THE ULTIMATE NUMBERS

The Ultimate Numbers and the 3/2 Ratio

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Abstract. According to a new mathematical definition, whole numbers are divided into two sets, one of which is the merger of the sequence of prime numbers and numbers zero and one. Three other definitions, deduced from this first, subdivide the set of whole numbers into four classes of numbers with own and unique arithmetic properties. The geometric distribution of these different types of whole numbers, in various closed matrices, is organized into exact value ratios to 3/2 or 1/1.

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1. Introduction

This study invests the whole numbers* set and proposes a mathematical definition to integrate the number *zero* (0) and the number *one* (1) into the thus called prime numbers sequence. This set is called the set of ultimate numbers. The study of many matrices of numbers such as, for example, the table of cross additions of the ten digit-numbers (from 0 to 9) highlights a non-random arithmetic and geographic organization of these ultimate numbers. It also appears that this distinction between ultimate and non-ultimate numbers (like also other proposed distinctions of different classes of whole numbers) is intimately linked to the decimal system, in particular and mainly by an almost systematic opposition of the entities in a ratio to 3/2. Indeed this ratio can only manifest itself in the presence of multiples of five (10/2) entities. Also, it is within matrices of ten times ten numbers that the majority of demonstrations validating an opposition of entities in ratios to value 3/2 or /and value to 1/1 are made.

* In statements, when this is not specified, the term "number" always implies a "whole number". Also, It is agreed that the number *zero* (0) is well integrated into the set of whole numbers.

2. The ultimate numbers

2.1 Definition of an ultimate number

Considering the set of whole numbers, these are organized into two sets: ultimate numbers and non-ultimate numbers.

Ultimate numbers definition:

> ***An ultimate number not admits any non-trivial divisor (whole number) being less than it.***

Non-ultimate numbers definition:

> ***A non-ultimate number admits at least one non-trivial divisor (whole number) being less than it.***

Note: a non-trivial divisor of a whole number *n* is a whole number which is a divisor of *n* but distinct from *n* and from 1 (which are its trivial divisors).

2.2. The first ten ultimate numbers and the first ten non-ultimate numbers

Considering the previous double definition, the sequence of ultimate numbers is initialized by these ten numbers:

0 1 2 3 5 7 11 13 17 19

Considering the previous double definition, the sequence of non-ultimate numbers is initialized by these ten numbers:

4 6 8 9 10 12 14 15 16 18

2.3 Development

2.3.1 Other definitions

Let *n* be a whole number (belonging to ℕ*), this one is ultimate if no divisor (whole number) lower than its value and other than 1 divides it.

Let *n* be a natural whole number (belonging to ℕ*), this one is non-ultimate if at least one divisor (whole number) lower than its value and other than 1 divides it.

2.3.2 Development

Below are listed, to illustration of definition, some of the first ultimate or non-ultimate numbers defined above, especially particular numbers *zero* (0) and *one* (1).

- 0 is ultimate: although it admits an infinite number of divisors superior to it, **since it is the first whole number**, the number 0 does not admit any divisor **being inferior to it**.
- 1 is ultimate: since the division by 0 has no defined result, the number 1 does not admit any divisor (whole number) being less than it.
- 2 is ultimate: since the division by 0 has no defined result, the number 2 does not admit any divisor* being less than it.
- 4 is non-ultimate: the number 4 admits the number 2 (number being less than it) as divisor *.
- 6 is non-ultimate: the number 6 admits numbers 2 and 3 (numbers being less than it) as divisors *.
- 7 is ultimate: since the division by 0 has no defined result, the number 7 does not admit any divisor* being less than it. The non-trivial divisors 2, 3, 4, 5 and 6 cannot divide it into whole numbers.
- 12 is non-ultimate: the number 6 admits numbers 2, 3, 4 and 6 (numbers being less than it) as divisors*.

Thus, by these previous definitions, the set of whole numbers is organized into these two entities:

- the set of ultimate numbers, which is the fusion of the prime numbers sequence with the numbers 0 and 1.

- the set of non-ultimate numbers identifying to the non-prime numbers sequence, deduced from the numbers 0 and 1.

* non-trivial divisor.

2.4 Conventional designations

As "primes" designates prime numbers, it is agree that designation "ultimates" designates ultimate numbers. Also it is agree that designation "non-ultimates" designates non-ultimate numbers. Other conventional designations will be applied to the different classes or types of whole numbers later introduced.

2.5 The ultimate numbers and the decimal system

It turns out that the tenth ultimate number is the number 19, a number located in twentieth place in the sequence of the whole numbers. This peculiarity undeniably links the ultimate numbers and the decimal system. So the first twenty numbers (twice ten numbers) are organized into different 1/1 and 3/2 ratios according to their different attributes.

By the nature of the decimal system, as shown in Figure 1, the ten *digit numbers* (digits confused as numbers) are opposed to the first ten *non-digit numbers* by a ratio of 1/1. Also, there are exactly the same quantity of ultimates and non-ultimates among these twenty numbers, so ten entities in each category. In a double 3/2 value ratio, six ultimates versus four are among the ten digit numbers and six non-ultimates versus four are among the first ten non-digit numbers.

10 digit numbers	← **1/1 ratio** →	10 non-digit numbers
6 ultimates	← **3/2 ratio** →	**4 ultimates**
0 **1** **2** **3** 4 **5** 6 **7** 8 9		10 **11** 12 **13** 14 15 16 **17** 18 **19**
4 non-ultimates	← **2/3 ratio** →	6 non-ultimates
3/2 ratio		**2/3 ratio**

Fig.1 Differentiation of the 20 fundamental numbers according to their digitality or non-digitality: the 10 digit numbers (digits confused as numbers) and the first 10 non-digit numbers.

As shown in Figure 2, it is also possible to describe this arithmetic phenomenon by crossing criteria. Thus, the first ten ultimates are opposed to the ten non-ultimates by a 1/1 value ratio. Also, there are exactly the same quantity of digit numbers and non-digit numbers among these twenty numbers. In a twice 3/2 ratio, six digits versus four are among the ten ultimates and six non-digits numbers versus four are among the first ten non-ultimates.

10 first ultimate numbers:	**← 1/1 ratio →**	10 first non-ultimate numbers:
6 digit numbers	**← 3/2 ratio →**	**4 digit numbers**
0 1 2 3 5 7		**4 6 8 9**
11 13 17 19		10 12 14 15 16 18
4 non-digit numbers	**← 2/3 ratio →**	6 non-digit numbers
3/2 ratio		**2/3 ratio**

Fig.2 Differentiation of the 20 fundamental numbers according to their ultimity or non-ultimity: 10 ultimates versus 10 non-ultimates.

Technical remark: due to a certain complexity of the phenomena presented and to clarify their understanding, no figure (table) has a title but just a legend in this paper.

2.6 The twenty fundamental numbers

Whole numbers sequence is therefore initialized by twenty numbers with symmetrically and asymmetrically complementary characteristics of reversible 1/1 and 3/2 ratios. This transcendent entanglement of the first twenty numbers according to their ultimate or non-ultimate nature (ultimate numbers or non-ultimate numbers) and according to their digit or non-digit nature (digits or non-digit numbers) allows, by convention, to qualify them as "fundamental numbers" among the whole numbers set. Figure 3 describes the total entanglement of these twenty fundamental numbers.

	digitality		
ultimity	**0 1 2 3 5 7**	**4 6 8 9**	non-ultimity
	11 13 17 19	*10 12 14 15 16 18*	
	non-digitality		

Fig. 3 Entanglement of the 20 fundamental numbers according to their ultimity or non-ultimity and their digitality or non-digitality.

Thus, the set of the first twenty whole numbers is simultaneously made up to a set of twenty entities including ten ultimate numbers and ten non-ultimate numbers and to a (same) set of twenty entities including ten digit numbers (10 digits) and ten non-digit numbers (not digits). Also, each of these four entangled subsets of ten entities with their own properties opposing two by two in 1/1 value ratio is composed of two opposing subsets in 3/2 value ratio according to the mixed properties of its components. This set of the first twenty numbers is defined as the set of fundamental numbers among the whole numbers. So it is agree that designation "fundamentals" designates these twenty fundamental numbers previously defined.

2.7 The thirty initial numbers

Also, according to the progressive consideration of three sets of 10, 20 and then 30 entities (the first thirty whole numbers), the ratio between the ultimate and non-ultimate numbers increases from 3/2 (10 numbers) to 1/1 (20 numbers) then switches to 2/3 (30 numbers). Thus (Figure 4), depending on whether we consider the first ten, the first twenty and then the first thirty whole numbers, 6 ultimates are opposed to 4 non-ultimates, then 10 ultimates are opposed to 10 non-ultimates then finally 12 ultimates are opposed to 18 non-ultimates. Beyond this triple set, no similar organization of (consecutive) groups of ten entities takes place. These thirty numbers are therefore here called "initials" among the set of natural numbers.

30 initial numbers		
20 fundamental numbers		
10 digits (digit numbers)		
0 1 2 3 4 **5** 6 **7** 8 9	10 **11** 12 **13** 14 15 16 **17** 18 **19**	20 21 22 **23** 24 25 26 27 28 **29**
6 ultimates / 4 non-ultimates: **3/2 ratio**		
10 ultimates / 10 non-ultimates: **1/1 ratio**		
12 ultimates / 18 non-ultimates: **3/2 ratio**		

Fig. 4 Switching from the 3/2 ratio to the 2/3 ratio according to the classification of the first thirty whole numbers and their degree of ultimity.

3. Addition matrix of the ten digits

The table in Figure 5 represents the matrix of the hundred different possible sums of additions (crossed) of the ten digit numbers (from 0 to 9) of the decimal system (ie the first ten whole numbers). Within this table operate multiple singular arithmetic phenomena depending on the ultimate or non-ultimate nature of the values of these hundred sums and their geographic distribution including mainly various 3/2 value ratios often transcendent.

3.1 Sixty versus forty numbers: 3/2 ratio

Among these hundred values, there are 40 ultimate numbers ($5x \longrightarrow x = 8$) and consecutively 60 non-ultimate numbers ($5y \longrightarrow y = 12$). These two sets therefore oppose each other in an exact 2/3 value ratio.

+	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	**0**	**1**	**2**	**3**	4	**5**	6	**7**	8	9
1	**1**	**2**	**3**	4	**5**	6	**7**	8	9	10
2	**2**	**3**	4	**5**	6	**7**	8	9	10	**11**
3	**3**	4	**5**	6	**7**	8	9	10	**11**	12
4	4	**5**	6	**7**	8	9	10	**11**	12	**13**
5	**5**	6	**7**	8	9	10	**11**	12	**13**	14
6	6	**7**	8	9	10	**11**	12	**13**	14	15
7	**7**	8	9	10	**11**	12	**13**	14	15	16
8	8	9	10	**11**	12	**13**	14	15	16	**17**
9	9	10	**11**	12	**13**	14	15	16	**17**	18

40 ultimates	**6**	**5 + 5**	**4 + 4 + 4**	**3 + 3 + 3 + 3**
	1(n)	**2(n-1)**	**3(n-2)**	**4(n-3)**
60 non-ultimates	4	5 + 5	6 + 6 + 6	7 + 7 + 7 + 7
	(2n/3)	2((2n/3) + 1)	3((2n/3)+2)	4((2n/3) + 3)

Fig. 5 Cross additions table of the ten digit numbers.

Also in this table, the quantities of the values equal to an ultimate number decrease regularly from 6 entities (n) to 3 from the first column to the tenth. This decrease is distinguished by a double arithmetic phenomenon: the first column, which represents the additions of the ten digit numbers with the first of these (0), therefore total a unique value number of 6 ultimates (n); the next two columns add up to two same ultimates quantities and that value (5) is just one unit less than the number in the first addition column; the following three columns total three same values (4) lower by one unit than the two preceding columns then finally, the four final columns continue and close this regular arithmetic arrangement with four same values of non-ultimate numbers (3) also lower by one unit to the three preceding columns. The same arithmetic arrangement is observed for the counting of sums equal to a non-ultimate number but in an increasing direction of the quantities of the non-ultimate numbers counted and with a source number (4) equal to $2n/3$. By the nature of this crosstab, the same phenomenon naturally occurs from line to line.

In this matrix, the addition columns are therefore grouped by one, two, three and then four arithmetic entities. Also, from the value n (6 ultimates in first column of additions), the complet sum of ultimates is obtained by this formula:

$$n + 2(n - 1) + 3(n - 2) + 4(n - 3)$$

The complet sum of non-ultimates is obtained by this formula:

$$(2n/3) + 2((2n/3) + 1) + 3((2n/3) + 2) + 4((2n/3) + 3)$$

This phenomenon is directly related to the decimal system organized from ten entities: the value 10 is indeed equal to the sum of four progressive values: 1 + 2 + 3 + 4 = 10.

3.2 Twenty-four versus sixteen ultimates: 3/2 ratio

Among the 50 sums equal to the addition of the 10 digit numbers (from 0 to 9) with the first 5 digits (from 0 to 4), there are 24 ultimates and among the 50 sums equal to the addition of the 10 digit numbers (from 0 to 9) with the last 5 digits (from 5 to 9), there are 16 ultimates. These two groups are therefore in opposition (Figure 6) in a ratio of 3/2.

Almong the first 50 values:

24 ultimates

0	**1**	**2**	**3**	4	**5**	6	**7**	8	9
1	**2**	**3**	4	**5**	6	**7**	8	9	10
2	**3**	4	**5**	6	**7**	8	9	10	**11**
3	4	**5**	6	**7**	8	9	10	**11**	12
4	**5**	6	**7**	8	9	10	**11**	12	**13**
5	6	**7**	8	9	10	**11**	12	**13**	14
6	**7**	8	9	10	**11**	12	**13**	14	15
7	8	9	10	**11**	12	**13**	14	15	16
8	9	10	**11**	12	**13**	14	15	16	**17**
9	10	**11**	12	**13**	14	15	16	**17**	18

Almong the last 50 values:

16 ultimates

Fig. 6 Cross addition semi tables of the 10 digits generating a 3/2 value ratio on the distribution of the ultimate numbers.

Among the 40 sums equal to an ultimate number, 24 correspond (Figure 7) to a digit of the decimal system (from 0 to 9) and 16 to a number greater than 9 (the last digit of the decimal system).

Among the digits:

24 ultimates

0	**1**	**2**	**3**	4	**5**	6	**7**	8	9	
1	**2**	**3**	4	**5**	6	**7**	8	9		10
2	**3**	4	**5**	6	**7**	8	9		10	**11**
3	4	**5**	6	**7**	8	9		10	**11**	12
4	**5**	6	**7**	8	9		10	**11**	12	**13**
5	6	**7**	8	9		10	**11**	12	**13**	14
6	**7**	8	9		10	**11**	12	**13**	14	15
7	8	9		10	**11**	12	**13**	14	15	16
8	9		10	**11**	12	**13**	14	15	16	**17**
9		10	**11**	12	**13**	14	15	16	**17**	18

Among the non-digits:

16 ultimates

Fig. 7 Cross addition tables of the 10 digits generating a 3/2 value ratio on the distribution of the ultimate numbers depending on the digital nature of values (digits or not digits).

3.3 Configurations at 3/2 transcendent double ratio

By symmetrically splitting the addition matrix of the ten digits into two sub-matrices on 60 *external* entities versus 40 *internal* entities, as presented in the left part of Figure 8, it appears that the non-ultimate numbers and the ultimate numbers always oppose in different sets of 3/2 value ratios according to their identical or opposite natures. This is therefore verified both inside the two sub-matrices and transversely to these two sub-matrices.

External sub-matrix
to 4 times 15 numbers
(60 sums)

0	**1**	**2**	**3**	4	**5**	6	**7**	8	9
1	**2**	**3**	4			**7**	8	9	10
2	**3**	4					9	10	**11**
3	4							**11**	12
4									**13**
5									14
6	**7**							14	15
7	8	9					14	15	16
8	9	10	**11**			14	15	16	**17**
9	10	**11**	12	**13**	14	15	16	**17**	18

36 non-ultimates
24 ultimates
3/2 ratio

← 3/2 ratio →

Internal sub-matrix
to 4 times 10 numbers
(40 sums)

				5	6				
			5	6	**7**	8			
		5	6	**7**	8	9	10		
	5	6	**7**	8	9	10	**11**	12	
	6	**7**	8	9	10	**11**	12	**13**	
		8	9	10	**11**	12	**13**		
			10	**11**	12	**13**			
				12	**13**				

24 non-ultimates
16 ultimates
3/2 ratio

← 3/2 ratio → (non-ultimates)
← 3/2 ratio → (ultimates)

Internal sub-matrix
to 4 times 15 numbers
(60 sums)

				4	**5**				
			4	**5**	6	**7**			
		4	**5**	6	**7**	8	9		
	4	**5**	6	**7**	8	9	10	**11**	
4	**5**	6	**7**	8	9	10	**11**	12	**13**
5	6	**7**	8	9	10	**11**	12	**13**	14
	7	8	9	10	**11**	12	**13**	14	
		9	10	**11**	12	**13**	14		
			11	12	**13**	14			
				13	14				

36 non-ultimates
24 ultimates
3/2 ratio

← 3/2 ratio →

External sub-matrix
to 4 times 10 numbers
(40 sums)

0	**1**	**2**	**3**			6	**7**	8	9
1	**2**	**3**					8	9	10
2	**3**							10	**11**
3									12
6									15
7	8							15	16
8	9	10					15	16	**17**
9	10	**11**	12			15	16	**17**	18

24 non-ultimates
16 ultimates
3/2 ratio

Fig.8 From the cross addition table of the 10 digits: external and internal sub-matrices on 60 versus 40 numbers generating opposite sets of numbers in transcendent ratios of value 3/2 according to the ultimity or not ultimity of their components.

Also, the same phenomena are observed by considering two sub-matrices on 60 *internal* entities versus 40 *external* entities as presented in right part of Figure 8. Finally, the same phenomena still occur by considering two sub-matrices on 60 geographically located entities *northwest* and 40 geographically located entities *southeast* as shown in Figure 9.

Northwest sub-matrix to 4 times 15 numbers (60 sums)	← 3/2 ratio →	*Southeast* sub-matrix to 4 times 10 numbers (40 sums)

0	**1**	**2**	**3**	4	**5**	6	**7**	8	9
1	**2**	**3**	4		6	**7**	8	9	
2	**3**	4			**7**	8	9		
3	4				8	9			
4					9				
5	6	**7**	8	9	10	**11**	12	**13**	14
6	**7**	8	9		**11**	12	**13**	14	
7	8	9			12	**13**	14		
8	9				**13**	14			
9					14				

← 3/2 ratio →

			5				10
		5	6			10	**11**
	5	6	**7**		10	**11**	12
5	6	**7**	8	10	**11**	12	**13**
			10				15
		10	**11**			15	16
	10	**11**	12		15	16	**17**
10	**11**	12	**13**	15	16	**17**	18

36 non-ultimates	← 3/2 ratio →	24 non-ultimates
24 ultimates	← 3/2 ratio →	**16 ultimates**
3/2 ratio		**3/2 ratio**

Fig.9 *Northwest* and *southeast* sub-matrices on 60 versus 40 numbers generating opposite sets of numbers in transcendent ratios of value 3/2 according to the ultimity or not ultimity of their components.

4. Addition matrix of the twenty fundamental numbers

The matrix of 100 numbers (Figure 10) of the additions of the ten digit numbers and of the following ten, that of the twenty fundamental numbers, generates 70 non-ultimates ($5x \rightarrow x = 14$) and 30 ultimates ($5y \rightarrow y = 6$). These two categories of numbers are not distributed randomly in this matrix but in singular arithmetic arrangements. Thus, the first two addition columns each total 6 non-ultimates and 4 ultimates; the last two total 8 non-ultimate and 2 ultimate each. The six central columns all have the same values of 7 and 3 numbers, respectively non-ultimate and ultimate. These six central columns therefore oppose, in 3/2 ratios, their global quantity of non-ultimate and ultimate numbers to that of the four peripheral columns with respectively 42 versus 28 non-ultimates and 18 versus 12 ultimates.

6 columns **(60 sums)**	← 3/2 ratio →	**4 columns** **(40 sums)**
42 non-ultimates **18 ultimates**	← 3/2 ratio → ← 3/2 ratio →	28 non-ultimates **12 ultimates**

+	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
0	10	**11**	12	**13**	14	15	16	**17**	18	**19**
1	**11**	12	**13**	14	15	16	**17**	18	**19**	20
2	12	**13**	14	15	16	**17**	18	**19**	20	21
3	**13**	14	15	16	**17**	18	**19**	20	21	22
4	14	15	16	**17**	18	**19**	20	21	22	**23**
5	15	16	**17**	18	**19**	20	21	22	**23**	24
6	16	**17**	18	**19**	20	21	22	**23**	24	25
7	**17**	18	**19**	20	21	22	**23**	24	25	26
8	18	**19**	20	21	22	**23**	24	25	26	27
9	**19**	20	21	22	**23**	24	25	26	27	28
70 non-ultimates	6	6	7	7	7	7	7	7	8	8
30 ultimates	**4**	**4**	**3**	**3**	**3**	**3**	**3**	**3**	**2**	**2**

42 non-ultimates
18 ultimates

28 non-ultimates
12 ultimates

Fig.10 Addition matrix of the twenty fundamental numbers.

4.1 Sub-matrices to sixty and forty numbers

In the addition matrix of the twenty fundamental numbers (to one hundred sums) two sub-matrices oppose, left part of Figure 11, their quantities of reciprocal non-ultimates and their quantities of reciprocal ultimates in ratios of 3/2 value. These sub-matrices on 60 versus 40 numbers are themselves each composed of two sub-zones with the numbers of entities opposing in 3/2 ratios: sub-matrix on 36 + 24 entities and sub-matrix on 24 + 16 entities. This arithmetic arrangement is a *geometric* variant of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* here have as values 6 and 4, values opposing in the 3/2 ratio. This remarkable identity will be more widely investigated in chapter 7.1 where very singular phenomena are presented.

