# On some possible mathematical connections between various equations concerning the Riemann zeta function, the Riemann's Hypothesis, the Einstein's type Universes, $\phi, \zeta(2)$ and some parameters of Particle Physics. 

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#### Abstract

In this paper we have described some possible mathematical connections between various equations concerning the Riemann zeta function, the Riemann's Hypothesis, the Einstein's type Universes, $\phi, \zeta(2)$ and some parameters of Particle Physics.


[^0]
# 1729 <br> $1^{3}+12^{3}=9^{3}+10^{3}$ 

https://funwithfunctions.es/post/168921716439/ramanujan

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation.

For more information on the data entered for the development of the various equations, see the "Observations" section.

From:
RIEMANN'S HYPOTHESIS AND SOME INFINITE SET OF MICROSCOPIC
UNIVERSES OF THE EINSTEIN'S TYPE IN THE EARLY PERIOD OF THE
EVOLUTION OF THE UNIVERSE - JAN MOSER - arXiv:1307.1095v2 [physics.gen-ph] 28 Jul 2013

We have that (Definition (4.5)-(4.6):

$$
\begin{aligned}
& E_{1}=2\left(\frac{2}{\mu^{2}}-1\right) \Lambda+\mathcal{O}\left\{\left(1+\frac{1}{\mu^{2}}\right) \frac{1}{t_{0}}\right\}, \\
& E_{2}=\frac{2}{\mu^{2}} \Lambda+\mathcal{O}\left\{\left(1+\frac{1}{\mu^{2}}\right) \frac{1}{t_{0}}\right\}, \\
& \kappa c^{2} \rho=\left(\frac{3}{\mu^{2}}-1\right) \Lambda+\mathcal{O}\left\{\left(1+\frac{1}{\mu^{2}}\right) \frac{1}{t_{0}}\right\},
\end{aligned}
$$

For $\Lambda=1.1056 \mathrm{e}-52$ and $\mu=\sqrt{ } 2$, we obtain:
From:

$$
\begin{equation*}
E_{2}=\frac{2}{\mu^{2}} \Lambda+\mathcal{O}\left\{\left(1+\frac{1}{\mu^{2}}\right) \frac{1}{t_{0}}\right\} \tag{a}
\end{equation*}
$$

$2 / 2 * 1.1056 \mathrm{e}-52+((1+1 / 2) * 1 /$ infinity $)=$
$1.1056 * 10^{-52}$
From:

$$
\begin{equation*}
\kappa c^{2} \rho=\left(\frac{3}{\mu^{2}}-1\right) \Lambda+\mathcal{O}\left\{\left(1+\frac{1}{\mu^{2}}\right) \frac{1}{t_{0}}\right\} \tag{b}
\end{equation*}
$$

$(3 / 2-1) * 1.1056 \mathrm{e}-52+((1+1 / 2) * 1 /$ infinity $)$
Input interpretation:
$\left(\frac{3}{2}-1\right) \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}$

## Result:

$5.528 \times 10^{-53}$
$5.528 * 10^{-53}$

From which, from the ratio between (a) and (b):

## Input interpretation:

$\frac{\frac{2}{2} \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}}{\left(\frac{3}{2}-1\right) \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}}$

## Result:

2
2
and the inverse:
$(((3 / 2-1) * 1.1056 \mathrm{e}-52+((1+1 / 2) * 1 /$ infinity $))) /(((2 / 2 * 1.1056 \mathrm{e}-$ $52+((1+1 / 2) * 1 /$ infinity $))))$

Input interpretation:
$\frac{\left(\frac{3}{2}-1\right) \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}}{\frac{2}{2} \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}}$

## Result:

0.5
0.5

## Rational form:

$\frac{1}{2}$

From:
$\frac{p\left(t ; t_{0}, \Lambda, \epsilon\right)}{c^{2} \rho\left(t ; t_{0}, \Lambda, \epsilon\right)}=-\frac{1}{3}+\frac{2 \epsilon^{2}}{9-3 \epsilon^{2}}+\mathcal{O}\left(\frac{1}{\Lambda t_{0}}\right)$
We obtain:
$-1 / 3+\left(2^{*}(1.2183 \mathrm{e}-60)^{\wedge} 2\right) /\left(9-3^{*}(1.2183 \mathrm{e}-60)^{\wedge} 2\right)+\mathrm{O}(1 /$ infinity $)$

## Input interpretation:

$-\frac{1}{3}+\frac{2\left(1.2183 \times 10^{-60}\right)^{2}}{9-3\left(1.2183 \times 10^{-60}\right)^{2}}+O\left(\frac{1}{\infty}\right)$

## Result:

$O(0)-0.333333$

## Alternate forms:

$O(0)-0.333333$
$0.333333(3 O(0)-1)$

That is:
$-\frac{1}{3}+\frac{2\left(1.2183 \times 10^{-60}\right)^{2}}{9-3\left(1.2183 \times 10^{-60}\right)^{2}}+\frac{1}{\infty}$
-0.33333333333333333333333333333333333333333333333333333333...
$-\frac{1}{3}$

From the previous expression

$$
\frac{\left(\frac{3}{2}-1\right) \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}}{\frac{2}{2} \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}}
$$

multiplied by
$-\frac{1}{3}+\frac{2\left(1.2183 \times 10^{-60}\right)^{2}}{9-3\left(1.2183 \times 10^{-60}\right)^{2}}+\frac{1}{\infty}$
we obtain:
$[(((3 / 2-1) * 1.1056 \mathrm{e}-52+((1+1 / 2) * 1 /$ infinity $))) /(((2 / 2 * 1.1056 \mathrm{e}-$
$52+\left((1+1 / 2)^{*} 1 /\right.$ infinity $\left.\left.\left.\left.)\right)\right)\right)\right]\left(\left(\left(-1 / 3+\left(2 *(1.2183 \mathrm{e}-60)^{\wedge} 2\right) /(9-3 *(1.2183 \mathrm{e}-\right.\right.\right.$ $\left.60)^{\wedge} 2\right)+(1 /$ infinity $\left.\left.\left.)\right)\right)\right)$

## Input interpretation:

$$
\frac{\left(\frac{3}{2}-1\right) \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}}{\frac{2}{2} \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}}\left(-\frac{1}{3}+\frac{2\left(1.2183 \times 10^{-60}\right)^{2}}{9-3\left(1.2183 \times 10^{-60}\right)^{2}}+\frac{1}{\infty}\right)
$$

## Result:

-0.16666666666666666666666666666666666666666666666666666666 .
-0.16666...

## Rational approximation:

$-\frac{1}{6}$
$-1 / 6$

While summing the two expressions, we obtain:
$[(((3 / 2-1) * 1.1056 \mathrm{e}-52+((1+1 / 2) * 1 /$ infinity $))) /(((2 / 2 * 1.1056 \mathrm{e}-$
$52+((1+1 / 2) * 1 /$ infinity $))))]+\left(\left(\left(-1 / 3+\left(2 *(1.2183 \mathrm{e}-60)^{\wedge} 2\right) /(9-3 *(1.2183 \mathrm{e}-\right.\right.\right.$ $\left.60)^{\wedge} 2\right)+(1 /$ infinity $\left.\left.)\right)\right)$ )

## Input interpretation:

$$
\frac{\left(\frac{3}{2}-1\right) \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}}{\frac{2}{2} \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}}+\left(-\frac{1}{3}+\frac{2\left(1.2183 \times 10^{-60}\right)^{2}}{9-3\left(1.2183 \times 10^{-60}\right)^{2}}+\frac{1}{\infty}\right)
$$

## Result:

$0.16666666666666666666666666666666666666666666666666666666 \ldots$
0.16666...

## Rational approximation: <br> $\frac{1}{6}$ <br> 1/6

And, dividing the expressions, we obtain:
$[(((3 / 2-1) * 1.1056 \mathrm{e}-52+((1+1 / 2) * 1 /$ infinity $))) /(((2 / 2 * 1.1056 \mathrm{e}-$
$52+\left((1+1 / 2)^{*} 1 /\right.$ infinity $\left.\left.\left.\left.)\right)\right)\right)\right] /\left(\left(\left(-1 / 3+\left(2^{*}(1.2183 \mathrm{e}-60)^{\wedge} 2\right) /\left(9-3^{*}(1.2183 \mathrm{e}-\right.\right.\right.\right.$ $\left.60)^{\wedge} 2\right)+(1 /$ infinity $\left.\left.\left.)\right)\right)\right)$

## Input interpretation:

$$
\frac{\frac{\left(\frac{3}{2}-1\right) \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}}{\frac{2}{2} \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{2}\right) \times \frac{1}{\infty}}}{-\frac{1}{3}+\frac{2\left(1.2183 \times 10^{-60}\right)^{2}}{9-3\left(1.2183 \times 10^{-60}\right)^{2}}+\frac{1}{\infty}}
$$

## Result:

-1.50000000000000000000000000000000000000000000000000000000...
-1.5

## Rational form:

$-\frac{3}{2}$
$-3 / 2$

Now, we have that:

$$
\begin{equation*}
V=V\left(t_{0}, \Lambda, \epsilon\right)=2 \pi^{2} R^{3}=\frac{2 \pi^{2} c^{3}}{\Lambda^{3 / 2}} \epsilon^{3}+\mathcal{O}\left(\frac{1}{t_{0}}\right) . \tag{5.2}
\end{equation*}
$$

that is the expression concerning the volume of the universes. We obtain:
$\left(\left(2^{*} \operatorname{Pi}^{\wedge} 2^{*}(3 \mathrm{e}+8)^{\wedge} 3^{*}(1.2183 \mathrm{e}-60)^{\wedge} 3\right)\right) /\left((1.1056 \mathrm{e}-52)^{\wedge} 1.5\right)+\mathrm{O}(1 /$ infinity $)$

## Input interpretation:

$$
\frac{2 \pi^{2}\left(3 \times 10^{8}\right)^{3}\left(1.2183 \times 10^{-60}\right)^{3}}{\left(1.1056 \times 10^{-52}\right)^{1.5}}+O\left(\frac{1}{\infty}\right)
$$

## Result:

$O(0)+8.29009 \times 10^{-76}$

That is:
$\frac{2 \pi^{2}\left(3 \times 10^{8}\right)^{3}\left(1.2183 \times 10^{-60}\right)^{3}}{\left(1.1056 \times 10^{-52}\right)^{1.5}}+\frac{1}{\infty}$
$8.29009 \ldots \times 10^{-76}$
$8.29009 \ldots * 10^{-76}$

Note that, from the above expression, we can to obtain also the Planck's volume:
$(73493+1729) * 4 / 3 * \mathrm{Pi}^{*} 1.616199 \mathrm{e}-35 *\left(\left(2 * \mathrm{Pi}^{\wedge} 2^{*}(3 \mathrm{e}+8)^{\wedge} 3^{*}(1.2183 \mathrm{e}-60)^{\wedge} 3\right)\right) /$ ((1.1056e-52)^1.5)

## Input interpretation:

$(73493+1729) \times \frac{4}{3} \pi \times 1.616199 \times 10^{-35} \times \frac{2 \pi^{2}\left(3 \times 10^{8}\right)^{3}\left(1.2183 \times 10^{-60}\right)^{3}}{\left(1.1056 \times 10^{-52}\right)^{1.5}}$

## Result:

$4.22170 \ldots \times 10^{-105}$
$4.22170 \ldots * 10^{-105}$
where 73493 is given by Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series:

$$
\begin{gathered}
\binom{\left.J_{21} \& \int_{-\infty}^{+\infty} \exp \left(-\left(\frac{t}{H}\right)^{2}\right)\right|_{\lambda \leqslant P^{1-\varepsilon_{2}}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda-\left.i(T+t)\right|^{2} d t \ll}{\& H\left\{\left(\frac{4}{\varepsilon_{2} \log T}\right)^{2 r}(\log T)(\log X)^{-2 \beta}+\left(\varepsilon_{2}^{-2 r}(\log T)^{-2 r}+\varepsilon_{2}^{-r} h_{1}^{r}(\log T)^{-r}\right) T^{-\varepsilon_{1}}\right\}} / \\
\quad /(26 \times 4)^{2}-24=\left(\frac{7.9313976505275 \times 10^{8}}{(26 \times 4)^{2}-24}\right)=73493.30662 \ldots
\end{gathered}
$$

Now, we have that:

$$
\begin{aligned}
& R\left(t ; t_{0}, \Lambda, \epsilon\right)=\epsilon \frac{c}{\sqrt{\Lambda}}+\mathcal{O}\left(\frac{1}{t_{0}}\right) \\
& \kappa c^{2} \rho\left(t ; t_{0}, \Lambda, \epsilon\right)=\left(\frac{3}{\epsilon^{2}}-1\right) \Lambda+\mathcal{O}\left\{\left(1+\frac{1}{\epsilon^{2}}\right) \frac{1}{t_{0}}\right\}, \\
& \kappa p\left(t ; t_{0}, \Lambda, \epsilon\right)=\left(1-\frac{1}{\epsilon^{2}}\right) \Lambda+\mathcal{O}\left\{\left(1+\frac{1}{\epsilon^{2}}\right) \frac{1}{t_{0}}\right\},
\end{aligned}
$$

From:

$$
R\left(t ; t_{0}, \Lambda, \epsilon\right)=\epsilon \frac{c}{\sqrt{\Lambda}}+\mathcal{O}\left(\frac{1}{t_{0}}\right)
$$

For: $\Lambda=2.036 \times 10^{-35} \mathrm{~s}^{-2} ; \quad \epsilon=1.2183 \mathrm{e}-60$; we obtain:
$((((1.2183 \mathrm{e}-60)) *(3 \mathrm{e}+8))) /(\operatorname{sqrt}(2.036 \mathrm{e}-35))+\mathrm{O}(1 /$ infinity $)$

## Input interpretation:

$\frac{1.2183 \times 10^{-60} \times 3 \times 10^{8}}{\sqrt{2.036 \times 10^{-35}}}+O\left(\frac{1}{\infty}\right)$

## Result:

$O(0)+8.10003 \times 10^{-35}$

## Alternate form:

$O(0)+8.10003 \times 10^{-35}$

## Property as a function: <br> Parity

even
$8.10003 \times 10^{-35}$
$8.10003 * 10^{-35}=$ Planck length

From:

$$
\kappa c^{2} \rho\left(t ; t_{0}, \Lambda, \epsilon\right)=\left(\frac{3}{\epsilon^{2}}-1\right) \Lambda+\mathcal{O}\left\{\left(1+\frac{1}{\epsilon^{2}}\right) \frac{1}{t_{0}}\right\}
$$

we obtain:
$\left(3 /\left(1.2183 \mathrm{e}-60^{\wedge} 2\right)-1\right)^{*}(1.1056 \mathrm{e}-52)+\left(\left(\left(1+1 /(1.2183 \mathrm{e}-60)^{\wedge} 2\right)\right)^{*} 1 /\right.$ infinity $)$
Input interpretation:
$\left(\frac{3}{\left(1.2183 \times 10^{-60}\right)^{2}}-1\right) \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{\left(1.2183 \times 10^{-60}\right)^{2}}\right) \times \frac{1}{\infty}$

## Result:

$2.2346566094183459284409027616543678693893337956258981 \ldots \times 10^{68}$
2.2346566 ... * $10^{68}$

And from:

$$
\kappa p\left(t ; t_{0}, \Lambda, \epsilon\right)=\left(1-\frac{1}{\epsilon^{2}}\right) \Lambda+\mathcal{O}\left\{\left(1+\frac{1}{\epsilon^{2}}\right) \frac{1}{t_{0}}\right\}
$$

we obtain:
$\left(\left(\left(1-1 /(1.2183 \mathrm{e}-60)^{\wedge} 2\right)\right) *(1.1056 \mathrm{e}-52)+\left(\left(\left(1+1 /(1.2183 \mathrm{e}-60)^{\wedge} 2\right)\right)^{*} 1 /\right.\right.$ infinity $)$

## Input interpretation:

$\left(1-\frac{1}{\left(1.2183 \times 10^{-60}\right)^{2}}\right) \times 1.1056 \times 10^{-52}+\left(1+\frac{1}{\left(1.2183 \times 10^{-60}\right)^{2}}\right) \times \frac{1}{\infty}$

