

Mathematical connections between some Ramanujan formulas, ϕ , $\zeta(2)$ and various topics and parameters of Open Strings and Particle Physics. V

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Abstract

In this paper we have described and analyzed some Ramanujan expressions. We have obtained several mathematical connections with ϕ , $\zeta(2)$ and various topics and parameters of Open Strings and Particle Physics.

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We want to highlight that the development of the various equations was carried out according to our possible logical and original interpretation

From:

RAMANUJAN-TYPE FORMULAE FOR $1/\pi$: q -ANALOGUES

VICTOR J. W. GUO AND WADIM ZUDILIN - arXiv:1802.04616v2 [math.NT] 21

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We have:

Theorem 1. *The following identities are true:*

$$\sum_{n=0}^{\infty} \frac{(-1)^n q^{2n^2} (q^2; q^4)_n^2 (q; q^2)_{2n}}{(q^4; q^4)_n^2 (q^4; q^4)_{2n}} \left([8n+1] + [4n+1] \frac{q^{4n+1}}{1+q^{4n+2}} \right) - \frac{(1+q)(q^2; q^4)_{\infty} (q^6; q^4)_{\infty}}{(q^4; q^4)_{\infty}^2}, \quad (4)$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n q^{2n^2} (q^2; q^4)_n (q^2; q^4)_{2n} (q; q^2)_{3n}}{(q^4; q^4)_n^2 (q^4; q^4)_{3n} (q; q^2)_n} \times \left([10n+1] + \frac{q^{6n+1} [4n+2] [6n+1]}{[12n+4]} + \frac{q^{6n+3} [6n+1] [6n+3] [8n+2] (1+q^{2n+1})}{[12n+4] [12n+8]} \right) - \frac{(1+q)(q^2; q^4)_{\infty} (q^6; q^4)_{\infty}}{(q^4; q^4)_{\infty}^2}, \quad (5)$$

$$\sum_{n=0}^{\infty} \frac{q^{4n^2} (q; q^2)_{2n}^2}{(q^4; q^4)_n^2 (q^4; q^4)_{2n}} \left([8n+1] - q^{8n+3} \frac{[4n+1]^2}{[8n+4]} \right) = \frac{(q^3; q^4)_{\infty} (q^5; q^4)_{\infty}}{(q^4; q^4)_{\infty}^2}. \quad (6)$$

Theorem 2. *We have*

$$\sum_{n=0}^{\infty} \frac{q^{2n^2} (q; q^2)_n^2 (q; q^2)_{2n}}{(q^2; q^2)_{2n} (q^6; q^6)_n^2} [8n+1] = \frac{(q^3; q^2)_{\infty} (q^3; q^6)_{\infty}}{(q^2; q^2)_{\infty} (q^6; q^6)_{\infty}}. \quad (7)$$

Observe that equations (4) and (7) are q -analogues of Ramanujan's

$$\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{1}{4}\right)_n \left(\frac{1}{2}\right)_n \left(\frac{3}{4}\right)_n}{n!^3 4^n} (20n+3) = \sum_{n=0}^{\infty} \frac{(-1)^n \binom{4n}{2n} \binom{2n}{n}^2}{2^{10n}} (20n+3) = \frac{8}{\pi},$$

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{4}\right)_n \left(\frac{1}{2}\right)_n \left(\frac{3}{4}\right)_n}{n!^3 9^n} (8n+1) = \sum_{n=0}^{\infty} \frac{\binom{4n}{2n} \binom{2n}{n}^2}{2^{8n} 3^{2n}} (8n+1) = \frac{2\sqrt{3}}{\pi}, \quad (8)$$

while the $q \rightarrow 1$ cases of (5) and (6) read

$$\sum_{n=0}^{\infty} \frac{(-1)^n \binom{6n}{3n} \binom{4n}{2n} \binom{2n}{n}}{2^{12n}} \frac{576n^3 + 624n^2 + 190n + 15}{(3n+1)(3n+2)} = \frac{16}{\pi},$$

$$\sum_{n=0}^{\infty} \frac{\binom{4n}{2n}^2 \binom{2n}{n}}{2^{12n}} \frac{48n^2 + 32n + 3}{2n+1} = \frac{8\sqrt{2}}{\pi}.$$

Now, we perform the following calculation. Multiplying the above results, we obtain:

$$8/\pi * (2\sqrt{3})/\pi * 16/\pi * (8\sqrt{2})/\pi$$

Input:

$$\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi}$$

Result:

$$\frac{2048\sqrt{6}}{\pi^4}$$

Decimal approximation:

51.49986454004408924624919821814606635242578249415040650654...

51.49986454...

Property:

$\frac{2048\sqrt{6}}{\pi^4}$ is a transcendental number

Series representations:

$$\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi} = \frac{2048 \sqrt{z_0}^{-2} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}}{\pi^4}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi} = \frac{1}{\pi^4} 2048 \exp\left(i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp\left(i \pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x}^{-2} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (2-x)^{k_1} (3-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi} = \frac{1}{\pi^4} 2048 \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2\pi)] + 1/2 [\arg(3-z_0)/(2\pi)]} z_0^{1+1/2 [\arg(2-z_0)/(2\pi)] + 1/2 [\arg(3-z_0)/(2\pi)]} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}$$

and performing the 8th root, we obtain:

$$\left(\left(\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi}\right)\right)^{1/8}$$

Input:

$$\sqrt[8]{\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi}}$$

Exact result:

$$\frac{2 \times 2^{7/16} \sqrt[16]{3}}{\sqrt{\pi}}$$

Decimal approximation:

1.636725168780632497073501021844070981246317179535061156051...

$$1.6367251687\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Property:

$\frac{2 \times 2^{7/16} \sqrt[16]{3}}{\sqrt{\pi}}$ is a transcendental number

All 8th roots of $(2048 \sqrt{6})/\pi^4$:

$$\frac{2 \times 2^{7/16} \sqrt[16]{3} e^0}{\sqrt{\pi}} \approx 1.6367 \text{ (real, principal root)}$$

$$\frac{2 \times 2^{7/16} \sqrt[16]{3} e^{(i\pi)/4}}{\sqrt{\pi}} \approx 1.1573 + 1.1573 i$$

$$\frac{2 \times 2^{7/16} \sqrt[16]{3} e^{(i\pi)/2}}{\sqrt{\pi}} \approx 1.6367 i$$

$$\frac{2 \times 2^{7/16} \sqrt[16]{3} e^{(3i\pi)/4}}{\sqrt{\pi}} \approx -1.1573 + 1.1573 i$$

$$\frac{2 \times 2^{7/16} \sqrt[16]{3} e^{i\pi}}{\sqrt{\pi}} \approx -1.6367 \text{ (real root)}$$

Series representations:

$$\sqrt[8]{\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}} = 2 \times 2^{3/8} \sqrt[8]{\frac{\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}}{\pi^4}}$$

for (not $(z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0)$)

$$\sqrt[8]{\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}} = 2 \times 2^{3/8} \left(\frac{1}{\pi^4} \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (2-x)^{k_1} (3-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} \right)^{\wedge}$$

(1/8) for $(x \in \mathbb{R}$ and $x < 0)$

$$\sqrt[8]{\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}} = 2 \times 2^{3/8} \left(\frac{1}{\pi^4} \left(\frac{1}{z_0} \right)^{1/2 [\text{arg}(2-z_0)/(2\pi)]+1/2 [\text{arg}(3-z_0)/(2\pi)]} z_0^{1+1/2 [\text{arg}(2-z_0)/(2\pi)]+1/2 [\text{arg}(3-z_0)/(2\pi)]} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!} \right)^{1/8}$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \text{ for } (0 < \gamma < -\text{Re}(a) \text{ and } |\text{arg}(z)| < \pi)$$

and subtracting 18, that is a Lucas number:

$$(((8/\pi*(2\sqrt{3})/\pi*16/\pi*(8\sqrt{2})/\pi)))^{1/8}-18/10^3$$

Input:

$$\sqrt[8]{\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi}} - \frac{18}{10^3}$$

Exact result:

$$\frac{2 \times 2^{7/16} \sqrt[16]{3}}{\sqrt{\pi}} - \frac{9}{500}$$

Decimal approximation:

1.618725168780632497073501021844070981246317179535061156051...

1.6187251687.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

Property:

$-\frac{9}{500} + \frac{2 \times 2^{7/16} \sqrt[16]{3}}{\sqrt{\pi}}$ is a transcendental number

Alternate form:

$$\frac{1000 \times 2^{7/16} \sqrt[16]{3} - 9 \sqrt{\pi}}{500 \sqrt{\pi}}$$

Series representations:

$$\sqrt[8]{\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}} - \frac{18}{10^3} = \frac{1}{500} \left(-9 + 1000 \times 2^{3/8} \sqrt[8]{\frac{\sqrt{z_0}^{-2} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}}{\pi^4}} \right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\sqrt[8]{\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}} - \frac{18}{10^3} = \frac{1}{500} \left(-9 + 1000 \times 2^{3/8} \left(\frac{1}{\pi^4} \exp\left(i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp\left(i \pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x}^{-2} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (2-x)^{k_1} (3-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} \right) \right) \wedge (1/8)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\sqrt[8]{\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}} - \frac{18}{10^3} = \frac{1}{500} \left(-9 + 1000 \times 2^{3/8} \left(\frac{1}{\pi^4} \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor + 1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} z_0^{1+1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor + 1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!} \right) \right) \wedge (1/8)$$

and also:

$$(8/\pi) * 1 / ((2\sqrt{3})/\pi) * 1 / (16/\pi) * 1 / ((8\sqrt{2})/\pi)$$

Input:

$$\frac{8}{\pi} \times \frac{1}{\frac{2\sqrt{3}}{\pi}} \times \frac{1}{\frac{16}{\pi}} \times \frac{1}{\frac{8\sqrt{2}}{\pi}}$$

Result:

$$\frac{\pi^2}{32\sqrt{6}}$$

Decimal approximation:

0.125914035134354664411453554599524826092096735547147858035...