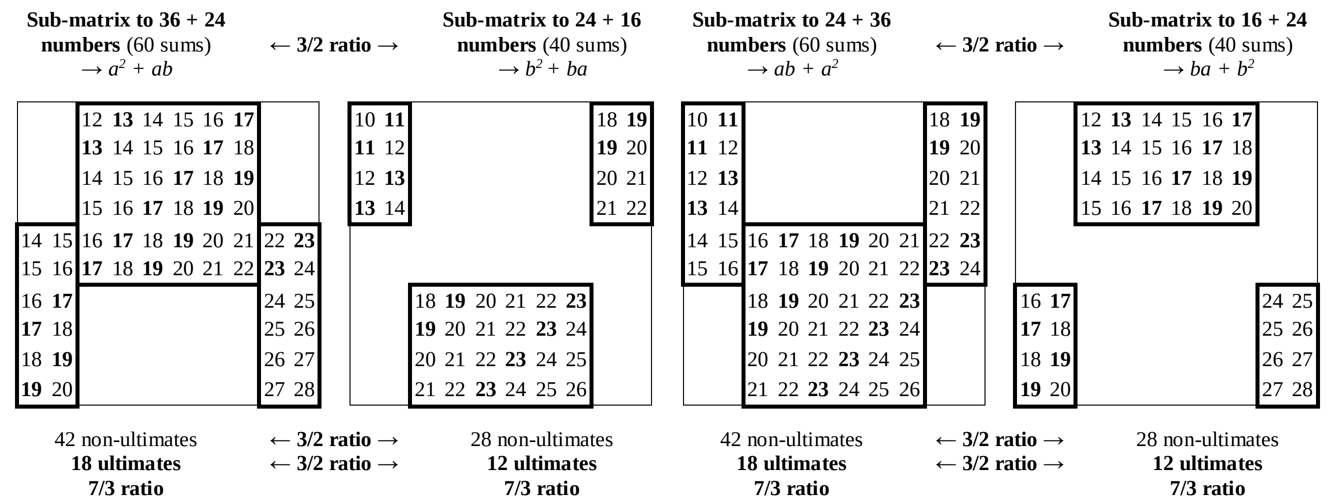


Fig.11 Addition sub-matrices of the twenty fundamental numbers on 60 versus 40 numbers. Geometric variant of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

This configuration contrasts the upper 3/5ths of the six central columns of the matrix and the lower 3/5ths of the four peripheral columns with the 2/5ths reciprocals of the columns considered. Also, in the right part of Figure 11, the vertical mirror configuration to this arrangement generates the same oppositions in 3/2 ratios of the reciprocal non-ultimate numbers and the reciprocal ultimate numbers of these other sub-matrices to 60 and 40 entities.

Also, by mixing, in Figure 12, the sub-matrices of 40 and 60 entities presented in Figure 11 and after having each split them vertically into two equal parts to 30 and 20 entities, we obtain new matrices of 50 entities each. In these horizontal mirror configurations, the non-ultimates and the ultimates are divided into exact ratios of value 1/1 with always 35 non-ultimates versus 35 and always 15 ultimates versus 15. Also, by this geometric rearrangement, in addition to being configurations horizontal mirror, the left configurations become vertical mirror of the right configurations and vice versa.

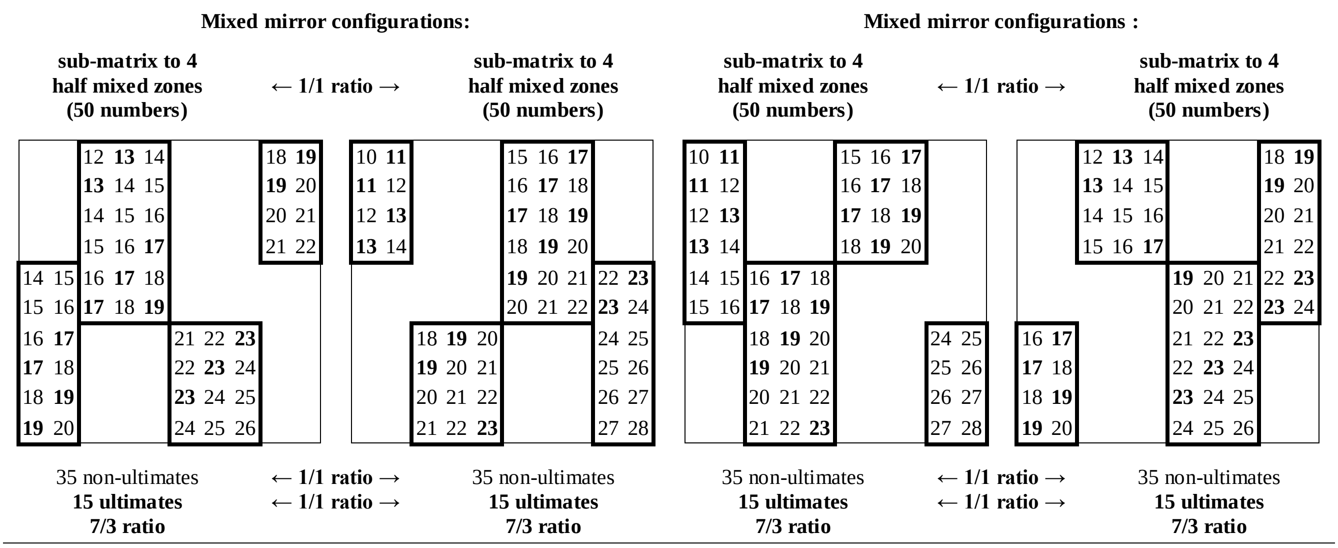


Fig.12 Mixed sub-matrices mirror of addition of the twenty fundamental numbers to 50 versus 50 numbers.

4.2 Concentric and eccentric matrices

In this matrix of the hundred additions of the twenty fundamentals, more sophisticated arrangements further oppose the ultimate numbers and the non-ultimate numbers in exact 3/2 ratios. Thus, as described in the left part of Figure 13, five concentric areas are opposed, three versus two, in the distribution of their ultimate numbers and their non-ultimate numbers in 3/2 ratios. The same phenomenon is reproduced by considering the five eccentric areas presented in the right part of Figure 13. The eccentricity of these five areas is in total asymmetry with respect to the five initial concentric areas. However, as shown in Figure 15, the same quantities of ultimates (and non-ultimates) are distributed in concentric or eccentric areas of the same size. These five concentric and eccentric rings are by sizes whose respective values increase regularly according to this arithmetic where $x = 1$:

$$4x \rightarrow 4(x+2) \rightarrow 4(x+4) \rightarrow 4(x+6) \rightarrow 4(x+8)$$

This arithmetic allows, in relation to the decimal system and by the interposition of the incorporated rings, the constitution of sub-matrices, which oppose in size and in categories of numbers , in 3/2 value ratios.

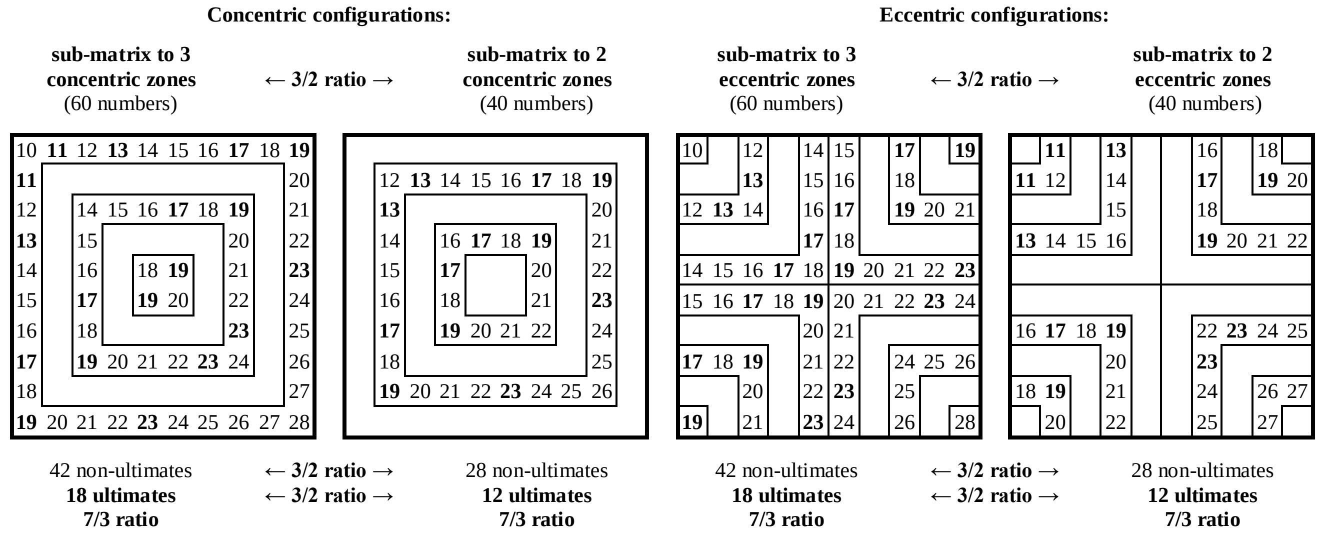


Fig.13 From the addition matrix of the twenty fundamental numbers, concentric and eccentric configurations of sub-matrices to 60 and 40 entities opposing their non-ultimates and their ultimates in 3/2 ratios.

Also, by mixing, in Figure 14, the sub-matrices of 40 and 60 entities presented in Figure 13 and after having each split them vertically into two equal parts to 30 and 20 entities, we obtain new matrices of 50 entities each. In these mixed configurations, the non-ultimates and the ultimates are divided into exact ratios of value 1/1 with always 35 non-ultimates versus 35 and always 15 ultimates versus 15. These reassemblings are exactly the same type as those proposed in Figure 12.

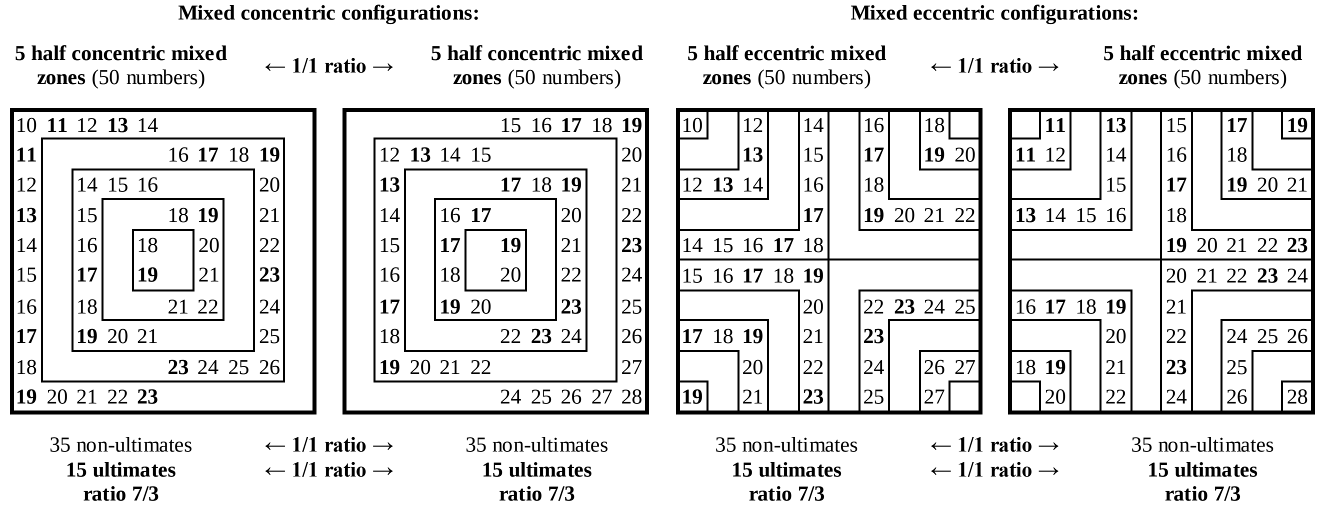


Fig.14 From the addition matrix of the twenty fundamental numbers, concentric and eccentric configurations of sub-matrices to 50 entities each opposing their non-ultimates and their ultimates in 1/1 ratios.

4.2.1 Arithmetic progressions

Other arrangements, as in the example in Figure 15, of three versus two concentric zones or three versus two eccentric zones generate the same arithmetic phenomena with a 3/2 ratio between the respective quantities of ultimates of these zones sets. This phenomenon is directly related to the regular progression of the value of quantities of ultimates from 2 to 10 depending on the size of the concentric or eccentric zones considered.

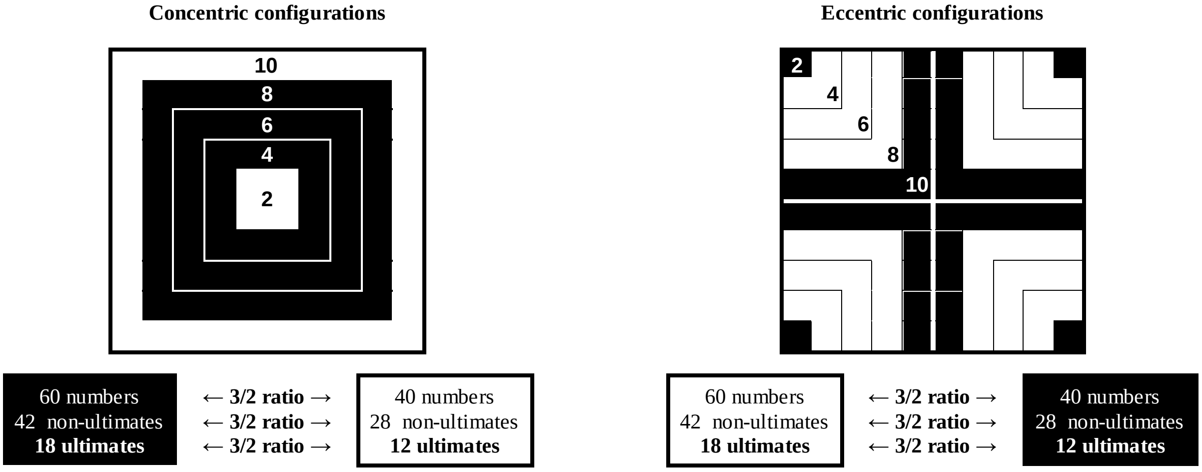


Fig.15 Regular arithmetic distribution of the ultimate numbers in the concentric and eccentric rings of the addition matrix of the twenty fundamental numbers. Example of a 3/2 ratio arrangement (see Fig.13 also).

In each of the five concentric zones of the addition matrix of the twenty fundamental numbers, the quantity of ultimate numbers (*x*) is linked to the whole quantity of numbers (*z*) of this zone by this formula:

$$x^2 - (x - 2)^2 = z$$

In these same areas, the quantity of non-ultimate numbers (y) is linked to the quantity of ultimate numbers (x) by this formula:

$$x^2 - (x - 2)^2 - x = y$$

As demonstrated in Figure 16, this phenomenon remains identical for the five symmetrically eccentric zones.

quantity of numbers by zones* (*z*)		**quantity of ultimate numbers (*x*)**	quantity of non-ultimate numbers (*y*)
$x^2 - (x - 2)^2$	$= z$	$\mathbf{x}$	$x^2 - (x - 2)^2 - x = y$
$2^2 - (2 - 2)^2$	= 4	**2**	2
$4^2 - (4 - 2)^2$	= 12	**4**	8
$6^2 - (6 - 2)^2$	= 20	**6**	14
$8^2 - (8 - 2)^2$	= 28	**8**	20
$10^2 - (10 - 2)^2$	= 36	**10**	26

Fig.16 Arithmetic relationship between the value of the quantity of **ultimates** (and of non-ultimates) and the dimension of the considered concentric * or eccentric * zone in the addition matrix of the twenty fundamentals.

5. Matrix of the first hundred numbers

The study of the matrix of the first hundred whole numbers, configured in ten lines of ten classified numbers from 0 to 9, reveals several singular phenomena according to the different classifications considered of the entities which compose it. These phenomena will be introduced in different chapters including, to begin, this chapter distinguishing couples of ultimate numbers from those without ultimates.

5. 1 Ultimate numbers and pairs of numbers

In the matrix of the first hundred numbers, 25 pairs of adjacent numbers, including at least one ultimate, are opposed, in an exact ratio of 1/1, to 25 other pairs not including any. From the couple of numbers 0-1 to the couple of numbers 98-99, these 50 couples are always formed of two consecutive numbers as illustrated in Figure 17. Although 27 ultimate numbers are present in the sequence of the first 100 numbers, only 25 couples integrating at least one ultimate emerge in this matrix. This is due to the fact that the first four ultimate numbers are also the first four whole numbers and therefore that they are consecutive, the first non-ultimate number (4) being in fifth position in the sequence of numbers. Also, only these last four are consecutive.

5. 1.1 Twice twenty-five pairs of numbers

As illustrated in Figure 17, it turns out that the 25 couples with ultimates and the 25 couples without ultimates oppose in 3/2 transcendent ratios according to whether they come from the upper part or from the lower part of the matrix of one hundred first numbers. Thus, among the first 25 couples, in a ratio to 3/2, 15 consist of ultimates and 10 of non-ultimates and among the last 25 couples, in a reverse ratio to 2/3, 10 consist of ultimates and 15 of non-ultimates.

15 couples with ultimates	**← 3/2 ratio →**	10 couples without ultimates
↑ 3/2 ratio ↓		**↑ 2/3 ratio ↓**
10 couples with ultimates	**← 2/3 ratio →**	15 couples without ultimates

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	8	9
10	**11**	**12**	**13**	14	15	**16**	**17**	**18**	**19**
20	21	**22**	**23**	24	25	26	27	**28**	**29**
30	**31**	32	33	34	35	**36**	**37**	38	39
40	**41**	**42**	**43**	44	45	**46**	**47**	48	49
50	51	**52**	**53**	54	55	56	57	**58**	**59**
60	**61**	62	63	64	65	**66**	**67**	68	69
70	**71**	**72**	**73**	74	75	76	77	**78**	**79**
80	81	**82**	**83**	84	85	86	87	**88**	**89**
90	91	92	93	94	95	**96**	**97**	98	99

25 couples
with ultimates
25 couples
without ultimates

Fig.17 Distribution of the 25 couples with ultimates and the 25 couples without ultimates in the matrix of the first hundred numbers.

5. 1.2 Entanglement of pairs of numbers

Also, for each of the two upper and lower parts of this matrix, their splitting into two sets of 3 and 2 alternating lines as illustrated in Figure 18 generates a multitude of entangled arithmetic phenomena always resulting in ratios of value 3/2 or opposite ratios of value 2/3 simultaneously according to the zone considered and the nature of the couple considered (with or without ultimates).

9 couples with ultimates 6 couples without ultimates **3/2 ratio**	**← 3/2 ratio →** **← 3/2 ratio →**	**6 couples with ultimates** 4 couples without ultimates **3/2 ratio**

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	8	9
20	21	**22**	**23**	24	25	26	27	**28**	**29**
40	**41**	**42**	**43**	44	45	**46**	**47**	48	49

← 3/2 ratio →

10	**11**	**12**	**13**	14	15	**16**	**17**	**18**	**19**
30	**31**	32	33	34	35	**36**	**37**	38	39

↑ **3/2 ratio ***↓	↑ 2/3 ratio** ↓	↑ **3/2 ratio*** ↓	↑ 2/3 ratio** ↓

50	51	**52**	**53**	54	55	56	57	**58**	**59**
70	**71**	**72**	**73**	74	75	76	77	**78**	**79**
90	91	92	93	94	95	**96**	**97**	98	99

← 3/2 ratio →

60	**61**	62	63	64	65	**66**	**67**	68	69
80	81	**82**	**83**	84	85	86	87	**88**	**89**

6 couples with ultimates 9 couples without ultimates **2/3 ratio**	**← 3/2 ratio →** **← 3/2 ratio →**	**4 couples with ultimates** 6 couples without ultimates **2/3 ratio**

Fig.18 Strong entanglement of the distribution of 25 couples with ultimates and of 25 couples without ultimate in the matrix of the first hundred numbers. Ratios between couples respectively **with ultimates*** and without ultimate**.

5. 2 Sub-matrices of thirty versus twenty pairs of numbers

From the matrix of the first 50 pairs of whole numbers, in the sub-matrices made up of five vertically alternating zones to 3/5th of column (30 pairs of numbers) such as those presented Figure 19 and in the complementary sub-matrices of five zones to 2/5th of column (20 couples), the quantities of couples with ultimates and those of couples without ultimate remain of equal values and oppose in 3/2 ratios to the respective values of the complementary sub matrices.

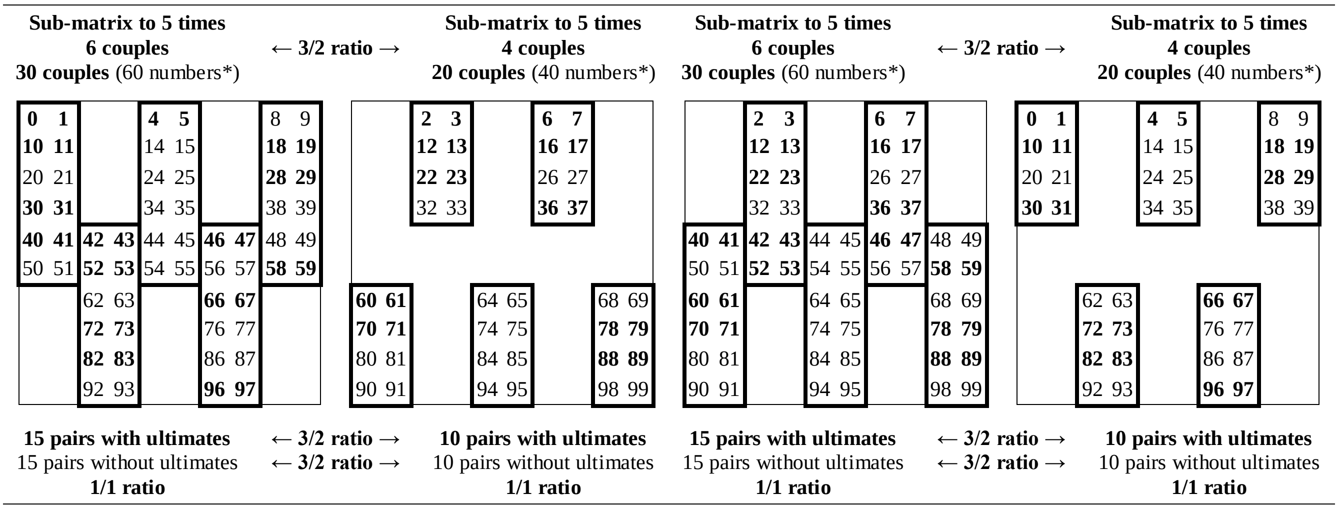


Fig.19 Equal distribution of **25 couples with ultimates** and of 25 couples without ultimate in sub-matrices to 30 versus 20 couples.

Also, still from this same matrix of 50 couples of numbers, in symmetrical sub-matrices made up of 10 zones of 3 pairs of numbers versus 10 zones of 2 pairs, as illustrated in Figure 20, the quantities of pairs of numbers with ultimates and those of pairs without ultimate remain of equal values and oppose in 3/2 ratios to the respective values of the complementary sub-matrices.