## Result:

$-7.448855364727819761469675872181226231297779318752993 \ldots \times 10^{67}$
$-7.44885536 \ldots * 10^{67}$

Dividing the following two results, we obtain:
(2.2346566094183459284409027616543678693893337956258981 $\times 10^{\wedge} 68 /-$ $7.448855364727819761469675872181226231297779318752993 \times 10^{\wedge} 67$ )

## Input interpretation:

$\underline{2.2346566094183459284409027616543678693893337956258981 \times 10^{68}}$
$7.448855364727819761469675872181226231297779318752993 \times 10^{67}$

## Result:

$-3.00000000000000000000000000000000000000000000000000026849 \ldots$
-3

Subtracting the two results, we obtain:
(2.2346566094183459284409027616543678693893337956258981 $\times 10^{\wedge} 68-$ $7.448855364727819761469675872181226231297779318752993 \times 10^{\wedge} 67$ )

## Input interpretation:

$2.2346566094183459284409027616543678693893337956258981 \times 10^{68}{ }_{-}$ $7.448855364727819761469675872181226231297779318752993 \times 10^{67}$

## Result:

148977107294556395229393517443624524625955586375059880000000 :
000000000

## Scientific notation:

$1.4897710729455639522939351744362452462595558637505988 \times 10^{68}$
$1.4897710729 \ldots * 10^{68}$

From which, we obtain also the Planck time:
$1 /(0.69897+1.61834) * 1 / 11468^{\wedge} 2$ *
$1 /(148977107294556395229393517443624524625955586375059880000000000000$ $000)^{\wedge} 1 / 2$

## Input interpretation:

$\frac{1}{0.69897+1.61834} \times \frac{1}{11468^{2}} \times 1 /$ ( $\sqrt{ } 148977107294556395229393517443624524625955586375059880$ : $000000000000000)$

## Result:

$2.68832 \ldots \times 10^{-43}$
$2.68832 \ldots * 10^{-43}=$ Planck time

From the following equation:

$$
\frac{p}{c^{2} \rho}=-\frac{1}{3}+\frac{2 \epsilon^{2}}{9-3 \epsilon^{2}}+\mathcal{O}\left(\frac{1}{\Lambda t_{0}}\right) \in\left(-\frac{1}{3},-\frac{1}{3}+\delta\right)
$$

we obtain, as above:
$-\frac{1}{3}+\frac{2\left(1.2183 \times 10^{-60}\right)^{2}}{9-3\left(1.2183 \times 10^{-60}\right)^{2}}+\frac{1}{\infty}$
-0.33333333333333333333333333333333333333333333333333333333...
$-\frac{1}{3}$

Now, from the various results, we obtain also:

$$
-1 /(2 \mathrm{Pi})(((((((1 / 2) /(-1 / 3))))+((((2) /(-3))))))))^{\wedge}
$$

## Input:

$-\frac{\left(-\frac{\frac{1}{2}}{1}+-\frac{2}{3}\right)^{3}}{2 \pi}$

## Result:

$\frac{2197}{432 \pi}$

## Decimal approximation:

1.618812083207842836501100130228768765693091859684179712493...
1.618812083.... result that is a very good approximation to the value of the golden ratio 1.618033988749 ...

## Property:

$\frac{2197}{432 \pi}$ is a transcendental number

## Alternative representations:

$$
\frac{\left(\frac{1}{\frac{2(-1)}{3}}+-\frac{2}{3}\right)^{3}(-1)}{2 \pi}=-\frac{\left(-\frac{2}{3}+\frac{1}{\frac{2(-1)}{3}}\right)^{3}}{360^{\circ}}
$$

$$
\frac{\left(\frac{\frac{1}{2(-1)}}{3}+-\frac{2}{3}\right)^{3}(-1)}{2 \pi}=\frac{-\left(-\frac{2}{3}+\frac{1}{\frac{2(-1)}{3}}\right)^{3}}{-2 i \log (-1)}
$$

$$
\frac{\left(\frac{\frac{1}{2(-1)}}{3}+-\frac{2}{3}\right)^{3}(-1)}{2 \pi}=-\frac{\left(-\frac{2}{3}+\frac{1}{\frac{2(-1)}{3}}\right)^{3}}{2 \cos ^{-1}(-1)}
$$

## Series representations:

$$
\frac{\left(\frac{1}{\frac{2(-1)}{3}}+-\frac{2}{3}\right)^{3}(-1)}{2 \pi}=\frac{2197}{1728 \sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}}
$$

$$
\frac{\left(\frac{1}{\frac{2(-1)}{3}}+-\frac{2}{3}\right)^{3}(-1)}{2 \pi}=\frac{2197}{1728 \sum_{k=0}^{\infty} \frac{(-1)^{1+k} 1195^{-1-2 k}\left(5^{1+2 k}-4 \times 239^{1+2 k}\right)}{1+2 k}}
$$

$$
\frac{\left(\frac{1}{\frac{2(-1)}{3}}+-\frac{2}{3}\right)^{3}(-1)}{2 \pi}=\frac{2197}{432 \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^{k}\left(\frac{1}{1+2 k}+\frac{2}{1+4 k}+\frac{1}{3+4 k}\right)}
$$

## Integral representations:

$$
\begin{aligned}
& \frac{\left(\frac{1}{\frac{2(-1)}{3}}+-\frac{2}{3}\right)^{3}(-1)}{2 \pi}=\frac{2197}{1728 \int_{0}^{1} \sqrt{1-t^{2}} d t} \\
& \frac{\left(\frac{1}{\frac{2(-1)}{3}}+-\frac{2}{3}\right)^{3}(-1)}{2 \pi}=\frac{2197}{864 \int_{0}^{\infty} \frac{1}{1+t^{2}} d t} \\
& \frac{\left(\frac{1}{\frac{2(-1)}{3}}+-\frac{2}{3}\right)^{3}(-1)}{2 \pi}=\frac{2197}{864 \int_{0}^{1} \frac{1}{\sqrt{1-t^{2}}} d t}
\end{aligned}
$$

Now, we have that:
1.2. In this paper we will study the following quotient

$$
0<\frac{Z(t)}{Z\left(t_{0}\right)}, \quad t \in\left(\gamma^{\prime}, \gamma^{\prime \prime}\right), \gamma^{\prime}<t_{0}<\gamma^{\prime \prime}, \quad \gamma^{\prime} \rightarrow \infty
$$

where $Z\left(t_{0}\right)$ is the local maximum or local minimum of the function $Z(t)$. We make use the estimate (1.2) to prove the following theorem.

Theorem. If

$$
\begin{align*}
& \Omega\left(t_{0}\right)=t_{0} \ln ^{4} t_{0}+\ln t_{0} \ln w\left(t_{0}\right) \\
& w\left(t_{0}\right)=\max \left\{\frac{2}{\omega\left(t_{0}\right)} t_{0} \ln ^{3} t_{0}, \frac{2}{m\left(t_{0}\right)}\right\}, \omega\left(t_{0}\right) \in(0,1),  \tag{1.4}\\
& \Delta_{1}\left(t_{0}\right)=\left\{1-\omega\left(t_{0}\right)\right\}\left(t_{0}-\gamma^{\prime}\right), \Delta_{2}\left(t_{0}\right)=\left\{1-\omega\left(t_{0}\right)\right\}\left(\gamma^{\prime \prime}-t_{0}\right), \\
& J\left(t_{0}\right)=\left[t_{0}-\Delta_{1}\left(t_{0}\right), t_{0}+\Delta_{2}\left(t_{0}\right)\right]
\end{align*}
$$

For: $\gamma^{\prime}=3 ; \mathrm{t}_{0}=5 ; \gamma^{\prime}=8 ; \quad \omega\left(t_{0}\right)=10^{-58}, \ldots$
From
$\Omega\left(t_{0}\right)=t_{0} \ln ^{4} t_{0}+\ln t_{0} \ln w\left(t_{0}\right)$,
we obtain:
$5 \ln \wedge 4(5)+\ln (5) * \ln \left(10^{\wedge}-58\right)$

## Input:

$5 \log ^{4}(5)+\log (5) \log \left(\frac{1}{10^{58}}\right)$

## Exact result:

$5 \log ^{4}(5)-\log (5) \log ($
10000000000000000000000000000000000000000000000000000000000 )

## Decimal approximation:

-181.392307599941110537532222328482066454668241682549582693.
-181.3923075...

## Alternate forms:

$5 \log ^{4}(5)-58 \log (5) \log (10)$
$\log (5)\left(5 \log ^{3}(5)-\log (\right.$
$10000000000000000000000000000000000000000000000000000000000)$
$5 \log ^{4}(5)-58 \log ^{2}(5)-58 \log (2) \log (5)$

## Alternative representations:

$5 \log ^{4}(5)+\log (5) \log \left(\frac{1}{10^{58}}\right)=\log _{e}(5) \log _{e}\left(\frac{1}{10^{58}}\right)+5 \log _{e}^{4}(5)$
$5 \log ^{4}(5)+\log (5) \log \left(\frac{1}{10^{58}}\right)=\log ^{2}(a) \log _{a}(5) \log _{a}\left(\frac{1}{10^{58}}\right)+5\left(\log (a) \log _{a}(5)\right)^{4}$
$5 \log ^{4}(5)+\log (5) \log \left(\frac{1}{10^{58}}\right)=\mathrm{Li}_{1}(-4) \mathrm{Li}_{1}\left(1-\frac{1}{10^{58}}\right)+5\left(-\mathrm{Li}_{1}(-4)\right)^{4}$

## Series representations:

$5 \log ^{4}(5)+\log (5) \log \left(\frac{1}{10^{58}}\right)=\left(\log (4)-\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}}{k}\right)\left(5 \log ^{3}(4)-\log (\right.$
9999999999999999999999999999999999999999999999999999 :

$$
\begin{gathered}
999999)-15 \log ^{2}(4) \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}}{k}+ \\
15 \log (4)\left(\sum_{k=1}^{\infty} \frac{\left.\left(-\frac{1}{4}\right)^{k}\right)^{2}-5\left(\sum_{k=1}^{\infty} \frac{\left.\left(-\frac{1}{4}\right)^{k}\right)^{3}}{k}\right)_{1}^{3}+}{\left.\sum_{k=1}^{\infty} \frac{\left(-\frac{90999999999999999999999999999999999999999999999999}{}\right)^{k}}{k}\right)}\right.
\end{gathered}
$$

$$
\begin{aligned}
& 5 \log ^{4}(5)+\log (5) \log \left(\frac{1}{10^{58}}\right)= \\
& 5\left(\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{4}- \\
& \left(\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right) \\
& \left(\log \left(z_{0}\right)+\left\lfloor\frac{1}{2 \pi} \arg ( \right.\right. \\
& 10000000000000000000000000000000000000000000000 \\
& \left.\left.000000000000-z_{0}\right)\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)- \\
& \sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k} \\
& (10000000000000000000000000000000000000000000000000
\end{aligned}
$$

$5 \log ^{4}(5)+\log (5) \log \left(\frac{1}{10^{58}}\right)=\left(\operatorname{Res}_{s=0} \frac{4^{-s} \Gamma(-s) \Gamma(1+s)}{s}+\sum_{j=1}^{\infty} \operatorname{Res}_{s=j} \frac{4^{-s} \Gamma(-s) \Gamma(1+s)}{s}\right)$

$$
\left(5\left(\operatorname{Res}_{s=0} \frac{4^{-s} \Gamma(-s) \Gamma(1+s)}{s}\right)^{3}-\operatorname{Res}_{s=0} \frac{1}{s}\right.
$$

9999999999999999999999999999999999999999999999999999 :

$$
999999^{-s} \Gamma(-s) \Gamma(1+s)+
$$

$$
15\left(\operatorname{Res}_{s=0} \frac{4^{-s} \Gamma(-s) \Gamma(1+s)}{s}\right)^{2} \sum_{j=1}^{\infty} \operatorname{Res}_{s=j} \frac{4^{-s} \Gamma(-s) \Gamma(1+s)}{s}+
$$

$$
15\left(\operatorname{Res}_{s=0} \frac{4^{-s} \Gamma(-s) \Gamma(1+s)}{s}\right)\left(\sum_{j=1}^{\infty} \operatorname{Res}_{s=j} \frac{4^{-s} \Gamma(-s) \Gamma(1+s)}{s}\right)^{2}+
$$

$$
5\left(\sum_{j=1}^{\infty} \operatorname{Res}_{s=j} \frac{4^{-s} \Gamma(-s) \Gamma(1+s)}{s}\right)^{3}-\sum_{j=1}^{\infty} \operatorname{Res}_{s=j} \frac{1}{s}
$$

9999999999999999999999999999999999999999999999999999 :

$$
\left.999999^{-s} \Gamma(-s) \Gamma(1+s)\right)
$$

## Integral representations:

$\begin{aligned} 5 \log ^{4}(5) & +\log (5) \log \left(\frac{1}{10^{58}}\right)=\left(\int_{1}^{5} \frac{1}{t} d t\right)\left(5\left(\int_{1}^{5} \frac{1}{t} d t\right)^{3}-\right. \\ & \left.\int_{1}^{1000000000000000000000000000000000000000000000000000000} \frac{1}{t} d t\right)\end{aligned}$

# $5 \log ^{4}(5)+\log (5) \log \left(\frac{1}{10^{58}}\right)=\frac{1}{16 \pi^{4}}\left(\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{4^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s\right)$ 

$$
\left(5\left(\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{4^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s\right)^{3}+4 \pi^{2} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{1}{\Gamma(1-s)}\right.
$$

9999999999999999999999999999999999999999999999999 :

$$
\left.999999999^{-s} \Gamma(-s)^{2} \Gamma(1+s) d s\right) \text { for }-1<\gamma<0
$$

From:
$\Delta_{1}\left(t_{0}\right)=\left\{1-\omega\left(t_{0}\right)\right\}\left(t_{0}-\gamma^{\prime}\right), \Delta_{2}\left(t_{0}\right)=\left\{1-\omega\left(t_{0}\right)\right\}\left(\gamma^{\prime \prime}-t_{0}\right)$,
we obtain (a):
(1-10^-58)(5-3)

## Input:

$\left(1-\frac{1}{10^{58}}\right)(5-3)$

## Exact result:

9999999999999999999999999999999999999999999999999999999999
5000000000000000000000000000000000000000000000000000000000

## Decimal approximation:

1.9999999999999999999999999999999999999999999999999999999998
1.9999999999 $\qquad$ $\approx 2$
and (b):
$\left(1-10^{\wedge}-58\right)(8-5)$

## Input:

$\left(1-\frac{1}{10^{58}}\right)(8-5)$

## Exact result:

29999999999999999999999999999999999999999999999999999999997
10000000000000000000000000000000000000000000000000000000000

## Decimal approximation:

2.999999999999999999999999999999999999999999999999999999999...
2.999999999999 $\qquad$ $\approx 3$

We have:

$$
\begin{align*}
& \sum_{\gamma^{\prime \prime}+1 \leq \gamma} \frac{1}{(t-\gamma)^{2}}<\sum_{\gamma^{\prime \prime}-1 \leq \gamma} \frac{1}{\left(\gamma-\gamma^{\prime \prime}\right)^{2}}= \\
& =\sum_{n=1}^{\infty}\left\{\sum_{\gamma^{\prime \prime}+n \leq \gamma \leq \gamma^{\prime \prime}+n+1} \frac{1}{\left(\gamma-\gamma^{\prime \prime}\right)^{2}}\right\}<  \tag{6.5}\\
& <A \sum_{n=1}^{\infty} \frac{\ln \left(\gamma^{\prime \prime}+n\right)}{n^{2}}<A \sum_{n \leq \gamma^{\prime \prime}} \frac{\ln 2 \gamma^{\prime \prime}}{n^{2}}+\sum_{\gamma^{\prime \prime}<n} \frac{\ln 2 n}{n^{2}}< \\
& <A \ln t_{0}, \quad t \in J\left(t_{0}\right) .
\end{align*}
$$

For: $\gamma^{\prime}=3 ; \mathrm{t}_{0}=5 ; \gamma^{\prime \prime}=8 ; \quad \omega\left(t_{0}\right)=10^{-58}, \ldots$
$\gamma=2 ; \mathrm{t}=6 ; \mathrm{A}=2.5$

From (6.5), we obtain:
$1 /(2-8)^{\wedge} 2<2.5 \ln (5)$

## Input:

$\frac{1}{(2-8)^{2}}<2.5 \log (5)$
$\log (x)$ is the natural logarithm

## Result:

True
$1 /(2-8)^{\wedge} 2$
Input:
$\frac{1}{(2-8)^{2}}$

Exact result:
$\frac{1}{36}$

Decimal approximation:
0.027777777777777777777777777777777777777777777777777777
0.02777777777
$2.5 \ln (5)$
Input:
$2.5 \log (5)$
$\log (x)$ is the natural logarithm

Result:
4.02359...
4.02359...

