25}$$

$$\frac{\left(\frac{3\pi 8}{-90 \times 8 + 20 \sqrt{(3+8)^3}}\right)^{0.25}}{10^{27}} = 2.21336 \times 10^{-27} \left( \frac{\pi}{-720 + \frac{10 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 1330^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}} \right)^{0.25}$$

From

$$L = 2 + 2\varphi'^2 - \frac{7}{3}\varphi'^4 + \frac{2}{27}\varphi'^6 - 3\varphi'^4 R,$$

For  $\varphi' = \sqrt{8}$  and  $R = 0.0270061728395$ ; we obtain:

$$(((2+2*8-7/3*(\text{sqrt}8)^4+2/27*(\text{sqrt}8)^6-3(\text{sqrt}8)^4*0.0270061728395)))$$

**Input interpretation:**

$$2 + 2 \times 8 - \frac{7}{3} \sqrt{8}^4 + \frac{2}{27} \sqrt{8}^6 + 3 \sqrt{8}^4 \times (-0.0270061728395)$$

**Result:**

-98.5925925925914074074074074074074074074074074074074...

**-98.5925925...**

**Repeating decimal:**

-98.592592592591407 (period 3)



From which:

$$1/((((-(\sqrt{8})^4)/9*(1+0.0270061728395)*((((81*(\sqrt{8})^4*(0.0270061728395)-(3+4*8)(8-6)^2))))))))))$$

**Input interpretation:**

$$\frac{1}{-\frac{\sqrt{8}^4}{9} (1 + 0.0270061728395) (81 \sqrt{8}^4 \times 0.0270061728395 - (3 + 4 \times 8) (8 - 6)^2)}$$

**Result:**

$\infty$

$\infty$  is complex infinity

**Decimal approximation:**

$$4.27897257700979281125515858628286254039721281960955911... \times 10^9$$

4278972577.00979281

From which:

$$[1/((((-(\sqrt{8})^4)/9*(1+0.0270061728395)*((((81*(\sqrt{8})^4*(0.0270061728395)-(3+4*8)(8-6)^2))))))))))]^{1/3+101+5}$$

where 101 and 5 are Eisenstein prime numbers

**Input interpretation:**

$$\sqrt[3]{\frac{1}{-\frac{\sqrt{8}^4}{9} (1 + 0.0270061728395) (81 \sqrt{8}^4 \times 0.0270061728395 - (3 + 4 \times 8) (8 - 6)^2)}} + 101 + 5$$

**Result:**

$\infty$

$\infty$  is complex infinity

**Decimal approximation:**

$$1729.478348603843624553986159567569867499212700015656817710...$$

1729.4783486...

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

and:

$$\left( \left( \left( \left( \left( \left( \left( \frac{1}{\left( \frac{\sqrt{8}^4}{9} (1 + 0.0270061728395) \left( 81 \sqrt{8}^4 \times 0.0270061728395 - (3 + 4 \times 8)(8 - 6)^2 \right) \right) \right) \right) \right) \right) \right) \right) \right)^{\frac{1}{3} + 101 + 5} \right)^{\frac{1}{15}}$$

**Input interpretation:**

$$\left( \left( \left( \left( \left( \left( \frac{1}{\left( -\frac{\sqrt{8}^4}{9} (1 + 0.0270061728395) \left( 81 \sqrt{8}^4 \times 0.0270061728395 - (3 + 4 \times 8)(8 - 6)^2 \right) \right) \right) \right) \right) \right) \right) \right)^{\frac{1}{3} + 101 + 5} \right)^{\frac{1}{15}}$$

**Result:**

$\infty$

$\infty$  is complex infinity

**Decimal approximation:**

1.643845543582629477072083116469313355230888635905780669974...

$$1.6438455435\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Furthermore, we have also:

**Input interpretation:**

$$\frac{1}{4} \sqrt[4]{ \frac{1}{-\frac{\sqrt{8}^4}{9} (1 + 0.0270061728395) \left( 81 \sqrt{8}^4 \times 0.0270061728395 - (3 + 4 \times 8)(8 - 6)^2 \right)}}$$

**Result:**

$\infty$

$\infty$  is complex infinity

**Decimal approximation:**

63.94033163551902909341386233815110797981409716818125925106...

63.94033163... ≈ 64

and again:

$$\frac{1}{2} \left[ \frac{1}{\left( \left( \left( \left( \left( \left( \sqrt{8}^4 \right) / 9 \cdot \left( 1 + 0.0270061728395 \right) \cdot \left( \left( \left( \left( 81 \cdot \left( \sqrt{8}^4 \cdot \left( 0.0270061728395 \right) - \left( 3 + 4 \cdot 8 \right) \cdot \left( 8 - 6^2 \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right]^{1/4} - \phi^2$$

**Input interpretation:**

$$\frac{1}{2} \left( \frac{1}{\left( -\frac{\sqrt{8}^4}{9} \left( 1 + 0.0270061728395 \right) \left( 81 \sqrt{8}^4 \times 0.0270061728395 - \left( 3 + 4 \times 8 \right) \left( 8 - 6^2 \right) \right) \right) \right)^{(1/4) - \phi^2}$$

φ is the golden ratio

**Result:**

∞

∞ is complex infinity

**Decimal approximation:**

125.2626292822881633386231378419365778419078851565567556399...