0.1259140351...

Property:

$\frac{\pi^2}{32\sqrt{6}}$ is a transcendental number

Series representations:

$$\frac{8}{(\pi \cdot 16(8\sqrt{2}))(2\sqrt{3})} = \frac{\pi^2}{32\sqrt{z_0}^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{8}{(\pi \pi) \pi} = \pi^2 / \left(32 \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x}^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{8}{(\pi 16(8\sqrt{2}))(2\sqrt{3})} = \frac{\pi^2 \left(\frac{1}{z_0}\right)^{-1/2 [\text{arg}(2-z_0)/(2\pi)] - 1/2 [\text{arg}(3-z_0)/(2\pi)]} z_0^{-1-1/2 [\text{arg}(2-z_0)/(2\pi)] - 1/2 [\text{arg}(3-z_0)/(2\pi)]}}{32 \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}}$$

From the expression

$$\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi}$$

we obtain also:

$$\left(\ln \left(\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi} \right) \right)^{1/e}$$

Input:

$$\sqrt[e]{\log \left(\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi} \right)}$$

log(x) is the natural logarithm

Exact result:

$$\sqrt[e]{\log \left(\frac{2048\sqrt{6}}{\pi^4} \right)}$$

Decimal approximation:

1.656284129688403132482441414156682262106568937036662945161...

1.6562841296.... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Alternate forms:

$$\sqrt[e]{\frac{\log(6)}{2} + \log(2048) - 4 \log(\pi)}$$

$$\sqrt[e]{\frac{1}{2} (23 \log(2) + \log(3) - 8 \log(\pi))}$$

$$\sqrt[e]{11 \log(2) + \frac{1}{2} (\log(2) + \log(3)) - 4 \log(\pi)}$$

Alternative representations:

$$\sqrt[e]{\log\left(\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}\right)} = \sqrt[e]{\log_e\left(2048 \left(\frac{1}{\pi}\right)^4 \sqrt{2} \sqrt{3}\right)}$$

$$\sqrt[e]{\log\left(\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}\right)} = \sqrt[e]{\log(a) \log_a\left(2048 \left(\frac{1}{\pi}\right)^4 \sqrt{2} \sqrt{3}\right)}$$

$$\sqrt[e]{\log\left(\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}\right)} = \sqrt[e]{-\text{Li}_1\left(1 - 2048 \left(\frac{1}{\pi}\right)^4 \sqrt{2} \sqrt{3}\right)}$$

Series representations:

$$\sqrt[e]{\log\left(\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}\right)} = \sqrt[e]{\log\left(-1 + \frac{2048 \sqrt{6}}{\pi^4}\right) - \sum_{k=1}^{\infty} \frac{\pi^{4k} \left(\frac{1}{-2048 \sqrt{6} + \pi^4}\right)^k}{k}}$$

$$\sqrt[e]{\log\left(\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}\right)} = \sqrt[e]{2i\pi \left[\frac{\arg\left(\frac{2048 \sqrt{6}}{\pi^4} - x\right)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{2048 \sqrt{6}}{\pi^4} - x\right)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\sqrt[e]{\log\left(\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}\right)} = \sqrt[e]{2i\pi \left[\frac{\arg\left(2048 \sqrt{6} - \pi^4 x\right)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{2048 \sqrt{6}}{\pi^4} - x\right)^k x^{-k}}{k}} \quad \text{for } x < 0$$

Integral representations:

$$\sqrt[e]{\log\left(\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}\right)} = \sqrt[e]{\int_1^{\frac{2048 \sqrt{6}}{\pi^4}} \frac{1}{t} dt}$$

$$e^{\sqrt{\log\left(\frac{(8 \times 16 (8 \sqrt{2})) (2 \sqrt{3})}{(\pi \pi \pi) \pi}\right)}} = (2\pi)^{-1/e} \sqrt{e^{-i} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1 + \frac{2048\sqrt{6}}{\pi^4}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

$$e^{*\left(\left(\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi}\right)\right)}$$

Input:

$$e^{\left(\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi}\right)}$$

Result:

$$\frac{2048 \sqrt{6} e}{\pi^4}$$

Decimal approximation:

139.9911459473041935556968745044990020859729381799610871010...

139.991145947... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternate form:

$$\frac{2048 e \sqrt{6}}{\pi^4}$$

Series representations:

$$\frac{e (2 \sqrt{3}) (8 \times 16 (8 \sqrt{2}))}{\pi (\pi \pi \pi)} = \frac{2048 e \sqrt{z_0}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}}{\pi^4}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{e(2\sqrt{3})(8 \times 16(8\sqrt{2}))}{\pi(\pi\pi\pi)} = \frac{1}{\pi^4} 2048 e \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x}^2$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (2-x)^{k_1} (3-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{e(2\sqrt{3})(8 \times 16(8\sqrt{2}))}{\pi(\pi\pi\pi)} =$$

$$\frac{1}{\pi^4} 2048 e \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor + 1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} z_0^{1+1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor + 1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor}$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}$$

$$e^{*(((8/\pi*(2\sqrt{3})/\pi*16/\pi*(8\sqrt{2})/\pi)))-11-\pi}$$

Input:

$$e\left(\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi}\right) - 11 - \pi$$

Result:

$$-11 + \frac{2048\sqrt{6} e}{\pi^4} - \pi$$

Decimal approximation:

125.8495532937144003172342311212194992017757687805859812800...

[125.84955329... result very near to the Higgs boson mass 125.18 GeV](#)

Alternate forms:

$$\frac{2048\sqrt{6} e - 11\pi^4 - \pi^5}{\pi^4}$$

$$\frac{2048 e \sqrt{6}}{\pi^4} - \pi - 11$$

$$\frac{2048\sqrt{6} e - \pi^4(11 + \pi)}{\pi^4}$$

Series representations:

$$\frac{e(2\sqrt{3})(8 \times 16(8\sqrt{2}))}{\pi(\pi\pi\pi)} - 11 - \pi =$$

$$-\frac{11\pi^4 + \pi^5 - 2048 e \sqrt{z_0}^{-2} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}}{\pi^4}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{e(2\sqrt{3})(8 \times 16(8\sqrt{2}))}{\pi(\pi\pi\pi)} - 11 - \pi =$$

$$-\frac{1}{\pi^4} \left(11\pi^4 + \pi^5 - 2048 e \exp\left(i\pi \left[\frac{\arg(2-x)}{2\pi} \right]\right) \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi} \right]\right) \sqrt{x}^{-2} \right.$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (2-x)^{k_1} (3-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} \right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{e(2\sqrt{3})(8 \times 16(8\sqrt{2}))}{\pi(\pi\pi\pi)} - 11 - \pi =$$

$$-\frac{1}{\pi^4} \left(11\pi^4 + \pi^5 - 2048 e \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2\pi)] + 1/2 [\arg(3-z_0)/(2\pi)]} z_0^{1+1/2 [\arg(2-z_0)/(2\pi)] + 1/2 [\arg(3-z_0)/(2\pi)]} \right.$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!} \right)$$

$$27 \cdot \frac{1}{2} \left(\left(e \left(\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi} \right) - 11 - 1 \right) \right) + 1$$

Input:

$$27 \times \frac{1}{2} \left(e \left(\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi} \right) - 11 - 1 \right) + 1$$

Result:

$$1 + \frac{27}{2} \left(\frac{2048 \sqrt{6} e}{\pi^4} - 12 \right)$$

Decimal approximation:

1728.880470288606613001907805810736528160634665429474675864...

1728.880470288...

This result is very near to the mass of candidate glueball **f₀(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

“The fundamental group of the complex form, compact real form, or any algebraic version of E₆ is the cyclic group Z/3Z, and its outer automorphism group is the cyclic group Z/2Z. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E₆ plays a role in some grand unified theories”.

Alternate forms:

$$\frac{27648 \sqrt{6} e - 161 \pi^4}{\pi^4}$$

$$\frac{27648 \sqrt{6} e}{\pi^4} - 161$$

$$\frac{27648 e \sqrt{6}}{\pi^4} - 161$$

Series representations:

$$\frac{27}{2} \left(\frac{e ((2 \sqrt{3}) 8 \times 16 (8 \sqrt{2}))}{\pi \pi \pi \pi} - 11 - 1 \right) + 1 = \frac{-161 \pi^4 + 27648 e \sqrt{z_0}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}}{\pi^4}$$

for (not (z₀ ∈ ℝ and -∞ < z₀ ≤ 0))

$$\frac{27}{2} \left(\frac{e \left((2\sqrt{3}) 8 \times 16 (8\sqrt{2}) \right)}{\pi \pi \pi \pi} - 11 - 1 \right) + 1 =$$

$$\frac{1}{\pi^4} \left(-161 \pi^4 + 27648 e \exp\left(i\pi \left[\frac{\arg(2-x)}{2\pi} \right]\right) \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi} \right]\right) \sqrt{x}^{-2} \right.$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (2-x)^{k_1} (3-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} \right)$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{27}{2} \left(\frac{e \left((2\sqrt{3}) 8 \times 16 (8\sqrt{2}) \right)}{\pi \pi \pi \pi} - 11 - 1 \right) + 1 =$$

$$\frac{1}{\pi^4} \left(-161 \pi^4 + 27648 e \left(\frac{1}{z_0} \right)^{1/2 [\arg(2-z_0)/(2\pi)]+1/2 [\arg(3-z_0)/(2\pi)]} \right.$$

$$\left. z_0^{1+1/2 [\arg(2-z_0)/(2\pi)]+1/2 [\arg(3-z_0)/(2\pi)]} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!} \right)$$

$$\left(\left(\left(27 \times \frac{1}{2} \left(\left(\frac{e \left(\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi} \right) - 11 - 1 \right) \right) + 1 \right) \right)^{1/15} \right)$$

Input:

$$\sqrt[15]{27 \times \frac{1}{2} \left(e \left(\frac{8}{\pi} \times \frac{2\sqrt{3}}{\pi} \times \frac{16}{\pi} \times \frac{8\sqrt{2}}{\pi} \right) - 11 - 1 \right) + 1}$$