## Alternative representations:

$2.5 \log (5)=2.5 \log _{e}(5)$
$2.5 \log (5)=2.5 \log (a) \log _{a}(5)$
$2.5 \log (5)=-2.5 \mathrm{Li}_{1}(-4)$

## Series representations:

$2.5 \log (5)=2.5 \log (4)-2.5 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}}{k}$
$2.5 \log (5)=5 i \pi\left\lfloor\frac{\arg (5-x)}{2 \pi}\right\rfloor+2.5 \log (x)-2.5 \sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}$ for $x<0$
$2.5 \log (5)=2.5\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor \log \left(\frac{1}{z_{0}}\right)+$
$2.5 \log \left(z_{0}\right)+2.5\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor \log \left(z_{0}\right)-2.5 \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}$

## Integral representations:

$2.5 \log (5)=2.5 \int_{1}^{5} \frac{1}{t} d t$
$2.5 \log (5)=\frac{1.25}{i \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{4^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s$ for $-1<\gamma<0$

From:

$$
\begin{equation*}
\sum_{\gamma \leq \gamma^{\prime}+1} \frac{1}{(t-\gamma)^{2}}<A \ln t_{0}, \quad t \in J\left(t_{0}\right) \tag{6.6}
\end{equation*}
$$

we obtain:
$1 /(6-2)^{\wedge} 2$

## Input:

$\frac{1}{(6-2)^{2}}$

## Exact result:

$\frac{1}{16}$

## Decimal form:

0.0625
0.0625

From these results, we obtain:
$1 /(2-8)^{\wedge} 2-1 /(6-2)^{\wedge} 2-2.5 \ln (5)$

## Input:

$\frac{1}{(2-8)^{2}}-\frac{1}{(6-2)^{2}}-2.5 \log (5)$

## Result:

-4.05832...
-4.05832...

## Alternative representations:

$\frac{1}{(2-8)^{2}}-\frac{1}{(6-2)^{2}}-2.5 \log (5)=-2.5 \log _{e}(5)+\frac{1}{(-6)^{2}}-\frac{1}{4^{2}}$
$\frac{1}{(2-8)^{2}}-\frac{1}{(6-2)^{2}}-2.5 \log (5)=-2.5 \log (a) \log _{a}(5)+\frac{1}{(-6)^{2}}-\frac{1}{4^{2}}$
$\frac{1}{(2-8)^{2}}-\frac{1}{(6-2)^{2}}-2.5 \log (5)=2.5 \mathrm{Li}_{1}(-4)+\frac{1}{(-6)^{2}}-\frac{1}{4^{2}}$

## Series representations:

$\frac{1}{(2-8)^{2}}-\frac{1}{(6-2)^{2}}-2.5 \log (5)=-0.0347222-2.5 \log (4)+2.5 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}}{k}$

$$
\begin{aligned}
& \frac{1}{(2-8)^{2}}-\frac{1}{(6-2)^{2}}-2.5 \log (5)= \\
& \quad-0.0347222-5 i \pi\left[\frac{\arg (5-x)}{2 \pi}\right]-2.5 \log (x)+2.5 \sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k} \text { for } x<0
\end{aligned}
$$

$$
\frac{1}{(2-8)^{2}}-\frac{1}{(6-2)^{2}}-2.5 \log (5)=-\frac{5}{144}-2.5\left[\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right] \log \left(\frac{1}{z_{0}}\right)-
$$

$$
2.5 \log \left(z_{0}\right)-2.5\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor \log \left(z_{0}\right)+2.5 \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}
$$

## Integral representations:

$\frac{1}{(2-8)^{2}}-\frac{1}{(6-2)^{2}}-2.5 \log (5)=-0.0347222-2.5 \int_{1}^{5} \frac{1}{t} d t$
$\frac{1}{(2-8)^{2}}-\frac{1}{(6-2)^{2}}-2.5 \log (5)=-0.0347222-\frac{1.25}{i \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{4^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s$
for $-1<\gamma<0$
$1 /(2-8)^{\wedge} 2+1 /(6-2)^{\wedge} 2+2.5 \ln (5)$
Input:
$\frac{1}{(2-8)^{2}}+\frac{1}{(6-2)^{2}}+2.5 \log (5)$
$\log (x)$ is the natural logarithm

## Result:

4.11387...
4.11387...

## Alternative representations:

$\frac{1}{(2-8)^{2}}+\frac{1}{(6-2)^{2}}+2.5 \log (5)=2.5 \log _{e}(5)+\frac{1}{(-6)^{2}}+\frac{1}{4^{2}}$
$\frac{1}{(2-8)^{2}}+\frac{1}{(6-2)^{2}}+2.5 \log (5)=2.5 \log (a) \log _{a}(5)+\frac{1}{(-6)^{2}}+\frac{1}{4^{2}}$
$\frac{1}{(2-8)^{2}}+\frac{1}{(6-2)^{2}}+2.5 \log (5)=-2.5 \mathrm{Li}_{1}(-4)+\frac{1}{(-6)^{2}}+\frac{1}{4^{2}}$

## Series representations:

$\frac{1}{(2-8)^{2}}+\frac{1}{(6-2)^{2}}+2.5 \log (5)=0.0902778+2.5 \log (4)-2.5 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}}{k}$
$\frac{1}{(2-8)^{2}}+\frac{1}{(6-2)^{2}}+2.5 \log (5)=$
$0.0902778+5 i \pi\left\lfloor\frac{\arg (5-x)}{2 \pi}\right\rfloor+2.5 \log (x)-2.5 \sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}$ for $x<0$

$$
\begin{aligned}
& \frac{1}{(2-8)^{2}}+\frac{1}{(6-2)^{2}}+2.5 \log (5)=\frac{13}{144}+2.5\left\lfloor\left.\frac{\arg \left(5-z_{0}\right)}{2 \pi} \right\rvert\, \log \left(\frac{1}{z_{0}}\right)+\right. \\
& 2.5 \log \left(z_{0}\right)+2.5\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor \log \left(z_{0}\right)-2.5 \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}
\end{aligned}
$$

## Integral representations:

$\frac{1}{(2-8)^{2}}+\frac{1}{(6-2)^{2}}+2.5 \log (5)=0.0902778+2.5 \int_{1}^{5} \frac{1}{t} d t$
$\frac{1}{(2-8)^{2}}+\frac{1}{(6-2)^{2}}+2.5 \log (5)=0.0902778+\frac{1.25}{i \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{4^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s$
for $-1<\gamma<0$
$1 /\left(\left(1 /(2-8)^{\wedge} 2 * 1 /(6-2)^{\wedge} 2 * 2.5 \ln (5)\right)\right)$

## Input:

$\frac{1}{\frac{1}{(2-8)^{2}} \times \frac{1}{(6-2)^{2}} \times 2.5 \log (5)}$
$\log (x)$ is the natural logarithm

## Result:

143.156...
143.156....

## Alternative representations:

$$
\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}=\frac{1}{\frac{2.5 \log _{e}(5)}{(-6)^{2} 4^{2}}}
$$

$$
\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}=\frac{1}{\frac{2.5 \log (a) \log _{a}(5)}{(-6)^{2} 4^{2}}}
$$

$$
\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}=-\frac{1}{\frac{2.5 \mathrm{Li}_{1}(-4)}{(-6)^{2} 4^{2}}}
$$

## Series representations:

$\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}=\frac{230.4}{\log (4)-\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}}{k}}$
$\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}=\frac{230.4}{2 i \pi\left\lfloor\frac{\operatorname{agg}(5-x)}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-x x^{k} x^{-k}\right.}{k}}$ for $x<0$
$\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}=\frac{230.4}{\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{k}}{k}}$

Integral representations:
$\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}=\frac{230.4}{\int_{1}^{5} \frac{1}{t} d t}$
$\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}=\frac{460.8 i \pi}{\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{4^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s}$ for $-1<\gamma<0$
$\left.\left(\left(\left(\left(1 /\left(\left(1 /(2-8)^{\wedge} 2 * 1 /(6-2)^{\wedge} 2 * 2.5 \ln (5)\right)\right)\right)\right)\right)\right)\right)^{\wedge} 1 / 10$
Input:
$\sqrt[10]{\frac{1}{\frac{1}{(2-8)^{2}} \times \frac{1}{(6-2)^{2}} \times 2.5 \log (5)}}$

## Result:

1.642785363126584884785003045638089762771246344213172677529...
$1.64278536 \ldots \approx \zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

## Input:

$\sqrt[10]{\frac{1}{\frac{1}{(2-8)^{2}} \times \frac{1}{(6-2)^{2}} \times 2.5 \log (5)}-\frac{24}{10^{3}}}$
$\log (x)$ is the natural logarithm

## Result:

1.618785...
$1.618785 \ldots$ result that is a very good approximation to the value of the golden ratio 1.618033988749...

## Alternative representations:

$\sqrt[10]{\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}}-\frac{24}{10^{3}}=-\frac{24}{10^{3}}+\sqrt[10]{\frac{1}{\frac{2.5 \log _{e}(5)}{(-6)^{2} 4^{2}}}}$
$\sqrt[10]{\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}}-\frac{24}{10^{3}}=-\frac{24}{10^{3}}+\sqrt[10]{-\frac{1}{\frac{2.5 \mathrm{Li}_{1}(-4)}{(-6)^{2} 4^{2}}}}$
$\sqrt[10]{\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}}-\frac{24}{10^{3}}=-\frac{24}{10^{3}}+\sqrt[10]{\frac{1}{\frac{2.5 \log (a) \log _{a}(5)}{(-6)^{2} 4^{2}}}}$

## Series representations:

$\sqrt[10]{\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}}-\frac{24}{10^{3}}=-0.024+1.72285 \sqrt[10]{\frac{1}{\log (4)-\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}}{k}}}$
$\sqrt[10]{\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}}-\frac{24}{10^{3}}=$
$-0.024+1.72285 \sqrt[10]{\frac{1}{2 i \pi\left[\frac{\arg (5-x)}{2 \pi}\right]+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}}}$ for $x<0$

$$
\begin{aligned}
& \sqrt[10]{\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}}-\frac{24}{10^{3}}= \\
& -\frac{3}{125}+1.72285 \sqrt[10]{\frac{1}{\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}}}
\end{aligned}
$$

## Integral representations:

$\sqrt[10]{\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}}-\frac{24}{10^{3}}=-0.024+1.72285 \sqrt[10]{\frac{1}{\int_{1}^{5} \frac{1}{t} d t}}$
$\sqrt[10]{\frac{1}{\frac{2.5 \log (5)}{(2-8)^{2}(6-2)^{2}}}}-\frac{24}{10^{3}}=-0.024+1.84651 \sqrt[10]{\frac{i \pi}{\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{4^{-s} \Gamma(-s)^{2} \Gamma(1+\mathrm{s})}{\Gamma(1-s)} d s}}$ for $-1<\gamma<0$

Now, we have that:
(A). If

$$
\gamma \leq \gamma^{\prime}-1,
$$

then we have for $t \in J\left(t_{0}\right)$ by (1.7) that

$$
\begin{equation*}
\left|\frac{t-t_{0}}{t_{u}-\gamma}\right| \leq \frac{\left|t-t_{0}\right|}{t_{0}-\gamma^{\prime}+1}<\gamma^{\prime \prime}-\gamma^{\prime}<\frac{A}{\ln \ln t_{0}} . \tag{8.2}
\end{equation*}
$$

Next,

$$
\begin{align*}
& \frac{t-t_{0}}{t_{0}-\gamma}-\ln \left|\frac{t-\gamma}{t_{0}-\gamma}\right|=\frac{t-t_{0}}{t_{0}-\gamma}-\ln \left(1+\frac{t-t_{0}}{t_{0}-\gamma}\right)= \\
& =\left(\frac{t-t_{0}}{t_{0}-\gamma}\right)^{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{k+2}\left(\frac{t-t_{0}}{t_{0}-\gamma}\right)^{k} .  \tag{8.3}\\
& \left.\sum_{k=0}^{\infty}(-1)^{k}\binom{t-t_{0}}{t_{0}-\gamma}^{k}={ }_{2}^{1} \right\rvert\, \mathcal{O}\left\{\sum_{k=1}^{\infty}\left|\begin{array}{c}
t-t_{0} \\
t_{0}-\gamma
\end{array}\right|^{k}\right\}=  \tag{8.4}\\
& =\frac{1}{2}+\mathcal{O}\left(\frac{1}{\ln \ln t_{0}}\right) .
\end{align*}
$$

Consoquently,

$$
\begin{aligned}
& \sum_{\gamma \leq \gamma^{\prime}-1}\left\{\frac{t-t_{0}}{t_{0}-\gamma}-\ln \left|\frac{t-\gamma}{t_{0}-\gamma}\right|\right\}= \\
& =\left(t-t_{0}\right)^{2} \sum_{\gamma \leq \gamma^{\prime}-1} \frac{1}{\left(t_{0}-\gamma\right)^{2}}\left\{\frac{1}{2}+\mathcal{O}\left(\frac{1}{\ln \ln t_{0}}\right)\right\},
\end{aligned}
$$

and from this (see (1.7), (6.6))

$$
\begin{equation*}
0 \leq \sum_{\gamma \leq \gamma^{\prime}-1}\left\{\frac{t-t_{0}}{t_{0}-\gamma}-\ln \left|\frac{t-\gamma}{t_{0}-\gamma}\right|\right\}<A \frac{\ln t_{0}}{\left(\ln \ln t_{0}\right)^{2}}, \quad t \in . J\left(t_{0}\right) . \tag{8.5}
\end{equation*}
$$

From (8.2), we obtain:

$$
\left|\frac{t-t_{0}}{t_{0}-\gamma}\right| \leq \frac{\left|t-t_{0}\right|}{t_{0}-\gamma^{\prime}+1}<\gamma^{\prime \prime}-\gamma^{\prime}<\frac{A}{\ln \ln t_{0}} .
$$

For: $\gamma^{\prime}=3 ; \mathrm{t}_{0}=5 ; \quad \gamma^{\prime \prime}=8 ; \quad \omega\left(t_{0}\right)=10^{-58}, \ldots$
$\gamma=2 ; \mathrm{t}=6 ; \mathrm{A}=2.5$
$2.5 /(\ln \ln (5))$

## Input: <br> $\frac{2.5}{\log (\log (5))}$

## Result:

5.25337.
5.25337...