125.26262928... result very near to the Higgs boson mass 125.18 GeV

$$\frac{1}{2} \left[ \frac{1}{\left( \left( \left( \left( \left( \left( \sqrt{8}^4 \right) / 9 \cdot \left( 1 + 0.0270061728395 \right) \cdot \left( \left( \left( \left( 81 \cdot \left( \sqrt{8}^4 \cdot \left( 0.0270061728395 \right) - \left( 3 + 4 \cdot 8 \right) \cdot \left( 8 - 6^2 \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right]^{1/4 + 13} - \phi$$

**Input interpretation:**

$$\frac{1}{2} \left( \frac{1}{\left( -\frac{\sqrt{8}^4}{9} \left( 1 + 0.0270061728395 \right) \left( 81 \sqrt{8}^4 \times 0.0270061728395 - \left( 3 + 4 \times 8 \right) \left( 8 - 6^2 \right) \right) \right) \right)^{(1/4) + 13 - \phi}$$



$\phi$  is the golden ratio

**Result:**

$\infty$

$\infty$  is complex infinity

**Decimal approximation:**

139.2626292822881633386231378419365778419078851565567556399...

139.26262982... result practically equal to the rest mass of Pion meson 139.57 MeV

Now, we have that:

$$\Delta = -27A_1^2A_4 + 9A_1A_2A_3 - 2A_2^3 + \sqrt{(-27A_1^2A_4 + 9A_1A_2A_3 - 2A_2^3)^2 - 4(A_2^2 - 3A_1A_3)^3}$$

(2.44)

We consider  $A_1=A_2=A_3=A_4=1$  and obtain:

$$-27+9-2+\text{sqrt}(((((-27+9-2)^2-4(1-3)^3))))$$

**Input:**

$$-27+9-2+\sqrt{(-27+9-2)^2-4(1-3)^3}$$

**Result:**

$$12\sqrt{3} - 20$$

**Decimal approximation:**

0.784609690826527522329356098070468403313663045724567536669...

0.7846096908...

**Alternate form:**

$$4(3\sqrt{3} - 5)$$

**Minimal polynomial:**

$$x^2 + 40x - 32$$

From which:

$$1 + ((((-27 + 9 - 2 + \sqrt{((-27 + 9 - 2)^2 - 4(1 - 3)^3)})))))^2$$

**Input:**

$$1 + \left( -27 + 9 - 2 + \sqrt{(-27 + 9 - 2)^2 - 4(1 - 3)^3} \right)^2$$

**Result:**

$$1 + \left( 12\sqrt{3} - 20 \right)^2$$

**Decimal approximation:**

1.615612366938899106825756077181263867453478171017298533212...

1.6156123669.... result that is a good approximation to the value of the golden ratio  
1.618033988749...

**Alternate form:**

$$833 - 480\sqrt{3}$$

**Minimal polynomial:**

$$x^2 - 1666x + 2689$$

Now, from

$$H^2 = \frac{1}{3A_1} \left( -A_2 + (A_2^2 - 3A_1A_3) \left( \frac{2}{\Delta} \right)^{1/3} + \left( \frac{\Delta}{2} \right)^{1/3} \right), \quad (2.43)$$

for  $\Delta = 0.7846096908$ , we obtain:

$$1/3(((((-1+(1-3)(2/0.7846096908)^(1/3)+(0.7846096908/2)^(1/3))))))$$

**Input interpretation:**

$$\frac{1}{3} \left( -1 + (1 - 3) \sqrt[3]{\frac{2}{0.7846096908}} + \sqrt[3]{\frac{0.7846096908}{2}} \right)$$

**Result:**

-1.00000000001301341064834616548313686330451781076745424525...

-1.0000.... ≈ -1

and:

$$1/\left(\left(\left(\left(\left(\left(-1+(1-3)\left(\frac{2}{0.7846096908}\right)^{1/3}+\left(\frac{0.7846096908}{2}\right)^{1/3}\right)\right)\right)\right)\right)\right)^{4096^2\left(\frac{64}{16}+\left(\frac{2\sqrt[4]{\frac{6079}{4029}}}{5^{3/4}}\right)/\pi^4\right)}$$

**Input interpretation:**

$$\frac{1}{\left(\frac{1}{3}\left(-\left(-1+(1-3)\sqrt[3]{\frac{2}{0.7846096908}}+\sqrt[3]{\frac{0.7846096908}{2}}\right)\right)\right)^{4096^2\left(\frac{64}{16}+\left(2\sqrt[4]{\frac{6079}{4029}}\right)5^{3/4}/\pi^4\right)}$$

where

$$\frac{2\sqrt[4]{\frac{6079}{4029}}5^{3/4}}{\pi^4} \approx 0.07608809784385273$$

**Result:**

0.999110468439405077608590335678712305851269034211146599641...

0.9991104684....result equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

= 0.9991104684

**Alternative representations:**

$$\frac{1}{\left(-\frac{1}{3} \left(-1 + (1-3) \sqrt[3]{\frac{2}{0.78461}} + \sqrt[3]{\frac{0.78461}{2}}\right)\right)^{4096^2} \left(64/16 + \left(2 \sqrt[4]{\frac{6079}{4029}} 5^{3/4}\right) / \pi^4\right)} =$$

$$\left( \frac{1}{\left(\frac{1}{3} \left(1 - \sqrt[3]{0.392305} + 2 \sqrt[3]{\frac{2}{0.78461}}\right)\right)^{4096^2} \left(64/16 + \left(2 \times 5^{3/4} \sqrt[4]{\frac{6079}{4029}}\right) / \cos^{-1}(-1)^4\right)} \right.$$

$$\left. 1. \quad \left(4 + \left(2 \sqrt[4]{\frac{6079}{4029}} 5^{3/4}\right) / \cos^{-1}(-1)^4\right) \right)$$

$$\frac{1}{\left(-\frac{1}{3} \left(-1 + (1-3) \sqrt[3]{\frac{2}{0.78461}} + \sqrt[3]{\frac{0.78461}{2}}\right)\right)^{4096^2} \left(64/16 + \left(2 \sqrt[4]{\frac{6079}{4029}} 5^{3/4}\right) / \pi^4\right)} =$$