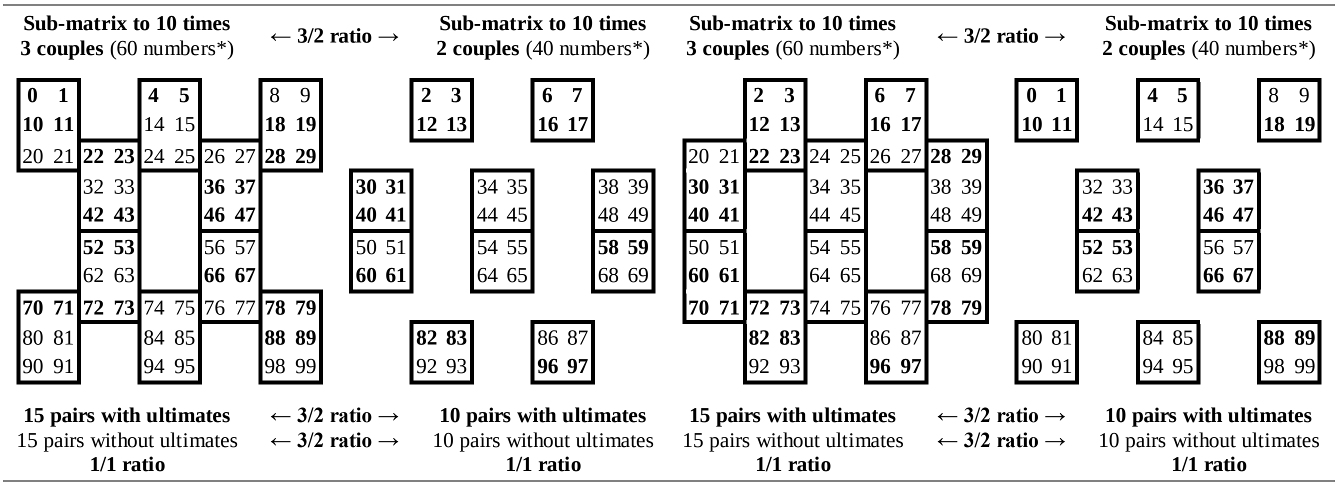


Fig.20 Equal distribution of **25 couples with ultimates** and of 25 couples without ultimate in sub-matrices to 30 versus 20 couples.

The existence of these singular arithmetic phenomena presented in this chapter greatly reinforce the main argument of this study about whole numbers to merge the special numbers zero (0) and one (1) and the sequence of prime numbers into the sequence of ultimate numbers. These phenomena indeed disappear completely without this fusion.

* Note: The configurations of the two types of sub-matrices presented in Figures 19 and 20 fits still in geometric variants of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ with *a* and *b* of respective value 6 and 4 (for entity quantities of 36 + 24 and 24 + 16).

6. The ten primordial ultimates

The identification of the first ten ultimate allows, with reference to the decimal system, to classify them as primordial. This notion of primordiality is developed in Chapter 9.

6.1 Matrix of additions of the ten primordial ultimates

The cross additions of the first ten ultimates generate, Figure 21, a matrix of one hundred values including 30 ultimate numbers ($5x \rightarrow x = 6$). Also these additions generate 30 digits.

30 digits including:

18 ultimates

↑ **3/2 ratio** ↓

12 non-ultimates

+	0	1	2	3	5	7					
0	**0**	**1**	**2**	**3**	**5**	**7**		**11**	**13**	**17**	**19**
1	**1**	**2**	**3**	4	6	8		**11**	**13**	**17**	**19**
2	**2**	**3**	4	**5**	**7**	9		12	14	18	20
3	**3**	4	**5**	6	8			**13**	15	**19**	21
5	**5**	6	**7**	8			10	14	16	20	22
7	**7**	8	9			10	12	16	18	22	24
					10	12	14	18	20	24	26
	11	**11**	12	**13**	14	16	18	22	24	28	30
	13	**13**	14	15	16	18	20	24	26	30	32
	17	**17**	18	**19**	20	22	24	28	30	34	36
	19	**19**	20	21	22	24	26	30	32	36	38

30 ultimates including:

18 digit
ultimates

↑ **3/2 ratio** ↓

12 non-digit
ultimates

Fig.21 Matrix of additions of the ten primordial ultimates.

In this matrix, it turns out that these two notions of ultimity and digitality fit into 3/2 entangled ratios. Thus, among the 30 digit numbers generated, are 18 ultimates versus 12 non-ultimates and among the 30 ultimates, 18 are found to be digit numbers and 12 non-digit numbers. Also, it turns out that all of these 30 ultimate numbers generated by these additions of the first ten primordial ultimates are all fundamental numbers, a concept introduced in Chapter 2.6.

6.2 Sub-matrices of sixty and forty entities

Illustrated in the left part of Figure 22, from the addition matrix of the first ten ultimates, in two sub-matrices to 60 versus 40 entities, the ultimates as well as the non-ultimates, are divided into 3/2 ratios with, in one and the other sub-matrix, 42 non-ultimates versus 28 and 18 ultimates versus 12.

These sub-matrices are made up of zones of four times 9, of (twice) four times 6 then four times 4 entities. These values (9 → 6 → 6 → 4) oppose in 3/2 transcendent ratios as will be more widely introduced in the next Chapter 7. Another arrangement such as that presented in the right part of Figure 22 generates the same phenomena. This last very particular arrangement of the considered zones of matrices of one hundred entities is more widely explained in the next Chapter 7 and illustrated Figure 27.

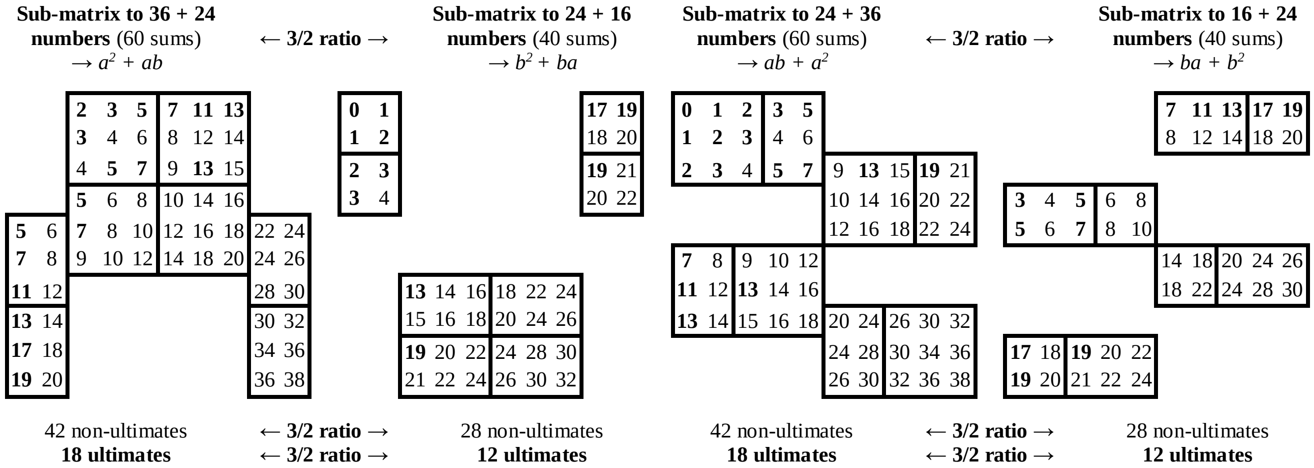


Fig.22 Addition sub-matrices of the ten primordial ultimates to 60 versus 40 numbers. Geometric variant of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

6.3 Pascal's triangle of the ten primordial ultimates

In a Pascal's triangle generated from the first ten ultimate numbers (from 0 to 19), it appears that out of the set of 55 values constituting it ($5x \longrightarrow x = 11$), there are 33 ultimates versus 22 non-ultimates. This therefore in an exact ratio of 3/2 value. Also, it turns out in this Pascal's triangle illustrated in Figure 23 that the distinction of odd and even columns still generates 3/2 ratios in these two subsets to 30 and 25 entities with respectively 18 ultimates versus 12 non-ultimates and 15 ultimates versus 10 non-ultimates.

15 ultimates **← 3/2 ratio →** 10 non-ultimates

0									
1	**0**								
2	**1**	**0**							
3	**3**	**1**	**0**						
5	6	4	**1**	**0**					
7	**11**	10	**5**	**1**	**0**				
11	18	21	15	6	**1**	**0**			
13	**29**	39	36	21	**7**	**1**	**0**		
17	42	68	75	57	28	8	**1**	**0**	
19	**59**	110	143	132	85	36	9	**1**	**0**

18 ultimates **← 3/2 ratio →** 12 non-ultimates

33 ultimates **← 3/2 ratio →** 22 non-ultimates

Fig.23 Pascal's triangle of the ten primordial ultimates generating sets and subsets opposing **the ultimates** and the non-ultimates in 3/2 ratios.

7. Fibonacci sequences and ultimate numbers

It has just been demonstrated in the preceding chapters, that the ultimate numbers (including also the numbers zero and one) are distributed non-randomly in more or less complex matrices of additions of types of numbers (digits numbers, fundamentals, ultimates primordial, etc.) Here, far more sophisticated arrangements of numbers will further demonstrate a remarkable organization of these whole numbers according to their ultimate or non-ultimate nature.

7.1 Matrix of the ten Fibonacci sequences

The creation of ten Fibonacci sequences of ten numbers and whose two initial numbers are, in first position, the first ten ultimates and in second position the following ultimate numbers respective to these first ten generate a matrix of 100 numbers with remarkable properties in relation to the differentiation of distribution of the ultimate or non-ultimate numbers thus created in this matrix.

It appears, Figure 24, in this matrix of one hundred entities, that exactly 50 ultimate numbers (5*x* → *x* = 10) are opposed to 50 other non-ultimates. Also, the set of six central sequences (sixty numbers) also totals an exact ratio of 1/1 between the quantity of ultimates and non-ultimates and this set opposes, in a perfect ratio of 3/2, to that of the four peripheral sequences totalling the same quantities of ultimates and non-ultimates also.

50 ultimates
↑ 1/1 ratio ↓
50 non-ultimates

0	**1**	**1**	**2**	**3**	**5**	8	**13**	21	34
1	**2**	**3**	**5**	8	**13**	21	34	55	**89**
2	**3**	**5**	8	**13**	21	34	55	**89**	144
3	**5**	8	**13**	21	34	55	**89**	144	**233**
5	**7**	12	**19**	**31**	50	81	**131**	212	343
7	**11**	18	**29**	**47**	76	123	**199**	322	**521**
11	**13**	24	**37**	**61**	98	159	**257**	416	**673**
13	**17**	30	**47**	77	124	201	325	526	851
17	**19**	36	55	91	146	237	**383**	620	1003
19	**23**	42	65	**107**	172	279	451	730	**1181**

30 ultimates
↑ 1/1 ratio ↓
30 non-ultimates

← 3/2 ratio →

← 3/2 ratio →

20 ultimates
↑ 1/1 ratio ↓
20 non-ultimates

Fig. 24 Matrix of the first 10 numbers of the 10 Fibonacci sequences generated from the first 10 ultimate numbers.

7.1.1 Four symmetrical areas and remarkable identity

A matrix of one hundred entities can be subdivided into four sub-matrices (areas) of 25 entities. The value 25 is the first that can be subdivided into four others (9 + 6 + 6 + 4) generating a triple value ratio to 3/2. As illustrated in Figure 25, the value 9 (3 × 3) is opposed in 3/2 ratio to the value 6 (3 × 2) then the value 6 (2 × 3) is opposed to the value 4 (2 × 2). These four values oppose themselves two by two in the 15/10 ratio, extension of the 3/2 ratio. This arithmetic demonstration is a *geometric* variant of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have here the values 3 and 2, values opposing in the 3/2 ratio.

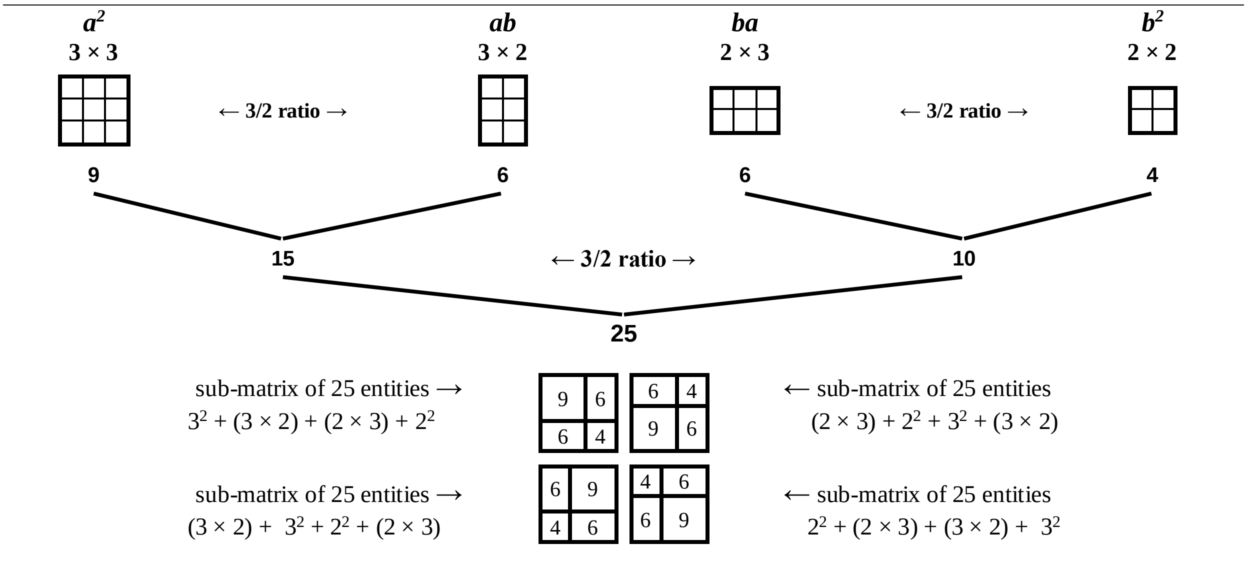


Fig. 25 Formation of four sub-matrices of 25 entities falling within the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

In this matrix of one hundred entities, it is then possible to arrange these four sub-matrices symmetrically and asymmetrically in 16 zones of four times (9 + 6 + 6 + 4) boxes (values) as described in the lower part of Figure 25.

Also, from these 16 zones (Figure 25), in this matrix of one hundred entities, we can redefine four other areas, schematically illustrated in Figure 26, including a first of four times nine (36) entities, a second of four times six (24) entities, a third of again four times six (24) entities then a fourth and final area of four times four (16) entities.

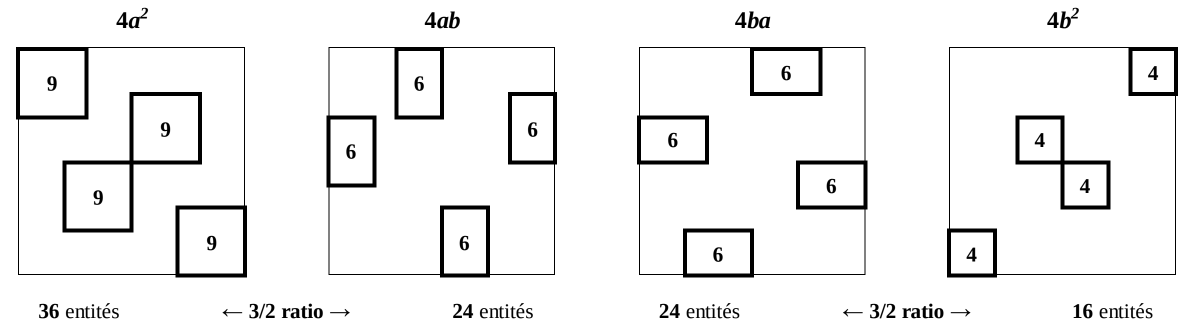


Fig. 26 Recombination of four sub-matrices of 36 → 24 → 24 → 16 entities deduced from the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

In the matrix of the ten Fibonacci sequences introduced in Figure 24, it is remarkable to note that the 50 ultimate numbers and the 50 non-ultimate numbers are always distributed in equal quantities in each of these last four defined areas. These four areas are all, and interactively, in a simultaneously symmetrical and asymmetrical geometric arrangement as introduced in Figure 25 and developed in Figure 27.

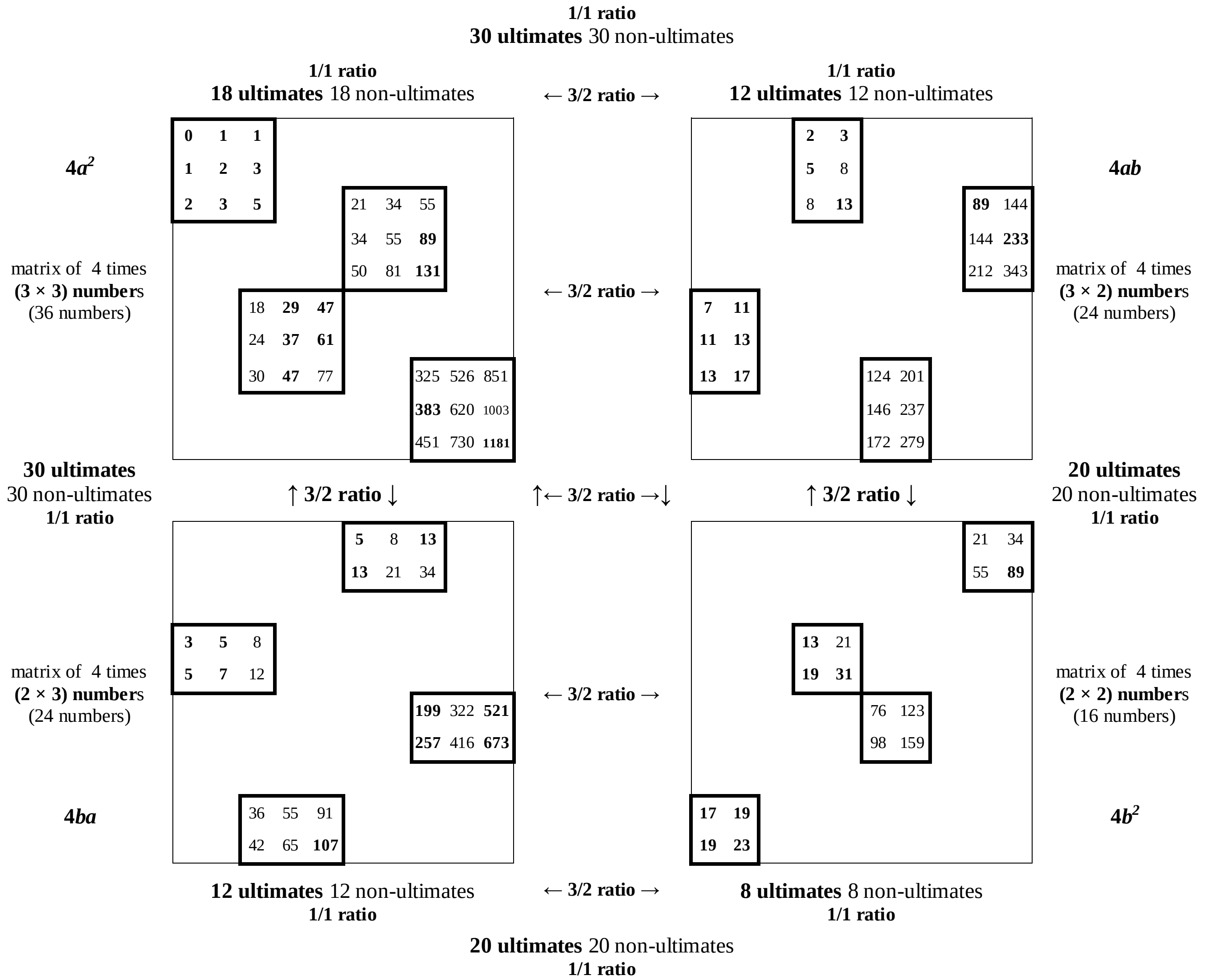


Fig. 27 Four sub-matrices of the ten Fibonacci sequences with equal distribution of the ultimate and non-ultimate numbers and 3/2 transcendent ratios. Sub-matrices organizing themselves from the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where a and b are 3 and 2 to value (see Figures 24, 25 and 26).

Also, due to their arithmetic and geometric particularities, these four areas are arranged two by two, in different configurations such as those presented in Figure 28 to generate sets of numbers always opposing in the ratio 1/1 according to their ultimity or non-ultimity and in a 3/2 ratio according to their geographic distribution.

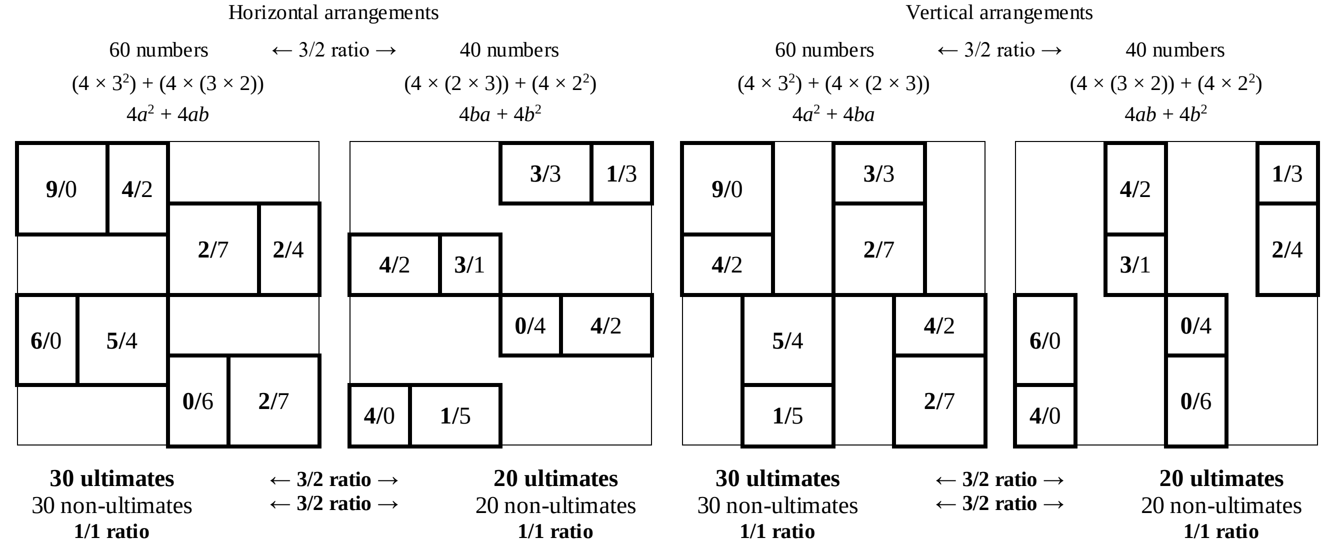


Fig. 28 From the matrix of the ten Fibonacci sequences: example of arrangements generating 1/1 and 3/2 ratios between the groups of **ultimate** and non-ultimate numbers (according to the values in Figure 27).

7.1.2 Alternative arrangement of four areas

From the matrix of the ten Fibonacci sequences, a slightly different rearrangement of the four sub-matrices of 25 entities generates similar arithmetic phenomena. In this new arrangement presented in Figure 29, the two upper sub-matrices and two lower are twin, therefore identical. These two double sub-matrices are still part of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* are 3 and 2 to value.

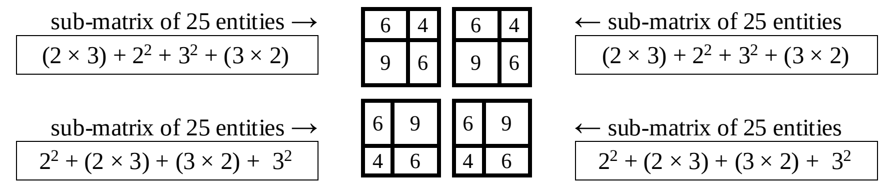


Fig. 29 Another arrangement of two double sub-matrices 25 entities falling within the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* are 3 and 2 to value.