## Alternative representations:

$\frac{2.5}{\log (\log (5))}=\frac{2.5}{\log _{e}(\log (5))}$
$\frac{2.5}{\log (\log (5))}=\frac{2.5}{\log (a) \log _{a}(\log (5))}$
$\frac{2.5}{\log (\log (5))}=-\frac{2.5}{\mathrm{Li}_{1}(1-\log (5))}$

## Series representations:

$\frac{2.5}{\log (\log (5))}=-\frac{2.5}{\sum_{k=1}^{\infty} \frac{(-1)^{k}(-1+\log (5))^{k}}{k}}$
$\frac{2.5}{\log (\log (5))}=\frac{2.5}{2 i \pi\left\lfloor\frac{\operatorname{agg}(-x+\log (5))}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k} x^{-k}(-x+\log (5))^{k}}{k}}$ for $x<0$
$\frac{2.5}{\log (\log (5))}=\frac{2.5}{\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(\log (5)-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\log (5)-z_{0}\right)^{k} z_{0}^{-k}}{k}}$

## Integral representations:

$\frac{2.5}{\log (\log (5))}=\frac{2.5}{\int_{1}^{\log (5)} \frac{1}{t} d t}$
$\frac{2.5}{\log (\log (5))}=\frac{5 i \pi}{\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\Gamma(-s)^{2} \Gamma(1+s)(-1+\log (5))^{-s}}{\Gamma(1-s)} d s}$ for $-1<\gamma<0$

Thence:
$5<5.25337$

From (8.3), we obtain:

$$
\begin{aligned}
& \frac{t-t_{0}}{t_{0}-\gamma}-\ln \left|\frac{t-\gamma}{t_{0}-\gamma}\right|=\frac{t-t_{0}}{t_{0}-\gamma}-\ln \left(1+\frac{t-t_{0}}{t_{0}-\gamma}\right)= \\
& =\left(\frac{t-t_{0}}{t_{0}-\gamma}\right)^{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{k+2}\left(\frac{t-t_{0}}{t_{0}-\gamma}\right)^{k}
\end{aligned}
$$

For: $\gamma^{\prime}=3 ; \mathfrak{t}_{0}=5 ; \gamma^{\prime \prime}=8 ; \quad \omega\left(t_{0}\right)=10^{-58}, \ldots$
$\gamma=2 ; \mathrm{t}=6 ; \mathrm{A}=2.5$
$(1 / 3)^{\wedge} 2 \operatorname{sum}\left((-1)^{\wedge} k /(k+2)(1 / 3)^{\wedge} k, k=0\right.$..infinity

## Input interpretation:

$\left(\frac{1}{3}\right)^{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{k+2}\left(\frac{1}{3}\right)^{k}$

## Result:

$\frac{1}{9}\left(3-9 \log \left(\frac{4}{3}\right)\right) \approx 0.0456513$
$\log (x)$ is the natural logarithm
0.0456513

Alternate forms:
$\frac{1}{3}-\log \left(\frac{4}{3}\right)$
$\frac{1}{3}\left(1-3 \log \left(\frac{4}{3}\right)\right)$
$\frac{1}{3}(1-6 \log (2)+3 \log (3))$

From (8.4), we obtain:

$$
\begin{aligned}
& \sum_{k=0}^{\infty} \frac{(-1)^{k}}{k+2}\left(\frac{t-t_{0}}{t_{0}-\gamma}\right)^{k}=\frac{1}{2}+\mathcal{O}\left\{\sum_{k=1}^{\infty}\left|\frac{t-t_{0}}{t_{0}-\gamma}\right|^{k}\right\}= \\
& =\frac{1}{2}+\mathcal{O}\left(\frac{1}{\ln \ln t_{0}}\right)
\end{aligned}
$$

$1 / 2+(1 /(\ln \ln (5))$
Input:
$\frac{1}{2}+\frac{1}{\log (\log (5))}$
$\log (x)$ is the natural logarithm

## Decimal approximation:

2.601348035385370238791927163840092024061482830296524086601...
2.60134803538......

## Alternate form:

$\frac{2+\log (\log (5))}{2 \log (\log (5))}$

Alternative representations:
$\frac{1}{2}+\frac{1}{\log (\log (5))}=\frac{1}{2}+\frac{1}{\log _{e}(\log (5))}$
$\frac{1}{2}+\frac{1}{\log (\log (5))}=\frac{1}{2}+\frac{1}{\log (a) \log _{a}(\log (5))}$

$$
\frac{1}{2}+\frac{1}{\log (\log (5))}=\frac{1}{2}+-\frac{1}{\mathrm{Li}_{1}(1-\log (5))}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1}{2}+\frac{1}{\log (\log (5))}=\frac{1}{2}-\frac{1}{\sum_{k=1}^{\infty} \frac{(1-\log (5))^{k}}{k}} \\
& \frac{1}{2}+\frac{1}{\log (\log (5))}=\frac{1}{2}+\frac{1}{2 i \pi\left\lfloor\frac{\arg (-x+\log (5))}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k} x^{-k}(-x+\log (5))^{k}}{k}} \text { for } x<0 \\
& \frac{1}{2}+\frac{1}{\log (\log (5))}= \\
& \frac{1}{2}+\frac{1}{\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(\log (5)-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\log (5)-z_{0}\right)^{k} z_{0}^{-k}}{k}}
\end{aligned}
$$

## Integral representations:

$\frac{1}{2}+\frac{1}{\log (\log (5))}=\frac{1}{2}+\frac{1}{\int_{1}^{\log (5) \frac{1}{t} d t}}$
$\frac{1}{2}+\frac{1}{\log (\log (5))}=\frac{1}{2}+\frac{2 i \pi}{\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\Gamma(-s)^{2} \Gamma(1+s)(-1+\log (5))^{-s}}{\Gamma(1-s)} d s}$ for $-1<\gamma<0$

We obtain also:
$(((1 / 2+(1 /(\ln \ln (5)))))))^{\wedge} 1 / 2$
Input:
$\sqrt{\frac{1}{2}+\frac{1}{\log (\log (5))}}$
$\log (x)$ is the natural logarithm

## Decimal approximation:

1.612869503520160876748055117184761672996826390050388052755...
$1.6128695035 \ldots$. result that is a good approximation to the value of the golden ratio 1.618033988749...

## Alternate form:

$\sqrt{\frac{2+\log (\log (5))}{2 \log (\log (5))}}$

## All 2nd roots of $1 / 2+1 / \log (\log (5))$ :

$e^{0} \sqrt{\frac{1}{2}+\frac{1}{\log (\log (5))}} \approx 1.6129$ (real. principal root)
$e^{i \pi} \sqrt{\frac{1}{2}+\frac{1}{\log (\log (5))}} \approx-1.6129$ (real root)

## Alternative representations:

$\sqrt{\frac{1}{2}+\frac{1}{\log (\log (5))}}=\sqrt{\frac{1}{2}+\frac{1}{\log _{e}(\log (5))}}$
$\sqrt{\frac{1}{2}+\frac{1}{\log (\log (5))}}=\sqrt{\frac{1}{2}+\frac{1}{\log (a) \log _{a}(\log (5))}}$
$\sqrt{\frac{1}{2}+\frac{1}{\log (\log (5))}}=\sqrt{\frac{1}{2}+-\frac{1}{\mathrm{Li}_{1}(1-\log (5))}}$

## Series representations:

$\sqrt{\frac{1}{2}+\frac{1}{\log (\log (5))}}=\sqrt{\frac{1}{2}-\frac{1}{\sum_{k=1}^{\infty} \frac{(1-\log (5))^{k}}{k}}}$
$\sqrt{\frac{1}{2}+\frac{1}{\log (\log (5))}}=\sqrt{\frac{1}{2}+\frac{1}{2 i \pi\left\lfloor\frac{\arg (-x+\log (5))}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k} x^{-k}(-x+\log (5))^{k}}{k}}}$
for $x<0$

$$
\sqrt{\frac{1}{2}+\frac{1}{\log (\log (5))}}=
$$

$$
\sqrt{\frac{1}{2}+\frac{1}{\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(\log (5)-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\log (5)-z_{0}\right)^{k} z_{0}^{-k}}{k}}}
$$

## Integral representations:

$\sqrt{\frac{1}{2}+\frac{1}{\log (\log (5))}}=\sqrt{\frac{1}{2}+\frac{1}{\int_{1}^{\log (5)} \frac{1}{t} d t}}$

$$
\sqrt{\frac{1}{2}+\frac{1}{\log (\log (5))}}=\sqrt{\frac{1}{2}+\frac{2 i \pi}{\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\Gamma(-s)^{2} \Gamma(1+s)(-1+\log (5))^{-s}}{\Gamma(1-s)} d s}} \text { for }-1<\gamma<0
$$

From (8.5), we obtain:

$$
0 \leq \sum_{\gamma \leq \gamma^{\prime}-1}\left\{\frac{t-t_{0}}{t_{0}-\gamma}-\ln \left|\frac{t-\gamma}{t_{0}-\gamma}\right|\right\}<A \frac{\ln t_{0}}{\left(\ln \ln t_{0}\right)^{2}},
$$

$\left(2.5^{*} \ln (5)\right) /(\ln \ln (5))^{\wedge} 2$

## Input:

$$
\frac{2.5 \log (5)}{\log ^{2}(\log (5))}
$$

## Result:

17.7668...
17.7668... result very near to the black hole entropy 17.7715

## Alternative representations:

$$
\begin{aligned}
& \frac{2.5 \log (5)}{\log ^{2}(\log (5))}=\frac{2.5 \log _{e}(5)}{\log _{e}^{2}(\log (5))} \\
& \frac{2.5 \log (5)}{\log ^{2}(\log (5))}=\frac{2.5 \log (a) \log _{a}(5)}{\left(\log (a) \log _{a}(\log (5))\right)^{2}}
\end{aligned}
$$

$$
\frac{2.5 \log (5)}{\log ^{2}(\log (5))}=-\frac{2.5 \mathrm{Li}_{1}(-4)}{\left(-\mathrm{Li}_{1}(1-\log (5))\right)^{2}}
$$

## Series representations:

$$
\begin{aligned}
& \frac{2.5 \log (5)}{\log ^{2}(\log (5))}=\frac{1.25\left(i \pi\left\lfloor\frac{\arg (5-x)}{2 \pi}\right\rfloor+0.5 \log (x)-0.5 \sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}\right)}{\left(i \pi\left\lfloor\frac{\arg (-x+\log (5))}{2 \pi}\right\rfloor+0.5 \log (x)-0.5 \sum_{k=1}^{\infty} \frac{(-1)^{k} x^{-k}(-x+\log (5))^{k}}{k}\right)^{2}} \text { for } x<0 \\
& \frac{2.5 \log (5)}{\log ^{2}(\log (5))}=\frac{1.25\left(i \pi\left\lfloor-\frac{-\pi+\arg \left(\frac{5}{z_{0}}\right)+\operatorname{targ}\left(z_{0}\right)}{2 \pi}\right\rfloor+0.5 \log \left(z_{0}\right)-0.5 \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{2 \pi}}{\left(i \pi\left\lfloor-\frac{-\pi+\arg \left(\frac{\log (5)}{z_{0}}\right)+\operatorname{targ}\left(z_{0}\right)}{2 \pi}\right\rfloor+0.5 \log \left(z_{0}\right)-0.5 \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\log (5)-z_{0}\right)^{k} z_{0}^{k}}{k}\right)^{2}} \\
& \frac{2.5 \log (5)}{\log ^{2}(\log (5))}= \\
& \frac{2.5\left(\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor \log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor \log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)}{\left(\left\lfloor\frac{\arg \left(\log (5)-z_{0}\right)}{2 \pi}\right\rfloor \log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(\log (5)-z_{0}\right)}{2 \pi}\right\rfloor \log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\log (5)-z_{0} k^{k} z_{0}^{-k}\right.}{k}\right)^{2}}
\end{aligned}
$$

## Integral representations:

$$
\frac{2.5 \log (5)}{\log ^{2}(\log (5))}=\frac{2.5 \int_{1}^{5} \frac{1}{t} d t}{\left(\int_{1}^{\log (5)} \frac{1}{t} d t\right)^{2}}
$$

$$
\frac{2.5 \log (5)}{\log ^{2}(\log (5))}=\frac{5 i \pi \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{4^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s}{\left(\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\Gamma(-s)^{2} \Gamma(1+s)(-1+\log (5))^{-s}}{\Gamma(1-s)} d s\right)^{2}} \text { for }-1<\gamma<0
$$

Now, from the sum of the above expressions, we obtain:
$2.5 /(\ln \ln (5))+\left(\left(\left((1 / 3)^{\wedge} 2 \operatorname{sum}\left((-1)^{\wedge} \mathrm{k} /(\mathrm{k}+2)(1 / 3)^{\wedge} \mathrm{k}, \mathrm{k}=0 .\right.\right.\right.\right.$. infinity $\left.\left.\left.)\right)\right)\right)+(((1 / 2+(1 /(\ln$ $\ln (5))))))+\left(\left(\left((2.5 * \ln (5)) /(\ln \ln (5))^{\wedge} 2\right)\right)\right)$

## Input interpretation:

$\frac{2.5}{\log (\log (5))}+\left(\frac{1}{3}\right)^{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{k+2}\left(\frac{1}{3}\right)^{k}+\left(\frac{1}{2}+\frac{1}{\log (\log (5))}\right)+\frac{2.5 \log (5)}{\log ^{2}(\log (5))}$

## Result:

25.6672
25.6672

From which:
$1+1 /\left(\left(\left(\left(2.5 /(\ln \ln (5))+\left(\left(\left((1 / 3)^{\wedge} 2 \operatorname{sum}\left((-1)^{\wedge} \mathrm{k} /(\mathrm{k}+2)(1 / 3)^{\wedge} \mathrm{k}, \mathrm{k}=0 .\right.\right.\right.\right.\right.\right.\right.\right.$. infinity $\left.\left.\left.)\right)\right)\right)+$ $\left.\left.\left.\left.(((1 / 2+(1 /(\ln \ln (5))))))+\left(\left(\left(\left(2.5^{*} \ln (5)\right) /(\ln \ln (5))^{\wedge} 2\right)\right)\right)\right)\right)\right)\right)^{\wedge} 1 / 7$

## Input interpretation:


$\log (x)$ is the natural logarithm

## Result:

1.62901
1.62901
and also:
$2.5 /(\ln \ln (5)) *\left(\left(\left((1 / 3)^{\wedge} 2 \operatorname{sum}\left((-1)^{\wedge} \mathrm{k} /(\mathrm{k}+2)(1 / 3)^{\wedge} \mathrm{k}, \mathrm{k}=0 .\right.\right.\right.\right.$. infinity $\left.\left.\left.)\right)\right)\right) *(((1 / 2+(1 /(\ln$ $\ln (5)))))) *\left(\left(\left((2.5 * \ln (5)) /(\ln \ln (5))^{\wedge} 2\right)\right)\right)$

## Input interpretation:

$\frac{2.5}{\log (\log (5))}\left(\left(\frac{1}{3}\right)^{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{k+2}\left(\frac{1}{3}\right)^{k}\right)\left(\frac{1}{2}+\frac{1}{\log (\log (5))}\right) \times \frac{2.5 \log (5)}{\log ^{2}(\log (5))}$

## Result:

11.0841
$11.0841 \approx 11$ (Lucas number)

Multiplying by 18 , that is a Lucas number, we obtain:
$18^{*} 1 /\left(\left(\left(\left(\left(2.5 /(\ln \ln (5)) *\left(\left(\left((1 / 3)^{\wedge} 2 \operatorname{sum}\left((-1)^{\wedge} \mathrm{k} /(\mathrm{k}+2)(1 / 3)^{\wedge} \mathrm{k}, \mathrm{k}=0 .\right.\right.\right.\right.\right.\right.\right.\right.\right.$. infinity $\left.\left.\left.)\right)\right)\right)$ * $\left.\left.\left.\left.\left.(((1 / 2+(1 /(\ln \ln (5)))))) *\left(\left(\left((2.5 * \ln (5)) /(\ln \ln (5))^{\wedge} 2\right)\right)\right)\right)\right)\right)\right)\right)$

## Input interpretation:

$18 \times \frac{1}{\frac{2.5}{\log (\log (5))}\left(\left(\frac{1}{3}\right)^{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{k+2}\left(\frac{1}{3}\right)^{k}\right)\left(\frac{1}{2}+\frac{1}{\log (\log (5))}\right) \times \frac{2.5 \log (5)}{\log ^{2}(\log (5))}}$

## Result:

1.62395
1.62395

Performing the $5^{\text {th }}$ root, we have:
$\left(\left(\left(2.5 /(\ln \ln (5)) *\left(\left(\left((1 / 3)^{\wedge} 2 \operatorname{sum}\left((-1)^{\wedge} \mathrm{k} /(\mathrm{k}+2)(1 / 3)^{\wedge} \mathrm{k}, \mathrm{k}=0 .\right.\right.\right.\right.\right.\right.\right.$. infinity $\left.\left.\left.)\right)\right)\right)$ * $\left.\left.\left.(((1 / 2+(1 /(\ln \ln (5)))))) *\left(\left(\left((2.5 * \ln (5)) /(\ln \ln (5))^{\wedge} 2\right)\right)\right)\right)\right)\right)^{\wedge} 1 / 5$