$$\left( \frac{1}{\left(\frac{1}{3} \left(1 - \sqrt[3]{0.392305} + 2 \sqrt[3]{\frac{2}{0.78461}}\right)\right)^{4096^2} \left(64/16 + \left(2 \times 5^{3/4} \sqrt[4]{\frac{6079}{4029}}\right) / (180^\circ)^4\right)} \right.$$

$$\left. 1. \quad \left(4 + \sqrt[4]{\frac{6079}{20145}} / (104976000^\circ)^4\right) \right)$$

$$\frac{1}{\left(-\frac{1}{3}\left(-1+(1-3)\sqrt[3]{\frac{2}{0.78461}}+\sqrt[3]{\frac{0.78461}{2}}\right)\right)^{4096^2}\left(64/16+\left(2\sqrt[4]{\frac{6079}{4029}}5^{3/4}\right)\right)/\pi^4} =$$

$$\left(\frac{1}{\left(\frac{1}{3}\left(1-\sqrt[3]{0.392305}+2\sqrt[3]{\frac{2}{0.78461}}\right)\right)^{4096^2}\left(64/16+\left(2\times 5^{3/4}\sqrt[4]{\frac{6079}{4029}}\right)\right)/(-i\log(-1))^4} =$$

$$1. \frac{16777216\left(4+\left(2\sqrt[4]{\frac{6079}{4029}}5^{3/4}\right)\right)/(i^4\log^4(-1))}{1}$$

### Integral representations:

$$\frac{1}{\left(-\frac{1}{3}\left(-1+(1-3)\sqrt[3]{\frac{2}{0.78461}}+\sqrt[3]{\frac{0.78461}{2}}\right)\right)^{4096^2}\left(64/16+\left(2\sqrt[4]{\frac{6079}{4029}}5^{3/4}\right)\right)/\pi^4} =$$

$$0.999127 e^{-0.000101134/\left(\int_0^\infty \frac{1}{1+t^2} dt\right)^4}$$

$$\frac{1}{\left(-\frac{1}{3}\left(-1+(1-3)\sqrt[3]{\frac{2}{0.78461}}+\sqrt[3]{\frac{0.78461}{2}}\right)\right)^{4096^2}\left(64/16+\left(2\sqrt[4]{\frac{6079}{4029}}5^{3/4}\right)\right)/\pi^4} =$$

$$0.999127 e^{-6.32089\times 10^{-6}/\left(\int_0^1 \sqrt{1-t^2} dt\right)^4}$$

$$\frac{1}{\left(-\frac{1}{3}\left(-1+(1-3)\sqrt[3]{\frac{2}{0.78461}}+\sqrt[3]{\frac{0.78461}{2}}\right)\right)^{4096^2}\left(64/16+\left(2\sqrt[4]{\frac{6079}{4029}}5^{3/4}\right)\right)/\pi^4} =$$

$$0.999127 e^{-0.000101134/\left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^4}$$

Now, from

## Further improvements in Waring's problem

by

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We have that:

LEMMA 6.1. *Let  $t$  be an integer with  $t \geq 3$ . Suppose that  $\phi_1 \geq \frac{1}{15}$ ,  $\phi_2 \leq 5\phi_1 - \frac{1}{3}$ ,*

$$U \leq \min\{M_2, PH_1 H_2^{-3}, Q_2^{1/4}, Q_2^{5/6} M_2^{-19/6}\}, \quad (6.1)$$

and

$$Z = PU^{1-1/t} (P^{1/3} M_1^{2t-10-\mu_t})^{1/t}. \quad (6.2)$$

Then

$$\int_0^1 |F_2(\alpha) f_2(\alpha)^{12}| d\alpha \ll P^{1+\epsilon} \tilde{M}_2 \tilde{H}_2 (Z^{-1/4} Q_2^{\lambda_6^*} + Q_2^{(3/4)\lambda_6^* - 5/4}).$$

$$\lambda_6^* \leq 7.541755. \quad \mu_{22} = 34.228489. \quad \Delta = \lambda_6^* - 7 \text{ and } \delta = \mu_{22} - 34.$$

From

$$\phi = \frac{289 + 105\theta + 6\delta\theta}{2136},$$

and:

$$\theta = \frac{3581 - 289\Delta}{20835 + 105\Delta - 6\delta(5 - \Delta)}.$$

we obtain:



**Repeating decimal:**

1.1307061962224 (period 2)

1.1307061962224

For

$$\lambda_6^* \leq 7.541755, \quad \mu_{22} = 34.228489, \quad \Delta = \lambda_6^* - 7 \text{ and } \delta = \mu_{22} - 34.$$

$$\lambda_7 = \frac{34}{41} \lambda_6^* + \frac{125}{41}, \quad \lambda_8 = \frac{34}{41} \lambda_7 + \frac{139}{41},$$

$$34/41 * 7.541755 + 125/41$$

**Input interpretation:**

$$\frac{34}{41} \times 7.541755 + \frac{125}{41}$$

**Result:**

9.3029187804878048780487804878048780487804878048780487804878048780487...

**Repeating decimal:**

9.3029187804 (period 5)

$$9.3029187804 = \lambda_7$$

and also:

$$\lambda_7^* = \lambda_6^*(1 - \theta) + 1 + 12\theta.$$

$$7.541755(1 - 0.163960075) + 1 + 12 * 0.163960075$$

**Input interpretation:**

$$7.541755(1 - 0.163960075) + 1 + 12 \times 0.163960075$$

**Result:**

9.272729184568375

$$9.272729184568375 = \lambda_7^*$$



$$34/41 \times 9.30291878 + 139/41$$

**Input interpretation:**

$$\frac{34}{41} \times 9.30291878 + \frac{139}{41}$$

**Result:**

11.1048594760975609756097560975609756097560975609756097560975609...