Since these new sub-matrices of 25 entities, the grouping, Figure 30, of the areas of equal configurations (zones of 9 ⟶ 6 ⟶ 6 ⟶ 4 entities) generates sets where the ultimate numbers and the non-ultimate numbers are found evenly distributed in equal quantities. Thus these sets also oppose in 3/2 ratios according to the ultimity and non-ultimity of their components.

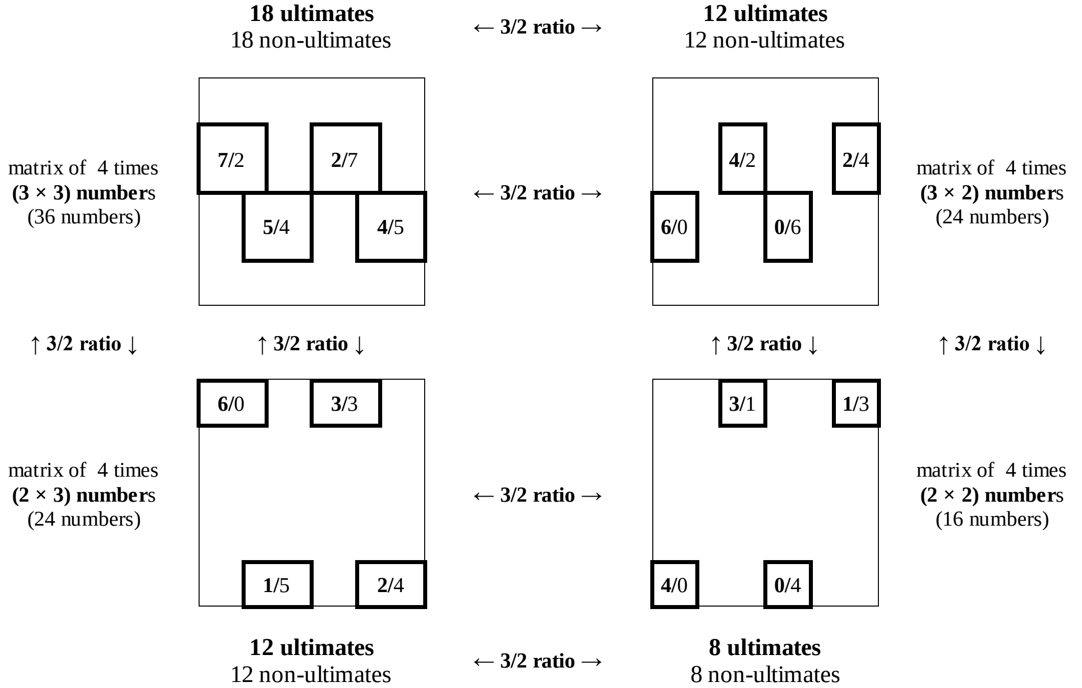


Fig. 30 Four sub-matrices of the ten Fibonacci sequences with equal distribution of the ultimate and non-ultimate numbers and 3/2 transcendent ratios. See Fig. 24 and 29.

The opposition, presented in Figure 24 on the introduction of the chapter, of the respective values of the ultimates and non-ultimates of the six central lines and four peripheral lines of the matrix of the ten Fibonacci sequences is the direct consequence of these other configurations described in Figure 30 (by opposition of central and peripheral configurations).

7.1.3 Redeployment of the Fibonacci matrix

A redeployment of the matrix of the ten Fibonacci sequences (introduced in Figure 24) in four rows of twenty five entities still generates a singular phenomenon of the same nature and always linked to the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have as the values 3 and 2. Here each line of 25 entities is split into seven symmetrical zones in which this remarkable identity is thus redeployed:

$$\rightarrow \quad \boldsymbol{(ab)^2 = b^2/2 + ba/2 + ab/2 + a^2 + ab/2 + ba/2 + b^2/2}$$

Thus, Figure 31, the quantities of respective entities of these seven zones are as follows:

$$\rightarrow \quad \mathbf{(3{\times}2)^2 = 2 + 3 + 3 + 9 + 3 + 3 + 2}$$

This particular arrangement and therefore the redeployment of this remarkable identity will also be made in Figures 43 and 49 then 69 and 70 with other values considered.

Sub-matrix of 4 times ($3^2 + (3 \times 2)$) = 60 numbers (→ 4 times ($a^2 + ab$))

30 ultimates **← 1/1 ratio →** 30 non-ultimates

0	**1**	**1**	**2**	**3**	**5**	8	**13**	21	34	**1**	**2**	**3**	**5**	8	**13**	21	34	55	**89**	**2**	**3**	**5**	8	**13**
21	34	55	**89**	144	**3**	**5**	8	**13**	21	34	55	**89**	144	**233**	**5**	**7**	12	**19**	**31**	50	81	**131**	212	343
7	**11**	18	**29**	**47**	76	123	**199**	322	**521**	**11**	**13**	24	**37**	**61**	98	159	**257**	416	**673**	**13**	**17**	30	**47**	77
124	201	325	526	851	**17**	**19**	36	55	91	146	237	**383**	620	1003	**19**	**23**	42	65	**107**	172	279	451	730	**1181**

20 ultimates **← 1/1 ratio →** 20 non-ultimates

Sub-matrix of 4 times ($2^2 + (2 \times 3)$) = 40 numbers (→ 4 times ($b^2 + ba$))

Fig. 31 Redeployment of the Fibonacci matrix inscribed in the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have as value 3 and 2. Equal distribution of the ultimates and non-ultimates in two sub-matrices entangled to 60 versus 40 entities.

By symmetrically regrouping and from their center these four rows of numbers, such that, from this center towards the edge of this new matrix, we isolate four times 9 + 6 values then four times 6 + 4 entities as illustrated in Figure 31, this forms two sub matrices to 60 versus 40 numbers. In these two 3/2 ratio sub-matrices, the 50 ultimates and the 50 non-ultimates are also distributed in equal quantities and thus, each of these two categories of numbers oppose in 3/2 ratios in one and the other sub-matrix.

7.2 Matrix of five Fibonacci sequences

The creation of five Fibonacci sequences of ten numbers, of which the two initial numbers are the first two digits and then, successively in each sequence, the following two, generates a matrix of 50 entities of which, very exactly, 25 are ultimate numbers and 25 are not ultimate. Also, as illustrated in Figure 32, these two categories of numbers are opposed in various 3/2 transcendent ratios according to their geographic distribution into this matrix.

Matrix of 25 + 25 = 50 numbers

25 ultimates **← 1/1 ratio →** 25 non-ultimates

	0	**1**	**1**	**2**	**3**	**5**	8	**13**	21	34	
	2	**3**	**5**	8	**13**	21	34	55	**89**	144	
25 numbers	4	**5**	9	14	**23**	**37**	60	**97**	**157**	254	25 numbers
	6	**7**	**13**	20	33	**53**	86	**139**	225	364	
	8	9	**17**	26	**43**	69	112	**181**	**293**	474	

15 ultimates
10 non-ultimates
3/2 ratio

← 3/2 ratio →
← 2/3 ratio →

10 ultimates
15 non-ultimates
2/3 ratio

Fig. 32 Distribution of ultimate and non-ultimate numbers in the matrix of five Fibonacci sequences.

Thus, in the first five columns of this matrix, in a ratio of 3/2, 15 ultimates are opposed to 10 non-ultimates and in an inverse ratio, 10 ultimates are opposed to 15 non-ultimates in the last five columns. These reciprocal values are themselves opposed in 3/2 and 2/3 crossed ratios according to the geographic areas considered in this matrix of five Fibonacci sequences initialized from the ten digit numbers.

Also, in sub-matrices of thirty versus twenty entities as presented in Figure 33, the ultimates and non-ultimates are distributed in equal quantity (1/1 ratio) and oppose to components of the complementary matrix in exact ratios of 3/2 .

Sub-matrix to 2 times 15 numbers (30 numbers) **← 3/2 ratio →** **Sub-matrix to 2 times 10 numbers (**20 numbers)

0	**1**	**1**	**2**	**3**	**5**	8	**13**	21	34
2	**3**	**5**	8			34	55	**89**	144
4	**5**	9					**97**	**157**	254
6	**7**							225	364
8									474

← 3/2 ratio →

			13	21			
		14	**23**	**37**	60		
	13	20	33	**53**	86	**139**	
9	**17**	26	**43**	69	112	**181**	**293**

15 ultimates
15 non-ultimates
1/1 ratio

← 3/2 ratio →
← 3/2 ratio →

10 ultimates
10 non-ultimates
1/1 ratio

Fig. 33 Distribution of ultimate and non-ultimate numbers in the matrix of five Fibonacci sequences.

The same phenomenon is observed in different configurations such as those presented in Figure 34 which opposing matrices of thirty versus twenty entities.

Sub-matrices to 2 times 15 numbers (30 numbers) ← **3/2 ratio** → **Sub-matrices to 2 times 10 numbers** (20 numbers)

0					**5**						**1**	**1**	**2**	**3**	8	**13**	21	34
2	**3**				21	34						**5**	8	**13**		55	**89**	144
4	**5**	9			**37**	60	**97**			← **3/2 ratio** →			14	**23**			**157**	254
6	**7**	**13**	20		**53**	86	**139**	225						33				364
8	9	**17**	26	**43**	69	112	**181**	**293**	474									

↑ **ratio 1/1** ↓ ↑ **ratio 1/1** ↓

0	**1**	**1**	**2**	**3**	**5**	8	**13**	21	34									
	3	**5**	8	**13**		34	55	**89**	144		**2**				21			
		9	14	**23**			**97**	**157**	254	← **3/2 ratio** →	4	**5**			**37**	60		
			20	33				225	364		6	**7**	**13**		**53**	86	**139**	
				43					474		8	9	**17**	26	69	112	**181**	**293**

15 ultimates	← **3/2 ratio** →	**10 ultimates**
15 non-ultimates	← 3/2 ratio →	10 non-ultimates
1/1 ratio		**1/1 ratio**

Fig. 34 Distribution of ultimate and non-ultimate numbers in the matrix of five Fibonacci sequences.

Finally, by alternately isolating the first six or the last six numbers of these five Fibonacci sequences, as illustrated Figure 35, it turns out that the ultimate and non-ultimate numbers are still distributed in equal quantities and these groups of 15 entities therefore oppose in exact ratios of 3/2 value to the sets of 10 ultimates and of 10 non-ultimates of the last four or the first four entities alternately isolated in these five sequences.

Sub-matrix to 2 times 15 numbers (30 numbers) ← **3/2 ratio** → **Sub-matrix to 2 times 10 numbers** (20 numbers)

0	**1**	**1**	**2**	**3**	**5**										8	**13**	21	34
				13	21	34	55	**89**	144		**2**	**3**	**5**	8				
4	**5**	9	14	**23**	**37**					← **3/2 ratio** →					60	**97**	**157**	254
				33	**53**	86	**139**	225	364		6	**7**	**13**	20				
8	9	**17**	26	**43**	69										112	**181**	**293**	474

15 ultimates	← **3/2 ratio** →	**10 ultimates**
15 non-ultimates	← 3/2 ratio →	10 non-ultimates
1/1 ratio		**1/1 ratio**

Fig. 35 Distribution of ultimate and non-ultimate numbers in the matrix of five Fibonacci sequences.

7.3 Fibonacci sequence and Pascal triangle

A Pascal triangle initialized with the first ten results of the Fibonacci sequence, generates 55 values including, in an exact ratio of 3/2, 33 non-ultimates opposing 22 ultimates. Also, it turns out, in this Pascal triangle illustrated in Figure 36, that the distinction of odd and even lines still generates a 3/2 ratio in these two subsets of 30 and 25 entities with respectively 18 non-ultimates versus 12 ultimates and 15 non-ultimates versus 10 ultimates. This configuration is very similar to that introduced in Figure 23 Chapter 6.3 concerning a Pascal triangle initialized with the ten primordial ultimates.

15 non-ultimates
10 ultimates

3/2 ratio

1									
2	**1**								
3	**3**	**1**							
5	6	4	**1**						
8	**11**	10	**5**	**1**					
13	**19**	21	15	6	**1**				
21	32	40	36	21	**7**	**1**			
34	**53**	72	76	57	28	8	**1**		
55	87	125	148	133	85	36	9	**1**	
89	142	212	273	**281**	218	121	45	10	**1**

12 non-ultimates
8 ultimates

3/2 ratio

33 non-ultimates ← **3/2 ratio** → **22 ultimates**

Fig. 36 Distribution of ultimates and non-ultimates in a Pascal triangle of the first 10 results of the Fibonacci sequence.

8. The four classes of whole numbers

The segregation of whole numbers into two sets of entities qualified as ultimate and non-ultimate is only a first step in the investigation of this type of numbers. Here is a further exploration of this set of numbers revealing its organization into four subsets of entities with their own but interactive properties and also the double concept of *ultimate divisor* and *ultimate algebra*.

8.1 Four different types of numbers

From the definition of ultimate numbers introduced above, it is possible to differentiate the set of whole numbers into four final classes, inferred from the three source classes and progressively defined according to these criteria:

Whole numbers are subdivided into these two categories:

- **ultimates**: an ultimate number not admits any non-trivial divisor (whole number) being less than it.
- **non-ultimates**: a non-ultimate number admits at least one non-trivial divisor (whole number) being less than it.

Non-ultimate numbers are subdivided into these two categories:

- **raiseds**: a raised number is a non-ultimate number, power of an ultimate number.
- **composites**: a composite number is a non-ultimate and not raised number admitting at least two different divisors.

Composite numbers are subdivided into these two categories:

- **pure composites**: a pure composite number is a non-ultimate and not raised number admitting no raised number as divisor.
- **mixed composites**: a mixed composite number is a non-ultimate and not raised number admitting at least a raised number as divisor.

The table in Figure 37 summarizes these different definitions. It is more fully developed in Figure 38 where the interactions of the four classes of whole numbers are highlighted.

<table>
<tr><th colspan="4">The whole numbers:</th></tr>
<tr><th rowspan="4">the ultimates:
an ultimate number not admits any non-trivial divisor (whole number) being less than it</th><th colspan="3">the non-ultimates:
A non-ultimate number admits at least one non-trivial divisor (whole number) being less than it</th></tr>
<tr><th rowspan="3">the raiseds:
a raised number is a non-ultimate number, power of an ultimate number</th><th colspan="2">the composites:
a composite number is a non-ultimate and not raised number admitting at least two different divisors</th></tr>
<tr><th>the pure composites:</th><th>the mixed composites:</th></tr>
<tr><td>a pure composite number is a non-ultimate and not raised number admitting no raised number as divisor</td><td>a mixed composite number is a non-ultimate and not raised number admitting at least a raised number as divisor</td></tr>
<tr><th>level 1</th><th>level 2</th><th>level 3</th><th>level 4</th></tr>
<tr><td colspan="4">degree of complexity of the final four classes of numbers</td></tr>
</table>

Fig.37 Classification of whole numbers from the definition of ultimate numbers (see Fig.38).

8.2 Ultimate divisor

The distinction of whole numbers into different classes deduced from the definition of ultimate numbers allows us to propose the double concept of *ultimate divisor* and *ultimate algebra*.

8.2.1 Ultimate divisor: definition

An ultimate divisor of a whole number is an ultimate number ***less than this whole number*** *and non-trivial divisor of this whole number.*

For example the number 12 has six divisors, the numbers 1, 2, 3, 4, 6 and 12 but only two ultimate divisors: 2 and 3. Also, the numbers zero (0) and one (1), although definite numbers as ultimate, are never ultimate divisors. As a reminder, the division by zero (0) is not defined and therefore this number is not an ultimate divisor. The number one (1) is a trivial divisor, it does not divide a number into some smaller part.

8.2.2 Concept of ultimate algebra

The ultimate algebra applies only to the set of whole numbers and is organized, on the one hand, around the definition of ultimate divisor (previously introduced), on the other hand around the definition of ultimate number (previously introduced). This algebra states that any whole number is either an ultimate number having no ultimate divisor, or a non-ultimate number (which can be either a raised, or a pure composite, or a mixed composite) breaking down into several ultimate divisors. In this algebra, no whole number x can be written in the form $x = x \times 1$ but only in the form $x = x$ (ultimate) or in the form $x = y \times y \times \ldots$ (raised) or $x = y \times z \times \ldots$ (composite) or $x = (y \times y \times \ldots) \times z \times \ldots$ (mixed). Also in this algebra, it is not allowed to write for example $0 = 0 \times y \times z \times \ldots$ but only $0 = 0$.

8.2.2.1 Specific features of the numbers zero and one

By these postulates proposing a concept of ultimate algebra, it is agreed and recalled that although defined as ultimate numbers, the numbers zero (0) and one (1) are neither ultimate divisors, nor composed of ultimate divisors.

8.2.3 Ultimate divisors and number classes

The table in Figure 38 synthesizes the four interactive definitions of the four classes of whole numbers by incorporating the double concept of ultimate divisor and ultimate algebra. It is also suggested here to name u an ultimate number, r a raised, c a pure composite and m a mixed composite number.

An ultimate number (***u***) not admits any non-trivial divisor (whole number) being less than it.
Can be written in the form:

$$u = u$$

→ without ultimate divisors

a raised number (***r***) is a non-ultimate number, power of an ultimate number.
Can be written in the form:

$$r = u \times u$$

→ only with identical ultimate divisors

$$\begin{tikzcd} u \arrow[r] \arrow[d] & r \arrow[d] \\ c \arrow[r] & m \end{tikzcd}$$

A pure composite number (***c***) is a non-ultimate and not raised number admitting no raised number as divisor.
Can be written in the form:

$$c = u \times u'$$

→ only at different ultimate divisors

A mixed composite number (***m***) is a non-ultimate and not raised number admitting at least a raised number as divisor
Can be written in the form:

$$m = u \times u \times u' = r \times u' = c \times u$$

→ simultaneously at identical ultimate divisors and at different ultimate divisors

Fig.38 Classification and interactions of the four classes of whole numbers (see Fig. 37).

8.3 The four classes of whole numbers

Thus, the classification of the set of whole numbers has just been proposed here in four classes of numbers:

- the ultimate numbers called ***ultimates*** (***u***),
- the raised numbers called ***raiseds*** (***r***),
- the pure composite numbers called ***composites*** (***c***),
- the mixed composite numbers called ***mixes*** (***m***).

So it is agree that designation "ultimates" designates ultimate numbers (as "primes" designates prime numbers). Also it is agree that designation "raiseds" designates raised numbers, designation "composites" designates pure composite numbers and designation "mixes" designates mixed composite numbers.

Below, Figure 39, a practical illustration of the concepts of ultimate numbers, ultimate divisors and ultimate algebra is developed, with for example ultimate numbers 2 and 3 (u and u'), ultimate divisors of non-ultimates 4, 6 and 12 (r, c and m).

$$\begin{tikzcd}
\begin{array}{c} u = u \\ u = 2 \end{array} & \begin{array}{c} u \\ 2 \end{array} \arrow[r] \arrow[d] & \begin{array}{c} r \\ 4 \end{array} \arrow[d] & \begin{array}{c} r = u \times u \\ r = 2 \times 2 \end{array} \\
\begin{array}{c} c = u \times u' \\ c = 2 \times 3 \end{array} & \begin{array}{c} c \\ 6 \end{array} \arrow[r] & \begin{array}{c} m \\ 12 \end{array} & \begin{array}{c} m = u \times u \times u' = r \times u' = c \times u \\ m = 2 \times 2 \times 3 = 4 \times 3 = 6 \times 2 \end{array}
\end{tikzcd}$$

Fig. 39 Illustration of the concepts of ultimate numbers, ultimate divisors and ultimate algebra (see Fig.38).

9. The forty primordial numbers

9.1 Numbers classes and 3/2 ratio

The progressive differentiation of source classes and final classes of whole numbers is organized (Figure 40) into a powerful arithmetic arrangement generating transcendent ratios of value 3/2. Thus, the source set of whole numbers includes, among its first ten numbers, 6 ultimate numbers against 4 non-ultimate numbers. The next source set, that of the non-ultimates, includes, among its first ten numbers, 4 raised numbers against 6 composite numbers. Finally, the source set of composites includes, among its first ten numbers, 6 pure composites against 4 mixed composites.

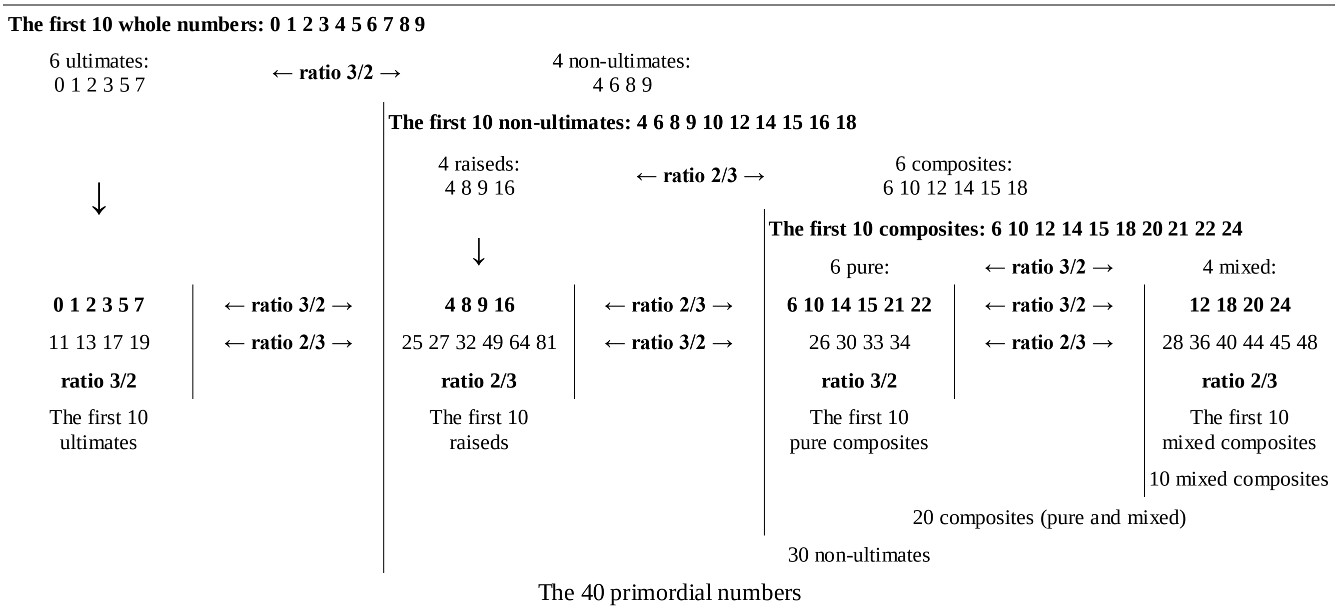


Fig. 40 From the first ten numbers of the three source classes of whole numbers, generation of the first ten numbers of each of the four final classes of numbers: the 40 primordials. See also Fig. 37 and Fig. 38.