Input interpretation:
$\sqrt[5]{\frac{2.5}{\log (\log (5))}\left(\left(\frac{1}{3}\right)^{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{k+2}\left(\frac{1}{3}\right)^{k}\right)\left(\frac{1}{2}+\frac{1}{\log (\log (5))}\right) \times \frac{2.5 \log (5)}{\log ^{2}(\log (5))}}$

## Result:

### 1.61786

1.61786 result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the algebraic sum, we obtain:
$\left(\left(\left(\left(\left(2.5 /(\ln \ln (5))-\left(\left(\left((1 / 3)^{\wedge} 2 \operatorname{sum}\left((-1)^{\wedge} \mathrm{k} /(\mathrm{k}+2)(1 / 3)^{\wedge} \mathrm{k}, \mathrm{k}=0 .\right.\right.\right.\right.\right.\right.\right.\right.\right.$. infinity $\left.\left.\left.)\right)\right)\right)-$ $\left.\left.\left.\left.\left.(((1 / 2+(1 /(\ln \ln (5))))))-\left(\left(\left(\left(2.5^{*} \ln (5)\right) /(\ln \ln (5))^{\wedge} 2\right)\right)\right)\right)\right)\right)\right)\right)$

## Input interpretation:

$$
\frac{2.5}{\log (\log (5))}-\left(\frac{1}{3}\right)^{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{k+2}\left(\frac{1}{3}\right)^{k}-\left(\frac{1}{2}+\frac{1}{\log (\log (5))}\right)-\frac{2.5 \log (5)}{\log ^{2}(\log (5))}
$$

## Result:

-15.1605
$-15.1605$

From which:
$1+1 /\left[-\left(\left(\left(\left(2.5 /(\ln \ln (5))-\left(\left(\left((1 / 3)^{\wedge} 2 \operatorname{sum}\left((-1)^{\wedge} \mathrm{k} /(\mathrm{k}+2)(1 / 3)^{\wedge} \mathrm{k}, \mathrm{k}=0 .\right.\right.\right.\right.\right.\right.\right.\right.\right.$. infinity $\left.\left.\left.)\right)\right)\right)-$ $\left.\left.\left.\left.\left.\left.(((1 / 2+(1 /(\ln \ln (5))))))-\left(\left(\left(\left(2.5^{*} \ln (5)\right) /(\ln \ln (5))^{\wedge} 2\right)\right)\right)\right)\right)\right)\right)\right)\right]^{\wedge} 1 / 6$

## Input interpretation:

$1+\frac{1}{\sqrt[6]{-\left(\frac{2.5}{\log (\log (5))}-\left(\frac{1}{3}\right)^{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{k+2}\left(\frac{1}{3}\right)^{k}-\left(\frac{1}{2}+\frac{1}{\log (\log (5))}\right)-\frac{2.5 \log (5)}{\log ^{2}(\log (5))}\right)}}$

## Result:

1.63564
$1.63564 \approx \zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

Now, we have that:
C. Let

$$
\gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right), \quad t \in J\left(t_{0}\right)
$$

and

$$
\begin{align*}
V & =\sum_{\gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right)}\left\{\frac{t-t_{0}}{t_{0}-\gamma}-\ln \left|\frac{t-\gamma}{t_{0}-\gamma}\right|\right\}= \\
& =\sum_{\gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right)} \frac{t-t_{0}}{t_{0}-\gamma}+\sum_{\gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right)} \ln \left|\frac{t_{0}-\gamma}{t-\gamma}\right|=V_{1}+V_{2} \tag{8.7}
\end{align*}
$$

Since

$$
\left|\frac{t-t_{0}}{t_{0}-\gamma}\right| \leq \frac{Q\left(t_{0}\right)}{m\left(t_{0}\right)}, \quad t \in J\left(t_{0}\right), \gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right)
$$

then (see (1.2), (6.3), (8.7))
(8.8) $\quad\left|V_{1}\right| \leq \sum_{\gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right)}\left|\frac{t-t_{0}}{t_{0}-\gamma}\right|<A \ln t_{0} \cdot t_{0} \ln ^{3} t_{0}=A t_{0} \ln ^{4} t_{0}$.

Next, by (1.2), (6.2) we have

$$
\begin{align*}
& \left|\frac{t_{0}-\gamma}{t-\gamma}\right|=\left|1+\frac{t_{0}-t}{t-\gamma}\right| \leq 1+\frac{\left|t-t_{0}\right|}{|t-\gamma|}<1+\frac{Q\left(t_{0}\right)}{\omega\left(t_{0}\right) m\left(t_{0}\right)}<  \tag{8.9}\\
& <\frac{2}{\omega\left(t_{0}\right)} \frac{Q\left(t_{0}\right)}{m\left(t_{0}\right)}<\frac{2}{\omega\left(t_{0}\right)} t_{0} \ln ^{3} t_{0}
\end{align*}
$$

For: $\gamma^{\prime}=3 ; \mathrm{t}_{0}=5 ; \gamma^{\prime}=8 ; \quad \omega\left(t_{0}\right)=10^{-58}, \ldots$

$$
\gamma=2 ; \mathrm{t}=6 ; \mathrm{A}=2.5
$$

From (8.7), we obtain:

$$
\begin{aligned}
V & =\sum_{\gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right)}\left\{\frac{t-t_{0}}{t_{0}-\gamma}-\ln \left|\frac{t-\gamma}{t_{0}-\gamma}\right|\right\}= \\
& =\sum_{\gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right)} \frac{t-t_{0}}{t_{0}-\gamma}+\sum_{\gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right)} \ln \left|\frac{t_{0}-\gamma}{t-\gamma}\right|=V_{1}+V_{2}
\end{aligned}
$$

$\ln ((5-2) /(6-2))$

## Input:

$\log \left(\frac{5-2}{6-2}\right)$
$\log (x)$ is the natural logarithm

## Exact result:

$-\log \left(\frac{4}{3}\right)$

## Decimal approximation:

-0.28768207245178092743921900599382743150350971089776105650...
$-0.28768207245 \ldots$.

## Property:

$-\log \left(\frac{4}{3}\right)$ is a transcendental number

## Alternate form:

$\log (3)-2 \log (2)$

## Alternative representations:

$\log \left(\frac{5-2}{6-2}\right)=\log _{e}\left(\frac{3}{4}\right)$
$\log \left(\frac{5-2}{6-2}\right)=\log (a) \log _{a}\left(\frac{3}{4}\right)$
$\log \left(\frac{5-2}{6-2}\right)=-\operatorname{Li}_{1}\left(1-\frac{3}{4}\right)$

## Series representations:

$\log \left(\frac{5-2}{6-2}\right)=\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{3}\right)^{k}}{k}$
$\log \left(\frac{5-2}{6-2}\right)=-2 i \pi\left[\frac{\arg \left(\frac{4}{3}-x\right)}{2 \pi}\right]-\log (x)+\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\frac{4}{3}-x\right)^{k} x^{-k}}{k}$ for $x<0$

$$
\begin{aligned}
& \log \left(\frac{5-2}{6-2}\right)= \\
& \quad-\left\lfloor\frac{\arg \left(\frac{4}{3}-z_{0}\right)}{2 \pi} \left\lvert\, \log \left(\frac{1}{z_{0}}\right)-\log \left(z_{0}\right)-\left\lfloor\frac{\arg \left(\frac{4}{3}-z_{0}\right)}{2 \pi} \left\lvert\, \log \left(z_{0}\right)+\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\frac{4}{3}-z_{0}\right)^{k} z_{0}^{-k}}{k}\right.\right.\right.\right.
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& \log \left(\frac{5-2}{6-2}\right)=-\int_{1}^{\frac{4}{3}} \frac{1}{t} d t \\
& \log \left(\frac{5-2}{6-2}\right)=\frac{i}{2 \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{3^{s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s \text { for }-1<\gamma<0
\end{aligned}
$$

From (8.8), we obtain:

$$
\left|V_{1}\right| \leq \sum_{\gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right)}\left|\frac{t-t_{0}}{t_{0}-\gamma}\right|<A \ln t_{0} \cdot t_{0} \ln ^{3} t_{0}=A t_{0} \ln ^{4} t_{0}
$$

$2.5 * 5 \ln \wedge 4(5)$

## Input:

$2.5 \times 5 \log ^{4}(5)$

## Result:

83.8701..
83.8701...

## Alternative representations:

$2.5 \times 5 \log ^{4}(5)=12.5 \log _{e}^{4}(5)$
$2.5 \times 5 \log ^{4}(5)=12.5\left(\log (a) \log _{a}(5)\right)^{4}$
$2.5 \times 5 \log ^{4}(5)=12.5\left(-\mathrm{Li}_{1}(-4)\right)^{4}$

## Series representations:

$2.5 \times 5 \log ^{4}(5)=12.5\left(\log (4)-\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}}{k}\right)^{4}$
$2.5 \times 5 \log ^{4}(5)=12.5\left(2 i \pi\left\lfloor\frac{\arg (5-x)}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}\right)^{4}$ for $x<0$
$2.5 \times 5 \log ^{4}(5)=$

$$
12.5\left(\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{k}}{k}\right)^{4}
$$

## Integral representations:

$2.5 \times 5 \log ^{4}(5)=12.5\left(\int_{1}^{5} \frac{1}{t} d t\right)^{4}$
$\left.\left.2.5 \times 5 \log ^{4}(5)=\frac{0.78125\left(\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{4}{}_{-s}^{\Gamma(-s)^{2} \Gamma(1+s)}\right.}{\Gamma(1-s)} d s\right)^{4}\right)$ for $-1<\gamma<0$

From (8.9), we obtain:

$$
\begin{aligned}
& \left|\frac{t_{0}-\gamma}{t-\gamma}\right|=\left|1+\frac{t_{0}-t}{t-\gamma}\right| \leq 1+\frac{\left|t-t_{0}\right|}{|t-\gamma|}<1+\frac{Q\left(t_{0}\right)}{\omega\left(t_{0}\right) m\left(t_{0}\right)}< \\
& <\frac{2}{\omega\left(t_{0}\right)} \frac{Q\left(t_{0}\right)}{m\left(t_{0}\right)}<\frac{2}{\omega\left(t_{0}\right)} t_{0} \ln ^{3} t_{0}
\end{aligned}
$$

$\left(2 / 10^{\wedge}(-58)\right) * 5 \ln \wedge 3(5)$

## Input:

$\frac{2}{\frac{1}{10^{58}}} \times 5 \log ^{3}(5)$

## Exact result:

100000000000000000000000000000000000000000000000000000000000 $\log ^{3}(5)$

## Decimal approximation:

$4.1689115642856527297917773543386666315782376035159757 \ldots \times 10^{59}$
4.168911564...*10 $0^{59}$

## Property:

100000000000000000000000000000000000000000000000000000000000 $\log ^{3}(5)$ is a transcendental number

## Alternative representations:

$\frac{\left(5 \log ^{3}(5)\right) 2}{\frac{1}{10^{58}}}=\frac{10 \log _{e}^{3}(5)}{\frac{1}{10^{58}}}$
$\frac{\left(5 \log ^{3}(5)\right) 2}{\frac{1}{10^{58}}}=\frac{10\left(\log (a) \log _{a}(5)\right)^{3}}{\frac{1}{10^{58}}}$
$\frac{\left(5 \log ^{3}(5)\right) 2}{\frac{1}{10^{58}}}=\frac{10\left(-\mathrm{Li}_{1}(-4)\right)^{3}}{\frac{1}{10^{58}}}$

## Series representations:

$$
\begin{aligned}
& \frac{\left(5 \log ^{3}(5)\right) 2}{\frac{1}{10^{58}}}= \\
& 100000000000000000000000000000000000000000000000000000000000 \\
& \left(\log (4)-\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}}{k}\right)^{3} \\
& \frac{\left(5 \log ^{3}(5)\right) 2}{\frac{1}{10^{58}}}= \\
& 100000000000000000000000000000000000000000000000000000000 \vdots \\
& 000\left(2 i \pi\left[\frac{\arg (5-x)}{2 \pi}\right]+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}\right)^{3} \text { for } x<0 \\
& \frac{\left(5 \log ^{3}(5)\right) 2}{\frac{1}{10^{58}}}= \\
& 100000000000000000000000000000000000000000000000000000000000 \\
& \left(\log \left(z_{0}\right)+\left[\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right]\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{3}
\end{aligned}
$$

From the three previous results, performing the $288^{\text {th }}$ root, we obtain:
$\left(-0.2876820724517809+83.8701+4.168911564 * 10^{\wedge} 59\right)^{\wedge} 1 / 288$

## Input interpretation:

$\sqrt[288]{-0.2876820724517809+83.8701+4.168911564 \times 10^{59}}$

## Result:

1.61069743711...
$1.61069743711 \ldots$ result that is a good approximation to the value of the golden ratio 1.618033988749...

Now, we have that:

Consequently,

$$
-\ln \frac{2}{m\left(t_{0}\right)}<\ln \left|\frac{t_{0}-\gamma}{t-\gamma}\right|<\ln \left\{\frac{2}{\omega\left(t_{0}\right)} t_{0} \ln ^{3} t_{0}\right\},
$$

(of course, $m\left(t_{0}\right) \in(0,1), t_{0} \rightarrow \infty$ ), and (see (1.4))

$$
\begin{equation*}
|\ln | \frac{t_{0}-\gamma}{t-\gamma}\left|\mid<\ln w\left(t_{0}\right), \quad t \in J\left(t_{0}\right), \gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right) .\right. \tag{8.11}
\end{equation*}
$$

Next, (see (6.3), (8.7), (8.11))

$$
\begin{equation*}
\left|V_{2}\right| \leq \sum_{\gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right)}|\ln | \frac{t_{0}-\gamma}{t-\gamma}| |<A \ln t_{0} \ln w\left(t_{0}\right) . \tag{8.12}
\end{equation*}
$$

Next, (see (8.7), (8.8), (8.12))

$$
\begin{equation*}
\left|V_{1}+V_{2}\right| \leq\left|V_{1}\right|+\left|V_{2}\right|<A t_{0} \ln ^{4} t_{0}+A \ln t_{0} \ln w\left(t_{0}\right), \tag{8.13}
\end{equation*}
$$

and, of course,

$$
\begin{equation*}
V_{1}+V_{2}>-A t_{0} \ln ^{4} t_{0}-A \ln t_{0} \ln w\left(t_{0}\right) \tag{8.14}
\end{equation*}
$$

Finally, the estimate (1.5) follows from (2.3) by (8.1), (8.5), (8.6), (8.13) and (8.14).

From (8.11), we obtain:
$\ln \left(10^{\wedge}-58\right)$

## Input:

$$
\log \left(\frac{1}{10^{58}}\right)
$$

## Exact result:

- $\log ($

10000000000000000000000000000000000000000000000000000000000 )

## Decimal approximation:

$-133.549935393654649673043504371693124040863886340468832609 \ldots$
-133.54993539...