Exact result:

$$\sqrt[15]{1 + \frac{27}{2} \left(\frac{2048 \sqrt{6} e}{\pi^4} - 12 \right)}$$

Decimal approximation:

1.643807652458036124381588984202009290007831989128043802140...

$$1.643807652\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Alternate forms:

$$\sqrt[15]{\frac{27648 \sqrt{6} e}{\pi^4} - 161}$$

$$\frac{\sqrt[15]{27648 \sqrt{6} e - 161 \pi^4}}{\pi^{4/15}}$$

All 15th roots of $1 + 27/2 ((2048 \sqrt{6} e)/\pi^4 - 12)$:

$$\sqrt[15]{1 + \frac{27}{2} \left(\frac{2048 \sqrt{6} e}{\pi^4} - 12 \right)} e^0 \approx 1.64381 \quad (\text{real, principal root})$$

$$\sqrt[15]{1 + \frac{27}{2} \left(\frac{2048 \sqrt{6} e}{\pi^4} - 12 \right)} e^{(2i\pi)/15} \approx 1.5017 + 0.6686 i$$

$$\sqrt[15]{1 + \frac{27}{2} \left(\frac{2048 \sqrt{6} e}{\pi^4} - 12 \right)} e^{(4i\pi)/15} \approx 1.0999 + 1.2216 i$$

$$\sqrt[15]{1 + \frac{27}{2} \left(\frac{2048 \sqrt{6} e}{\pi^4} - 12 \right)} e^{(6i\pi)/15} \approx 0.5080 + 1.5634 i$$

$$\sqrt[15]{1 + \frac{27}{2} \left(\frac{2048 \sqrt{6} e}{\pi^4} - 12 \right)} e^{(8i\pi)/15} \approx -0.17182 + 1.63480 i$$

Series representations:

$$\sqrt[15]{\frac{27}{2} \left(\frac{e((2\sqrt{3})8 \times 16(8\sqrt{2}))}{\pi \pi \pi \pi} - 11 - 1 \right) + 1} = \left(\frac{1}{\pi^4} \left(-161 \pi^4 + 27648 e \sqrt{z_0}^{-2} \right. \right. \\ \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!} \right) \right)^{\wedge}$$

(1/15) for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\sqrt[15]{\frac{27}{2} \left(\frac{e((2\sqrt{3})8 \times 16(8\sqrt{2}))}{\pi \pi \pi \pi} - 11 - 1 \right) + 1} = \\ \left(\frac{1}{\pi^4} \left(-161 \pi^4 + 27648 e \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x}^{-2} \right. \right. \\ \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (2-x)^{k_1} (3-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} \right) \right)^{\wedge}$$

(1/15) for ($x \in \mathbb{R}$ and $x < 0$)

$$\begin{aligned}
& \sqrt[15]{\frac{27}{2} \left(\frac{e((2\sqrt{3})8 \times 16(8\sqrt{2}))}{\pi \pi \pi \pi} - 11 - 1 \right) + 1} = \\
& \left(\frac{1}{\pi^4} \left[-161 \pi^4 + 27648 e \left(\frac{1}{z_0} \right)^{1/2 [\arg(2-z_0)/(2\pi)] + 1/2 [\arg(3-z_0)/(2\pi)]} \right. \right. \\
& \quad \left. \left. z_0^{1+1/2 [\arg(2-z_0)/(2\pi)] + 1/2 [\arg(3-z_0)/(2\pi)]} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!} \right] \right)^{1/15}
\end{aligned}$$

Integral representation:

$$(1+z)^\alpha = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-\alpha-s)}{z^s} ds}{(2\pi i)\Gamma(-\alpha)} \quad \text{for } (0 < \gamma < -\text{Re}(\alpha) \text{ and } |\arg(z)| < \pi)$$

From:

Open Strings

C. Angelantonj and A. Sagnotti - arXiv:hep-th/0204089v2 4 Jul 2002

Now, we have that:

These can be associated to the standard bosonic O25 planes, whose contribution scales proportionally to the length of the circle, consistently with the fact that they invade the whole internal space, and to new bosonic O24 planes, whose contribution scales inversely with it, consistently with the fact that they are localized at the two fixed points. Thus, both D25 and D24 branes would be needed in this case if one insisted on cancelling all tadpoles, as can be seen from the transverse-channel open-string amplitudes

$$\begin{aligned}
\tilde{\mathcal{A}} = & \frac{2^{-13}}{4} \left[N^2 v W_n + \frac{D^2}{v} P_n + 2ND \sqrt{\frac{2\eta}{\vartheta_2}} \right. \\
& \left. + 2(R_N^2 + R_D^2) \sqrt{\frac{\eta}{\vartheta_4}} + 2\sqrt{2} R_N R_D \sqrt{\frac{\eta}{\vartheta_3}} \right]
\end{aligned} \tag{258}$$

and

$$\tilde{\mathcal{M}} = \frac{2}{4} \epsilon \left[N v W_{2n} + \frac{D}{v} P_{2m} + (N+D) \sqrt{\frac{2\hat{\eta}}{\hat{\vartheta}_2}} \right], \tag{259}$$

that in terms of the ϕ 's would become

$$\begin{aligned}
\tilde{\mathcal{A}} = & \frac{2^{-13}}{4} \left\{ \left(N\sqrt{v} + \frac{D}{\sqrt{v}} \right)^2 \phi_{++} \right. \\
& + \left(N\sqrt{v} - \frac{D}{\sqrt{v}} \right)^2 \phi_{+-} \\
& + 2 \left[\left(\frac{R_N}{\sqrt{2}} + R_D \right)^2 + \left(\frac{R_N}{\sqrt{2}} \right)^2 \right] \phi_{-+} \\
& \left. + 2 \left[\left(\frac{R_N}{\sqrt{2}} - R_D \right)^2 + \left(\frac{R_N}{\sqrt{2}} \right)^2 \right] \phi_{--} \right\} + \dots
\end{aligned} \tag{260}$$

and

$$\begin{aligned}
\tilde{\mathcal{M}} = & \frac{2}{4} \epsilon \left[\left(\sqrt{v} + \frac{1}{\sqrt{v}} \right) \left(N\sqrt{v} + \frac{D}{\sqrt{v}} \right) \hat{\phi}_{++} \right. \\
& \left. + \left(\sqrt{v} - \frac{1}{\sqrt{v}} \right) \left(N\sqrt{v} - \frac{D}{\sqrt{v}} \right) \hat{\phi}_{+-} \right] + \dots
\end{aligned} \tag{261}$$

These expressions match precisely the Klein-bottle amplitude (255), while the tadpole conditions would lead to

$$\begin{aligned}
N = -8192 \epsilon, & \quad R_N = 0, \\
D = -8192 \epsilon, & \quad R_D = 0,
\end{aligned} \tag{262}$$

thus also requiring that $\epsilon = -1$.

For:

$$v = R/\sqrt{\alpha'}.$$

$R = 1$ and $\alpha' \approx 1 = 0.9991104684$, value that is equal to the following Rogers-Ramanujan continued fraction

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\varphi^5 4 \sqrt{5^3} - 1}}}{\sqrt{5}} - \varphi + 1$$

and from

$$\phi_{+\pm} = \frac{1}{2} \left(\frac{1}{\eta} \pm \sqrt{\frac{2\eta}{\vartheta_2}} \right), \quad \phi_{-\pm} = \frac{1}{2} \left(\sqrt{\frac{\eta}{\vartheta_4}} \pm \sqrt{\frac{\eta}{\vartheta_3}} \right), \quad (250)$$

where $\eta = 2, 3, 7$ and $\vartheta = 1, 2, 2$, we obtain:

$$1/2(1/2+\text{sqrt}(4))$$

Input:

$$\frac{1}{2} \left(\frac{1}{2} + \sqrt{4} \right)$$

Exact result:

$$\frac{5}{4}$$

Decimal form:

$$1.25$$

$$1.25$$

$$1/2(1/2-\text{sqrt}(4))$$

Input:

$$\frac{1}{2} \left(\frac{1}{2} - \sqrt{4} \right)$$

Exact result:

$$-\frac{3}{4}$$

Decimal form:

$$-0.75$$

$$-0.75$$

$$\frac{1}{2}(\sqrt{3/2} + \sqrt{7/2})$$

Input:

$$\frac{1}{2} \left(\sqrt{\frac{3}{2}} + \sqrt{\frac{7}{2}} \right)$$

Decimal approximation:

1.547786782389279870945258201755610173430491822108849268684...

1.5477867823...

Alternate forms:

$$\frac{1}{4} (\sqrt{6} + \sqrt{14})$$

$$\frac{\sqrt{5 + \sqrt{21}}}{2}$$

$$\frac{\sqrt{3} + \sqrt{7}}{2\sqrt{2}}$$

Minimal polynomial:

$$4x^4 - 10x^2 + 1$$

$$\frac{1}{2}(\sqrt{3/2} - \sqrt{7/2})$$

Input:

$$\frac{1}{2} \left(\sqrt{\frac{3}{2}} - \sqrt{\frac{7}{2}} \right)$$

Decimal approximation:

-0.32304191099769082184661616440266447744751808178051420446...

-0.32304191099...