A very strong entanglement links all these sets of numbers which oppose in multiple ways in ratios of value 3/2 (or reversibly of ratios 2/3). For example, the first 6 ultimates (0-1-2-3-5-7) are simultaneously opposed to the 4 non-ultimates (4-6-8-9) among the first 10 natural numbers, to the 4 raiseds of the first 10 non-ultimates (4-8-9-16) and to the 4 ultimates beyond the first 10 whole numbers (11-13-17-19).

This entangled classification of whole numbers makes it possible to define (Figure 40) a set of forty primordial numbers. These forty primordial numbers are the set of first ten numbers in each of the four final classes of whole numbers. It is understood that the term "primordials" designates these forty primordial numbers.

9.2 Matrix of the first hundred numbers and primordial numbers

Therefore, in the matrix of the first 100 whole numbers and in a ratio of 3/2, 60 non-primordial numbers oppose to the 40 primordials previously defined in Figure 40. The position differentiation of the 40 primordials generates singular phenomena of 3/2 ratio depending on the different areas considered to 60 versus 40 entities or to 50 versus 50 entities.

Thus, in this matrix, it turns out in Figure 41, that the distinction of two sub-matrices of twice 3 columns against twice 2 columns generates sets of primordial numbers and non-primordial numbers which are opposed in 3/2 transcendent ratios to 36 versus 24 entities and 24 versus 16 entities.

sub-matrix of twice
3 columns
(60 numbers)

← 3/2 ratio →

sub-matrix of twice
2 columns
(40 numbers)

36 non-primordials
24 primordials
3/2 ratio

← 3/2 ratio →
← 3/2 ratio →

24 non-primordials
16 primordials
3/2 ratio

36 non-primordial numbers
24 primordial numbers
3/2 ratio

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
20	**21**	**22**	23	**24**	**25**	**26**	**27**	**28**	29
30	31	**32**	**33**	**34**	35	**36**	37	38	39
40	41	42	43	**44**	**45**	46	47	**48**	**49**
50	51	52	53	54	55	56	57	58	59
60	61	62	63	**64**	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	**81**	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

24 non-primordial numbers
16 primordial numbers
3/2 ratio

Fig. 41 Distinction and distribution of the **40 primordial** and 60 non-primordial numbers in the matrix of the first hundred numbers.

In the sub-matrices of equal sizes and alternately made up of the upper and lower quarters of the complete matrix of the first hundred numbers as illustrated in Figure 42, the 60 non-primordial numbers are distributed in values of equal quantities and these sets of twice 30 non-primordial oppose in 3/2 value ratios to the 40 primordials also distributed in two equal sets of 20 entities.

Matrix to 2 times 25 numbers (50 numbers) **← 1/1 ratio →** **Matrix to 2 times 25 numbers** (50 numbers)

0	**1**	**2**	**3**	**4**					
10	**11**	**12**	**13**	**14**					
20	**21**	**22**	23	**24**					
30	31	**32**	**33**	**34**					
40	41	42	43	**44**					
					55	56	57	58	59
					65	66	67	68	69
					75	76	77	78	79
					85	86	87	88	89
					95	96	97	98	99

← 1/1 ratio →

					5	**6**	**7**	**8**	**9**
					15	**16**	**17**	**18**	**19**
					25	**26**	**27**	**28**	29
					35	**36**	37	38	39
					45	46	47	**48**	**49**
50	51	52	53	54					
60	61	62	63	**64**					
70	71	72	73	74					
80	**81**	82	83	84					
90	91	92	93	94					

30 non-primordials
20 primordials
3/2 ratio

← 1/1 ratio →
← 1/1 ratio →

30 non-primordials
20 primordials
3/2 ratio

Fig. 42 Equal distribution of the 60 non-primordials and **40 primordials** in two sub-matrices of the first hundred numbers.

A redeployment of the matrix of the first hundred numbers introduced in Figure 41 on four rows of 25 entities still generates a remarkable phenomenon in the distinction of the 40 primordials and the 60 non-primordials. This matrix, of the same type as that presented in Chapter 7 in Figure 31, is also linked to the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where a and b have as the values 3 and 2.

By symmetrically regrouping and from their center these four rows of numbers, such that, from this center towards the edge of this new matrix, we isolate four times 9 + 6 values then four times 6 + 4 entities as illustrated in Figure 43, this forms two sub-matrices to 60 versus 40 numbers. In these two 3/2 ratio sub-matrices, the non-primordials and the primordials are also distributed in sets of numbers which are opposed in transcendent 3/2 ratios to 36 versus 24 entities and to 24 versus 16 entities.

Matrix to 4 times $(3^2 + (3 \times 2)) = 60$ numbers (→ 4 times $(a^2 + ab)$)

36 non-primordials **← ratio 3/2 →** **24 primordials**

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	23	**24**
25	**26**	**27**	**28**	29	**30**	31	**32**	**33**	**34**	35	**36**	37	38	39	**40**	41	42	43	**44**	**45**	46	47	**48**	**49**
50	51	52	53	54	55	56	57	58	59	60	61	62	63	**64**	65	66	67	68	69	70	71	72	73	74
75	76	77	78	79	80	**81**	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99

24 non-primordials **← ratio 3/2 →** **16 primordials**

Matrix to 4 times $(2^2 + (2 \times 3)) = 40$ numbers (→ 4 times $(b^2 + ba)$)

Fig. 43 Redeployment of the matrix of the first 100 numbers registering in the remarkable identity *(a + b…* . Distribution of the 60 non-primordials and the **40 primordials** in sets opposing in 3/2 transcendent ratios into two entangled sub-matrices of 60 versus 40 entities.

9.3 Linear sub-matrices of sixty and forty numbers

In the sub-matrix of 60 entities made up alternately of the first six numbers then of the last six numbers of each of the ten lines of the matrix of the first hundred numbers introduced Figure 41, the non-primordial and primordial numbers are opposed, left part of Figure 44, into two sets of 3/2 value ratio and these sets are themselves opposed to the two reciprocal sets of the complementary sub-matrix of 40 entities in 3/2 value transcendent ratios.

Also, exactly the same phenomena occur inside and between the two sub-matrices, of 60 versus 40 entities where the alternation of the considered numbers applies two by two lines as illustrated in the right part of Figure 44.

Sub-matrix to 10 times
6 numbers (60 numbers)

0	**1**	**2**	**3**	**4**	**5**				
				14	**15**	**16**	**17**	**18**	**19**
20	**21**	**22**	23	**24**	**25**				
				34	35	**36**	37	38	39
40	41	42	43	**44**	**45**				
				54	55	56	57	58	59
60	61	62	63	**64**	65				
				74	75	76	77	78	79
80	**81**	82	83	84	85				
				94	95	96	97	98	99

36 non-primordials
24 primordials
3/2 ratio

← **3/2 ratio** →

Sub-matrix to 10 times
4 numbers (40 numbers)

						6	**7**	**8**	**9**
10	**11**	**12**	**13**						
						26	**27**	**28**	29
30	31	**32**	**33**						
						46	47	**48**	**49**
50	51	52	53						
						66	67	68	69
70	71	72	73						
						86	87	88	89
90	91	92	93						

24 non-primordials
16 primordials
3/2 ratio

← **3/2 ratio** →
← **3/2 ratio** →

Sub-matrix to 5 times
12 numbers (60 numbers)

0	**1**	**2**	**3**	**4**	**5**				
10	**11**	**12**	**13**	**14**	**15**				
				24	**25**	**26**	**27**	**28**	29
				34	35	**36**	37	38	39
40	41	42	43	**44**	**45**				
50	51	52	53	54	55				
				64	65	66	67	68	69
				74	75	76	77	78	79
80	**81**	82	83	84	85				
90	91	92	93	94	95				

36 non-primordials
24 primordials
3/2 ratio

← **3/2 ratio** →

Sub-matrix to 5 times
8 numbers (40 numbers)

						6	**7**	**8**	**9**
						16	**17**	**18**	**19**
20	**21**	**22**	23						
30	31	**32**	**33**						
						46	47	**48**	**49**
						56	57	58	59
60	61	62	63						
70	71	72	73						
						86	87	88	89
						96	97	98	99

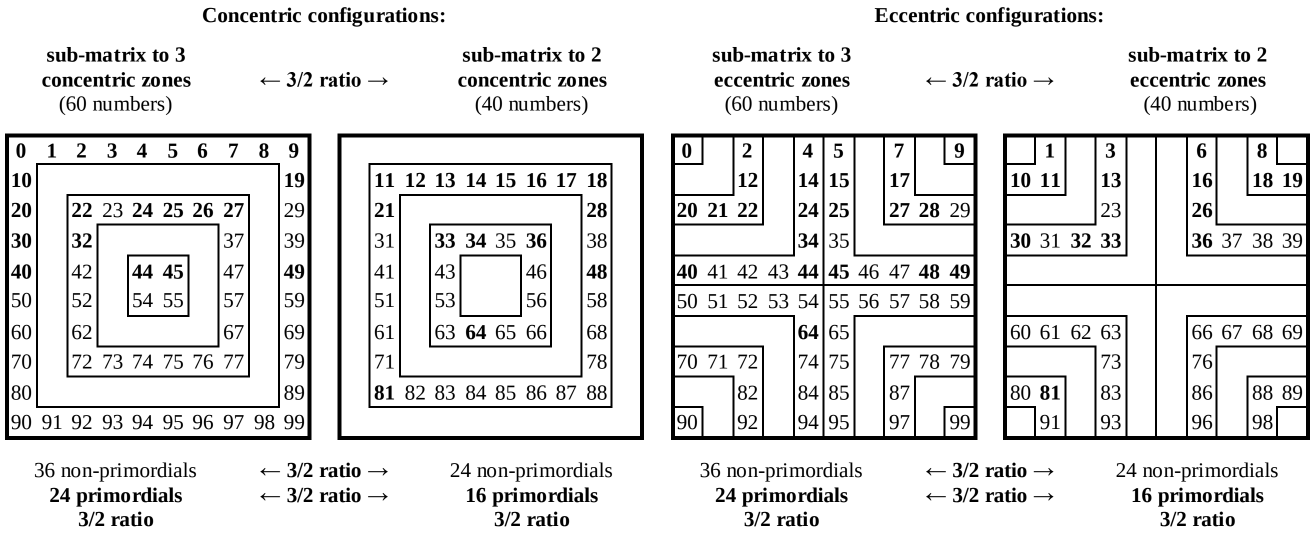
24 non-primordials
16 primordials
3/2 ratio

← **3/2 ratio** →
← **3/2 ratio** →

Fig. 44 According to different alternatively linear sub-matrices: distribution of the 60 non-primordials and the **40 primordials** in opposing sets to 3/2 transcendent ratios.

9.4 Concentric and eccentric matrices

In this matrix of the first hundred numbers, more sophisticated arrangements identical to the configurations introduced in Chapter 4 Figure 13 bring into opposition sets of non-primordial and of primordial numbers in exact 3/2 ratios. Thus, as described in the left part of Figure 45, five concentric zones are opposed, three versus two, in the distribution of their non-primordial and primordial numbers in 3/2 ratios. The same phenomenon is reproduced by considering the five eccentric zones presented in the right part of this Figure 45.



Concentric configurations:

sub-matrix to 3
concentric zones
(60 numbers)

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10									**19**
20		**22**	23	**24**	**25**	**26**	**27**		29
30		**32**					37		39
40		42		**44**	**45**		47		**49**
50		52		54	55		57		59
60		62					67		69
70		72	73	74	75	76	77		79
80									89
90	91	92	93	94	95	96	97	98	99

36 non-primordials
24 primordials
3/2 ratio

← **3/2 ratio** →

sub-matrix to 2
concentric zones
(40 numbers)

	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	
	21							**28**	
	31		**33**	**34**	35	**36**		38	
	41		43			46		**48**	
	51		53			56		58	
	61		63	**64**	65	66		68	
	71							78	
	81	82	83	84	85	86	87	88	

24 non-primordials
16 primordials
3/2 ratio

← **3/2 ratio** →
← **3/2 ratio** →

Eccentric configurations:

sub-matrix to 3
eccentric zones
(60 numbers)

0		**2**		**4**	**5**		**7**		**9**
		12		**14**	**15**		**17**		
20	**21**	**22**		**24**	**25**		**27**	**28**	29
				34	35				
40	41	42	43	**44**	**45**	46	47	**48**	**49**
50	51	52	53	54	55	56	57	58	59
				64	65				
70	71	72		74	75		77	78	79
		82		84	85		87		
90		92		94	95		97		99

36 non-primordials
24 primordials
3/2 ratio

← **3/2 ratio** →

sub-matrix to 2
eccentric zones
(40 numbers)

	1		**3**			**6**		**8**	
10	**11**		**13**			**16**		**18**	**19**
			23			**26**			
30	31	**32**	**33**			**36**	37	38	39
60	61	62	63			66	67	68	69
			73			76			
80	**81**		83			86		88	89
	91		93			96		98	

24 non-primordials
16 primordials
3/2 ratio

← **3/2 ratio** →
← **3/2 ratio** →

Fig. 45 From the matrix of the first hundred numbers, concentric and eccentric configurations of sub-matrices to 60 and 40 entities opposing their non-primordials and their primordials in 3/2 ratios.

Also, in configurations identical to those introduced Chapter 4 in Figure 14, by mixing these sub-matrices of 40 and 60 entities (presented in Figure 45) and after having each split them vertically into two equal parts of 30 and 20 entities, we obtain, Figure 46, new matrices of 50 entities each. In these mixed configurations, the non-primordials and the primordials are divided into exact ratios of value 1/1 with always 30 non-primordials versus 30 and always 20 non-primordials versus 20.

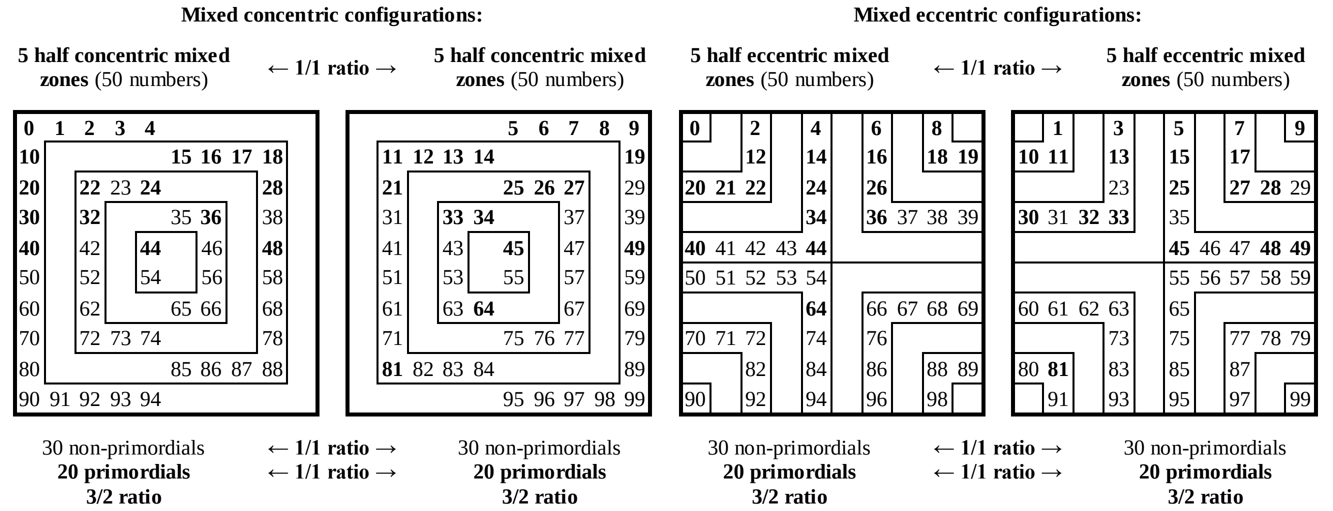


Fig. 46 From the matrix of the first hundred numbers, concentric and eccentric mixed configurations of sub- matrices to 50 entities each opposing their non-ultimates and their ultimates in 1/1 ratios.

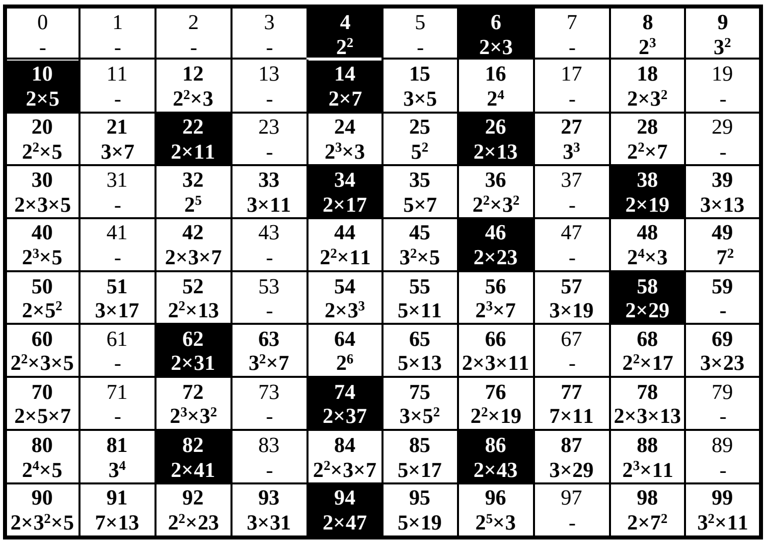
It is important to underline here the total similarity of these arithmetic phenomena with those operating in the sub-matrices introduced Chapter 4.2 where the source data (table of crossed additions of the twenty fundamentals) are however completely different.

10. Ultimate divisors and matrix of the first hundred numbers.

In the matrix of the first hundred whole numbers, 27 are ultimate numbers and 73 are non-ultimate numbers. The tables in Figure 47 demonstrate that these 73 non-ultimates are compositions of 15 different ultimate divisors (ultimate numbers from 2 to 47) and locate their first appearance within this matrix. For example, the *ultimate divisor* 5 appears to the first time as a ultimate divisor of the *non-ultimate 10*. In the right part of Figure 47, the total of the ultimate divisors individually composing these 73 non-ultimate is counted**.**

As a reminder (see Chapter 8.2), the ultimate numbers are not composed of ultimate divisors and the numbers *zero* (0) and *one* (1) are neither ultimate divisors, nor composed of ultimate divisors.

Matrix of the first hundred whole numbers:
27 ultimates - **73 non-ultimates**



0 -	1 -	2 -	3 -	**4** 2^2	5 -	**6** 2×3	7 -	**8** 2^3	**9** 3^2
10 2×5	11 -	**12** $2^2\times3$	13 -	**14** 2×7	**15** 3×5	**16** 2^4	17 -	**18** 2×3^2	19 -
20 $2^2\times5$	**21** 3×7	**22** 2×11	23 -	**24** $2^3\times3$	**25** 5^2	**26** 2×13	**27** 3^3	**28** $2^2\times7$	29 -
30 2×3×5	31 -	**32** 2^5	**33** 3×11	**34** 2×17	**35** 5×7	**36** $2^2\times3^2$	37 -	**38** 2×19	**39** 3×13
40 $2^3\times5$	41 -	**42** 2×3×7	43 -	**44** $2^2\times11$	**45** $3^2\times5$	**46** 2×23	47 -	**48** $2^4\times3$	**49** 7^2
50 2×5^2	**51** 3×17	**52** $2^2\times13$	53 -	**54** 2×3^3	**55** 5×11	**56** $2^3\times7$	**57** 3×19	**58** 2×29	59 -
60 $2^2\times3\times5$	61 -	**62** 2×31	**63** $3^2\times7$	**64** 2^6	**65** 5×13	**66** 2×3×11	67 -	**68** $2^2\times17$	**69** 3×23
70 2×5×7	71 -	**72** $2^3\times3^2$	73 -	**74** 2×37	**75** 3×5^2	**76** $2^2\times19$	**77** 7×11	**78** 2×3×13	79 -
80 $2^4\times5$	**81** 3^4	**82** 2×41	83 -	**84** $2^2\times3\times7$	**85** 5×17	**86** 2×43	**87** 3×29	**88** $2^3\times11$	89 -
90 $2\times3^2\times5$	**91** 7×13	**92** $2^2\times23$	**93** 3×31	**94** 2×47	**95** 5×19	**96** $2^5\times3$	97 -	**98** 2×7^2	**99** $3^2\times11$

210 ultimate divisors:

0	0	0	0	**2**	0	**2**	0	**3**	**2**
2	0	**3**	0	**2**	**2**	**4**	0	**3**	0
3	**2**	**2**	0	**4**	**2**	**2**	**3**	**3**	0
3	0	**5**	**2**	**2**	**2**	**4**	0	**2**	**2**
4	0	**3**	0	**3**	**3**	**2**	0	**5**	**2**
3	**2**	**3**	0	**4**	**2**	**4**	**2**	**2**	0
4	0	**2**	**3**	**6**	**2**	**3**	0	**3**	**2**
3	0	**5**	0	**2**	**3**	**3**	**2**	**3**	0
5	**4**	**2**	0	**4**	**2**	**2**	**2**	**4**	0
4	**2**	**3**	**2**	**2**	**2**	**6**	0	**3**	**3**

Fig. 47 Distribution of the 15 ultimate divisors (from 2 to 47) in the matrix of the first hundred numbers and distinction of their first appearance. Individual statement of the total quantity of ultimate divisors constituting the 73 non-ultimates.

10.1 Fifteen ultimate divisors and matrix of the first hundred numbers.

These fifteen ultimate divisors are grouped, Figure 48, into three sets whose size increases regularly according to whether they make up more or less categories of numbers (classes). Among these fifteen ultimate divisors, 4 are found to be divisors of the three classes of non-ultimate numbers (the raiseds, the composites and the mixes). Then 5 are only divisors of two classes (composites and mixes) and finally 6 ultimate divisors are only of one class of numbers, that of composites. Also, in a ratio of

3/2 value, 9 ultimate divisors (from 2 to 23) composing more than one class of numbers oppose to 6 divisors (from 29 to 47) composing only one.

The 15 ultimate divisors of the first 100 numbers:

2 3 5 7	11 13 17 19 23	29 31 37 41 43 47
4 divisors of raiseds, of composites, of mixes	**5 divisors** of composites, of mixes	**6 divisors** of composites

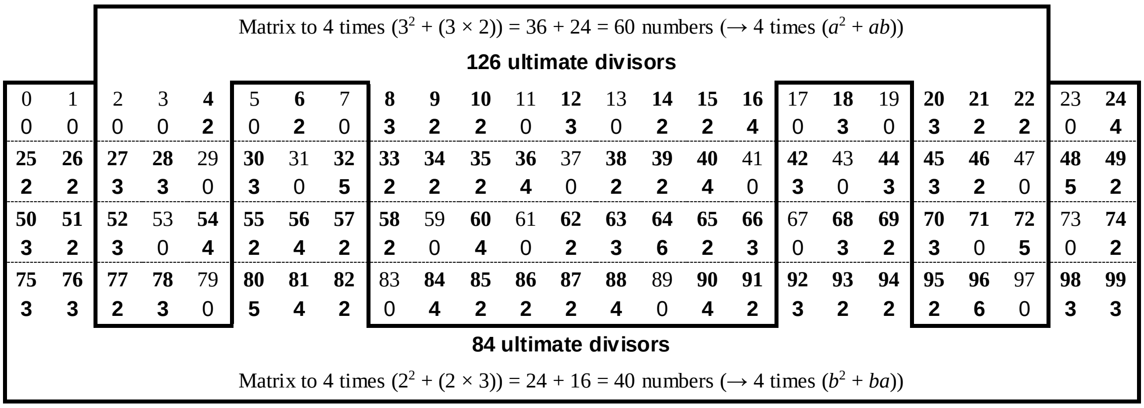
9 divisors of 3 classes or of 2 classes ← 3/2 ratio → 6 divisors of 1 class only

Fig. 48 Distinction of the 15 ultimate divisors (from 2 to 47) according to the types of classes of whole numbers that they compose.