**Repeating decimal:**

11.1048594760975 (period 5)

$$11.1048594760975 = \lambda_8$$

From

$$(4\lambda_6^* + 5 - 3\lambda_8)(1 - \theta - \phi)$$

$$(4 \times 7.541755 + 5 - 3 \times 11.1048594)(1 - 0.163960075 - 0.14346469)$$

**Input interpretation:**

$$(4 \times 7.541755 + 5 + 3 \times (-11.1048594))(1 - 0.163960075 - 0.14346469)$$

**Result:**

1.282955314958823

$$1.282955314958823$$

From which:

$$[(4 \times 7.541755 + 5 - 3 \times 11.1048594)(1 - 0.163960075 - 0.14346469)]^2$$

**Input interpretation:**

$$((4 \times 7.541755 + 5 + 3 \times (-11.1048594))(1 - 0.163960075 - 0.14346469))^2$$

**Result:**

1.645974340181092722990185545329

$$1.645974340181092722990185545329 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Thence:

$$(4\lambda_6^* + 5 - 3\lambda_8)(1 - \theta - \phi) > 1 + \frac{21(5 - 24\phi - 5\theta)}{132} + \frac{1 - 3\delta\theta}{66}.$$

$$1.282955314958823 > 1.1307061962224$$

Now, we have that:

Write  $\mathcal{E} = \lambda_8 - 2\lambda_7^* + \lambda_6^*$ . Also, as in case (i), write  $\Delta = \lambda_6^* - 7$  and  $\delta = \mu_{22} - 34$ . Suppose that our ultimate choices for  $\theta$  and  $\phi$  imply that (6.1) holds when  $U$  satisfies (6.19). The equations (6.2), (6.22) and (6.23) then yield

$$\begin{aligned} 4 - 20\phi &= 1 + \frac{21(5 - 24\phi - 5\theta)}{132} + \frac{1 - 3\delta\theta}{66}, \\ 10\theta &= 1 + \mathcal{E}(1 - \theta) + (5 - \Delta)\phi. \end{aligned}$$

Therefore

$$\phi = \frac{289 + 105\theta + 6\delta\theta}{2136}, \quad (6.25)$$

and hence

$$\theta = \frac{3581 + 2136\mathcal{E} - 289\Delta}{20835 + 2136\mathcal{E} + 105\Delta - 6\delta(5 - \Delta)}. \quad (6.26)$$

Given an iterate for  $\lambda_8$ , we therefore obtain the next iterate as follows. We compute  $\theta$  and  $\phi$  from (6.25) and (6.26). We then check that the choice of  $U$  given by (6.19) is indeed permissible, and check that  $U_1$  is the dominating contribution. The latter follows provided that (6.21) holds. The next iterate for  $\lambda_8$  is then given by (6.18), that is, by

$$\lambda_8' = \lambda_7^*(1 - \theta) + 1 + 14\theta. \quad (6.27)$$

To succeed with this iteration process, we need to start with an initial iterate for  $\lambda_8$  reasonably close to  $\lambda_8^*$ . For this purpose we can use inequality  $(k-2)$  of §4 of Vaughan [8] once again. We therefore take

$$\lambda_8 = \frac{34}{41}\lambda_7^* + \frac{139}{41}.$$

A computation now shows that  $\lambda_8^* \leq 11.077363$ . We note that  $\lambda_8^*$  can be calculated directly

For

$$\lambda_6^* \leq 7.541755, \quad \mu_{22} = 34.228489, \quad \Delta = \lambda_6^* - 7 \text{ and } \delta = \mu_{22} - 34.$$

$$\mathcal{E} = \lambda_8 - 2\lambda_7^* + \lambda_6^*$$

$$= 11.1048594760975 - 2 \times 9.272729184568375 + 7.541755 =$$

$$11.1048594760975 + 2 \times (-9.272729184568375) + 7.541755$$

$$0.10115610696075$$

$$0.10115610696075$$

$$0.14346469 = \phi$$

From

$$\theta = \frac{3581 + 2136\mathcal{E} - 289\Delta}{20835 + 2136\mathcal{E} + 105\Delta - 6\delta(5 - \Delta)}$$

we obtain:

$$(3581 + 2136 \times 0.10115610696075 - 289 \times 0.541755) / (20835 + 2136 \times 0.10115610696075 + 105 \times 0.541755 - 6 \times 0.228489(5 - 0.541755))$$

**Input interpretation:**

$$\frac{3581 + 2136 \times 0.10115610696075 + 289 \times (-0.541755)}{20835 + 2136 \times 0.10115610696075 + 105 \times 0.541755 + 6(5 - 0.541755) \times (-0.228489)}$$

**Result:**

$$0.172520592794914219451746158418676890280406636420239240741\dots$$

$$0.1725205927949\dots = \theta$$

From which:

$$1 + [(3581 + 2136 \times 0.10115610696075 - 289 \times 0.541755) / (20835 + 2136 \times 0.10115610696075 + 105 \times 0.541755 - 6 \times 0.228489(5 - 0.541755))]^{1/4}$$

**Input interpretation:**

$$1 + ((3581 + 2136 \times 0.10115610696075 + 289 \times (-0.541755)) / (20835 + 2136 \times 0.10115610696075 + 105 \times 0.541755 + 6(5 - 0.541755) \times (-0.228489)))^{(1/4)}$$

**Result:**

1.6444812...

$$1.6444812\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

From

$$\lambda'_8 = \lambda_7^*(1-\theta) + 1 + 14\theta. \tag{6.27}$$

we obtain:

$$9.272729184568375(1-0.1725205927949) + 1 + 14 * 0.1725205927949$$

**Input interpretation:**

$$9.272729184568375 (1 - 0.1725205927949) + 1 + 14 \times 0.1725205927949$$

**Result:**

11.0882807479486692517085987125

**Repeating decimal:**

11.0882807479486692517085987125

$$11.0882807479486692517085987125 = \lambda'_8$$

From which:

$$1/2 * \text{sqrt}[9.272729184568375(1-0.1725205927949) + 1 + 14 * 0.1725205927949]$$

**Input interpretation:**

$$\frac{1}{2} \sqrt{9.272729184568375 (1 - 0.1725205927949) + 1 + 14 \times 0.1725205927949}$$

**Result:**

1.6649535089567...