Alternate forms:

$$\frac{1}{4} (\sqrt{6} - \sqrt{14})$$

$$-\frac{1}{2} \sqrt{5 - \sqrt{21}}$$

$$\frac{\sqrt{\frac{3}{2}}}{2} - \frac{\sqrt{\frac{7}{2}}}{2}$$

Minimal polynomial:

$$4x^4 - 10x^2 + 1$$

Thence:

$$\begin{aligned} N &= -8192\epsilon, & R_N &= 0, \\ D &= -8192\epsilon, & R_D &= 0, \end{aligned}$$

thus also requiring that $\epsilon = -1$.

$$\phi_{++} = 1.25 \quad \phi_{+-} = -0.75$$

$$\phi_{-+} = 1.5477867 \quad \phi_{--} = -0.32304191 \quad v = 1 / \text{sqrt}(0.9991104684)$$

From (261):

$$\begin{aligned} \tilde{\mathcal{M}} &= \frac{2}{4}\epsilon \left[\left(\sqrt{v} + \frac{1}{\sqrt{v}} \right) \left(N\sqrt{v} + \frac{D}{\sqrt{v}} \right) \hat{\phi}_{++} \right. \\ &\quad \left. + \left(\sqrt{v} - \frac{1}{\sqrt{v}} \right) \left(N\sqrt{v} - \frac{D}{\sqrt{v}} \right) \hat{\phi}_{+-} \right] + \dots \end{aligned}$$

we obtain:

$$\begin{aligned} &2/4*(-1/2) [((1/\text{sqrt}(0.9991104684))^0.5+1/((1/\text{sqrt}(0.9991104684))^0.5)) \\ &(8192*(1/\text{sqrt}(0.9991104684))^0.5+8192/(((1/\text{sqrt}((0.9991104684))))^0.5)*1.25] \end{aligned}$$

Input interpretation:

$$\frac{1}{2} \times \frac{2}{4} \times (-1) \left(\left(\sqrt{\frac{1}{\sqrt{0.9991104684}}} + \frac{1}{\sqrt{\frac{1}{\sqrt{0.9991104684}}}} \right) \left(8192 \sqrt{\frac{1}{\sqrt{0.9991104684}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9991104684}}}} \times 1.25 \right) \right)$$

Result:

-9215.77...

-9215.77...

or, for $v = 1/\sqrt{0.927}$, where 0.927 is the maximum value of Kaon meson Regge slope:

$$K^* \quad | \quad 5 \quad | \quad m_{u/d} = 0 - 240 \quad m_s = 0 - 390 \quad | \quad 0.848 - 0.927$$

$$2/4 * (-1/2) [((1/\sqrt{0.927})^{0.5} + 1/((1/\sqrt{0.927})^{0.5})) (8192 * (1/\sqrt{0.927})^{0.5} + 8192/(((1/\sqrt{0.927})))^{0.5} * 1.25)]$$

Input:

$$\frac{1}{2} \times \frac{2}{4} \times (-1) \left(\left(\sqrt{\frac{1}{\sqrt{0.927}}} + \frac{1}{\sqrt{\frac{1}{\sqrt{0.927}}}} \right) \left(8192 \sqrt{\frac{1}{\sqrt{0.927}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.927}}}} \times 1.25 \right) \right)$$

Result:

-9199.90...

-9199.90...

$$-9215.77 + 2/4*(-1/2) [((1/\sqrt{0.9991104684})^0.5-1/((1/\sqrt{0.9991104684})^0.5)) (8192*(1/\sqrt{0.9991104684})^0.5-8192/(((1/\sqrt{0.9991104684})))^0.5)*-0.75]$$

Input interpretation:

$$-9215.77 + \frac{1}{2} \times \frac{2}{4} \times (-1) \left(\left(\sqrt{\frac{1}{\sqrt{0.9991104684}}} - \frac{1}{\sqrt{\frac{1}{\sqrt{0.9991104684}}}} \right) \left(8192 \sqrt{\frac{1}{\sqrt{0.9991104684}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9991104684}}}} \times (-0.75) \right) \right)$$

Result:

-9217.36...

-9217.36...

or:

$$-9199.90 + 2/4*(-1/2) [((1/\sqrt{0.927})^0.5-1/((1/\sqrt{0.927})^0.5)) (8192*(1/\sqrt{0.927})^0.5-8192/(((1/\sqrt{0.927})))^0.5)*-0.75]$$

Input interpretation:

$$-9199.90 + \frac{1}{2} \times \frac{2}{4} \times (-1) \left(\left(\sqrt{\frac{1}{\sqrt{0.927}}} - \frac{1}{\sqrt{\frac{1}{\sqrt{0.927}}}} \right) \left(8192 \sqrt{\frac{1}{\sqrt{0.927}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.927}}}} \times (-0.75) \right) \right)$$

Result:

-9336.14...

-9336.14...

$$(2^{-13})/4$$

$$[(((8192*((1/\sqrt{0.9991104684})^{0.5})+8192/((1/\sqrt{0.9991104684})^{0.5}))))^2*1.25+(((8192*((1/\sqrt{0.9991104684})^{0.5})-8192/((1/\sqrt{0.9991104684})^{0.5}))))^2*(-0.75)$$

Input interpretation:

$$\frac{1}{2^{13} \times 4} \left[\left(8192 \sqrt{\frac{1}{\sqrt{0.9991104684}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9991104684}}}} \right)^2 \times 1.25 + \left(8192 \sqrt{\frac{1}{\sqrt{0.9991104684}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9991104684}}}} \right)^2 \times (-0.75) \right]$$

Result:

10240.0...

$$10240 = 8^3 * 2^2 * 5$$

or:

$$(2^{-13})/4$$

$$[(((8192*((1/\sqrt{0.927})^{0.5})+8192/((1/\sqrt{0.927})^{0.5}))))^2*1.25+(((8192*((1/\sqrt{0.927})^{0.5})-8192/((1/\sqrt{0.927})^{0.5}))))^2*(-0.75)$$

Input:

$$\frac{1}{2^{13} \times 4} \left[\left(8192 \sqrt{\frac{1}{\sqrt{0.927}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.927}}}} \right)^2 \times 1.25 + \left(8192 \sqrt{\frac{1}{\sqrt{0.927}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.927}}}} \right)^2 \times (-0.75) \right] \quad (B)$$

Result:

10241.5...

$$10241.5....$$

Summing the two results, we obtain:

$$-9217.36 + (2^{(-13)})/4$$

$$[(((8192*((1/\sqrt{0.9991104684})^{0.5})+8192/((1/\sqrt{0.9991104684})^{0.5}))))^2*1.25+(((8192*((1/\sqrt{0.9991104684})^{0.5})-8192/((1/\sqrt{0.9991104684})^{0.5}))))^2*(-0.75)$$

Input interpretation:

$$-9217.36 + \frac{1}{2^{13} \times 4} \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.9991104684}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9991104684}}}} \right)^2 \times 1.25 + \left(8192 \sqrt{\frac{1}{\sqrt{0.9991104684}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9991104684}}}} \right)^2 \times (-0.75) \right)$$

Result:

1022.64...

1022.64... (note that $1022.64 + \sqrt{2} = 1024.0542 \approx 1024$)

or for $v = 1/\sqrt{0.927}$

$$-9336.14 + (2^{(-13)})/4$$

$$[(((8192*((1/\sqrt{0.927})^{0.5})+8192/((1/\sqrt{0.927})^{0.5}))))^2*1.25+(((8192*((1/\sqrt{0.927})^{0.5})-8192/((1/\sqrt{0.927})^{0.5}))))^2*(-0.75)$$

Input interpretation:

$$-9336.14 + \frac{1}{2^{13} \times 4} \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.927}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.927}}}} \right)^2 \times 1.25 + \left(8192 \sqrt{\frac{1}{\sqrt{0.927}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.927}}}} \right)^2 \times (-0.75) \right)$$

Result:

905.331...

905.331...

We note that $9336.14 - 9217.36 + 905.331 = 1024.111$

Thence, we can to obtain also:

$$(9336.14 - 9217.36) - 9336.14 + (2^{-13})/4$$

$$[\left(\left(\left(8192 \cdot \left(\frac{1}{\sqrt{0.927}}\right)^{0.5} + 8192 / \left(\left(\frac{1}{\sqrt{0.927}}\right)^{0.5}\right)\right)\right)^2 \cdot 1.25 + \left(\left(\left(8192 \cdot \left(\frac{1}{\sqrt{0.927}}\right)^{0.5} - 8192 / \left(\left(\frac{1}{\sqrt{0.927}}\right)^{0.5}\right)\right)\right)^2 \cdot (-0.75)\right)$$

Input interpretation:

$$(9336.14 - 9217.36) - 9336.14 + \frac{1}{2^{13} \times 4} \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.927}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.927}}}} \right)^2 \times 1.25 + \left(8192 \sqrt{\frac{1}{\sqrt{0.927}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.927}}}} \right)^2 \times (-0.75) \right)$$

Result:

1024.11...

1024.11 \approx 1024

Thence, we have a sort of “mean” between the two values of α' , i.e. 0.9991104684 and 0.927 (mean = 0.9630552342) that is near to the following value of Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

If we take instead of 9336.14 or 9217.36, the value 9216.36, (i.e. 9217.36 - 1), and the mean calculated for $\alpha' = 0.9630552342$, we note that obtain:

$$[-9216.36 + (2^{(-13)})/4 \\ [((((8192*((1/\sqrt{0.9630552342}))^0.5)+8192/((1/\sqrt{0.9630552342}))^0.5))))^2*1.2 \\ 5+((((8192*((1/\sqrt{0.9630552342}))^0.5)-8192/((1/\sqrt{0.9630552342}))^0.5))))^2*(- \\ 0.75)))]$$

Input interpretation:

$$-9216.36 + \frac{1}{2^{13} \times 4} \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.9630552342}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \times 1.25 + \left(8192 \sqrt{\frac{1}{\sqrt{0.9630552342}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \times (-0.75) \right)$$

Result:

1024.00...