As shown in the right table of Figure 47, the set of the first hundred numbers (deduced from the 27 ultimates not being composed of them) total 210 ultimate divisors. This value allows the appearance of 3/2 ratios and indeed, in several symmetrical configurations opposing, in a 3/2 ratio, matrices to 60 and 40 numbers, these ultimate divisors also oppose in a 3/2 ratio with respectively for each double matrix, 126 ultimate divisors versus 84 (3 times 42 versus 2 times 42).

10.2 Ultimate divisors and linear matrix

Figure 49, in a reorganization of the matrix of the first hundred numbers into four lines of 25 entities (→ $25 = 3^2 + (3 \times 2) + (2 \times 3) + 2^2$, see chapter 7.1.1), the 210 ultimate divisors oppose, in a 3/2 ratio, in two symmetrically opposed matrices to 60 versus 40 entities with respectively, in each matrix, 126 versus 84 ultimate divisors.



Matrix to 4 times $(3^2 + (3 \times 2)) = 36 + 24 = 60$ numbers (→ 4 times $(a^2 + ab)$)

126 ultimate divisors

0	1	2	3	**4**	5	**6**	7	**8**	**9**	**10**	11	**12**	13	**14**	**15**	**16**	17	**18**	19	**20**	**21**	**22**	23	**24**
0	0	0	0	**2**	0	**2**	0	**3**	**2**	**2**	0	**3**	0	**2**	**2**	**4**	0	**3**	0	**3**	**2**	**2**	0	**4**
25	**26**	**27**	**28**	29	**30**	31	**32**	**33**	**34**	**35**	**36**	37	**38**	**39**	**40**	41	**42**	43	**44**	**45**	**46**	47	**48**	**49**
2	**2**	**3**	**3**	0	**3**	0	**5**	**2**	**2**	**2**	**4**	0	**2**	**2**	**4**	0	**3**	0	**3**	**3**	**2**	0	**5**	**2**
50	**51**	**52**	53	**54**	**55**	**56**	**57**	**58**	59	**60**	61	**62**	**63**	**64**	**65**	**66**	67	**68**	**69**	**70**	**71**	**72**	73	**74**
3	**2**	**3**	0	**4**	**2**	**4**	**2**	**2**	0	**4**	0	**2**	**3**	**6**	**2**	**3**	0	**3**	**2**	**3**	0	**5**	0	**2**
75	**76**	**77**	**78**	79	**80**	**81**	**82**	83	**84**	**85**	**86**	**87**	**88**	89	**90**	**91**	**92**	**93**	**94**	**95**	**96**	97	**98**	**99**
3	**3**	**2**	**3**	0	**5**	**4**	**2**	0	**4**	**2**	**2**	**2**	**4**	0	**4**	**2**	**3**	**2**	**2**	**2**	**6**	0	**3**	**3**

84 ultimate divisors

Matrix to 4 times $(2^2 + (2 \times 3)) = 24 + 16 = 40$ numbers (→ 4 times $(b^2 + ba)$)

Fig. 49 Counting of the ultimate divisors in a redeployment of the matrix of the first 100 numbers registering in the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where a and b have as 3 and 2 to value. Distribution of the 210 divisors in 3/2 ratio in two entangled sub-matrices to 60 versus 40 entities.

10.3 Symmetric sub-matrices

Inside symmetrical sub-matrices made up of 10 microzones to 6 versus 4 entities, such as those in Figure 50, the quantities of ultimate divisors constituting the first hundred numbers are opposed in 3/2 value ratios with 126 ultimate divisors in sub-matrices of 60 entities versus 84 ultimate divisors in sub-matrices of 40 entities respectively.

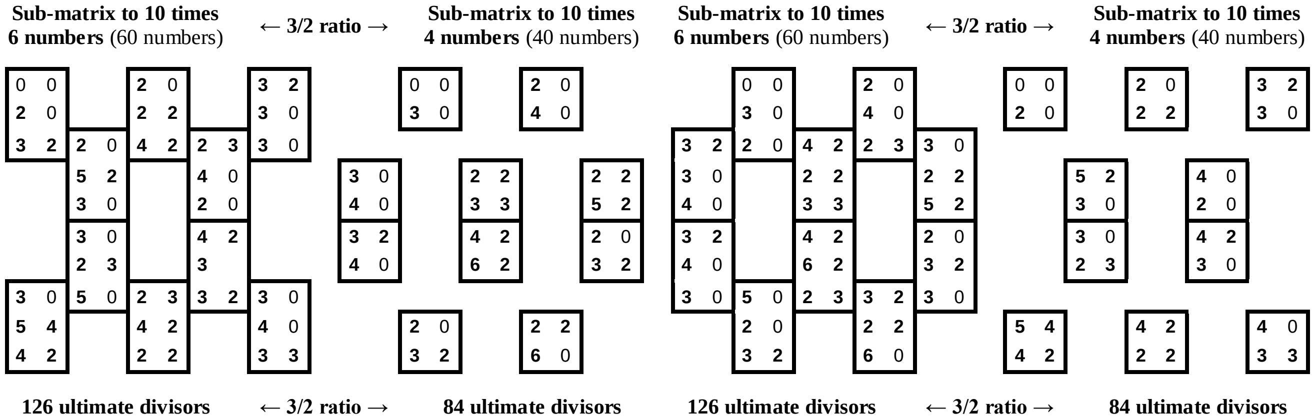
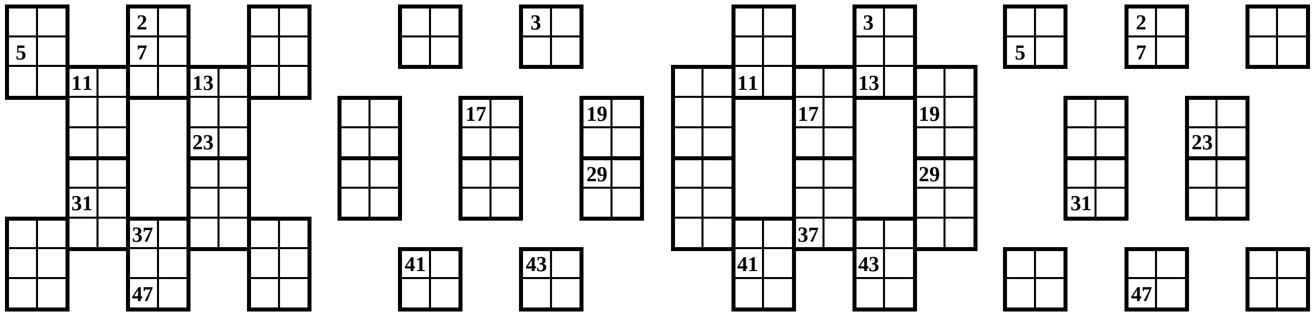


Fig. 50 Respective quantities of the ultimate divisors constituting the first hundred numbers. See matrix in Figure 47.

10.4 First appearance of the fifteen ultimate divisors

In these same two double geometrically symmetric sub-matrices, it turns out, Figure 51, that the first appearance of the fifteen ultimate divisors (see Figure 47) is also organized in a perfect ratio of 3/2 value with nine first appearances in sub-matrices of 60 numbers versus six first appearances in sub-matrices of 40 numbers.

Sub-matrix to 10 times 6 numbers (60 numbers)	**← 3/2 ratio →**	**Sub-matrix to 10 times 4 numbers** (40 numbers)	**Sub-matrix to 10 times 6 numbers** (60 numbers)	**← 3/2 ratio →**	**Sub-matrix to 10 times 4 numbers** (40 numbers)



9 ultimate divisors appearances	**← 3/2 ratio →**	**6 ultimate divisors appearances**	**9 ultimate divisors appearances**	**← 3/2 ratio →**	**6 ultimate divisors appearances**

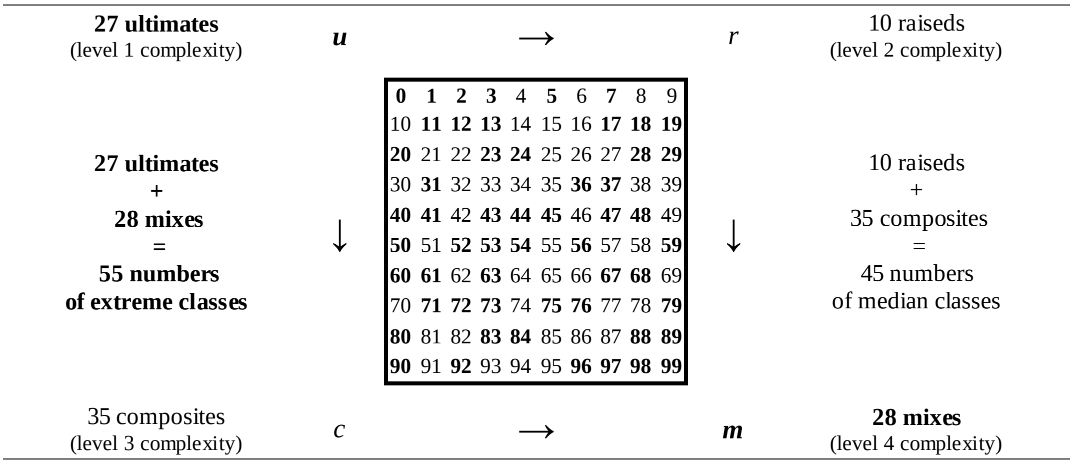
Fig. 51 Distribution of the first appearance of the 15 ultimate divisors in the matrix of the first 100 numbers.

11. Interactions of the four classes of numbers

The distinction of the four classes of numbers introduced in Chapter 8 generate singular arithmetic phenomena in different interactions of entities such as the opposition of the numbers of so-called *extreme* and *median* classes or the study of the different possible associations of pairs of numbers according to their belonging to such or such class. Finally, the crossing of these different criteria directly towards the twenty fundamental numbers (concept introduced in Chapter 2.6), then indirectly from a matrix of which they are a source, still reveals the same types of arithmetic phenomena as those presented many times upstream this study. These latest investigations undoubtedly legitimize all the new mathematical classifications proposed to whole numbers.

11.1 Association of opposing classes

Depending on their degree of complexity, the four classes of numbers can be grouped into two sets of extreme or median classes. Thus, the ultimate numbers, of level 1 complexity and the mixed numbers, of level 4 complexity form a set of entities of extreme classes and the raised and composite numbers, of level 2 and 3 complexity, form a second set of median classes. So it is agree that designation "extremes" designates numbers of extreme classes and designation "medians" designates numbers of medians classes.



0	**1**	**2**	**3**	4	**5**	6	**7**	8	9
10	**11**	**12**	**13**	14	15	16	**17**	**18**	**19**
20	21	22	**23**	**24**	25	26	27	**28**	**29**
30	**31**	32	33	34	35	**36**	**37**	38	39
40	**41**	42	**43**	**44**	**45**	46	**47**	**48**	49
50	51	**52**	**53**	**54**	55	**56**	57	58	**59**
60	**61**	62	**63**	64	65	66	**67**	**68**	69
70	**71**	**72**	**73**	74	**75**	**76**	77	78	**79**
80	81	82	**83**	**84**	85	86	87	**88**	**89**
90	91	**92**	93	94	95	**96**	**97**	**98**	**99**

Fig. 52 Counting of the four classes of numbers in the matrix of the first 100 numbers according to their degree of complexity (see Fig. 37 and 47).

Figure 52, in the matrix of the first hundred numbers, these two sets are made up of 55 numbers to extreme classes and 45 numbers to median classes. In increasingly diluted sub-matrices of 60 versus 40 entities, these two families of numbers are always distributed in 3/2 value ratios.

11.1.1 Dilution of sub-matrices

Thus, in the left part of Figure 53, in two compact blocks of 60 versus 40 entities made up respectively of the first 60 and the following 40, the extreme numbers and the median numbers are distributed in ratios of values 3/2 with, respectively for each set of numbers, 33 extremes versus 22 and 27 medians versus 18. Splitting the matrix of the first hundred numbers into 10 blocks of 5 times 12 and 5 times 8 entities as illustrated in the right part of Figure 53 generates the same arithmetic phenomena.

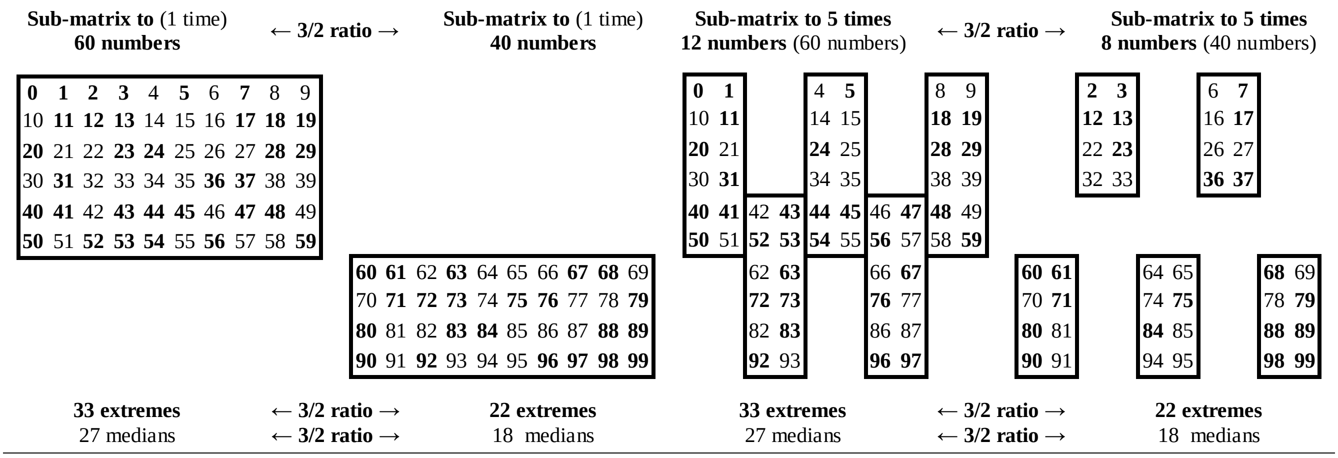


Fig. 53 Distribution of the numbers to **extreme** and median classes in two little diluted and more diluted double sub-matrices of the first 100 numbers.

Also, the two sets of extreme and median numbers are further divided into 3/2 value ratios in a more diluted fractionation of this matrix into 20 blocks of 10 times 6 and 10 times 4 entities as described in the left part of Figure 54. Lastly, on the right side of Figure 54, in a final fractionation of this matrix into 40 blocks of 20 times 3 entities versus 20 times 2 entities, the same partitions of the two families of numbers in 3/2 ratios are still observed with always 33 extremes versus 22 and 27 medians versus 18.

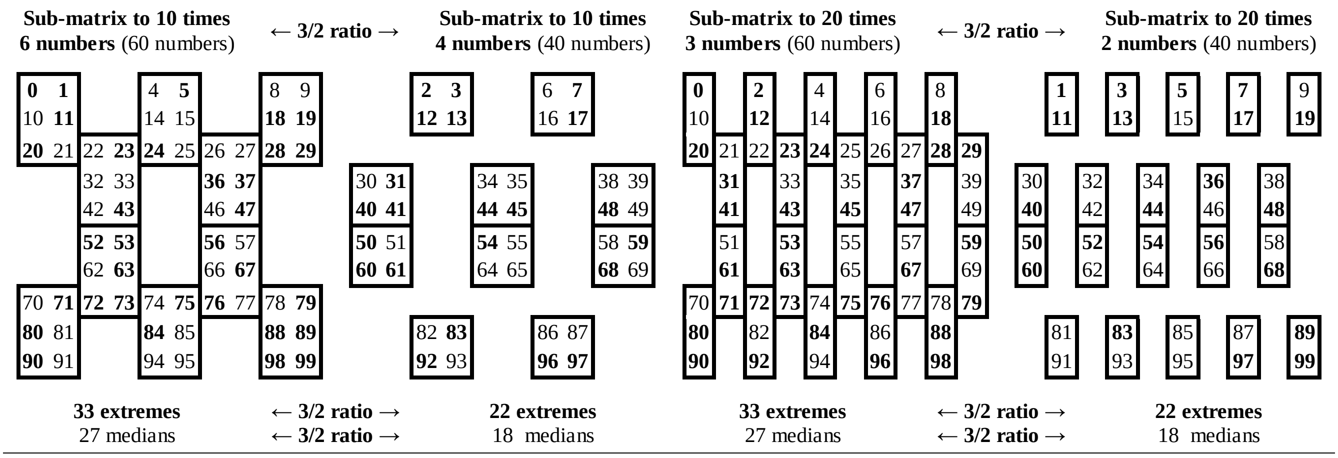


Fig. 54 Distribution of the numbers to **extreme** and median classes in two diluted and very diluted double sub-matrices of the first 100 numbers.

11.1.2 Matrix to twenty-five entities

As it was presented in Chapter 7.1.1, it is necessary to have a minimum to 25 entities so that two double sets to 9 versus 6 then to 6 versus 4 numbers (and globally to 15 versus 10 entities) can be opposed in transcendent ratios of 3/2 value. Also these arithmetic arrangements are organized from the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$. It so happens that the matrix of the first 25 numbers (matrix configured from this remarkable identity) generates these phenomena by the opposition of the numbers of extreme classes to those of median classes of which it is made up.

$(a + b)^2 = a^2 + 2ab + b^2 \rightarrow (3 + 2)^2 = 3^2 + 2(3 \times 2) + 2^2 \rightarrow 9 + 6 + 6 + 4 = 25$ **entities**

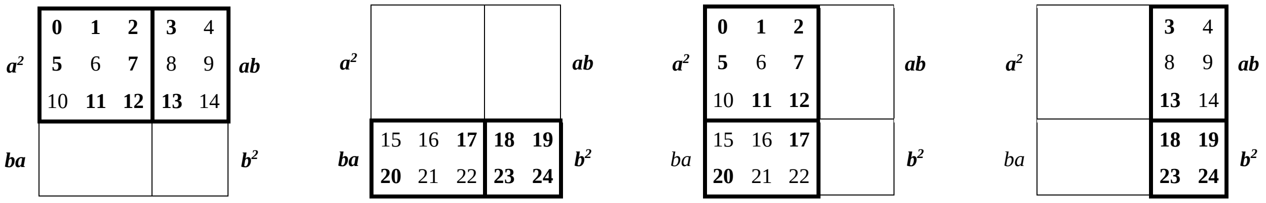
	0	**1**	**2**	**3**	4	
9 *a^2*	**5**	6	**7**	8	9	**6** *ab*
	10	**11**	**12**	**13**	14	
6 *ba*	15	16	**17**	**18**	**19**	**4** *b^2*
	20	21	22	**23**	**24**	

15 extremes ← **3/2 ratio** → 10 medians

Fig. 55 Opposition in 3/2 ratio of the first 15 extremes and the first 10 medians in a matrix of 25 entities deduced from the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have the values 3 and 2.

It thus appears, Figure 55, that among the first 25 numbers are 15 entities of extreme classes which oppose in a ratio of value 3/2 to 10 entities of median classes. Also, these two types of classes of numbers are opposed, Figure 56, in different transcendent ratios of value 3/2 in and between the sub-matrices whose configuration is deduced from the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have 3 and 2 to respective value.

Sub-matrix to 15 numbers ($a^2 + ab$)	← **3/2 ratio** →	**Sub-matrix to 10 numbers ($ba + b^2$)**	**Sub-matrix to 15 numbers ($a^2 + ba$)**	← **3/2 ratio** →	**Sub-matrix to 10 numbers ($ab + b^2$)**

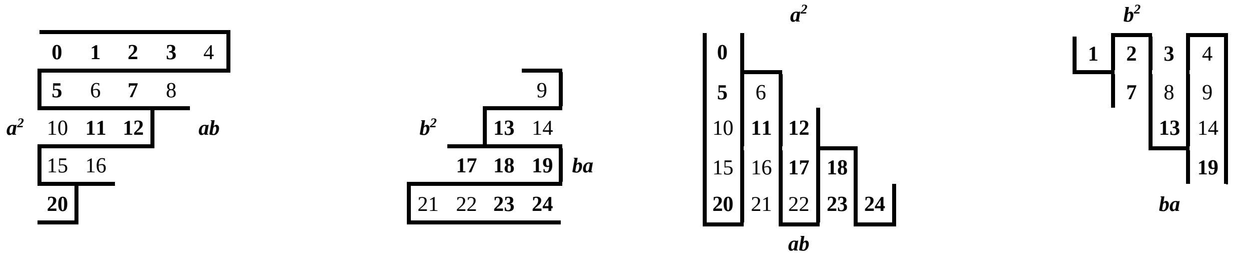


9 extremes 6 medians **3/2 ratio**	← **3/2 ratio** → ← **3/2 ratio** →	**6 extremes** 4 medians **3/2 ratio**	**9 extremes** 6 medians **3/2 ratio**	← **3/2 ratio** → ← **3/2 ratio** →	**6 extremes** 4 medians **3/2 ratio**

Fig. 56 Distribution of the numbers to extreme and median classes in two double sub-matrices of the first 25 numbers. Configurations inferred from the identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have the values 3 and 2.

Also, in this matrix of the first 25 numbers, other configurations of the same arithmetic arrangements, but which are geographically redeployed in various other ways still generate the same phenomena opposing the numbers to extreme classes and those to median classes in transcendent ratios of 3/2 value. Figures 57 and 58 illustrate these singular phenomena.

Sub-matrix to 9 + 6 numbers ($a^2 + 2ab/2$)	← **3/2 ratio** →	**Sub-matrix to 6 + 4 numbers ($2ba/2 + b^2$)**	**Sub-matrix to 9 + 6 numbers ($a^2 + ba$)**	← **3/2 ratio** →	**Sub-matrix to 6 + 4 numbers ($ab + b^2$)**



9 extremes 6 medians **3/2 ratio**	← **3/2 ratio** → ← **3/2 ratio** →	**6 extremes** 4 medians **3/2 ratio**	**9 extremes** 6 medians **3/2 ratio**	← **3/2 ratio** → ← **3/2 ratio** →	**6 extremes** 4 medians **3/2 ratio**

Fig. 57 Distribution of the numbers to extreme and median classes in two double sub-matrices of the first 25 numbers. Configurations inferred from the identity *(a + b… .*

These phenomena are always and again inscribed in different geometric variables of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have 3 and 2 to value.