1.6649535089567.... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

Note that, summing the following two results, and performing the 6<sup>th</sup> root, we obtain:

$$(11.0882807479486692517085987125 + 9.272729184568375)^{1/6}$$

**Input interpretation:**

$$\sqrt[6]{11.0882807479486692517085987125 + 9.272729184568375}$$

**Result:**

1.6524686131945259...

1.6524686131945259.... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

From:

$$\lambda_8 = \frac{34}{41}\lambda_7^* + \frac{139}{41}.$$

we obtain:

$$34/41 * 9.272729184568375 + 139/41$$

**Input interpretation:**

$$\frac{34}{41} \times 9.272729184568375 + \frac{139}{41}$$

**Result:**

11.0798242018371890243902439024390243902439024390243902439024390...

**Repeating decimal:**

11.0798242018371890243 (period 5)

$$11.0798242018371890243 = \lambda_8$$

Now, we have that:

The equations for  $\lambda_9$ ,  $\theta$ ,  $\phi$  and  $\psi$  are now determined by

$$\begin{aligned} P\tilde{H}_2\tilde{M}_2M_3Q_3^{\lambda_6^*} &\approx P^{1/2}(\tilde{H}_3\tilde{M}_3)^{3/4}Q_3^{3\lambda_6^*/4}, \\ PH_1M_1M_2Q_2^{\lambda_7^*} &\approx (P(\tilde{H}_2\tilde{M}_2)^2M_3^{12}Q_2^{\lambda_8^*}Q_3^{\lambda_6^*})^{1/2}, \\ PM_1Q_1^{\lambda_8^*} &\approx (P(H_1M_1)^2M_2^{14}Q_1^{\lambda_9}Q_2^{\lambda_7^*})^{1/2}, \\ P^{\lambda_9} &\approx PM_1^{16}Q_1^{\lambda_8^*}. \end{aligned}$$

Let

$$\begin{aligned} \delta &= 3\lambda_8^* - 4\lambda_6^*, \\ \alpha_2 &= \frac{\lambda_6^* - 12}{3k + 1 + \delta}, \\ \mathcal{E}_2 &= \lambda_8^* - 2\lambda_7^* + \lambda_6^*, \\ \alpha_1 &= \frac{\lambda_7^* - 14}{2k + \mathcal{E}_2 + \alpha_2(k - 1 - \delta)}, \\ \mathcal{E}_1 &= \lambda_9 - 2\lambda_8^* + \lambda_7^*. \end{aligned}$$

Then, arguing as in previous cases we obtain

$$\begin{aligned} \psi &= \frac{(k-1-\delta)(\theta+\phi)-1+\delta}{3k+1+\delta}, \\ \phi &= \frac{1 + \mathcal{E}_2(1-\theta) + \alpha_2(1-\delta-(k-1-\delta)\theta)}{2k + \mathcal{E}_2 + \alpha_2(k-1-\delta)}, \\ \theta &= \frac{1 + \mathcal{E}_1 - \alpha_1(1 + \mathcal{E}_2 + \alpha_2(1-\delta))}{2k + \mathcal{E}_1 - \alpha_1(\mathcal{E}_2 + \alpha_2(k-1-\delta))}. \end{aligned}$$

The next iterate for  $\lambda_9$  is given by

$$\lambda_9' = \lambda_8^*(1-\theta) + 1 + 16\theta.$$

From

$$\begin{aligned}\delta &= 3\lambda_8^* - 4\lambda_6^*, \\ \alpha_2 &= \frac{\lambda_6^* - 12}{3k + 1 + \delta}, \\ \mathcal{E}_2 &= \lambda_8^* - 2\lambda_7^* + \lambda_6^*, \\ \alpha_1 &= \frac{\lambda_7^* - 14}{2k + \mathcal{E}_2 + \alpha_2(k - 1 - \delta)}, \\ \mathcal{E}_1 &= \lambda_9 - 2\lambda_8^* + \lambda_7^*.\end{aligned}$$

$$e = \begin{cases} 0 & \text{when } k = 6, 7, 9, \\ 1 & \text{when } k = 8. \end{cases}$$

For:

$$\lambda_6^* \leq 7.541755. \quad 9.272729184568375 = \lambda_7^*$$

$$11.0798242018371890243 = \lambda_8$$

we obtain:

$$11.0798242 - 2 \times (9.27272918) + 7.541755$$

**Input interpretation:**

$$11.0798242 + 2 \times (-9.27272918) + 7.541755$$

**Result:**

$$0.07612084$$

$$0.07612084 = \mathcal{E}_2$$

$$3 \times 11.0798242 - 4 \times 7.541755$$

**Input interpretation:**

$$3 \times 11.0798242 + 4 \times (-7.541755)$$

**Result:**

3.0724526

$$3.0724526 = \delta$$

$$(7.541755-12)/(3*8+1+3.0724526)$$

**Input interpretation:**

$$\frac{7.541755 - 12}{3 \times 8 + 1 + 3.0724526}$$

**Result:**

-0.15881209467248330129872585482609381981822279397133971828...

$$-0.158812094672\dots = \alpha_2$$

$$(9.272729184-14)/(((2*8+0.07612084-0.158812094672)(8-1-3.0724526)))$$

**Input interpretation:**

$$\frac{9.272729184 - 14}{(2 \times 8 + 0.07612084 - 0.158812094672)(8 - 1 - 3.0724526)}$$

**Result:**

-0.07561699376459127217554319459879408407234177270847183247...