1024

and, for the previous expression (B), for the value 0.9630552342, we obtain:

$$(2^{(-13)})/4 \\ [((((8192*((1/\sqrt{0.9630552342}))^0.5)+8192/((1/\sqrt{0.9630552342}))^0.5))))^2*1.2 \\ 5+((((8192*((1/\sqrt{0.9630552342}))^0.5)-8192/((1/\sqrt{0.9630552342}))^0.5))))^2*(- \\ 0.75)$$

Input interpretation:

$$\frac{1}{2^{13} \times 4} \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.9630552342}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \times 1.25 + \left(8192 \sqrt{\frac{1}{\sqrt{0.9630552342}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \times (-0.75) \right)$$

Result:

10240.4...

$$10240.4... \approx 10240 = 64 \cdot 32 \cdot 5$$

Indeed:

$$\frac{1}{160} \cdot \left(\frac{(2^{-13})}{4} \left[\left(\left(\frac{8192 \cdot \left(\frac{1}{\sqrt{\sqrt{0.9630552}}} \right)^{0.5} + 8192 / \left(\frac{1}{\sqrt{\sqrt{0.9630552}}} \right)^{0.5} \right)^2 \cdot 1.25 + \left(\left(\frac{8192 \cdot \left(\frac{1}{\sqrt{\sqrt{0.9630552}}} \right)^{0.5} - 8192 / \left(\frac{1}{\sqrt{\sqrt{0.9630552}}} \right)^{0.5} \right)^2 \cdot (-0.75) \right) \right] \right) \right]$$

Input interpretation:

$$\frac{1}{160} \left(\frac{1}{2^{13} \times 4} \left(\left(8192 \sqrt{\frac{1}{\sqrt{\sqrt{0.9630552}}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{\sqrt{0.9630552}}}}} \right)^2 \times 1.25 + \left(8192 \sqrt{\frac{1}{\sqrt{\sqrt{0.9630552}}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{\sqrt{0.9630552}}}}} \right)^2 \times (-0.75) \right) \right)$$

Result:

64.0023...

$$64.0023... \approx 64$$

$$\left[27 \cdot \frac{1}{160} \cdot \left(\frac{(2^{-13})}{4} \left[\left(\left(\frac{8192 \cdot \left(\frac{1}{\sqrt{\sqrt{0.9630552}}} \right)^{0.5} + 8192 / \left(\frac{1}{\sqrt{\sqrt{0.9630552}}} \right)^{0.5} \right)^2 \cdot 1.25 + \left(\left(\frac{8192 \cdot \left(\frac{1}{\sqrt{\sqrt{0.9630552}}} \right)^{0.5} - 8192 / \left(\frac{1}{\sqrt{\sqrt{0.9630552}}} \right)^{0.5} \right)^2 \cdot (-0.75) \right) \right] \right) \right] + 1 \right]$$

Input interpretation:

$$27 \times \frac{1}{160} \left(\frac{1}{2^{13} \times 4} \left(\left(8192 \sqrt{\frac{1}{\sqrt{\sqrt{0.9630552}}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{\sqrt{0.9630552}}}}} \right)^2 \times 1.25 + \left(8192 \sqrt{\frac{1}{\sqrt{\sqrt{0.9630552}}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{\sqrt{0.9630552}}}}} \right)^2 \times (-0.75) \right) \right) + 1$$

Result:

1729.06...

1729.06...

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Thence, we obtain from (260):

$$\begin{aligned} \tilde{A} = & \frac{2^{-13}}{4} \left\{ \left(N\sqrt{v} + \frac{D}{\sqrt{v}} \right)^2 \phi_{++} \right. \\ & + \left(N\sqrt{v} - \frac{D}{\sqrt{v}} \right)^2 \phi_{+-} \\ & + 2 \left[\left(\frac{R_N}{\sqrt{2}} + R_D \right)^2 + \left(\frac{R_N}{\sqrt{2}} \right)^2 \right] \phi_{-+} \\ & \left. + 2 \left[\left(\frac{R_N}{\sqrt{2}} - R_D \right)^2 + \left(\frac{R_N}{\sqrt{2}} \right)^2 \right] \phi_{--} \right\} + \dots \end{aligned} \quad (260)$$

$$\begin{aligned} & (((([27*1/160*((2^(-13))/4 \\ & (((8192*((1/\sqrt{0.9630552})^0.5)+8192/((1/\sqrt{0.9630552})^0.5))))^2*1.25+(((81 \\ & 92*((1/\sqrt{0.9630552})^0.5)-8192/((1/\sqrt{0.9630552})^0.5))))^2*(- \\ & 0.75))))]+1))))^1/15 \end{aligned}$$

Input interpretation:

$$\begin{aligned} & \left(27 \times \frac{1}{160} \left(\frac{1}{2^{13} \times 4} \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.9630552}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9630552}}}} \right)^2 \times 1.25 + \right. \right. \right. \\ & \left. \left. \left(8192 \sqrt{\frac{1}{\sqrt{0.9630552}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9630552}}}} \right)^2 \times (-0.75) \right) + 1 \right)^{1/15} \end{aligned}$$

Result:

1.643819...

$$1.643819\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

and:

$$\left(\left(\left(\frac{27}{160} \cdot (2^{-13}) \right) / 4 \right. \right. \\ \left. \left. \left(\left(\left(8192 \cdot \left(\frac{1}{\sqrt{0.9630552}} \right)^{0.5} \right) + 8192 / \left(\left(\frac{1}{\sqrt{0.9630552}} \right)^{0.5} \right) \right) \right)^2 \cdot 1.25 + \left(\left(\left(8192 \cdot \left(\frac{1}{\sqrt{0.9630552}} \right)^{0.5} \right) - 8192 / \left(\left(\frac{1}{\sqrt{0.9630552}} \right)^{0.5} \right) \right) \right)^2 \cdot (-0.75) \right) \right) \right) + 1 \right)^{1/15} - \frac{26}{10^3}$$

Input interpretation:

$$\left(\frac{27}{160} \left(\frac{1}{2^{13} \times 4} \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.9630552}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9630552}}}} \right)^2 \times 1.25 + \left(8192 \sqrt{\frac{1}{\sqrt{0.9630552}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9630552}}}} \right)^2 \times (-0.75) \right) \right) + 1 \right)^{\frac{1}{15}} - \frac{26}{10^3}$$

Result:

1.617819109004131817616245021316754788981404304100154636369...

1.617819109.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

Now for

$$W_{n_1, n_2} = q^{\frac{Y_2}{4X_2} [(n_1 + X_1 n_2)^2 + n_2^2 X_2^2]},$$

$$P_{\epsilon_1, \epsilon_2} = \sum_{m_1, m_2} q^{\frac{1}{2X_2 Y_2} [(m_2 - \epsilon_2 - X_1(m_1 - \epsilon_1))^2 + (m_1 - \epsilon_1)^2 X_2^2]}.$$
(236)

we obtain:

$$0.5^{((((1/(4*3))(((1+2*2)^2+2^2*3^2))))))}$$

Input:

$$0.5^{1/(4*3)((1+2*2)^2+2^2*3^2)}$$

Result:

$$0.0294961\dots$$

$$0.0294961 = W_n$$

$$0.5^{((((1/(2*3))((2-0.8-2(1-0.5))^2+(1-0.5)^2*3^2))))))}$$

Input:

$$0.5^{1/(2*3)((2-0.8-2(1-0.5))^2+(1-0.5)^2*3^2)}$$

Result:

$$0.767550\dots$$

$$0.767550\dots = P_m$$

$$0.5^{((((1/(2*3))((2-1.3-2(1))^2+(1)^2*3^2))))))}$$

Input:

$$0.5^{1/(2*3)((2-1.3-2*1)^2+1^2*3^2)}$$

Result:

$$0.290847\dots$$

$$0.290847\dots = P_{m+1/2}$$

Now, we have that:

The Klein bottle amplitude completes the projection of the closed sector, and thus receives contributions from all modes mapped onto themselves by Ω . The relevant lattice states, defined by the condition $p_L = p_R$, have zero winding number, and therefore the resulting amplitude is

$$\mathcal{K}_{\text{KK}} = \frac{1}{2}(V_8 - S_8) P_{2m}, \quad (269)$$

while the corresponding transverse-channel amplitude is

$$\tilde{\mathcal{K}}_{\text{KK}} = \frac{2^5}{4} v (V_8 - S_8) W_n, \quad (270)$$

where $v = \frac{R}{\sqrt{\alpha'}}$.

From (270):

$$\tilde{\mathcal{K}}_{\text{KK}} = \frac{2^5}{4} v (V_8 - S_8) W_n,$$

For

$$v = \frac{R}{\sqrt{\alpha'}},$$

$V_8 - S_8 = 5 - 3 = 2$; we obtain:

$$(2^5)/4 * 1/(\text{sqrt}0.9630552) * 2 * 0.0294961$$

Input interpretation:

$$\frac{2^5}{4} \times \frac{1}{\sqrt{0.9630552}} \times 2 \times 0.0294961$$

Result:

0.480905...

0.480905...

From which:

$$2 * (((2^5)/4 * 1/(\text{sqrt}0.9630552) * 2 * 0.0294961)))^{1/4}$$

Input interpretation:

$$2 \sqrt[4]{\frac{2^5}{4} \times \frac{1}{\sqrt{0.9630552}} \times 2 \times 0.0294961}$$

Result:

1.665500...

1.66550.... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Now, we have:

$$n_1 + n_2 + n_3 + n_4 = 32,$$

$$n_1 = 16 = n_4, \quad n_2 = 0 = n_3, \quad (281) \text{ and } (283)$$

and the following direct-channel open string amplitude

$$\mathcal{M}_W = -\frac{1}{2}(n_1 + n_4) \left[(\hat{V}_8 - \hat{S}_8) P_m + (\hat{V}_8 + \hat{S}_8) P_{m+\frac{1}{2}} \right] \quad (285)$$

we obtain:

$$-1/2(16) [2*0.767550+8*0.290847]$$

Input interpretation:

$$-\frac{1}{2} \times 16 (2 \times 0.767550 + 8 \times 0.290847)$$

Result:

-30.895008

-30.895008

From which:

$$18 - (2\sqrt{5})/3 * (((-1/2)(16) [2*0.767550 + 8*0.290847])))$$

Input interpretation:

$$18 - \left(\frac{1}{3} (2\sqrt{5})\right) \left(-\frac{1}{2} \times 16 (2 \times 0.767550 + 8 \times 0.290847)\right)$$

Result:

64.0556...