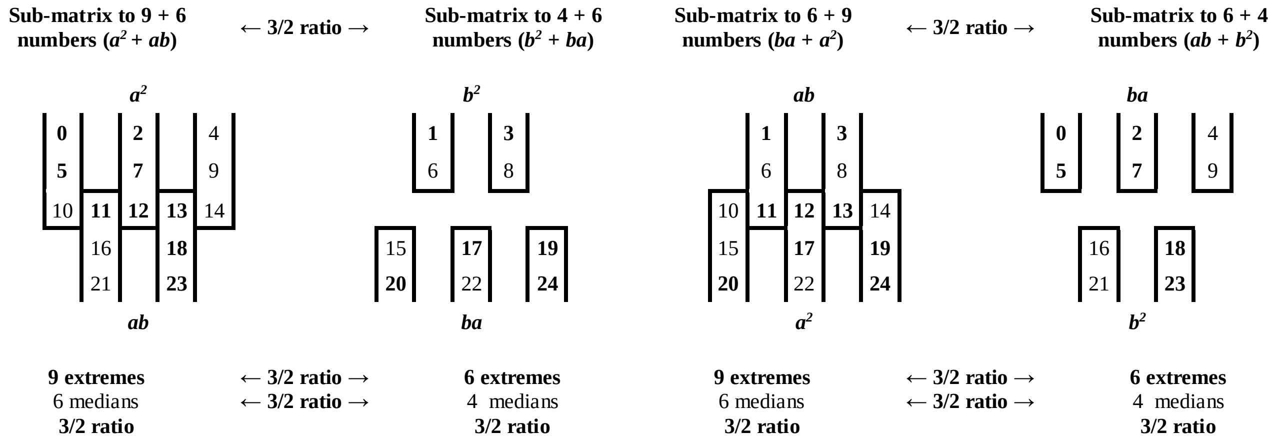


Fig. 58 Distribution of the numbers to extreme and median classes in two double sub-matrices of the first 25 numbers. Configurations inferred from the identity *(a + b…* .

11.2 Classes of numbers and pairs of numbers

According to their classification into four classes as defined in Chapter 8 (*u* = ultimate, *r* = raised, *c* = composite and *m* = mix, see Figure 38 Chapter 8), whole numbers can be associated two by two in ten different configurations.

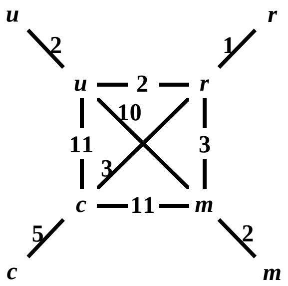
11.2.1 The ten associations of number classes

In the matrix of the first hundred whole numbers classified linearly into ten lines of ten consecutive entities, it is possible to form 50 pairs of consecutive numbers. These couples can be arranged in ten different ways according to the respective class of the two entities constituting them. It turns out Figure 59 that, in this matrix, all the ten possible associations are represented including only one but very ever-present association of two raised class numbers: the couple 8-9 (2^3 and 3^2).

For fun (but maybe not) it's nice to note that these two numbers are the last two of the digit numbers. Also, their respective root values are in a ratio of 2/3 and they are respectively raised by 3 and 2 powers: another ratio of 3/2. Also, (the demonstration will not be done here) it would seem that it is the only pair of consecutive raised class numbers among the set of whole numbers.

30 couples ***r-r*** ***r-m*** ***m-u*** ***u-c*** ***c-c***

0-1	2-3	4-5	**6-7**	**8-9**
10-11	**12-13**	**14-15**	16-17	**18-19**
20-21	**22-23**	**24-25**	26-27	**28-29**
30-31	32-33	**34-35**	**36-37**	**38-39**
40-41	**42-43**	44-45	**46-47**	**48-49**
50-51	**52-53**	54-55	56-57	**58-59**
60-61	62-63	64-65	**66-67**	68-69
70-71	**72-73**	74-75	76-77	**78-79**
80-81	**82-83**	84-85	**86-87**	**88-89**
90-91	92-93	**94-95**	**96-97**	98-99

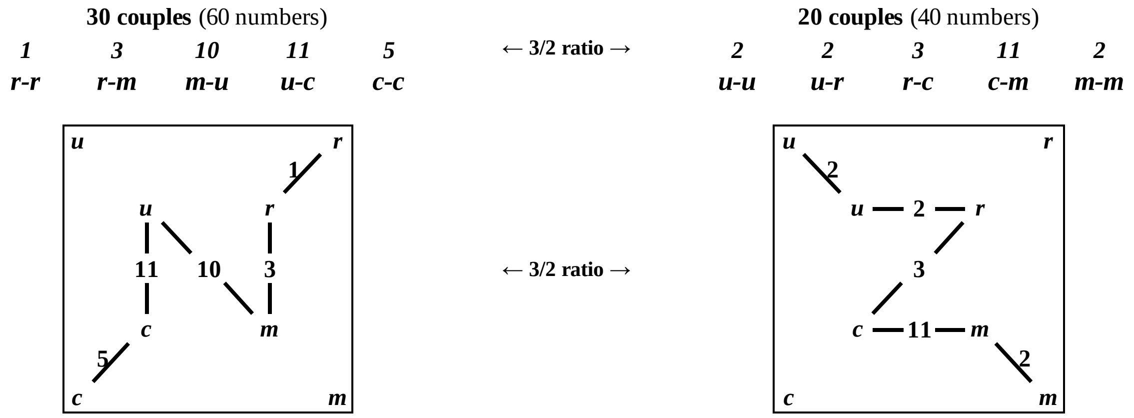


20 couples *u-u* *u-r* *r-c* *c-m* *m-m*

Fig. 59 Count of the associations of classes of numbers of the pairs of adjacent numbers of the matrix of the first 100 numbers (see also Fig. 60).

11.2.2 Symmetric associations of number classes

By grouping together five particular associations of pairs of numbers and five others, it turns out that, in a ratio of 3/2, 30 couples are made up of these first five associations considered and 20 couples are made up of the other five possible associations. As shown in Figure 60, these two groups of five associations are not arbitrary but are organized in two sub symmetrical hyper configurations which can be called configuration *N* and configuration *Z*. This, with reference to the image released from these hyper configurations of twice five associations of numbers in the schematization of these configurations.



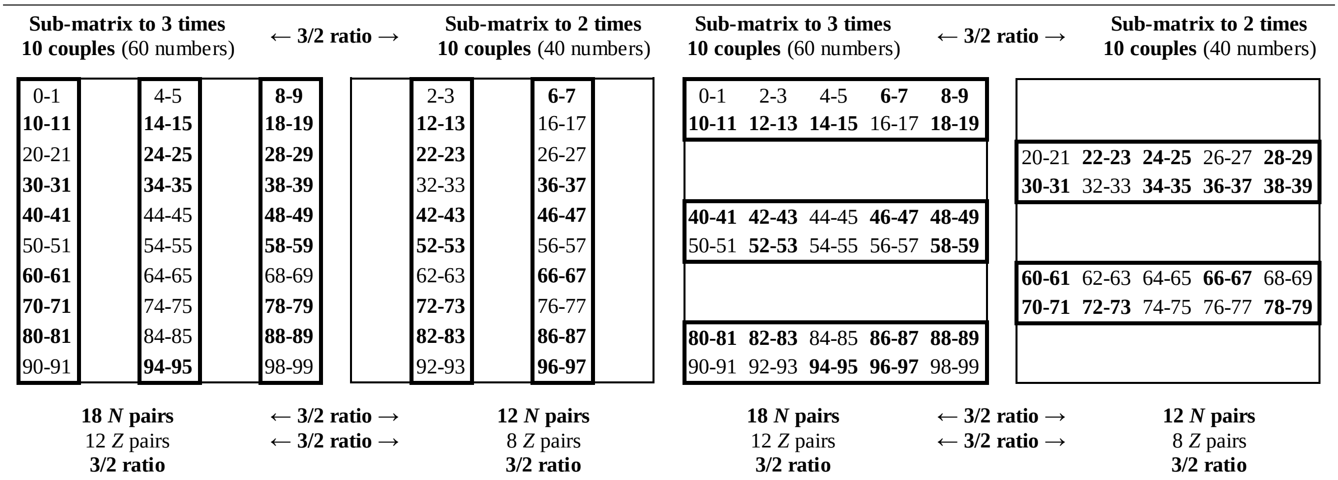
'N' configuration ***'Z' configuration***

Fig. 60 Classification of the 50 pairs of numbers according to two symmetrical configurations of associations of couples. In a 3/2 value ratio: 30 pairs to N configuration versus 20 couples to Z configuration (see Figure 59).

The N-type configuration has two *protuberances* made up of associations of two raiseds (*r-r*) and two composites (*c-c*), namely types of numbers of median classes. The Z-type configuration has its two similar and symmetrical protuberances made up of associations of two ultimates (*u-u*) and two mixes (*m-m*), namely types of numbers of extreme classes. Also, among the 30 couples of configuration *N*, 6 couples are formed of two numbers of the same classes (5 *c-c* couples and 1 *r-r* couple) and among the 20 couples of configuration *Z*, 4 couples are formed of two numbers of the same classes (2 *u-u* couples and 2 *m-m* couples). Again, these sets of couples are in opposition in a 3/2 value ratio.

11.2.3 Associations of classes of numbers and 3/2 transcendent ratios

In two double vertical and then horizontal matrices opposing sets of 3 times 10 couples and sets of 2 times 10 couples as presented in Figure 61, the pairs of numbers of *configuration N* and those of *configuration Z* are opposed in transcendent ratios of values 3/2. In these configurations, 18 *N* pairs are simultaneously opposed to 12 *Z* pairs and 12 other *N* pairs and 8 *Z* pairs are simultaneously opposed to 12 *N* pairs and 12 other *Z* pairs.



Sub-matrix to 3 times 10 couples (60 numbers) **← 3/2 ratio →** **Sub-matrix to 2 times 10 couples** (40 numbers)

0-1		4-5		**8-9**		2-3		**6-7**	
10-11		**14-15**		**18-19**		**12-13**		16-17	
20-21		**24-25**		**28-29**		**22-23**		26-27	
30-31		**34-35**		**38-39**		32-33		**36-37**	
40-41		44-45		**48-49**		**42-43**		**46-47**	
50-51		54-55		**58-59**		**52-53**		56-57	
60-61		64-65		68-69		62-63		**66-67**	
70-71		74-75		**78-79**		**72-73**		76-77	
80-81		84-85		**88-89**		**82-83**		**86-87**	
90-91		**94-95**		98-99		92-93		**96-97**	

18 *N* pairs **← 3/2 ratio →** **12 *N* pairs**
12 *Z* pairs **← 3/2 ratio →** 8 *Z* pairs
3/2 ratio **3/2 ratio**

Sub-matrix to 3 times 10 couples (60 numbers) **← 3/2 ratio →** **Sub-matrix to 2 times 10 couples** (40 numbers)

0-1	2-3	4-5	**6-7**	**8-9**					
10-11	**12-13**	**14-15**	16-17	**18-19**					
					20-21	**22-23**	**24-25**	26-27	**28-29**
					30-31	32-33	**34-35**	**36-37**	**38-39**
40-41	**42-43**	44-45	**46-47**	**48-49**					
50-51	**52-53**	54-55	56-57	**58-59**					
					60-61	62-63	64-65	**66-67**	68-69
					70-71	**72-73**	74-75	76-77	**78-79**
80-81	**82-83**	84-85	**86-87**	**88-89**					
90-91	92-93	**94-95**	**96-97**	98-99					

18 *N* pairs **← 3/2 ratio →** **12 *N* pairs**
12 *Z* pairs **← 3/2 ratio →** 8 *Z* pairs
3/2 ratio **3/2 ratio**

Fig. 61 Distribution of couples of ***configuration N*** (***r-r r-m m-u u-c c-c***) and of *configuration Z* (*u-u u-r r-c c-m m-m*) in vertical and horizontal sub matrices to 30 versus 20 pairs of adjacent numbers.

In this matrix of the first hundred numbers, the same arithmetic phenomena appear, Figure 62, both in the opposition of the 30 vertically peripheral couples to the 20 vertically central couples as well as in the opposition of the 20 peripheral couples to the 30 central couples.

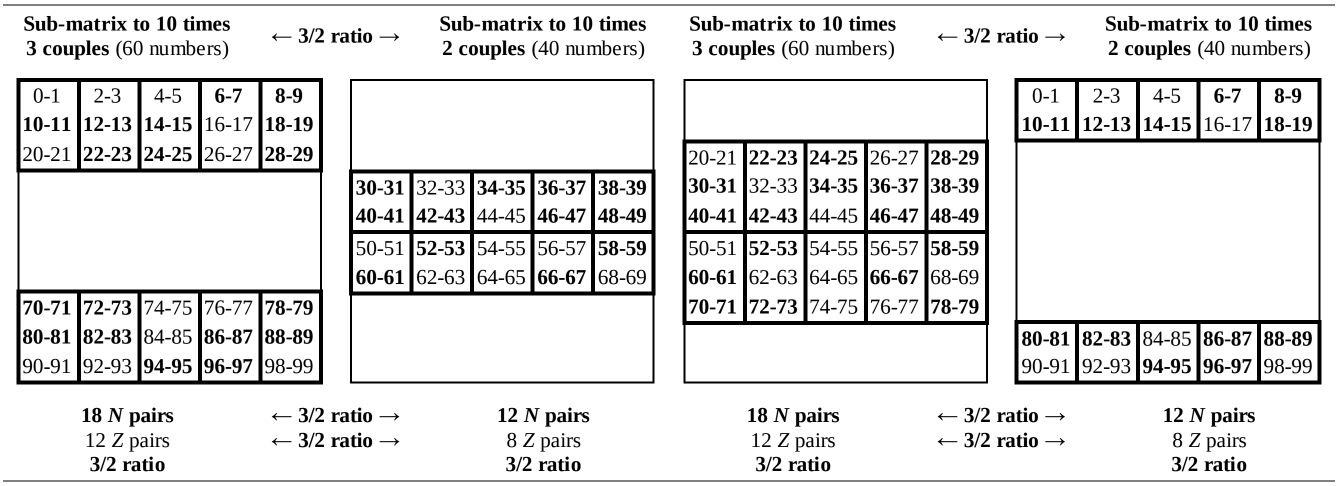


Fig. 62 Distribution of couples of ***configuration N*** (***r-r r-m m-u u-c c-c***) and of *configuration Z* (*u-u u-r r-c c-m m-m*) in horizontal sub-matrices to 30 versus 20 pairs of adjacent numbers.

Also, these same arithmetic phenomena still occur in more diluted sub-matrices such as those illustrated in Figure 63 and with configurations identical to those already introduced above in this study of whole numbers.

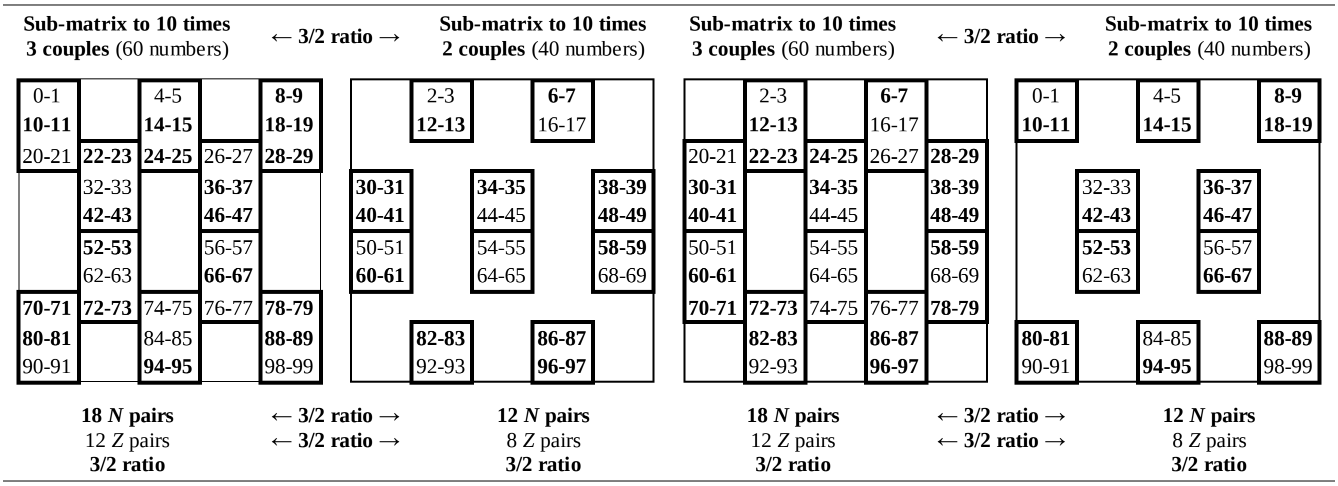


Fig. 63 Distribution of couples of ***configuration N*** (***r-r r-m m-u u-c c-c***) and of *configuration Z* (*u-u u-r r-c c-m m-m*) in symmetrical sub-matrices to 30 versus 20 pairs of adjacent numbers.

11.3 Interactions of the twenty fundamentals

To close this series of investigations on whole numbers and their interactions dependent on their various natures, attention is paid here to the first twenty of these called (see Chapter 2) *the twenty fundamentals*. Depending on the interaction of pairs of fundamentals which may be adjacent, sub-adjacent or distant, they always oppose in 3/2 value ratios with 6 couples versus 4 couples of different considered configurations.

11.3.1 Interactions of ultimate and non-ultimate numbers

Considering the double criterion of ultimity or non-ultimity, Figure 64, both in the configurations of adjacent couples as well as sub-adjacent or distant, the opposition of pairs composed of two ultimate or two non-ultimate to mixed pairs (one ultimate + one non-ultimate) generates, in a 3/2 value ratio, always sets of 6 versus 4 pairs.

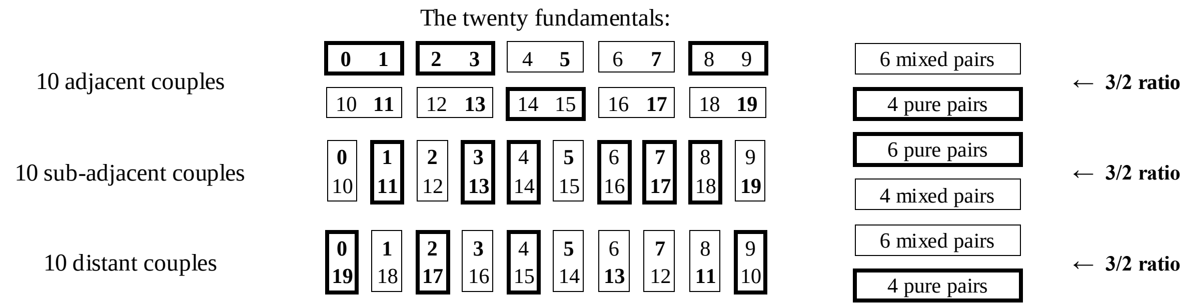


Fig. 64 Distinction between pure couples (composed of 2 **ultimate** or 2 non-ultimate) and mixed couples (composed of a single **ultimate** and a single non-ultimate).

11.3.2 Interactions of extreme and median numbers

Considering the double criterion of extreme number or median number, Figure 65, both in the configurations of adjacent couples as well as sub-adjacent or distant, the opposition of pairs composed of two extreme or two median to mixed pairs (one extreme + one median) generates, in a 3/2 value ratio, always sets of 6 versus 4 pairs.

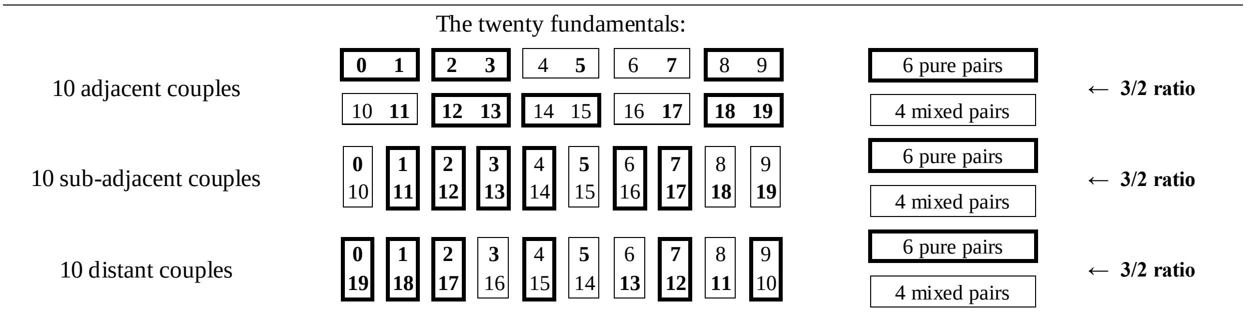


Fig. 65 Distinction between pure couples (composed of 2 **extreme** or 2 median) and mixed couples (composed of a single **extreme** and a single median).

11.3.3 Interactions of pairs of numbers to configurations N and Z

Considering the double criterion of couple of numbers by configuration *N* or by configuration *Z*, Figure 66, both in the configurations of adjacent couples as well as sub-adjacent or distant, the opposition of pairs by configuration *N* to pairs by configuration *Z* generates, in a 3/2 value ratio, always sets of 6 versus 4 pairs.

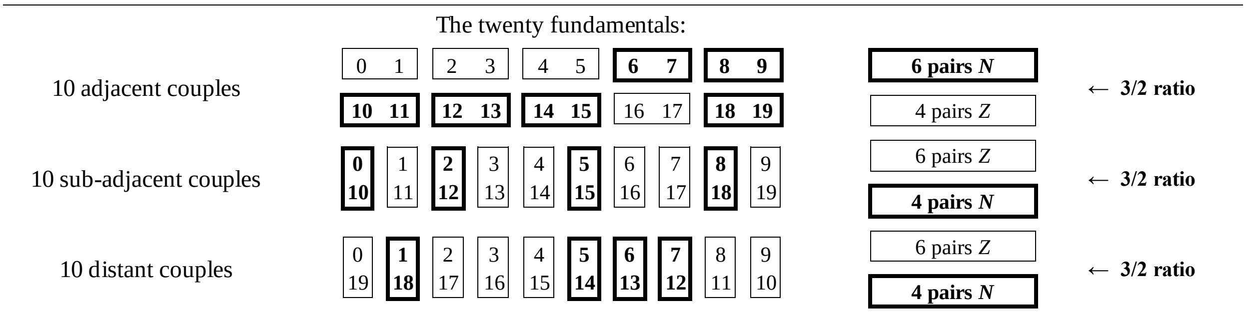


Fig. 66 Distinction of the pairs to **configuration *N* (*e-e e-m m-u u-c c-c*)** and to configuration *Z* (*u-u u-e e-c c-m m-m*). (See Fig. 60.)

11.4 Matrix of the twenty fundamentals and classes of numbers

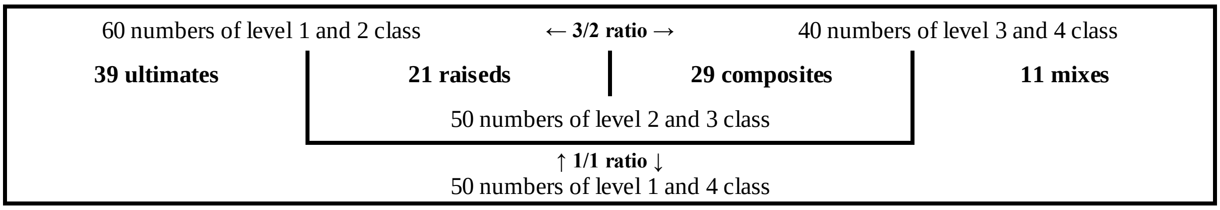
Once again, the undeniable demonstration of the great importance of organization of the interactions of the twenty fundamental numbers, the concept of which was introduced in Chapter 2, is made here.