## Property:

- $\log ($

10000000000000000000000000000000000000000000000000000000 :
000 ) is a transcendental number

## Alternate forms:

-58 $\log (10)$
$-58(\log (2)+\log (5))$
$-58 \log (2)-58 \log (5)$

## Alternative representations:

$\log \left(\frac{1}{10^{58}}\right)=\log _{e}\left(\frac{1}{10^{58}}\right)$
$\log \left(\frac{1}{10^{58}}\right)=\log (a) \log _{a}\left(\frac{1}{10^{58}}\right)$
$\log \left(\frac{1}{10^{58}}\right)=-\mathrm{Li}_{1}\left(1-\frac{1}{10^{58}}\right)$

## Series representations:

$$
\log \left(\frac{1}{10^{58}}\right)=-\log (
$$

$$
9999999999999999999999999999999999999999999999999999999 \text { : }
$$

999)     + 

$$
\left.\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{9999999999999999} 999999999999999999999999999999999999999990\right.}{}\right)^{k}
$$

$$
\log \left(\frac{1}{10^{58}}\right)=-2 i \pi\left[\frac{1}{2 \pi} \arg (\right.
$$

10000000000000000000000000000000000000000000000000 :

$$
000000000-x)]-\log (x)+\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}
$$

(10000000000000000000000000000000000000000000000000000:

$$
000000-x)^{k} x^{-k} \text { for } x<0
$$

$$
\log \left(\frac{1}{10^{58}}\right)=-2 i \pi\left[\frac{\pi-\arg \left(\frac{1}{z_{0}}\right)-\arg \left(z_{0}\right)}{2 \pi}\right]-\log \left(z_{0}\right)+\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}
$$

(10000000000000000000000000000000000000000000000000000:

$$
\left.000000-z_{0}\right)^{k} z_{0}^{-k}
$$

## Integral representations:

$$
\begin{aligned}
\log \left(\frac{1}{10^{58}}\right)= & -\int_{1}^{10000000000000000000000000000000000000000000000000000000000} \frac{1}{t} d t \\
\log \left(\frac{1}{10^{58}}\right)= & \frac{i}{2 \pi} \int_{-i \infty 0+\gamma}^{i \infty>\gamma} \frac{1}{\Gamma(1-s)} \\
& 999999999999999999999999999999999999999999999999999: \\
& 999999^{-5} \Gamma(-s)^{2} \Gamma(1+s) d s \text { for }-1<\gamma<0
\end{aligned}
$$

From which:
$-\ln \left(10^{\wedge}-58\right)+$ sqrt2

## Input:

$$
-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}
$$

## Exact result:

$\sqrt{2}+\log ($
10000000000000000000000000000000000000000000000000000000000 )

## Decimal approximation:

134.9641489560277447218451930959028221194335582158457806831...
$134.9641489 \ldots$ result practically equal to the rest mass of Pion meson 134.9766 MeV

## Property:

$\sqrt{2}+\log ($
10000000000000000000000000000000000000000000000000000000 :
000 ) is a transcendental number

## Alternate forms:

$\sqrt{2}+58 \log (10)$
$\sqrt{2}+58 \log (2)+58 \log (5)$

## Alternative representations:

$-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}=-\log _{e}\left(\frac{1}{10^{58}}\right)+\sqrt{2}$
$-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}=-\log (a) \log _{a}\left(\frac{1}{10^{58}}\right)+\sqrt{2}$
$-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}=\operatorname{Li}_{1}\left(1-\frac{1}{10^{58}}\right)+\sqrt{2}$

## Series representations:

$$
\begin{aligned}
& -\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}=\sqrt{2}+\log ( \\
& 9999999999999999999999999999999999999999999999999999999999 \\
& )-\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{999999999999999999999999999999999999990999999999999999999}\right)^{k}}{k}
\end{aligned}
$$

$$
\begin{aligned}
& -\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}=\sqrt{2}+2 i \pi\left[\frac{1}{2 \pi} \arg ( \right. \\
& 10000000000000000000000000000000000000000000000000 \\
& 000000000-x)]+\log (x)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k} \\
& (10000000000000000000000000000000000000000000000000000 \\
& 000000-x)^{k} x^{-k} \text { for } x<0
\end{aligned}
$$

$$
-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}=\sqrt{2}+2 i \pi\left\lfloor\left.\frac{\pi-\arg \left(\frac{1}{z_{0}}\right)-\arg \left(z_{0}\right)}{2 \pi} \right\rvert\,+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}\right.
$$

(10000000000000000000000000000000000000000000000000000:

$$
\left.000000-z_{0}\right)^{k} z_{0}^{-k}
$$

## Integral representations:

$$
\begin{aligned}
& -\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}= \\
& \sqrt{2}+\int_{1}^{10000000000000000000000000000000000000000000000000000000000} \frac{1}{t} d t \\
& -\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}=\sqrt{2}-\frac{i}{2 \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{1}{\Gamma(1-s)} \\
& 9999999999999999999999999999999999999999999999999999: \\
& \\
& 999999^{-s} \Gamma(-s)^{2} \Gamma(1+s) d s \text { for }-1<\gamma<0
\end{aligned}
$$

$-\ln \left(10^{\wedge}-58\right)+$ sqrt2 $-11+$ golden ratio

## where 11 is a Lucas number

## Input:

$$
-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-11+\phi
$$

## Exact result:

$\phi-11+\sqrt{2}+\log ($
10000000000000000000000000000000000000000000000000000000000 )

## Decimal approximation:

125.5821829447776395700497799302684602371538673956515435452...
$125.5821829 \ldots$ result very near to the Higgs boson mass 125.18 GeV

## Property:

## $-11+\sqrt{2}+\phi+\log ($

10000000000000000000000000000000000000000000000000000000 : 000 ) is a transcendental number

## Alternate forms:

$\phi-11+\sqrt{2}+58 \log (10)$
$-\frac{21}{2}+\sqrt{2}+\frac{\sqrt{5}}{2}+\log ($
10000000000000000000000000000000000000000000000000000000000 )
$-11+\sqrt{2}+\frac{1}{2}(1+\sqrt{5})+\log ($
10000000000000000000000000000000000000000000000000000000000 )

## Alternative representations:

$-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-11+\phi=-11+\phi-\log _{e}\left(\frac{1}{10^{58}}\right)+\sqrt{2}$

$$
\begin{aligned}
& -\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-11+\phi=-11+\phi-\log (a) \log _{a}\left(\frac{1}{10^{58}}\right)+\sqrt{2} \\
& -\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-11+\phi=-11+\phi+\operatorname{Li}_{1}\left(1-\frac{1}{10^{58}}\right)+\sqrt{2}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& -\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-11+\phi=-\frac{21}{2}+\sqrt{2}+\frac{\sqrt{5}}{2}+\log ( \\
& 9999999999999999999999999999999999999999999999999999999999 \\
& )-\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{999999999999999999999999999999999999999999999999909999999}\right)^{k}}{k}
\end{aligned}
$$

$$
\left.-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-11+\phi=-\frac{21}{2}+\sqrt{2}+\frac{\sqrt{5}}{2}+2 i \pi \right\rvert\, \frac{1}{2 \pi} \arg (
$$

10000000000000000000000000000000000000000000000000 :

$$
000000000-x)]+\log (x)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}
$$

(10000000000000000000000000000000000000000000000000000:

$$
000000-x)^{k} x^{-k} \text { for } x<0
$$

$$
\begin{aligned}
& -\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-11+\phi= \\
& -\frac{21}{2}+\sqrt{2}+\frac{\sqrt{5}}{2}+2 i \pi\left\lfloor\frac{\pi-\arg \left(\frac{1}{z_{0}}\right)-\arg \left(z_{0}\right)}{2 \pi}\right\rfloor+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}
\end{aligned}
$$

(10000000000000000000000000000000000000000000000000000:

$$
\left.000000-z_{0}\right)^{k} z_{0}^{k}
$$

## Integral representations:

$$
\begin{aligned}
& -\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-11+\phi= \\
& -11+\sqrt{2}+\phi+\int_{1}^{10000000000000000000000000000000000000000000000000000000000} \frac{1}{t} d t \\
& -\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-11+\phi=-\frac{21}{2}+\sqrt{2}+\frac{\sqrt{5}}{2}-\frac{i}{2 \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{1}{\Gamma(1-s)} \\
& 9999999999999999999999999999999999999999999999999999: \\
& \quad 999999^{-5} \Gamma(-s)^{2} \Gamma(1+s) d s \text { for }-1<\gamma<0
\end{aligned}
$$

$27^{*} 1 / 2\left[-\ln \left(10^{\wedge}-58\right)+\right.$ sqrt2-7]+ golden ratio

## Input:

$27 \times \frac{1}{2}\left(-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-7\right)+\phi$

## Exact result:

$\phi+\frac{27}{2}(-7+\sqrt{2}+\log ($
$10000000000000000000000000000000000000000000000000000000000)$ )

## Decimal approximation:

1729.134044895124448593114693629053736730073345093723802084...
1729.13404489...

This result is very near to the mass of candidate glueball $\mathbf{f}_{\mathbf{0}}(\mathbf{1 7 1 0})$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the $j$-invariant of an elliptic curve. The number 1728 is one less than the Hardy-Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):
"The fundamental group of the complex form, compact real form, or any algebraic version of $E_{6}$ is the cyclic group $\boldsymbol{Z} / 3 \boldsymbol{Z}$, and its outer automorphism group is the cyclic group $\boldsymbol{Z} / 2 \boldsymbol{Z}$. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, $E_{6}$ plays a role in some grand unified theories ".

## Property:

$$
\begin{aligned}
& \phi+\frac{27}{2}(-7+\sqrt{2}+\log ( \\
& \quad 10000000000000000000000000000000000000000000000000 \\
& \quad 000000000) \text { is a transcendental number }
\end{aligned}
$$

## Alternate forms:

$\phi-\frac{189}{2}+\frac{27}{\sqrt{2}}+783 \log (10)$
$\phi+\frac{27}{2}(-7+\sqrt{2}+58 \log (10))$
$\frac{1}{2}(-188+27 \sqrt{2}+\sqrt{5}+27 \log ($
10000000000000000000000000000000000000000000000000000 : 000000 )

## Alternative representations:

$$
\begin{aligned}
& \frac{27}{2}\left(-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-7\right)+\phi=\phi+\frac{27}{2}\left(-7-\log _{e}\left(\frac{1}{10^{58}}\right)+\sqrt{2}\right) \\
& \frac{27}{2}\left(-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-7\right)+\phi=\phi+\frac{27}{2}\left(-7-\log (a) \log _{a}\left(\frac{1}{10^{58}}\right)+\sqrt{2}\right) \\
& \frac{27}{2}\left(-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-7\right)+\phi=\phi+\frac{27}{2}\left(-7+\operatorname{Li}_{1}\left(1-\frac{1}{10^{58}}\right)+\sqrt{2}\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{27}{2}\left(-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-7\right)+\phi=-94+\frac{27}{\sqrt{2}}+\frac{\sqrt{5}}{2}+\frac{1}{2} 27 \log ( \\
& 9999999999999999999999999999999999999999999999999999999: \\
& 999)-\frac{27}{2} \\
& \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{99999999999999999999999999999999999999999999999999999999}\right)^{k}}{k} \\
& \frac{27}{2}\left(-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-7\right)+\phi=-94+\frac{27}{\sqrt{2}}+\frac{\sqrt{5}}{2}+27 i \pi\left[\frac{1}{2 \pi} \arg ( \right. \\
& 10000000000000000000000000000000000000000000000000 \\
& 000000000-x)]+\frac{27 \log (x)}{2}-\frac{27}{2} \sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}
\end{aligned}
$$

(10000000000000000000000000000000000000000000000000:

$$
000000000-x)^{k} x^{-k} \text { for } x<0
$$

$$
\begin{aligned}
& \frac{27}{2}\left(-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-7\right)+\phi= \\
& -94+\frac{27}{\sqrt{2}}+\frac{\sqrt{5}}{2}+27 i \pi\left|-\frac{-\pi+\arg \left(\frac{1}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right|+\frac{27 \log \left(z_{0}\right)}{2}-\frac{27}{2} \sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k} \\
& (10000000000000000000000000000000000000000000000000000 \\
& \left.000000-z_{0}\right)^{k} z_{0}^{-k}
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& \frac{27}{2}\left(-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-7\right)+\phi=-94+\frac{27}{\sqrt{2}}+\frac{\sqrt{5}}{2}+ \\
& \quad \frac{27}{2} \int_{1}^{10000000000000000000000000000000000000000000000000000000000} \frac{1}{t} d t
\end{aligned}
$$

$$
\begin{gathered}
\frac{27}{2}\left(-\log \left(\frac{1}{10^{58}}\right)+\sqrt{2}-7\right)+\phi=-94+\frac{27}{\sqrt{2}}+\frac{\sqrt{5}}{2}-\frac{27 i}{4 \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{1}{\Gamma(1-s)} \\
9999999999999999999999999999999999999999999999999999: \\
999999^{-s} \Gamma(-s)^{2} \Gamma(1+s) d s \text { for }-1<\gamma<0
\end{gathered}
$$

From (8.12), we obtain:

$$
\left|V_{2}\right| \leq \sum_{\gamma \in\left(\gamma^{\prime}-1, \gamma^{\prime \prime}+1\right)}|\ln | \frac{t_{0}-\gamma}{t-\gamma}| |<A \ln t_{0} \ln w\left(t_{0}\right)
$$

$2.5 * \ln (5) * \ln \left(10^{\wedge}-58\right)$

## Input:

$2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)$

## Result:

-537.351...
$-537.351 \ldots$

## Alternative representations:

$2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=2.5 \log _{e}(5) \log _{e}\left(\frac{1}{10^{58}}\right)$
$2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=2.5 \log ^{2}(a) \log _{a}(5) \log _{a}\left(\frac{1}{10^{58}}\right)$
$2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=2.5 \mathrm{Li}_{1}(-4) \mathrm{Li}_{1}\left(1-\frac{1}{10^{58}}\right)$

## Series representations:

$2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=2.5\left(\log \left(z_{0}\right)+\right.$
$\left\lfloor\frac{\arg \left(\frac{1}{10000000000000000000000000000000000000000000000000000000000}-z_{0}\right)}{2 \pi}\right\rfloor$
$\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}(1 /$
10000000000000000000000000000000000000000000 :

$$
\left.\left.000000000000000-z_{0}\right)^{k} z_{0}^{-k}\right)
$$

$$
\left(\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)
$$

$2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=$

$$
\left.\begin{array}{l}
10\left(i^{2} \pi^{2}\left[\frac{\arg \left(\frac{1}{1000000000000000000000000000000000000000000000000000000000}-x\right)}{2 \pi}\right]\right. \\
{\left[\frac{\arg (5-x)}{2 \pi}\right\rfloor+0.5 i \pi}
\end{array}\right]
$$

$2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=10\left\{i^{2} \pi^{2} \left\lvert\,-\frac{1}{2 \pi}(-\pi+\arg (1 /\right.\right.$
(10000000000000000000000000000000000000000000: $\left.\left.000000000000000 z_{0}\right)\right)+$
$\left.\left.\arg \left(z_{0}\right)\right)\right]\left[-\frac{-\pi+\arg \left(\frac{5}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right]+0.5 i \pi\left[-\frac{1}{2 \pi}(-\pi+\arg (1 /\right.$
(10000000000000000000000000000000000000000000:

$$
\left.\left.000000000000000 z_{0}\right)\right)+
$$

$\left.\arg \left(z_{0}\right)\right) \left\lvert\, \log \left(z_{0}\right)+0.5 i \pi\left\lfloor-\frac{-\pi+\arg \left(\frac{5}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right\rfloor \log \left(z_{0}\right)+\right.$
$0.25 \log ^{2}\left(z_{0}\right)-0.5 i \pi\left[\left.-\frac{-\pi+\arg \left(\frac{5}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi} \right\rvert\, \sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}(1 /\right.$
10000000000000000000000000000000000000000 :
$\left.000000000000000000-z_{0}\right)^{k}$
$z_{0}^{-k}-0.25 \log \left(z_{0}\right) \sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}(1 /$
10000000000000000000000000000000000000000 :
$\left.000000000000000000-z_{0}\right)^{k}$
$z_{0}^{-k}-0.5 i \pi \left\lvert\,-\frac{1}{2 \pi}(-\pi+\arg (1 /\right.$
(10000000000000000000000000000000000000000000:

$$
\left.\left.000000000000000 z_{0}\right)\right)+
$$

$$
\left.\left.\arg \left(z_{0}\right)\right)\right] \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}-
$$

$0.25 \log \left(z_{0}\right) \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}+0.25 \sum_{k_{1}=1}^{\infty} \sum_{k_{2}=1}^{\infty} \frac{1}{k_{1} k_{2}}(-1)^{k_{1}+k_{2}}(1 /$
10000000000000000000000000000000000000000 : 000000000000000000 -
$\left.\left.z_{0}\right)^{k_{1}}\left(5-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}\right)$

## Integral representation:

```
\(2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=\)
\(-10000000000000000000000000000000000000000000000000000000000\)
    \(\int_{0}^{1} \int_{0}^{1}(1 /\)
        ( -10000000000000000000000000000000000000000000000 :
                                    000000000000 +
                                    9999999999999999999999999999999999999999999 :
                                    \(\left.\left.999999999999999 t_{1}\right)\left(1+4 t_{2}\right)\right)\) )
\(d t_{2} d t_{1}\)
```