$$-0.075616993764\dots = \alpha_1$$

From

$$\theta = \frac{1 + \mathcal{E}_1 - \alpha_1(1 + \mathcal{E}_2 + \alpha_2(1 - \delta))}{2k + \mathcal{E}_1 - \alpha_1(\mathcal{E}_2 + \alpha_2(k - 1 - \delta))}$$

we obtain:

$$\frac{(((1+(x-2*11.0798242+9.272729184)+0.075616993764(((1+0.07612084-0.158812094672)(1-3.0724526))))))) / (((2*8+(x-2*11.0798242+9.272729184)+0.075616993764((0.07612084-0.158812094672)(8-1-3.0724526))))))$$

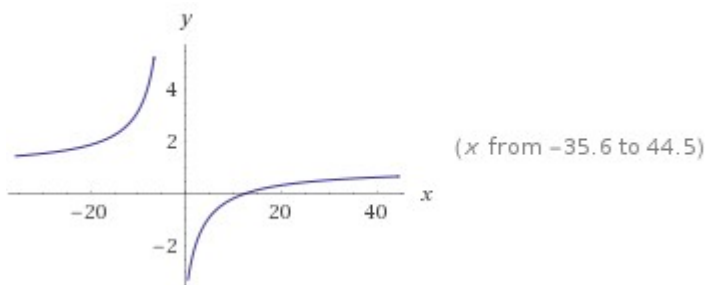
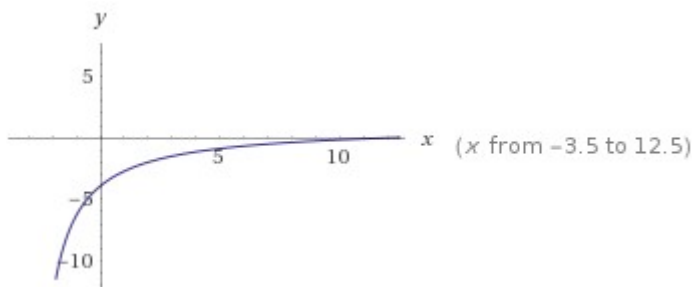


**Input interpretation:**

$$\frac{(1 + (x + 2 \times (-11.0798242) + 9.272729184) + 0.075616993764 ((1 + 0.07612084 - 0.158812094672) (1 - 3.0724526))) / (2 \times 8 + (x + 2 \times (-11.0798242) + 9.272729184) + 0.075616993764 (0.07612084 + (8 - 1 - 3.0724526) \times (-0.158812094672)))}{x - 12.0307}$$

**Result:**

$$\frac{x - 12.0307}{x + 3.07167}$$

**Plots:****Alternate forms:**

$$1 - \frac{15.1023}{x + 3.07167}$$

$$\frac{x - 12.0307}{x + 3.07167}$$

**Expanded form:**

$$\frac{x}{x + 3.07167} - \frac{12.0307}{x + 3.07167}$$

**Root:**

$$x \approx 12.0307$$

$$12.0307 = \varepsilon_1$$

**Properties as a real function:****Domain**

$$\left\{ x \in \mathbb{R} : x \neq -\frac{239\,974\,321\,556\,815\,221\,855\,716\,643\,411}{78\,125\,000\,000\,000\,000\,000\,000\,000\,000} \right\}$$

**Range**

$$\left\{ y \in \mathbb{R} : y \neq 1 \text{ and } y \neq \frac{1\,179\,870\,656\,470\,248\,815\,605\,716\,643\,411}{1\,393\,730\,716\,643\,411} \right\}$$

$\mathbb{R}$  is the set of real numbers

**Series expansion at  $x = 0$ :**

$$-3.91665 + 1.60064x - 0.521099x^2 + 0.169647x^3 - 0.0552294x^4 + O(x^5)$$

(Taylor series)

**Series expansion at  $x = \infty$ :**

$$1 - \frac{15.1023}{x} + \frac{46.3894}{x^2} - \frac{142.493}{x^3} + O\left(\left(\frac{1}{x}\right)^4\right)$$

(Laurent series)

**Derivative:**

$$\frac{d}{dx} \left( \frac{x - 12.0307}{x + 3.07167} \right) = \frac{15.1023}{(x + 3.07167)^2}$$

**Indefinite integral:**

$$\int \frac{1+(x-2 \times 11.0798242+9.272729184)+0.075616993764 \left( (1+0.07612084-0.158812094672)(1-3.0724526) \right)}{2 \times 8+(x-2 \times 11.0798242+9.272729184)+0.075616993764 \left( 0.07612084-0.158812094672 \left( 8-1-3.0724526 \right) \right)} dx = x - 15.1023 \log(x + 3.07167) + \text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$  is the natural logarithm

**Limit:**

$$\lim_{x \rightarrow \pm\infty} \frac{-12.0307 + x}{3.07167 + x} = 1$$

Thence:

$$\theta = \frac{1 + \mathcal{E}_1 - \alpha_1(1 + \mathcal{E}_2 + \alpha_2(1 - \delta))}{2k + \mathcal{E}_1 - \alpha_1(\mathcal{E}_2 + \alpha_2(k - 1 - \delta))}$$

$$\frac{(((1+(12.0307-2*11.0798242+9.272729184)+0.0756169(((1+0.07612084-0.15881209)(1-3.0724526)))))))/((2*8+(12.0307-2*11.0798242+9.272729)+0.0756169((0.07612084-0.15881209(8-1-3.0724526))))))$$

**Input interpretation:**

$$\frac{(1 + (12.0307 + 2 \times (-11.0798242) + 9.272729184) + 0.0756169 ((1 + 0.07612084 - 0.15881209) (1 - 3.0724526))) / (2 \times 8 + (12.0307 + 2 \times (-11.0798242) + 9.272729) + 0.0756169 (0.07612084 + (8 - 1 - 3.0724526) \times (-0.15881209)))$$