64.0556... \approx 64

and:

$$27 * (((18 - (2\sqrt{5})/3 * (((-1/2)(16) [2*0.767550 + 8*0.290847]))))) - 1/2$$

Input interpretation:

$$27 \left(18 - \left(\frac{1}{3} (2\sqrt{5})\right) \left(-\frac{1}{2} \times 16 (2 \times 0.767550 + 8 \times 0.290847)\right)\right) - \frac{1}{2}$$

Result:

1729.00...

1729

This result is very near to the mass of candidate glueball **f₀(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$$((((27 * (((18 - (2\sqrt{5})/3 * (((-1/2)(16) [2*0.767550 + 8*0.290847]))))) - 1/2))))^{1/15}$$

Input interpretation:

$$\sqrt[15]{27 \left(18 - \left(\frac{1}{3} (2\sqrt{5})\right) \left(-\frac{1}{2} \times 16 (2 \times 0.767550 + 8 \times 0.290847)\right)\right) - \frac{1}{2}}$$

Result:

1.643815...

$$1.643815\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

$$\left(\left(\left(\left(27 \cdot \left(\left(18 - (2\sqrt{5})/3 \cdot \left(-\frac{1}{2}(16) [2 \cdot 0.767550 + 8 \cdot 0.290847]\right)\right)\right)\right) - \frac{1}{2}\right)\right)\right)^{1/15} - \frac{26}{10^3}$$

Input interpretation:

$$\sqrt[15]{27 \left(18 - \left(\frac{1}{3} (2\sqrt{5})\right) \left(-\frac{1}{2} \times 16 (2 \times 0.767550 + 8 \times 0.290847)\right)\right) - \frac{1}{2} - \frac{26}{10^3}}$$

Result:

1.617815234133748603421777970284434772806294362656246121270...

1.6178152341... result that is a very good approximation to the value of the golden ratio 1.618033988749...

We observe that the results of the two expression, are practically equal. Indeed, we have:

$$\begin{aligned} \tilde{A} = & \frac{2^{-13}}{4} \left\{ \left(N\sqrt{v} + \frac{D}{\sqrt{v}} \right)^2 \phi_{++} \right. \\ & + \left(N\sqrt{v} - \frac{D}{\sqrt{v}} \right)^2 \phi_{+-} \\ & + 2 \left[\left(\frac{R_N}{\sqrt{2}} + R_D \right)^2 + \left(\frac{R_N}{\sqrt{2}} \right)^2 \right] \phi_{-+} \\ & \left. + 2 \left[\left(\frac{R_N}{\sqrt{2}} - R_D \right)^2 + \left(\frac{R_N}{\sqrt{2}} \right)^2 \right] \phi_{--} \right\} + \dots \end{aligned} \quad (260)$$

$$\left(\left(\left(\left(\left[27 \cdot \frac{1}{160} \cdot (2^{-13})/4 \cdot \left(\left(8192 \cdot \left(\frac{1}{\sqrt{0.9630552}}\right)^{0.5} + 8192 / \left(\frac{1}{\sqrt{0.9630552}}\right)^{0.5}\right)\right)^2 \cdot 1.25 + \left(\left(8192 \cdot \left(\frac{1}{\sqrt{0.9630552}}\right)^{0.5} - 8192 / \left(\frac{1}{\sqrt{0.9630552}}\right)^{0.5}\right)\right)^2 \cdot (-0.75)\right)\right)\right] + 1\right)\right)^{1/15}$$

Now, we have that:

$$\begin{aligned} Q_o &= V_4 O_4 - C_4 C_4, & Q_v &= O_4 V_4 - S_4 S_4, \\ Q_s &= O_4 C_4 - S_4 O_4, & Q_c &= V_4 S_4 - C_4 V_4, \end{aligned} \quad (294)$$

that are eigenvectors of the \mathbb{Z}_2 generator [47, 48].

The partition function clearly encodes the massless string excitations, that can be identified using the standard $SO(4) \sim SU(2) \times SU(2)$ decompositions. For instance

$$\begin{aligned} V_4 \times \bar{V}_4 &= (2, 2) \times (2, 2) = (3, 3) + (3, 1) + (1, 3) + (1, 1), \\ C_4 \times \bar{C}_4 &= (2, 1) \times (2, 1) = (3, 1) + (1, 1), \\ S_4 \times \bar{S}_4 &= (1, 2) \times (1, 2) = (1, 3) + (1, 1), \\ V_4 \times \bar{C}_4 &= (2, 2) \times (2, 1) = (3, 2) + (1, 2), \\ V_4 \times \bar{S}_4 &= (2, 2) \times (1, 2) = (2, 3) + (2, 1). \end{aligned} \quad (295)$$

for: $\text{Sqrt}(v_4) = (1/\text{sqrt}(0.9630552342))^{0.5}$; $Q_o = Q_v = 8$

From:

$$\tilde{\mathcal{K}}_0 = \frac{2^5}{4} \left[Q_o \left(\sqrt{v_4} + \frac{1}{\sqrt{v_4}} \right)^2 + Q_v \left(\sqrt{v_4} - \frac{1}{\sqrt{v_4}} \right)^2 \right]. \quad (298)$$

that is the massless tadpole contributions, we obtain:

$(2^5)/4$

$[8((((1/\text{sqrt}(0.9630552342))^{0.5}+1/(((1/\text{sqrt}(0.9630552342))^{0.5}))))^2+8((((1/\text{sqrt}(0.9630552342))^{0.5}-1/(((1/\text{sqrt}(0.9630552342))^{0.5}))))^2]$

Input interpretation:

$$\frac{2^5}{4} \left(8 \left(\left(\sqrt{\frac{1}{\sqrt{0.9630552342}}} + \frac{1}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 + \left(\sqrt{\frac{1}{\sqrt{0.9630552342}}} - \frac{1}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \right) \right)$$

Result:

256.2040698...

256.2040698.... $\approx 256 = 64*4$

From which:

$$\frac{1}{4} * (2^5)/4$$

$$[8((((1/\sqrt{0.9630552342})^{0.5}+1/(((1/\sqrt{0.9630552342})^{0.5}))))^2+8((((1/\sqrt{0.9630552342})^{0.5}-1/(((1/\sqrt{0.9630552342})^{0.5}))))^2]$$

Input interpretation:

$$\frac{1}{4} \times \frac{2^5}{4} \left(\left(\left(\sqrt{\frac{1}{\sqrt{0.9630552342}}} + \frac{1}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 + \left(\sqrt{\frac{1}{\sqrt{0.9630552342}}} - \frac{1}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \right) \right)$$

Result:

64.05101744...

64.05101744... \approx 64

From which:

$$27 * ((1/4 * (2^5)/4$$

$$[8((((1/\sqrt{0.9630552342})^{0.5}+1/(((1/\sqrt{0.9630552342})^{0.5}))))^2+8((((1/\sqrt{0.9630552342})^{0.5}-1/(((1/\sqrt{0.9630552342})^{0.5}))))^2]]))$$

Input interpretation:

$$27 \left(\frac{1}{4} \times \frac{2^5}{4} \left(\left(\left(\sqrt{\frac{1}{\sqrt{0.9630552342}}} + \frac{1}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 + \left(\sqrt{\frac{1}{\sqrt{0.9630552342}}} - \frac{1}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \right) \right) \right)$$

Result:

1729.377471...

1729.377471...

This result is very near to the mass of candidate glueball **f₀(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

and:

$$\left(\left(\left(\left(27 \cdot \left(\frac{1}{4} \cdot \frac{2^5}{4} \right) \right) \left[8 \left(\left(\frac{1}{\sqrt{0.9630552342}} \right)^{0.5} + \frac{1}{\left(\frac{1}{\sqrt{0.9630552342}} \right)^{0.5}} \right) \right]^2 + 8 \left(\left(\frac{1}{\sqrt{0.9630552342}} \right)^{0.5} - \frac{1}{\left(\frac{1}{\sqrt{0.9630552342}} \right)^{0.5}} \right) \right]^2 \right) \right)^{1/15}$$

Input interpretation:

$$\left(27 \left[\frac{1}{4} \times \frac{2^5}{4} \left(8 \left(\sqrt{\frac{1}{\sqrt{0.9630552342}}} + \frac{1}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 + 8 \left(\sqrt{\frac{1}{\sqrt{0.9630552342}}} - \frac{1}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \right) \right] \right)^{1/15}$$

Result:

1.6437712400...

$$1.6437712400.... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

For the previous expressions:

$$n_1 + n_2 + n_3 + n_4 = 32,$$

$$n_1 = 16 = n_4, \quad n_2 = 0 = n_3,$$

and

$$R_N = i(n - \bar{n}),$$

$$R_D = i(d_i - \bar{d}_i), \quad (\text{see 315})$$

For: $N = D = 8192$; $\text{Sqrt}(v_4) = (1/\text{sqrt}(0.9630552342))^{0.5}$; $Q_0 = Q_v = 8$;

$$R_N = R_D = 16$$

We have that:

It is instructive to compare these results with a more general case, where the D5 branes are distributed over the 16 fixed points, whose coordinates are denoted concisely by x . The direct-channel amplitude now reads

$$\begin{aligned}
\mathcal{A} = \frac{1}{4} & \left[(Q_o + Q_v) \left(N^2 \sum_m \frac{q^{\frac{\alpha'}{2} m^T g^{-1} m}}{\eta^4} \right. \right. \\
& + \left. \sum_{i,j=1}^{16} D_i D_j \sum_n \frac{q^{\frac{1}{2\alpha'} (n+x_i-x_j)^T g (n+x_i-x_j)}}{\eta^4} \right) \\
& + \left(R_N^2 + \sum_{i=1}^{16} R_{D,i}^2 \right) (Q_o - Q_v) \left(\frac{2\eta}{\vartheta_2} \right)^2 \\
& + 2N \sum_{i=1}^{16} D_i (Q_s + Q_c) \left(\frac{\eta}{\vartheta_4} \right)^2 \\
& \left. + 2R_N \sum_{i=1}^{16} R_{D,i} (Q_s - Q_c) \left(\frac{\eta}{\vartheta_3} \right)^2 \right], \tag{309}
\end{aligned}$$

while the corresponding tadpole contributions

$$\begin{aligned}
\tilde{\mathcal{A}}_0 = \frac{2^{-5}}{4} & \left[Q_o \left(N\sqrt{v_4} + \sum_{i=1}^{16} \frac{D_i}{\sqrt{v_4}} \right)^2 + Q_v \left(N\sqrt{v_4} - \sum_{i=1}^{16} \frac{D_i}{\sqrt{v_4}} \right)^2 \right. \\
& \left. + Q_s \sum_{i=1}^{16} (R_N - 4R_{D,i})^2 + Q_c \sum_{i=1}^{16} (R_N + 4R_{D,i})^2 \right] \tag{310}
\end{aligned}$$

reflect again the distribution of the D5 branes among the fixed points.