From the previous expression:
$2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)$
we obtain:
$1-1 /\left(\left(\left(2.5 * \ln (5) * \ln \left(10^{\wedge}-58\right)\right)\right)\right)$

## Input:

$1-\frac{1}{2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)}$
$\log (x)$ is the natural logarithm

## Result:

1.001860981610296258552531154164899137363877843001952936657...
$1.0018609816 \ldots$. result practically equal to the following Rogers-Ramanujan continued fraction:

$$
\frac{\mathrm{e}^{-\frac{2 \pi}{5}}}{\sqrt{\varphi \sqrt{5}}-\varphi}=1+\frac{\mathrm{e}^{-2 \pi}}{1+\frac{\mathrm{e}^{-4 \pi}}{1+\frac{\mathrm{e}^{-6 \pi}}{1+\frac{\mathrm{e}^{-8 \pi}}{1+\ldots}}}} \approx 1.0018674362
$$

$$
=1.0018674362
$$

## Alternative representations:

$$
\begin{aligned}
& 1-\frac{1}{2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)}=1-\frac{1}{2.5 \log _{e}(5) \log _{e}\left(\frac{1}{10^{58}}\right)} \\
& 1-\frac{1}{2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)}=1-\frac{1}{2.5 \log ^{2}(a) \log _{a}(5) \log _{a}\left(\frac{1}{10^{58}}\right)} \\
& 1-\frac{1}{2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)}=1-\frac{1}{2.5 \operatorname{Li}_{1}(-4) \mathrm{Li}_{1}\left(1-\frac{1}{10^{58}}\right)}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& 1-\frac{1}{2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)}=1-0.4 /\left(\left(2 i \pi \left\lfloor\frac{1}{2 \pi} \arg (1 /\right.\right.\right. \\
& 10000000000000000000000000000000000000000 \text { : } \\
& 000000000000000000 \text { - } \\
& x)\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}(1 / \\
& 10000000000000000000000000000000000000 \text { : } \\
& 000000000000000000000 \text { - } \\
& \left.x^{k} x^{-k}\right) \\
& \left.\left(2 i \pi\left[\frac{\arg (5-x)}{2 \pi}\right]+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}\right)\right) \text { for } x<0 \\
& 1-\frac{1}{2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)}=1-0.4 / \int\left(\log \left(z_{0}\right)+\left\lfloor\frac{1}{2 \pi} \arg (1 /\right.\right. \\
& 10000000000000000000000000000000000000000 \text { : } \\
& \left.000000000000000000-z_{0}\right) \text { ] } \\
& \left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}(1 / \\
& 10000000000000000000000000000000000000 \text { : } \\
& 000000000000000000000 \text { - } \\
& \left.\left.z_{0}\right)^{k} z_{0}^{-k}\right) \\
& \left.\left(\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)\right)
\end{aligned}
$$

$1-\frac{1}{2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)}=1-0.4 /\left(\left(2 i \pi\left[-\frac{1}{2 \pi}(-\pi+\arg (1 /\right.\right.\right.$
(10000000000000000000000000000000000000:
000000000000000000000

$$
\left.\left.\left.\left.z_{0}\right)\right)+\arg \left(z_{0}\right)\right)\right]+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}(1 /
$$

10000000000000000000000000000000000000 : 000000000000000000000 -
$\left.\left.z_{0}\right)^{k} z_{0}^{-k}\right)$

$$
\left.\left(2 i \pi\left[-\frac{-\pi+\arg \left(\frac{5}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right]+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)\right)
$$

## Integral representation:

$1-\frac{1}{2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)}=(-0.4-$
10000000000000000000000000000000000000000000000000000 :
$000000 \int_{0}^{1} \int_{0}^{1}(1 /$
( $(-10000000000000000000000000000000000000000000$ :
$000000000000000+$ 9999999999999999999999999999999999999 : 999999999999999999999
$\left.\left.\left.\left.t_{1}\right)\left(1+4 t_{2}\right)\right)\right) d t_{2} d t_{1}\right) /$
$\left(\left(\int_{1} \frac{1}{10000000000000000000000000000000000000000000000000000000000} \frac{1}{t} d t\right) \int_{1}^{5} \frac{1}{t} d t\right)$

From (8.13), we obtain:

$$
\left|V_{1}+V_{2}\right| \leq\left|V_{1}\right|+\left|V_{2}\right|<A t_{0} \ln ^{4} t_{0}+A \ln t_{0} \ln w\left(t_{0}\right),
$$

$\left(\left(2.5 * 5^{*} \ln \wedge 4(5)+2.5^{*} \ln (5) \ln \left(10^{\wedge}-58\right)\right)\right)$

## Input:

$2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)$

## Result:

-453.481...
$-453.481 \ldots$

## Alternative representations:

$2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=2.5 \log _{e}(5) \log _{e}\left(\frac{1}{10^{58}}\right)+12.5 \log _{e}^{4}(5)$
$2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=$
$2.5 \log ^{2}(a) \log _{a}(5) \log _{a}\left(\frac{1}{10^{58}}\right)+12.5\left(\log (a) \log _{a}(5)\right)^{4}$
$2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=2.5 \mathrm{Li}_{1}(-4) \mathrm{Li}_{1}\left(1-\frac{1}{10^{58}}\right)+12.5\left(-\mathrm{Li}_{1}(-4)\right)^{4}$

## Series representations:

$$
\left.\begin{array}{c}
2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=2.5\left(2 i \pi \left[\frac{1}{2 \pi} \arg (1 /\right.\right. \\
10000000000000000000000000000000000000000000 \\
00000000000000-x)]+ \\
\log (x)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}(1 / \\
10000000000000000000000000000000000000000 \\
000000000000000000-x)^{k}
\end{array}\right] \begin{aligned}
& \left.x^{-k}\right)\left(2 i \pi\left[\frac{\arg (5-x)}{2 \pi}\right]^{\left.+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}\right)+}\right. \\
& 12.5\left(2 i \pi\left\lfloor\frac{\arg (5-x)}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}\right)^{4} \text { for } x<0
\end{aligned}
$$

$2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=2.5\left(\log \left(z_{0}\right)+\right.$

$$
\left\lfloor\frac{\arg \left(\frac{1}{10000000000000000000000000000000000000000000000000000000000}-z_{0}\right)}{2 \pi}\right\rfloor
$$

$$
\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}(1 /
$$

10000000000000000000000000000000000000000 :

$$
\left.000000000000000000-z_{0}\right)^{k}
$$

$$
\left.z_{0}^{-k}\right)\left(\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{k}}{k}\right)+
$$

$$
12.5\left(\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{4}
$$

$2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=2.5\left(2 i \pi \left\lvert\,-\frac{1}{2 \pi}(-\pi+\arg (1 /\right.\right.$
(10000000000000000000000000000000000000000: $\left.000000000000000000 z_{0}\right)$ ) +
$\left.\arg \left(z_{0}\right)\right) \left\lvert\,+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}(1 /\right.$
10000000000000000000000000000000000000000 :

$$
\left.000000000000000000-z_{0}\right)^{k}
$$

$$
\left.z_{0}^{-k}\right)\left(2 i \pi\left[-\frac{-\pi+\arg \left(\frac{5}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right]+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)+
$$

$12.5\left(2 i \pi\left[-\frac{-\pi+\arg \left(\frac{5}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right]+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{4}$

## Integral representation:

$2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)=2.5\left(5\left(\int_{1}^{5} \frac{1}{t} d t\right)^{4}-\right.$ 10000000000000000000000000000000000000000000000000000 : $000000 \int_{0}^{1} \int_{0}^{1}(1 /$ $((-10000000000000000000000000000000000000000000$ : 000000000000000 + 9999999999999999999999999999999999999 : 999999999999999999999

$$
\left.\left.\left.\left.t_{1}\right)\left(1+4 t_{2}\right)\right)\right) d t_{2} d t_{1}\right)
$$

From which:
$\left(\left(2.5^{*} 5^{*} \ln 4(5)+2.5^{*} \ln (5) \ln \left(10^{\wedge}-58\right)\right)\right)-47+3$

## Input:

$\left(2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)\right)-47+3$
$\log (x)$ is the natural logarithm

## Result:

-497.481..
$-497.481 \ldots$ result practically equal to the rest mass of Kaon meson 497.614 with minus sign

## Alternative representations:

$\left(2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)\right)-47+3=$
$-44+2.5 \log _{e}(5) \log _{e}\left(\frac{1}{10^{58}}\right)+12.5 \log _{e}^{4}(5)$
$\left(2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)\right)-47+3=$
$-44+2.5 \log ^{2}(a) \log _{a}(5) \log _{a}\left(\frac{1}{10^{58}}\right)+12.5\left(\log (a) \log _{a}(5)\right)^{4}$
$\left(2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)\right)-47+3=$
$-44+2.5 \operatorname{Li}_{1(-4)} \mathrm{Li}_{1}\left(1-\frac{1}{10^{58}}\right)+12.5\left(-\mathrm{Li}_{1}(-4)\right)^{4}$

## Series representations:

$$
\left.\begin{array}{c}
\left(2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)\right)-47+3=-44+2.5\left(2 i \pi \left\lvert\, \frac{1}{2 \pi} \arg (1 /\right.\right. \\
10000000000000000000000000000000000000000000 \\
00000000000000-x)]+ \\
\log (x)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}(1 / \\
10000000000000000000000000000000000000000: \\
000000000000000000-x)^{k}
\end{array}\right] \begin{aligned}
& \left.x^{-k}\right)\left(2 i \pi\left\lfloor\frac{\arg (5-x)}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}\right)+ \\
& 12.5\left(2 i \pi\left\lfloor\frac{\arg (5-x)}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}\right)^{4} \text { for } x<0
\end{aligned}
$$

$\left(2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)\right)-47+3=-44+2.5\left(\log \left(z_{0}\right)+\right.$


$$
\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}(1 /
$$

$$
10000000000000000000000000000000000000000 \text { : }
$$

$$
\left.000000000000000000-z_{0}\right)^{k}
$$

$$
\left.z_{0}^{-k}\right)\left(\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)+
$$

$$
12.5\left(\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{4}
$$

$\left(2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)\right)-47+3=-44+2.5\left(2 i \pi\left[-\frac{1}{2 \pi}(-\pi+\arg (1 /\right.\right.$
(10000000000000000000000000000000000000000: $\left.\left.000000000000000000 z_{0}\right)\right)+$
$\left.\left.\arg \left(z_{0}\right)\right)\right]+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{1}{k}(-1)^{k}(1 /$

$$
10000000000000000000000000000000000000000 \text { : }
$$

$$
\left.000000000000000000-z_{0}\right)^{k}
$$

$$
\left.z_{0}^{-k}\right)\left(2 i \pi\left[-\frac{-\pi+\arg \left(\frac{5}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right]+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)+
$$

$$
12.5\left(2 i \pi\left[-\frac{-\pi+\arg \left(\frac{5}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right]+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{4}
$$

## Integral representation:

```
\(\left(2.5 \times 5 \log ^{4}(5)+2.5 \log (5) \log \left(\frac{1}{10^{58}}\right)\right)-47+3=2.5\left(-17.6+5\left(\int_{1}^{5} \frac{1}{t} d t\right)^{4}-\right.\)
10000000000000000000000000000000000000000000000000000 :
\(000000 \int_{0}^{1} \int_{0}^{1}(1 /\)
( -10000000000000000000000000000000000000000000 :
                                    000000000000000 +
                9999999999999999999999999999999999999 :
                        999999999999999999999
                    \(\left.\left.\left.\left.t_{1}\right)\left(1+4 t_{2}\right)\right)\right) d t_{2} d t_{1}\right)\)
```

From (8.14), we obtain:

$$
V_{1}+V_{2}>-A t_{0} \ln ^{4} t_{0}-A \ln t_{0} \ln w\left(t_{0}\right) .
$$

$-2.5 * 5^{*} \ln \wedge 4(5)-2.5^{*} \ln (5) \ln \left(10^{\wedge}-58\right)$
$-2.5 \times 5 \log ^{4}(5)-(2.5 \log (5)) \log \left(\frac{1}{10^{58}}\right)$

## Result:

453.481..
$453.481 \ldots$ result equal to the previous expression in absolute value.

From the various results, we obtain the following expression:
$-8(-133.54993539-537.351-453.481+453.481)$

## Input interpretation:

$-8(-133.54993539-537.351-453.481+453.481)$

## Result:

5367.20748312
5367.20748312 result very near to the rest mass of strange B meson 5366.3

From:

ON SOME PROPERTIES OF RIEMANN ZETA FUNCTION ON
CRITICAL LINE - JAN MOSER - arXiv:0710.0943v1 [math.CA] 4 Oct 2007

For: $\quad \gamma^{\prime}=3 ; \mathrm{t}_{0}=5 ; \gamma^{\prime \prime}=8 ; \quad \omega\left(t_{0}\right)=10^{-58}, \ldots$
$\gamma=2 ; \mathrm{t}=6 ; \mathrm{A}=2.5 ; \quad \mathrm{Z}(\mathrm{t})=2$
from:

$$
\begin{equation*}
\frac{\kappa c^{2}}{3} \rho(t)=\frac{c^{2}}{Z^{2}(t)}+\left\{\frac{Z^{\prime}(t)}{Z(t)}\right\}^{2} \tag{17}
\end{equation*}
$$

we obtain:
$((1.866 \mathrm{e}-26 *(9 \mathrm{e}+16) * 8.5 \mathrm{e}-27)) / 3=(9 \mathrm{e}+16) / 4+(\mathrm{x} / 2)^{\wedge} 2$

## Input interpretation:

$\frac{1}{3}\left(1.866 \times 10^{-26} \times 9 \times 10^{16} \times 8.5 \times 10^{-27}\right)=\frac{9 \times 10^{16}}{4}+\left(\frac{x}{2}\right)^{2}$

## Result:

$4.7583 \times 10^{-36}=\frac{x^{2}}{4}+22500000000000000$

## Alternate forms:

$4.7583 \times 10^{-36}=\frac{1}{4}\left(x^{2}+90000000000000000\right)$
$-\frac{x^{2}}{4}-2.25 \times 10^{16}=0$

## Complex solutions:

$x=-300000000 i$
$x=300000000 i$
300.000.000 $i \approx \mathrm{c}=$ speed of light (with imaginary unit)

Thence:
$((1.866 \mathrm{e}-26 *(9 \mathrm{e}+16) * 8.5 \mathrm{e}-27)) / 3$

## Input interpretation:

$\frac{1}{3}\left(1.866 \times 10^{-26} \times 9 \times 10^{16} \times 8.5 \times 10^{-27}\right)$

## Result:

$4.7583 \times 10^{-36}$
$4.7583 * 10^{-36}$

From the right-hand side of (17), we obtain:
$(9 \mathrm{e}+16) / 4+((300000000 \mathrm{i}) / 2)^{\wedge} 2$

## Input interpretation:

$\frac{9 \times 10^{16}}{4}+\left(\frac{300000000 i}{2}\right)^{2}$

## Result:

0
0

From:

$$
\begin{equation*}
\frac{\kappa}{2} p(t)=\sum_{\gamma} \frac{1}{(t-\gamma)^{2}}-\frac{3}{2}\left\{\frac{Z^{\prime}(t)}{Z(t)}\right\}^{2}-\frac{c^{2}}{2} \frac{1}{Z^{2}(t)}+\mathcal{O}\left(\frac{1}{t}\right) \tag{18}
\end{equation*}
$$

we obtain:
$1 /(6-2)^{\wedge} 2-3 / 2((300000000 i) / 2)^{\wedge} 2-((9 \mathrm{e}+16) /(2 * 4))+1 / 6$

## Input interpretation:

$\frac{1}{(6-2)^{2}}-\frac{3}{2}\left(\frac{300000000 i}{2}\right)^{2}-\frac{9 \times 10^{16}}{2 \times 4}+\frac{1}{6}$
$i$ is the imaginary unit

## Exact result:

1080000000000000011

## Decimal approximation:

$2.2500000000000000229166666666666666666666666666666666 \ldots \times 10^{16}$
$2.25 \ldots * 10^{16}$

From:

$$
\begin{equation*}
\left|Z\left(\tilde{t}_{0}\right)\right|>A_{2} \exp \left(\ln ^{\beta} \tilde{t}_{0}\right), \quad \beta<\frac{1}{2} \tag{20}
\end{equation*}
$$

we obtain:
$2.5^{*} \exp \left(\ln ^{\wedge}(1 / 3)(5)\right)$

## Input:

$2.5 \exp (\sqrt[3]{\log (5)})$

## Result:

8.07032...
8.07032...