**Result:**

$$1.7937996827957222556979514824297601461309054719969714... \times 10^{-6}$$

$$1.79379968... * 10^{-6} = \theta$$

From which:

$$1 + [ \frac{(((1+(12.0307-2*11.0798242+9.272729184)+0.0756169(((1+0.07612084-0.15881209)(1-3.0724526)))))))/((2*8+(12.0307-2*11.0798242+9.272729)+0.0756169((0.07612084-0.15881209(8-1-3.0724526)))))))]^{1/27}$$

**Input interpretation:**

$$1 + ((1 + (12.0307 + 2 \times (-11.0798242) + 9.272729184) + 0.0756169 ((1 + 0.07612084 - 0.15881209) (1 - 3.0724526))) / (2 \times 8 + (12.0307 + 2 \times (-11.0798242) + 9.272729) + 0.0756169 (0.07612084 + (8 - 1 - 3.0724526) \times (-0.15881209))))^{(1/27)}$$

**Result:**

$$1.612599746421253920885223011370189261556471306960044267389...$$

1.6125997464..... result that is a good approximation to the value of the golden ratio  
1.618033988749...

From

$$\lambda'_9 = \lambda_8^*(1-\theta) + 1 + 16\theta.$$

we obtain:

$$11.0798242018371890243(1-1.79379968279572225569795148242976e-6)+1+16*1.79379968279572225569795148242976e-6$$

**Input interpretation:**

$$11.0798242018371890243 \left(1 - 1.79379968279572225569795148242976 \times 10^{-6}\right) + 1 + 16 \times 1.79379968279572225569795148242976 \times 10^{-6}$$

**Result:**

$$12.07983302764697506793989698135052626551575229413063231683...$$

$$12.0798330276... = \lambda_9$$

From which, we obtain:

$$\left(\left(\left(11.0798242018371890243(1-1.7937996827957e-6)+1+16*1.7937996827957e-6\right)\right)^{1/5}\right)$$

**Input interpretation:**

$$\left(11.0798242018371890243 \left(1 - 1.7937996827957 \times 10^{-6}\right) + 1 + 16 \times 1.7937996827957 \times 10^{-6}\right)^{(1/5)}$$

**Result:**

$$1.6459331273156222758...$$

$$1.6459331273..... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

or:

from

$$\mathcal{E}_1 = \lambda_9 - 2\lambda_8^* + \lambda_7^*.$$

$$12.0798330276 - 2*11.0798242 + 9.27272918$$

**Input interpretation:**

$$12.0798330276 + 2 \times (-11.0798242) + 9.27272918$$

**Result:**

$$-0.8070861924$$

$$-0.8070861924$$

From which, we obtain:

$$-2 * [(((11.0798242018371890243(1 - 1.7937996827957e-6) + 1 + 16 * 1.7937996827957e-6))) - 2 * 11.0798242 + 9.27272918]$$

**Input interpretation:**

$$-2 \left( \left( 11.0798242018371890243 \left( 1 - 1.7937996827957 \times 10^{-6} \right) + 1 + 16 \times 1.7937996827957 \times 10^{-6} \right) + 2 \times (-11.0798242) + 9.27272918 \right)$$

**Result:**

$$1.61417238470604986433920993016315847102$$

1.61417238470604986433920993016315847102 result that is a good approximation to the value of the golden ratio 1.618033988749...

From:

$$\theta = \frac{1 + \mathcal{E}_1 - \alpha_1(1 + \mathcal{E}_2 + \alpha_2(1 - \delta))}{2k + \mathcal{E}_1 - \alpha_1(\mathcal{E}_2 + \alpha_2(k - 1 - \delta))}$$

$$\frac{(((1 + (-0.8070861924) + 0.0756169(((1 + 0.07612084 - 0.15881209)(1 - 3.0724526)))))) / ((2 * 8 + (-0.8070861924) + 0.0756169((0.07612084 - 0.15881209)(8 - 1 - 3.0724526))))))$$

**Input interpretation:**

$$(1 - 0.8070861924 + 0.0756169 ((1 + 0.07612084 - 0.15881209) (1 - 3.0724526))) / (2 \times 8 - 0.8070861924 + 0.0756169 (0.07612084 + (8 - 1 - 3.0724526) \times (-0.15881209)))$$

**Result:**

0.003244569843094083179492999064449781156313766381088917108...

0.003244569843.....

From which:

$$\sqrt{31} \times 1 / [(((1 + (-0.8070861924) + 0.0756169((1 + 0.07612084 - 0.15881209)(1 - 3.0724526)))))) * 1 / ((2 * 8 + (-0.8070861924) + 0.0756169((0.07612084 - 0.15881209)(8 - 1 - 3.0724526)))))] + 13$$

**Input interpretation:**

$$\sqrt{31} \times 1 / ((1 - 0.8070861924 + 0.0756169((1 + 0.07612084 - 0.15881209)(1 - 3.0724526))) \times 1 / (2 \times 8 - 0.8070861924 + 0.0756169(0.07612084 + (8 - 1 - 3.0724526) \times (-0.15881209)))) + 13$$

**Result:**

1729.03...

1729.03...

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

and:

$$(((\sqrt{31} \times 1 / [(((1 + (-0.8070861924) + 0.0756169((1 + 0.07612084 - 0.15881209)(1 - 3.0724526)))))) * 1 / ((2 * 8 + (-0.8070861924) + 0.0756169((0.07612084 - 0.15881209)(8 - 1 - 3.0724526)))))] + 13))^{1/15} - 26/10^3$$

**Input interpretation:**

$$\left( \sqrt{31} \times 1 / ((1 - 0.8070861924 + 0.0756169((1 + 0.07612084 - 0.15881209)(1 - 3.0724526))) \times 1 / (2 \times 8 - 0.8070861924 + 0.0756169(0.07612084 + (8 - 1 - 3.0724526) \times (-0.15881209)))) + 13 \right)^{1/15} - \frac{26}{10^3}$$

**Result:**

1.617817...