One can actually consider a more general situation, where pairs of image D5 branes are moved away from the fixed points, to generic positions denoted concisely by y , as first shown in [142]. The main novelty is that the R_D terms are absent for the pairs of displaced branes. This reflects the fact that the projection interchanges the images in each pair, consistently with the structure of the conformal field theory, and this more general configuration thus results in the annulus amplitude

$$\begin{aligned}
\mathcal{A} = \frac{1}{4} & \left[(Q_o + Q_v) \left(N^2 \sum_m \frac{q^{\frac{\alpha'}{2} m^T g^{-1} m}}{\eta^4} \right. \right. \\
& \left. + \sum_{i,j=1}^{16} D_i D_j \sum_n \frac{q^{\frac{1}{2\alpha'} (n+x_i-x_j)^T g (n+x_i-x_j)}}{\eta^4} \right) \\
& \left. \right],
\end{aligned}$$

$$\begin{aligned}
& + \sum_{i=1}^{16} \sum_{k=1}^{2p} D_i D_k \sum_n \frac{q^{\frac{1}{2\alpha'}(n+x_i-y_k)^T g(n+x_i-y_k)}}{\eta^4} \\
& + \sum_{k,l=1}^{2p} D_k D_l \sum_n \frac{q^{\frac{1}{2\alpha'}(n+y_k-y_l)^T g(n+y_k-y_l)}}{\eta^4} \\
& + \left(R_N^2 + \sum_{i=1}^{16} R_{D,i}^2 \right) (Q_o - Q_v) \left(\frac{2\eta}{\vartheta_2} \right)^2 \\
& + 2N \left(\sum_{i=1}^{16} D_i + \sum_{k=1}^{2p} D_k \right) (Q_s + Q_c) \left(\frac{\eta}{\vartheta_4} \right)^2 \\
& + 2R_N \sum_{i=1}^{16} R_{D,i} (Q_s - Q_c) \left(\frac{\eta}{\vartheta_3} \right)^2 \Big], \tag{311}
\end{aligned}$$

where the indices i, j refer to the D5 branes at the 16 fixed points x , while the indices k, l refer to the p image pairs of D5 branes away from the fixed points, at generic positions y .

In this case the tadpole contributions may be read from

$$\begin{aligned}
\tilde{\mathcal{A}}_0 = & \frac{2^{-5}}{4} \left\{ Q_o \left[N\sqrt{v_4} + \frac{1}{\sqrt{v_4}} \left(\sum_{i=1}^{16} D_i + \sum_{k=1}^{2p} D_k \right) \right]^2 \right. \\
& + Q_v \left[N\sqrt{v_4} - \frac{1}{\sqrt{v_4}} \left(\sum_{i=1}^{16} D_i + \sum_{k=1}^{2p} D_k \right) \right]^2 \\
& \left. + Q_s \sum_{i=1}^{16} (R_N - 4R_{D,i})^2 + Q_c \sum_{i=1}^{16} (R_N + 4R_{D,i})^2 \right\}, \tag{312}
\end{aligned}$$

and, while the untwisted exchanges are sensitive to all branes, the twisted ones feel only the branes that touch the fixed points, consistently with the fact that twisted closed-strings states are confined to them. In general, some of these can be “fractional branes” [145], peculiar branes stuck at the fixed points that are responsible for the generalized Green-Schwarz couplings of [49, 66]. While they are not present in this model, for a reason that will soon be evident, we shall meet them in the next subsections.

We obtain from (310):

$$\begin{aligned}
& (2^{(-5)})/4 \\
& [8(((8192*(1/\text{sqrt}(0.9630552342))^0.5+8192/((1/\text{sqrt}(0.9630552342))^0.5))))^2+8(((8192*(1/\text{sqrt}(0.9630552342))^0.5-8192/((1/\text{sqrt}(0.9630552342))^0.5))))^2+8(16-64)^2+8(16+64)^2]
\end{aligned}$$

Result:

64.003352838...

64.003352838... \approx 64

$$27 * \left(\left(\left(\left(\left(2^{-5} \right) / 4 \right) \left[8 \left(\left(8192 * \left(1 / \sqrt{0.9630552342} \right) \right)^{0.5} + 8192 / \left(\left(1 / \sqrt{0.9630552342} \right) \right)^{0.5} \right) \right]^2 + 8 \left(\left(8192 * \left(1 / \sqrt{0.9630552342} \right) \right)^{0.5} - 8192 / \left(\left(1 / \sqrt{0.9630552342} \right) \right)^{0.5} \right) \right]^2 + 8 \left(16 - 64 \right)^2 + 8 \left(16 + 64 \right)^2 \right) \right) \right)^{1/4} + 1$$

Input interpretation:

$$27 \left(\frac{1}{2^5 \times 4} \left(8 \left(8192 \sqrt{\frac{1}{\sqrt{0.9630552342}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 + 8 \left(8192 \sqrt{\frac{1}{\sqrt{0.9630552342}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 + 8 \left(16 - 64 \right)^2 + 8 \left(16 + 64 \right)^2 \right) \right)^{(1/4) + 1}$$

Result:

1729.0905266...

1729.0905266...

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

and:

$$\left(\left(27 \cdot \left(\left(\left(\left(\left(2^{-5} \right) / 4 \right) \left[8 \left(\left(8192 \cdot \left(1 / \sqrt{0.9630552} \right) \right)^{0.5} + 8192 / \left(\left(1 / \sqrt{0.9630552} \right) \right)^{0.5} \right) \right)^2 + 8 \left(\left(8192 \cdot \left(1 / \sqrt{0.9630552} \right) \right)^{0.5} - 8192 / \left(\left(1 / \sqrt{0.9630552} \right) \right)^{0.5} \right) \right)^2 + 8 \left(16 - 64 \right)^2 + 8 \left(16 + 64 \right)^2 \right) \right) \right)^{1/4 + 1} \right)^{1/15}$$

Input interpretation:

$$\left(27 \left[\frac{1}{2^5 \times 4} \left(8 \left[8192 \sqrt{\frac{1}{\sqrt{0.9630552}}} + \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9630552}}}} \right]^2 + 8 \left[8192 \sqrt{\frac{1}{\sqrt{0.9630552}}} - \frac{8192}{\sqrt{\frac{1}{\sqrt{0.9630552}}}} \right]^2 + 8 \left(16 - 64 \right)^2 + 8 \left(16 + 64 \right)^2 \right] \right)^{(1/4) + 1} \right)^{(1/15)}$$

Result:

1.64382097...

$$1.64382097 \dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Now, we have that:

$$\begin{aligned} \tilde{\mathcal{A}}_0 = & \frac{2^{-5}}{4} \left\{ Q_o \left[N \sqrt{v_4} + \frac{1}{\sqrt{v_4}} \left(\sum_{i=1}^{16} D_i + \sum_{k=1}^{2p} D_k \right) \right]^2 \right. \\ & + Q_v \left[N \sqrt{v_4} - \frac{1}{\sqrt{v_4}} \left(\sum_{i=1}^{16} D_i + \sum_{k=1}^{2p} D_k \right) \right]^2 \\ & \left. + Q_s \sum_{i=1}^{16} (R_N - 4R_{D,i})^2 + Q_c \sum_{i=1}^{16} (R_N + 4R_{D,i})^2 \right\}, \end{aligned} \quad (312)$$

from the previous expression

$$\tilde{\mathcal{A}}_0 = \frac{2^{-5}}{4} \left[Q_o \left(N\sqrt{v_4} + \sum_{i=1}^{16} \frac{D_i}{\sqrt{v_4}} \right)^2 + Q_v \left(N\sqrt{v_4} - \sum_{i=1}^{16} \frac{D_i}{\sqrt{v_4}} \right)^2 + Q_s \sum_{i=1}^{16} (R_N - 4R_{D,i})^2 + Q_c \sum_{i=1}^{16} (R_N + 4R_{D,i})^2 \right]$$

we obtain from

$$\tilde{\mathcal{A}}_0 = \frac{2^{-5}}{4} \left\{ Q_o \left[N\sqrt{v_4} + \frac{1}{\sqrt{v_4}} \left(\sum_{i=1}^{16} D_i + \sum_{k=1}^{2p} D_k \right) \right]^2 + Q_v \left[N\sqrt{v_4} - \frac{1}{\sqrt{v_4}} \left(\sum_{i=1}^{16} D_i + \sum_{k=1}^{2p} D_k \right) \right]^2 + Q_s \sum_{i=1}^{16} (R_N - 4R_{D,i})^2 + Q_c \sum_{i=1}^{16} (R_N + 4R_{D,i})^2 \right\}, \quad (312)$$

$$(2^{(-5)})/4 * [8 * (((8192 * (1/\sqrt{0.9630552342}))^{0.5} + 1/((1/\sqrt{0.9630552342}))^{0.5})) * (2 * 8192)))^2 + 8 * (((8192 * (1/\sqrt{0.9630552342}))^{0.5} - 1/((1/\sqrt{0.9630552342}))^{0.5})) * (2 * 8192)))^2 + 8 * (16 - 64)^2 + 8 * (16 + 64)^2]$$

Input interpretation:

$$\frac{1}{2^5 \times 4} \left(\left(\left(8192 \sqrt{\frac{1}{\sqrt{0.9630552342}}} + \frac{1}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right) (2 \times 8192) \right)^2 + \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.9630552342}}} - \frac{1}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right) (2 \times 8192) \right)^2 + 8(16 - 64)^2 + 8(16 + 64)^2 \right)$$

Result:

$$2.2945851949117085360534015202976144377149294233052234... \times 10^{15}$$

[2294585194911708.536053](#)

From which:

$$(2294585194911708.536053)/(64^6 \cdot 64 \cdot 8) - 64^{(1/22)}$$

Input interpretation:

$$\frac{2.294585194911708536053 \times 10^{15}}{64^6 \times 64 \times 8} - \sqrt[22]{64}$$

Result:

64.00794422868307668189...