Alternative representations:
$2.5 \exp (\sqrt[3]{\log (5)})=2.5 \exp \left(\sqrt[3]{\log _{e}(5)}\right)$
$2.5 \exp (\sqrt[3]{\log (5)})=2.5 \exp \left(\sqrt[3]{\log (a) \log _{a}(5)}\right)$

## Series representations:

$2.5 \exp (\sqrt[3]{\log (5)})=2.5 \exp \left(\sqrt[3]{\log (4)-\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}}{k}}\right)$
$2.5 \exp (\sqrt[3]{\log (5)})=2.5 \exp \left(\sqrt[3]{2 i \pi\left[\frac{\arg (5-x)}{2 \pi}\right]+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}}\right)$
for $x<0$

$$
\begin{aligned}
& 2.5 \exp (\sqrt[3]{\log (5)})= \\
& 2.5 \exp \left(\sqrt[3]{\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}}\right)
\end{aligned}
$$

## Integral representations:

$2.5 \exp (\sqrt[3]{\log (5)})=2.5 \exp \left(\sqrt[3]{\int_{1}^{5} \frac{1}{t} d t}\right)$
$2.5 \exp (\sqrt[3]{\log (5)})=2.5 \exp \left(\frac{\sqrt[3]{\frac{1}{i \pi} \int_{-i \infty+\gamma+\gamma}^{i \infty+\gamma} \frac{4^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s}}{\sqrt[3]{2}}\right)$ for $-1<\gamma<0$

Now, from the ln of:
$\frac{1}{(6-2)^{2}}-\frac{3}{2}\left(\frac{300000000 i}{2}\right)^{2}-\frac{9 \times 10^{16}}{2 \times 4}+\frac{1}{6}$
and
$2.5 \exp (\sqrt[3]{\log (5)})$
and performing the $6^{\text {th }}$ root of ratio, we obtain:

## Input interpretation:

$\sqrt[6]{\frac{\log \left(\frac{1}{(6-2)^{2}}-\frac{3}{2}\left(\frac{300000000 i}{2}\right)^{2}-\frac{9 \times 10^{16}}{2 \times 4}+\frac{1}{6}\right)}{\log (2.5 \exp (\sqrt[3]{\log (5)}))}}$
$\log (x)$ is the natural logarithm
$i$ is the imaginary unit

## Result:

1.619335344627602926796650649895990688508615502404709139582...
$1.619335344 \ldots$. result that is a good approximation to the value of the golden ratio 1.618033988749...

From

$$
\begin{equation*}
\sum_{\gamma} \frac{1}{(t-\gamma)^{2}}>\frac{1}{\left(\gamma^{\prime \prime}-\gamma^{\prime}\right)^{2}}>A_{1}\left(\ln \ln \ln t_{0}\right)^{2} \tag{19}
\end{equation*}
$$

we obtain:
$2.5(((\ln \ln \ln (5))))^{\wedge} 2$

## Input:

$2.5 \log ^{2}(\log (\log (5)))$

## Result:

1.378559152353602395171788571615483901642497379628422811416...
1.37855915...

## Alternative representations:

$2.5 \log ^{2}(\log (\log (5)))=2.5 \log _{e}^{2}(\log (\log (5)))$
$2.5 \log ^{2}(\log (\log (5)))=2.5\left(\log (a) \log _{a}(\log (\log (5)))\right)^{2}$
$2.5 \log ^{2}(\log (\log (5)))=2.5\left(-\mathrm{Li}_{1}(1-\log (\log (5)))\right)^{2}$

## Series representations:

$2.5 \log ^{2}(\log (\log (5)))=2.5\left(\sum_{k=1}^{\infty} \frac{(-1)^{k}(-1+\log (\log (5)))^{k}}{k}\right)^{2}$

$$
\begin{aligned}
& 2.5 \log ^{2}(\log (\log (5)))=2.5 \\
& \qquad\left(2 i \pi\left[\frac{\arg (-x+\log (\log (5)))}{2 \pi}\right]+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k} x^{-k}(-x+\log (\log (5)))^{k}}{k}\right)^{2} \text { for } x<0
\end{aligned}
$$

$2.5 \log ^{2}(\log (\log (5)))=2.5\left(\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(\log (\log (5))-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\right.$

$$
\left.\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\log (\log (5))-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{2}
$$

## Integral representation:

$2.5 \log ^{2}(\log (\log (5)))=2.5\left(\int_{1}^{\log (\log (5))} \frac{1}{t} d t\right)^{2}$

From the ratio of two previous expressions, performing the $4^{\text {th }}$ root and adding 1 , we obtain:
$\left.1+\left[\left(\left((2.5(((\ln \ln \ln (5)))))^{\wedge} 2\right)\right)\right) /\left(\left(\left(2.5^{*} \exp (\ln \wedge(1 / 3)(5))\right)\right)\right)\right]^{\wedge} 1 / 4$

## Input:

$1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}$

## Result:

1.642886...
$1.642886 \ldots \approx \zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

## Alternative representations:

$$
1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}=1+\sqrt[4]{\frac{2.5 \log _{e}^{2}(\log (\log (5)))}{2.5 \exp \left(\sqrt[3]{\log _{e}(5)}\right)}}
$$

$$
1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}=1+\sqrt[4]{\frac{2.5\left(\log (a) \log _{a}(\log (\log (5)))\right)^{2}}{2.5 \exp \left(\sqrt[3]{\log (a) \log _{a}(5)}\right)}}
$$

## Series representations:

$$
\begin{aligned}
& 1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}=} \\
& 1+\sqrt[4]{\frac{\left(2 i \pi\left\lfloor\frac{\arg (-x+\log (\log (5)))}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k} x^{-k}(-x+\log (\log (5)))^{k}}{k}\right)^{2}}{\exp \left(\sqrt[3]{2 i \pi\left\lfloor\frac{\arg (5-x)}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}}\right)}} \text { for } x<0
\end{aligned}
$$

$$
1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}=
$$

$$
1+\frac{\left(2 i \pi\left[-\frac{-\pi+\arg \left(\frac{\log (\log (5))}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right\rfloor+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\log (\log (5))-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{2}}{\left.\exp \left[\sqrt[3]{2 i \pi\left\lfloor-\frac{-\pi+\arg \left(\frac{5}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right.}\right]+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)}
$$

$$
\begin{aligned}
& 1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}= \\
& 1+\left(\left(\frac{\arg \left(\log (\log (5))-z_{0}\right)}{2 \pi}\right\rfloor \log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(\log (\log (5))-z_{0}\right)}{2 \pi}\right] \log \left(z_{0}\right)-\right. \\
& \left.\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\log (\log (5))-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{2} / \\
& \left.\exp \left(\sqrt[3]{\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}}\right)\right)
\end{aligned}
$$

(1/4)
$1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}=$
$\frac{\left(2 i \pi\left[\frac{\pi-\arg \left(\frac{\log (\log (5))}{z_{0}}\right)-\arg \left(z_{0}\right)}{2 \pi}\right\rfloor+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\log (\log (5))-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{2}}{\exp \left[\sqrt[3]{2 i \pi\left\lfloor\frac{\pi-\arg \left(\frac{5}{z_{0}}\right)-\arg \left(z_{0}\right)}{2 \pi}\right\rfloor+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}}\right)}$

## Integral representation:

$1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}=1+\sqrt[4]{\frac{\left(\int_{1}^{\log (\log (5))} \frac{1}{t} d t\right)^{2}}{\exp \left(\sqrt[3]{\int_{1}^{5} \frac{1}{t} d t}\right)}}$

From which, we obtain also:
$1+\left[\left(\left(\left(2.5(((\ln \ln \ln (5))))^{\wedge} 2\right)\right)\right) /\left(\left(\left(2.5^{*} \exp (\ln \wedge(1 / 3)(5))\right)\right)\right)\right]^{\wedge} 1 / 4-24 / 10^{\wedge} 3$

## Input:

$1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}-\frac{24}{10^{3}}$

## Result:

1.618885562847996063837818877502534188801742125658190687290...
$1.6188855628 \ldots$. result that is a very good approximation to the value of the golden ratio 1.618033988749...

## Alternative representations:

$$
\begin{aligned}
& 1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}-\frac{24}{10^{3}}=1-\frac{24}{10^{3}}+\sqrt[4]{\frac{2.5 \log _{e}^{2}(\log (\log (5)))}{2.5 \exp \left(\sqrt[3]{\log _{e}(5)}\right)}} \\
& 1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}-\frac{24}{10^{3}}=1-\frac{24}{10^{3}}+\sqrt[4]{\frac{2.5(\log (a) \log (\log (\log (5))))^{2}}{2.5 \exp \left(\sqrt[3]{\log (a) \log _{a}(5)}\right)}}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& 1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}-\frac{24}{10^{3}}=} \\
& 0.976+\sqrt[4]{\frac{\left(2 i \pi\left\lfloor\frac{\arg (-x+\log (\log (5)))}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k} x^{-k}(-x+\log (\log (5)))^{k}}{k}\right)^{2}}{\exp \left(\sqrt[3]{2 i \pi\left\lfloor\frac{\arg (5-x)}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}}{k}}\right)} \text { for } x<0} \text { f }
\end{aligned}
$$

$$
1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}-\frac{24}{10^{3}}=
$$

$$
0.976+\sqrt{\frac{\left(2 i \pi\left\lfloor-\frac{-\pi+\arg \left(\frac{\log (\log (5))}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right\rfloor+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\log (\log (5))-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{2}}{\exp \left(\sqrt[3]{2 i \pi\left\lfloor-\frac{-\pi+\arg \left(\frac{5}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right\rfloor+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}}\right)}}
$$

$$
\left.\begin{array}{l}
1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}-\frac{24}{10^{3}}=} \\
0.976+\left(\left(\frac{\arg \left(\log (\log (5))-z_{0}\right)}{2 \pi}\right\rfloor \log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(\log (\log (5))-z_{0}\right)}{2 \pi}\right] \log \left(z_{0}\right)-\right. \\
\left.\quad \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\log (\log (5))-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)^{2} / \\
\exp \left(\sqrt[3]{\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(5-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k}}\right)
\end{array}\right)
$$

(1/4)

$$
1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}-\frac{24}{10^{3}}=
$$



## Integral representation:

$1+\sqrt[4]{\frac{2.5 \log ^{2}(\log (\log (5)))}{2.5 \exp (\sqrt[3]{\log (5)})}}-\frac{24}{10^{3}}=0.976+\sqrt[4]{\frac{\left(\int_{1}^{\log (\log (5))} \frac{1}{t} d t\right)^{2}}{\exp \left(\sqrt[3]{\int_{1}^{5} \frac{1}{t} d t}\right)}}$

From

$$
\begin{aligned}
& \frac{1}{3}\left(1.866 \times 10^{-26} \times 9 \times 10^{16} \times 8.5 \times 10^{-27}\right) \\
& \frac{1}{(6-2)^{2}}-\frac{3}{2}\left(\frac{300000000 i}{2}\right)^{2}-\frac{9 \times 10^{16}}{2 \times 4}+\frac{1}{6}
\end{aligned}
$$

$2.5 \log ^{2}(\log (\log (5)))$
performing the following calculations, we obtain:
$\left(\left(\left(\left(\left(\left(((()(((1.866 \mathrm{e}-26 *(9 \mathrm{e}+16) * 8.5 \mathrm{e}-27)) / 3))) *\left(\left(\left(1 /(6-2)^{\wedge} 2-3 / 2((300000000 \mathrm{i}) / 2)^{\wedge} 2-\right.\right.\right.\right.\right.\right.\right.\right.\right.$ $((9 \mathrm{e}+16) /(2 * 4))+1 / 6)))))))+\left(\left(\left(\left(\operatorname{sqrt}\left(\left(\left(2.5(((\ln \ln \ln (5))) / 2)^{\wedge} 2\right)\right)\right)\right)\right)\right)\right)^{*} 10^{\wedge}-$ $19))))^{*} 1 / 10^{\wedge} 8+(11+3) / 10^{\wedge} 30$

## Input interpretation:

$$
\left(\begin{array}{l}
\left(\frac{1}{3}\left(1.866 \times 10^{-26} \times 9 \times 10^{16} \times 8.5 \times 10^{-27}\right)\right) \\
\left.\frac{\left(\frac{1}{(6-2)^{2}}-\frac{3}{2}\left(\frac{300000000 i}{2}\right)^{2}-\frac{9 \times 10^{16}}{2 \times 4}+\frac{1}{6}\right)+}{10^{19}}\right) \times \frac{1}{10^{8}}+\frac{11+3}{10^{30}}
\end{array}\right.
$$

## Result:

$1.67168 \ldots \times 10^{-27}$
$1.67168 \ldots * 10^{-27}$ result practically equal to the value of the formula:
$m_{p^{\prime}}=2 \times \frac{\eta}{R} m_{P}=1.6714213 \times 10^{-27} \mathrm{~kg}$
that is the holographic proton mass (N. Haramein)

## Observations

## From:

https://www.scientificamerican.com/article/mathematicsramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8m pSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions-the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that $p(9)=30, p(9+5)=135, p(9+10)=490, p(9+15)=1,575$ and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11 -there are infinite sequences of $p(n)$ that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11... and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by $5^{\wedge} 3=125$ units, saying that the corresponding $p(n)$ 's should all be divisible by 125 .
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

## From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field $\phi$ and a Dirac field $\psi$. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for $T=0$ and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$
g_{22}=\sqrt{(1+\sqrt{2})}
$$

Hence

$$
\begin{array}{rlr}
64 g_{22}^{24} & = & e^{\pi \sqrt{22}}-24+276 e^{-\pi \sqrt{22}}-\cdots \\
64 g_{22}^{-24} & = & 4096 e^{-\pi \sqrt{22}}+\cdots
\end{array}
$$

so that

$$
64\left(g_{22}^{24}+g_{22}^{-24}\right)=e^{\pi \sqrt{22}}-24+4372 e^{-\pi \sqrt{22}}+\cdots=64\left\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\right\}
$$

Hence

$$
e^{\pi \sqrt{22}}=2508951.9982 \ldots
$$

Thence:

$$
64 g_{22}^{-24}=\quad 4096 e^{-\pi \sqrt{22}}+\cdots
$$

And

$$
64\left(g_{22}^{24}+g_{22}^{-24}\right)=e^{\pi \sqrt{22}}-24+4372 e^{-\pi \sqrt{22}}+\cdots=64\left\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\right\}
$$

That are connected with $64,128,256,512,1024$ and $4096=64^{2}$
(Modular equations and approximations to $\pi-S$. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350-372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants $\pi, \phi, 1 / \phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted $F_{n}$, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of $n$ and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as $n$ increases.
Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:
$0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987,1597,2584,4181,6765$, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842-91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio. ${ }^{[1]}$ The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:
$2,1,3,4,7,11,18,29,47,76,123,199,322,521,843,1364,2207,3571,5778$, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803......

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is $\varphi$, the golden ratio. ${ }^{[1]}$ That is, a golden spiral gets wider (or further from its origin) by a factor of $\varphi$ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies ${ }^{[3]}$ - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball $\mathbf{f}_{\mathbf{0}}(\mathbf{1 7 1 0})$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the $\mathrm{j}-$ invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross-Zagier theorem. The number 1728 is one less than the HardyRamanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV . In conclusion we obtain also many results that are very good approximations to the value of the golden ratio $1.618033988749 \ldots$ and to $\zeta(2)=$ $\frac{\pi^{2}}{6}=1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 and 134.9766 (mass of the two Pions - mesons Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to $\mathbf{1 3 7 . 5 0 8}$ to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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