1.617817.... result that is a very good approximation to the value of the golden ratio  
 1.618033988749...

and again:

$$\lambda'_9 = \lambda_8^*(1 - \theta) + 1 + 16\theta.$$

11.0798242018371890243(1-0.003244569843)+1+16\*0.003244569843

**Input interpretation:**

11.0798242018371890243 (1 - 0.003244569843) + 1 + 16 × 0.003244569843

**Result:**

12.0957880558541665355958656258151

12.095788....

From which, we obtain:

**Input interpretation:**

$\sqrt[5]{11.079824201837 (1 - 0.003244569843) + 1 + 16 \times 0.003244569843}$

**Result:**

1.646367687069...

1.646367687069....  $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

We have also:

$((((1+0.07612084(1-0.003244569843)-0.158812094672(1-3.0724526-(8-1-3.0724526)* 0.003244569843)))) / (((2*8+0.07612084-0.158812094672(8-1-3.0724526))))))$

**Input interpretation:**

$$(1 + 0.07612084 (1 - 0.003244569843) + (1 - 3.0724526 - (8 - 1 - 3.0724526) \times 0.003244569843) \times (-0.158812094672)) / (2 \times 8 + 0.07612084 + (8 - 1 - 3.0724526) \times (-0.158812094672))$$

**Result:**

0.091055765003433847971283458193121798624264217165528493666...

0.0910557650034.....

From:

$$\lambda'_9 = \lambda_8^*(1 - \phi_1) + 1 + 16\phi$$

we obtain:

$$11.0798242018371890243(1-0.0910557650034)+1+16*0.0910557650034$$

**Input interpretation:**

$$11.0798242018371890243 (1 - 0.0910557650034) + 1 + 16 \times 0.0910557650034$$

**Result:**

12.52783457309011796995651346781738

12.52783457.....result very near to the value of  $4\pi = 12.56637$  and to the Bekenstein-Hawking black hole entropy

From which:

$$(((11.0798242018371890243(1-0.0910557650034)+1+16*0.0910557650034)))^{1/5}$$

**Input interpretation:**

$$\sqrt[5]{11.0798242018371890243 (1 - 0.0910557650034) + 1 + 16 \times 0.0910557650034}$$

**Result:**

1.657964403461...

1.657964403461..... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...



From

# On Broken Supersymmetry and Vacuum Stability in Supergravity and String Theory

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Based on: 1811.11448, 1711.11494, 1612.08566 [hep-th]  
and refs therein  
AS and J. Mourad, to appear

Julia Fest ENS – Paris, December 16–17, 2019



## AdS<sub>3</sub> × S<sup>7</sup> (& AdS<sub>7</sub> × S<sup>3</sup>) Vacua

$$R_{AdS}^2 \mathcal{M}^2 = \frac{L_3}{3} (\sigma_3 - 1) \mathbf{1}_3 + \begin{pmatrix} 4 + 3\sigma_3 & -\frac{7}{2} \sigma_3 & \frac{L_3}{2} (\sigma_3 - 1) \\ -2\sigma_3 & 2\sigma_3 + \tau_3 & -\frac{L_3}{3} (\sigma_3 - 1) \\ 8\sigma_3 & -4\sigma_3 & 0 \end{pmatrix},$$

$$\sigma_3 \rightarrow \frac{3}{2}, \quad \tau_3 \rightarrow \frac{9}{2}, \quad L_3 \rightarrow \ell(\ell + 6)$$

$$[\mathcal{M}^2 R_{AdS}^2 \geq -1?]$$

- Breitenlohner-Freedman Bound Violated for  $\ell = 2, 3, 4$  :

*(Gubser, Mitra, 2002)*

- [Similar result for heterotic SO(16) × SO(16) for  $\ell = 1$ ]

We have that:

$$7/3*(3/2-1) + \det(\{\{4+3*3/2, -7/2*3/2, 7/2*(3/2-1)\}, \{-2*3/2, 2*3/2+9/2, -7/3*(3/2-1)\}, \{8*3/2, -4*3/2, 0\}\})$$













## Observations

*From:*

[https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn\\_RpOSvJlQxWsVLBcJ6KVgd\\_Af\\_hrmDYBNyU8mpSjRs1BDeremA](https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJlQxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA)

*Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that  $p(9) = 30$ ,  $p(9 + 5) = 135$ ,  $p(9 + 10) = 490$ ,  $p(9 + 15) = 1,575$  and so on are all divisible by 5. Note that here the  $n$ 's come at intervals of five units.*

*Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of  $p(n)$  that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.*

*Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of  $n$ 's separated by  $5^3 = 125$  units, saying that the corresponding  $p(n)$ 's should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

*From Wikipedia*

*In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field  $\phi$  and a Dirac field  $\psi$ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.*

*Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:*



125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and  $4096 = 64^2$

(Modular equations and approximations to  $\pi$  - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

*In mathematics, the Fibonacci numbers, commonly denoted  $F_n$ , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the  $n$ th Fibonacci number in terms of  $n$  and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as  $n$  increases.*

*Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences*

*The beginning of the sequence is thus:*

*0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...*

*The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

*The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.<sup>[1]</sup> The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

*The sequence of Lucas numbers is:*

*2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....*

*All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

*A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:*

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

*In geometry, a golden spiral is a logarithmic spiral whose growth factor is  $\varphi$ , the golden ratio.<sup>[1]</sup> That is, a golden spiral gets wider (or further from its origin) by a factor of  $\varphi$  for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies<sup>[3]</sup> - golden spirals are one special case of these logarithmic spirals*

We observe that 1728 and 1729 are results very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to  $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

**We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson  $\pi$ ) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.**

## References

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Nov 2017

### Further improvements in Waring's problem

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