64.00794422868... \approx 64

or, considering $0.9630552342 \approx 0.963$:

Input:

$$\frac{1}{2^5 \times 4} \times \frac{1}{64^6 \times 64 \times 8} \left(8 \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.963}}} + \frac{1}{\sqrt{\frac{1}{\sqrt{0.963}}}} \right) (2 \times 8192) \right)^2 + \right. \\ \left. 8 \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.963}}} - \frac{1}{\sqrt{\frac{1}{\sqrt{0.963}}}} \right) (2 \times 8192) \right)^2 + \right. \\ \left. 8 (16 - 64)^2 + 8 (16 + 64)^2 \right) - \sqrt[22]{64}$$

Result:

64.0098...

64.0098... \approx 64

and:

$$27[(2^{(-5)})/4*[8*(((8192*(1/\sqrt{0.963}))^{0.5}+1/((1/\sqrt{0.963}))^{0.5}))(2*8192)))^2+8*(((8192*(1/\sqrt{0.963}))^{0.5}-1/((1/\sqrt{0.963}))^{0.5}))(2*8192)))^2+8(16-64)^2+8(16+64)^2]/((64^6*64*8)-64^{(1/22)}]$$

Input:

$$27 \left(\frac{1}{2^5 \times 4} \times \frac{1}{64^6 \times 64 \times 8} \left[8 \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.963}}} + \frac{1}{\sqrt{\frac{1}{\sqrt{0.963}}}} \right) (2 \times 8192) \right)^2 + 8 \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.963}}} - \frac{1}{\sqrt{\frac{1}{\sqrt{0.963}}}} \right) (2 \times 8192) \right)^2 + 8(16 - 64)^2 + 8(16 + 64)^2 - \sqrt[22]{64} \right] \right)$$

Result:

1728.26...

1728.26...

This result is very near to the mass of candidate glueball **f₀(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$$(27[(2^{-5})/4[8(((8192(1/\sqrt{0.963})^{0.5}+1/((1/\sqrt{0.963})^{0.5}))(2*8192)))^2+8((((8192(1/\sqrt{0.963})^{0.5}-1/((1/\sqrt{0.963})^{0.5}))(2*8192)))^2+8(16-64)^2+8(16+64)^2]/((64^6*512))-64^{(1/22)}])^{1/15}$$

Input:

$$\left(27 \left[\frac{1}{2^5 \times 4} \times \frac{1}{64^6 \times 512} \left(8 \left(\left(8192 \sqrt{\frac{1}{\sqrt{0.963}}} + \frac{1}{\sqrt{\frac{1}{\sqrt{0.963}}}} \right) (2 \times 8192) \right)^2 + \left(8192 \sqrt{\frac{1}{\sqrt{0.963}}} - \frac{1}{\sqrt{\frac{1}{\sqrt{0.963}}}} \right) (2 \times 8192) \right)^2 + 8(16 - 64)^2 + 8(16 + 64)^2 - \sqrt[22]{64} \right] \right)^{1/15}$$

Result:

1.643768633074275164116393204490819455428812998513433084314...

$$1.643768633\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Now, from:

As is typically the case for non-supersymmetric models, a dilaton potential, here localized on the $\overline{D5}$ branes, is generated. This can be easily deduced from the transverse-channel amplitudes, that in general encode the one-point functions of bulk fields on branes and orientifold planes, and in this case the uncancelled tadpoles

$$\left[(N - 32)\sqrt{v_4} + \frac{D + 32}{\sqrt{v_4}} \right]^2 V_4 O_4 + \left[(N - 32)\sqrt{v_4} - \frac{D + 32}{\sqrt{v_4}} \right]^2 O_4 V_4 \quad (353)$$

are associated to the characters $V_4 O_4$ and $O_4 V_4$, and thus to the deviations of the six-dimensional dilaton φ_6 and of the internal volume v_4 with respect to their background values. Proceeding as in subsection 5.6, factorization and the R-R tadpole conditions

$$\left[(N - 32)\sqrt{v_4} + \frac{D + 32}{\sqrt{v_4}} \right]^2 V_4 O_4 + \left[(N - 32)\sqrt{v_4} - \frac{D + 32}{\sqrt{v_4}} \right]^2 O_4 V_4 \quad (353)$$

$$N = D = 8192; \quad \text{Sqrt}(v_4) = (1/\text{sqrt}(0.9630552342))^{0.5}; \quad Q_0 = Q_v = 8;$$

$V_4 O_4 = O_4 V_4 = 12$; we obtain from (353):

$$[(8192 - 32) * (1/\text{sqrt}(0.9630552342))^{0.5} + (8192 + 32) / ((1/\text{sqrt}(0.9630552342))^{0.5})]^2 * 12 + [(8192 - 32) * (1/\text{sqrt}(0.9630552342))^{0.5} - (8192 + 32) / ((1/\text{sqrt}(0.9630552342))^{0.5})]^2 * 12$$

Input interpretation:

$$\left((8192 - 32) \sqrt{\frac{1}{\sqrt{0.9630552342}}} + \frac{8192 + 32}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \times 12 + \left((8192 - 32) \sqrt{\frac{1}{\sqrt{0.9630552342}}} - \frac{8192 + 32}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \times 12$$

Result:

$$3.221371548... \times 10^9$$

$$3.221371548... * 10^9$$

From which:

$$\left(\left(\left(\left((8192 - 32) * (1/\text{sqrt}(0.9630552342))^{0.5} + (8192 + 32) / ((1/\text{sqrt}(0.9630552342))^{0.5}) \right)^2 * 12 + \left((8192 - 32) * (1/\text{sqrt}(0.9630552342))^{0.5} - (8192 + 32) / ((1/\text{sqrt}(0.9630552342))^{0.5}) \right)^2 * 12 \right) \right) \right) * 1 / (64^4 * 3)$$

Input interpretation:

$$\left(\left(\left((8192 - 32) \sqrt{\frac{1}{\sqrt{0.9630552342}}} + \frac{8192 + 32}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \times 12 + \left((8192 - 32) \sqrt{\frac{1}{\sqrt{0.9630552342}}} - \frac{8192 + 32}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \times 12 \right) \right) \times \frac{1}{64^4 \times 3}$$

Result:

64.00290226...

64.00290226.... ≈ 64

and:

$$27 \left(\left((8192 - 32) \left(\frac{1}{\sqrt{0.9630552342}} \right)^{0.5} + \frac{8192 + 32}{\left(\frac{1}{\sqrt{0.9630552342}} \right)^{0.5}} \right)^2 \times 12 + \left((8192 - 32) \left(\frac{1}{\sqrt{0.9630552342}} \right)^{0.5} - \frac{8192 + 32}{\left(\frac{1}{\sqrt{0.9630552342}} \right)^{0.5}} \right)^2 \times 12 \right) \times \frac{1}{64^4 \times 3}$$

Input interpretation:

$$27 \left(\left((8192 - 32) \sqrt{\frac{1}{\sqrt{0.9630552342}}} + \frac{8192 + 32}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \times 12 + \left((8192 - 32) \sqrt{\frac{1}{\sqrt{0.9630552342}}} - \frac{8192 + 32}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \times 12 \right) \times \frac{1}{64^4 \times 3}$$

Result:

1728.078361...

1728.078361...

This result is very near to the mass of candidate glueball **f₀(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$$[27(((([(8192-32)*(1/\sqrt{0.9630552342})^{0.5}+(8192+32)/((1/\sqrt{0.9630552342})^{0.5})]^2*12+[(8192-32)*(1/\sqrt{0.9630552342})^{0.5}-(8192+32)/((1/\sqrt{0.9630552342})^{0.5})]^2*12)))) * 1/(64^4*3)]^{1/15}$$

Input interpretation:

$$\left(27 \left(\left((8192 - 32) \sqrt{\frac{1}{\sqrt{0.9630552342}}} + \frac{8192 + 32}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \times 12 + \left((8192 - 32) \sqrt{\frac{1}{\sqrt{0.9630552342}}} - \frac{8192 + 32}{\sqrt{\frac{1}{\sqrt{0.9630552342}}}} \right)^2 \times 12 \right) \times \frac{1}{64^4 \times 3} \right)^{(1/15)}$$

Result:

1.6437567988...

$$1.6437567988.... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

We note that the various values, very close to $\zeta(2)$, are recurrent and have, although with slight variations, a "repetitive" trend, typical of fractal mathematics

Observations

From:

https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJlQxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that $p(9) = 30$, $p(9 + 5) = 135$, $p(9 + 10) = 490$, $p(9 + 15) = 1,575$ and so on are all divisible by 5. Note that here the n 's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of $p(n)$ that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n 's separated by $5^3 = 125$ units, saying that the corresponding $p(n)$'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and $4096 = 64^2$

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π , ϕ , $1/\phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the n th Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.^[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ , the golden ratio.^[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies^[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

8 64 512

$8 + 56 (8 \times 7) =$

$64 + 448 (64 \times 7) =$

8

64

512

$8; 8 + (8 \times 7) = 64; 64 + (64 \times 7) = 512$

$512 + 64 = 576; \sqrt{576} = 24$

The Radial Octavisation of the Vacuum Geometry

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RAMANUJAN-TYPE FORMULAE FOR $1/\pi$: q -ANALOGUES

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