The addition matrix of the first ten ultimate with the first ten non-ultimate numbers generates a hundred values that can be distinguished according to the four classes of numbers defined in Chapter 8. As shown in Figure 67, in this addition matrix of the twenty fundamentals, the number classes oppose two by two in ratios of value 3/2 or value 1/1 depending on the considered configurations.

the 10 first non-ultimates

the 10 first ultimates

+	***4***	***6***	***8***	***9***	***10***	***12***	***14***	***15***	***16***	***18***
0	4	6	8	9	10	12	14	15	16	18
1	5	7	9	10	11	13	15	16	17	19
2	6	8	10	11	12	14	16	17	18	20
3	7	9	11	12	13	15	17	18	19	21
5	9	11	13	14	15	17	19	20	21	23
7	11	13	15	16	17	19	21	22	23	25
11	15	17	19	20	21	23	25	26	27	29
13	17	19	21	22	23	25	27	28	29	31
17	21	23	25	26	27	29	31	32	33	35
19	23	25	27	28	29	31	33	34	35	37



60 numbers of level 1 and 2 class ← **3/2 ratio** → 40 numbers of level 3 and 4 class

39 ultimates **21 raiseds** **29 composites** **11 mixes**

50 numbers of level 2 and 3 class

↑ **1/1 ratio** ↓

50 numbers of level 1 and 4 class

Fig. 67 Distribution of the 4 classes of numbers generated from the additions matrix of the 20 fundamentals segregated into 10 ultimates versus 10 non-ultimates. (See also Fig. 37 and 47).

Thus, the opposition of the extreme classes to the median classes is organized in a ratio of 1/1 and the opposition of the first two classes of 1st and 2nd level of complexity to the last two classes of 3rd and 4th level of complexity is organized into a 3/2 value ratio.

11.4.1 Matrix of the twenty fundamentals

These two types of oppositions generate sets of opposing entities in transcendent ratios of value 3/2 or / and values 1/1 in the two double configurations of four sub-matrices of 60 versus 40 numbers as illustrated in Figure 68. These sub-matrices are inscribed in the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where a and b have the values 3 and 2.

60 ultimates and raiseds versus 40 composites and mixes (**3/2 ratio**)

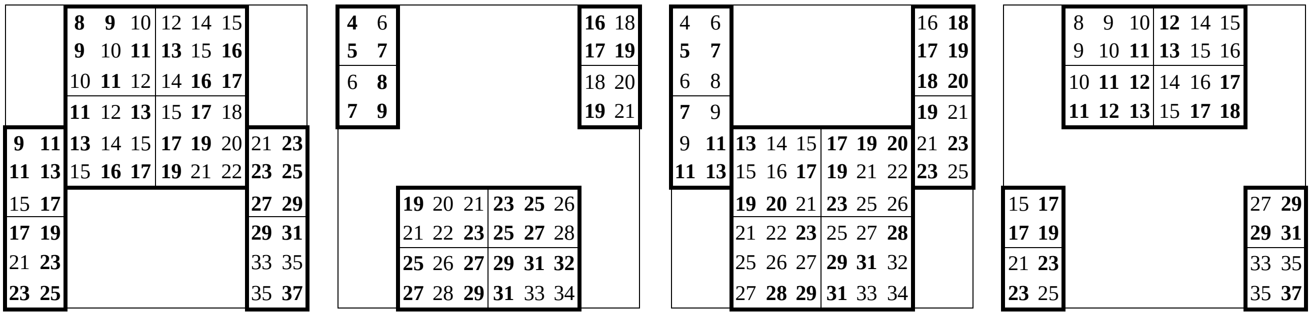
level 1 and 2 classes versus level 3 and 4 classes

Sub-matrix to 36 + 24 numbers (60 numbers) → **4(a^2 + ab)** ← **3/2 ratio** → **Sub-matrix to 24 + 16 numbers** (40 numbers) → **4(ba + b^2)**

50 ultimates and mixes versus 50 raiseds and composites (**1/1 ratio**)

level 1 and 4 classes versus level 2 and 3 classes

Sub-matrix to 36 + 24 numbers (60 numbers) → **4(a^2 + ab)** ← **3/2 ratio** → **Sub-matrix to 24 + 16 numbers** (40 numbers) → **4(ba + b^2)**



		8	**9**	10	12	14	15		
		9	10	**11**	**13**	15	**16**		
		10	**11**	12	14	**16**	**17**		
		11	12	**13**	15	**17**	18		
9	**11**	**13**	14	15	**17**	**19**	20	21	**23**
11	**13**	15	**16**	**17**	**19**	21	22	**23**	**25**
15	**17**							**27**	**29**
17	**19**							**29**	**31**
21	**23**							33	35
23	**25**							35	**37**

4	6							**16**	18
5	**7**							**17**	**19**
6	**8**							18	20
7	**9**							**19**	21
		19	20	21	**23**	**25**	26		
		21	22	**23**	**25**	**27**	28		
		25	26	**27**	**29**	**31**	**32**		
		27	28	**29**	**31**	33	34		

4	6							16	**18**
5	**7**							**17**	**19**
6	8							**18**	**20**
7	9							**19**	21
9	**11**	**13**	14	15	**17**	**19**	**20**	21	**23**
11	**13**	15	16	**17**	**19**	21	22	**23**	25
		19	**20**	21	**23**	25	26		
		21	22	**23**	25	27	**28**		
		25	26	27	**29**	**31**	32		
		27	**28**	**29**	**31**	33	34		

		8	9	10	**12**	14	15		
		9	10	**11**	**13**	15	16		
		10	**11**	**12**	14	16	**17**		
		11	**12**	**13**	15	**17**	**18**		
15	**17**							27	**29**
17	**19**							**29**	**31**
21	**23**							33	35
23	25							35	**37**

36 ultimates and raiseds
24 composites and mixes
3/2 ratio

← **3/2 ratio** →
← **3/2 ratio** →

24 ultimates and raiseds
16 composites and mixes
3/2 ratio

30 ultimates and mixes
30 raiseds and composites
1/1 ratio

← **3/2 ratio** →
← **3/2 ratio** →

20 ultimates and mixes
20 raiseds and composites
1/1 ratio

Fig. 68 Distribution of the 4 classes of numbers generated from the additions matrix of the 20 fundamentals (see Fig. 67). Sub-matrices inscribed in the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

11.4.2 Redeployment of the matrix of twenty fundamentals

In a redeployment of this matrix of additions of the twenty fundamentals (introduced in Figure 67) as illustrated in Figure 69, it appears that the opposition of the numbers of the first two classes of 1st and 2nd level of complexity to the numbers of the last two classes of 3rd and 4th level of complexity is also organized into transcendent ratios of value 3/2. This linear matrix has the same configuration as that introduced Chapter 7.1 in Figure 31.

Matrix to 4 times (3^2 + (3 × 2)) = 60 numbers (→ 4 times ($a^2 + ab$))

36 ultimates or raiseds **← 3/2 ratio →** 24 composites or mixes

4	6	**8**	**9**	10	12	14	15	**16**	18	**5**	**7**	**9**	10	**11**	**13**	15	**16**	**17**	**19**	6	**8**	10	**11**	12
14	**16**	**17**	18	20	**7**	**9**	**11**	12	**13**	15	**17**	18	**19**	21	**9**	**11**	**13**	14	15	**17**	**19**	20	21	**23**
11	**13**	15	**16**	**17**	**19**	21	22	**23**	**25**	15	**17**	**19**	20	21	**23**	**25**	26	**27**	**29**	**17**	**19**	21	22	**23**
25	**27**	28	**29**	**31**	21	**23**	**25**	26	**27**	**29**	**31**	**32**	33	35	**23**	**25**	**27**	28	**29**	**31**	33	34	35	**37**

24 ultimates or raiseds **← 3/2 ratio →** 16 composites or mixes

Matrix to 4 times (2^2 + (2 × 3)) = 40 numbers (→ 4 times ($b^2 + ba$))

Fig. 69 Redeployment of the matrix (see Fig. 67) of addition of the twenty fundamentals inscribed in the remarkable identity (*a* + *b*… . Distribution of classes of numbers in sets opposing in 3/2 transcendent ratios in two entangled sub-matrices of 60 versus 40 entities.

In another redeployment of this matrix of additions of the twenty fundamentals (introduced in Figure 67) as illustrated in Figure 70, it appears that the opposition of the numbers of extreme classes to the numbers of median classes is organized into 1/1 value ratios within of the two sub-matrices of 60 and 40 entities and in 3/2 value ratios transversely to these two sub-matrices.

Matrix to 4 times (3^2 + (3 × 2)) = 60 numbers (→ 4 times ($a^2 + ab$))

30 ultimates or mixes **← 1/1 ratio →** 30 raiseds or composites

4	6	8	9	10	**12**	14	15	16	**18**	**5**	**7**	9	10	**11**	**13**	15	16	**17**	**19**	6	8	10	**11**	**12**
14	16	**17**	**18**	**20**	**7**	9	**11**	**12**	**13**	15	**17**	**18**	**19**	21	9	**11**	**13**	14	15	**17**	**19**	**20**	21	**23**
11	**13**	15	16	**17**	**19**	21	22	**23**	25	15	**17**	**19**	**20**	21	**23**	25	26	27	**29**	**17**	**19**	21	22	**23**
25	27	**28**	**29**	**31**	21	**23**	25	26	27	**29**	**31**	32	33	35	**23**	25	27	**28**	**29**	**31**	33	34	35	**37**

20 ultimates or mixes **← 1/1 ratio →** 20 raiseds or composites

Matrix to 4 times (2^2 + (2 × 3)) = 40 numbers (→ 4 times ($b^2 + ba$))

Fig. 70 Redeployment of the matrix (see Fig. 67) of addition of the twenty fundamentals inscribed in the remarkable identity (*a* + *b*… .Equal distribution of **ultimates and mixes** and raiseds and composites in two entangled sub matrices of 60 versus 40 entities and opposition in 3/2 value ratios transversely to these two sub-matrices.

12 Sophie Germain ultimate numbers

Here is applied to ultimate numbers (u) the concept of safe prime numbers and numbers of Sophie Germain. As a reminder, if p and $2p + 1$ are both prime, then p is a prime number of Sophie Germain and $2p + 1$ is a safe prime number.

12.1 Concept of safe ultimate number

So we can agree that if u and $2u + 1$ are ultimate, then u is **an ultimate number of Sophie Germain** and $2u + 1$ **a safe ultimate number**.

From this convention, it follows that the two particular numbers *zero* (0) and *one* (1), recognized as ultimate since the definition of this type of number (definition introduced in Chapter 2.1), are both ultimate numbers of Sophie Germain:

0 and [(2 × 0) + 1] are both ultimates → 0 is Sophie Germain ultimate
1 and [(2 × 1) + 1] are both ultimates → 1 is Sophie Germain ultimate

12.2 Concept of safe non-ultimate number

So we can extend this concept *of safety* to non-ultimate numbers (*ʉ*) and agree that if *ʉ* and 2*ʉ* + 1 are non-ultimate, then *ʉ* is a non-ultimate number of Sophie Germain and 2*ʉ* + 1 **a safe non-ultimate number**.

12.3 Concept of fertile number

Thus, according to these new conventions and the ultimity degree of whole numbers these can only belong to one of four different types of numbers including two types of *Sophie Germain numbers* (which can be ultimates or non-ultimates) and two types of *no Sophie Germain numbers* (which can be ultimates or non-ultimates). We propose here to qualify these Sophie Germain numbers as *fertile* and these no Sophie Germain numbers as *sterile*. A whole number is therefore:

- either a fertile ultimate (*u*): 2*u* + 1 = *u'* → *u* is fertile ultimate, *u'* is safe* ultimate,
- or a sterile ultimate (*u*): 2*u* + 1 = *ʉ* → *u* is sterile ultimate, *ʉ* is unsafe* non-ultimate,
- or a fertile non-ultimate (*ʉ*): 2*ʉ* + 1 = *ʉ'* → *ʉ* is fertile non-ultimate, *ʉ'* is safe* non-ultimate,
- or a sterile non-ultimate (*ʉ*): 2*ʉ* + 1 = *u* → *ʉ* is sterile non-ultimate, *u* is unsafe* ultimate.

So it is agree that designation "fertiles" designates fertile numbers (which can be ultimates or non-ultimates) and designation "steriles" designates sterile numbers (which can be ultimates or non-ultimates).

* At the conclusion of this article, a more appropriate term will be proposed to describe the safety or non-safety of these numbers.

12.4 Security entanglement

It turns out that among the twenty fundamental numbers (concept introduced in Chapter 2.6), which are simultaneously the first 20 numbers but also the first 10 ultimates and the first ten non-ultimates, are 50% of fertile numbers of which, in a ratio of value 3/2, 6 fertile ultimates versus 4 fertile non-ultimates. As illustrated in the upper part of Figure 71, many transcendent ratios of value 3/2 (or 2/3) thus operate according to the different natures of the entities constituting this set of twenty fundamental numbers.

Also (lower part of Figure 71), the group of twenty other numbers made up of the 10 ultimates and the 10 non-ultimates directly following of the entities of the previous group of the twenty fundamentals is organized in exactly inverse ratios according to the same natures considered: fertile or sterile ultimates, fertile or sterile non-ultimates. This is all the more remarkable since, differently from the first group of the twenty fundamentals made up of the first 20 numbers (from 0 to 19), this last group is not made up of the following twenty numbers (from 20 to 40) but just by the next 10 ultimates and the next 10 non-ultimates.

The 10 + 10 first ultimates (*u*) :

10 fertiles → 2*u* + 1 → 6 + 4 safe ultimates

10 steriles → 2*u* + 1 → 4 + 6 unsafe non-ultimates

The 10 + 10 first non-ultimates (~~*u*~~) :

10 fertiles → 2~~*u*~~ + 1 → 4 + 6 safe non-ultimates

10 steriles → 2~~*u*~~ + 1 → 6 + 4 unsafe ultimates

0	→	**1**
1	→	**3**
2	→	**5**
3	→	**7**
5	→	**11**
7	→	15
11	→	**23**
13	→	27
17	→	35
19	→	39

10 fertile numbers

6 fertile ultimates **← 3/2 ratio →** **4 fertile non-ultimates**

↑ 3/2 ratio ↓ **↑ 2/3 ratio ↓**

4 sterile ultimates ← 2/3 ratio → 6 sterile non-ultimates

10 sterile numbers

4	→	**9**
6	→	13
8	→	17
9	→	19
10	→	**21**
12	→	**25**
14	→	29
15	→	31
16	→	**33**
18	→	37

↑ 3/2 ratio ↓ **↑ 2/3 ratio ↓**

↓ 2/3 ratio ↑ ↑ 3/2 ratio ↓

23	→	**47**
29	→	**59**
31	→	63
37	→	75
41	→	**83**
43	→	87
47	→	95
53	→	**107**
59	→	119
61	→	123

10 fertile numbers

4 fertile ultimates **← 2/3 ratio →** **6 fertile non-ultimates**

↑ 2/3 ratio ↓ **↑ 3/2 ratio ↓**

6 sterile ultimates ← 3/2 ratio → 4 sterile non-ultimates

10 sterile numbers

20	→	41
21	→	43
22	→	**45**
24	→	**49**
25	→	**51**
26	→	53
27	→	**55**
28	→	**57**
30	→	61
32	→	**65**

Fig. 71 Safety entanglement (of Sophie Germain) of twice 10 first ultimates and twice 10 first non-ultimates. (See also Fig. 72).

Thus, according to the concept of safety of Sophie Germain applied here as well to the ultimate numbers as to the non-ultimate numbers, we can observe a very strong entanglement between the four double groups of whole numbers made up of twice the first 10 ultimates and twice first 10 non-ultimates and, transversely, of twice the first 10 fertile or sterile ultimates and of twice first 10 fertile or sterile non-ultimates.

Figure 72, which complements the previous one, illustrates this entanglement of the safety of the first 20 ultimates and 20 first non-ultimates from another angle.

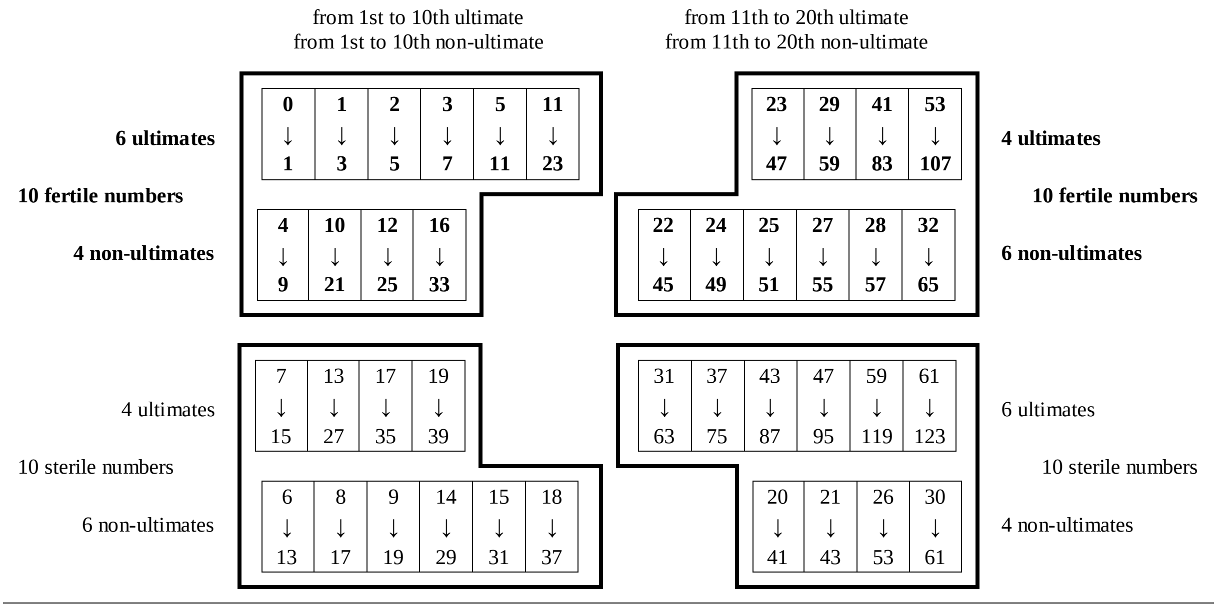


Fig. 72 Safety entanglement (of Sophie Germain) of twice 10 first ultimates and twice 10 first non-ultimates. (See also Fig. 71).

12.5 Fertility of the first hundred numbers

In this article investigating whole numbers, many arithmetic demonstrations were made inside the matrix of the first hundred numbers. The last demonstration that follows, both closes this one, but also can be like the introduction of another article yet to be published. It turns out that the distribution of fertile and non-fertile numbers (which can, recall, be both ultimate and non-ultimate, as specified in Chapter 12.3) is by no means random in this matrix of the first hundred numbers. Many arrangements of the same nature as many of those already introduced in different chapters of this study generate oppositions of entities in various value ratios 3/2 or 1/1 depending on whether these numbers are qualified as fertile or sterile.

At the conclusion of this article, presented in Figure 73, an example of these remarkable arrangements demonstrating and confirming the non-random nature of the concepts of fertility and sterility (and therefore also of the concepts of ultimity and non-ultimity) of the whole numbers as previously defined in Chapter 12.3.

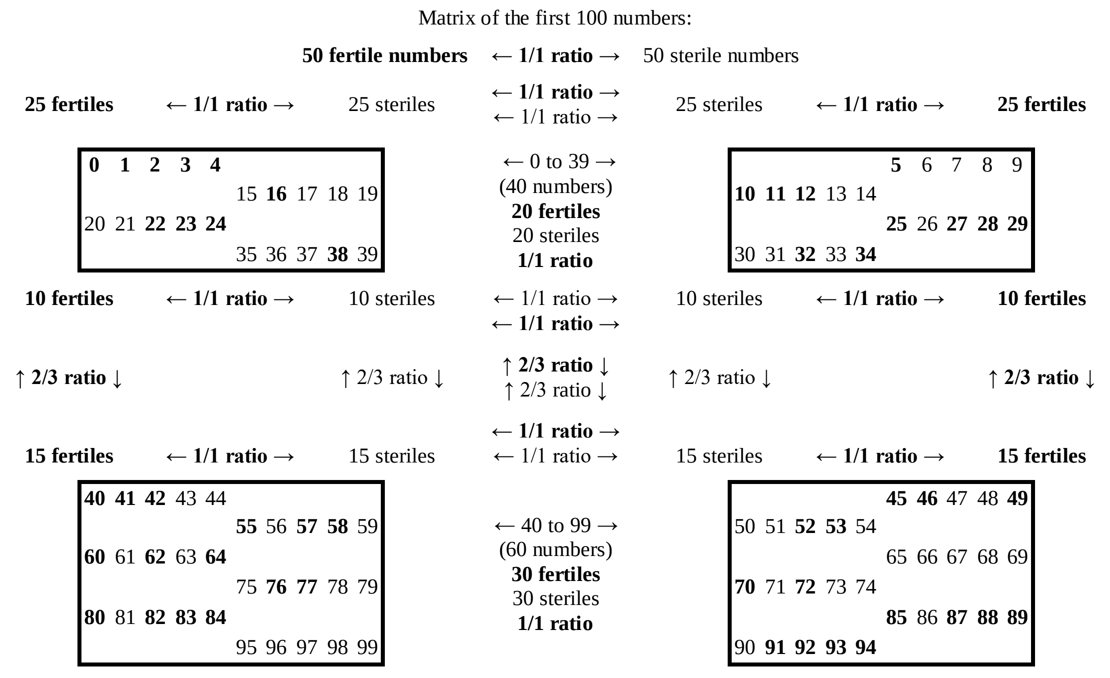


Fig. 73 Distribution of the **50 fertile numbers** and the **50 sterile numbers** in sub-matrices from the matrix of the first 100 numbers.

So, of the first 100 whole numbers, exactly 50 are fertile numbers and another 50 are sterile numbers. Also, isolation, in a 2/3 value ratio, of the first 40 numbers and the next 60, shows the same split in groups with the same amount of entities. Finally, very sophisticated intricacies of these subsets of numbers appear in the interlacing of packets of always 5 consecutive entities. These intricacies generate numerous oppositions of the groups considered in various ratios of always 3/2 or 1/1 value.

All this remarkable arithmetic mechanics (like all the other demonstrations of this study of whole numbers) manifests itself only if it is well agreed that the numbers zero (0) and one (1) are merged with the set of prime numbers by creating a new set called the set of ultimate numbers as defined in the article introduction. Without this consideration, all of the demonstrations from this study of whole numbers would be destroyed.

13 Discussions and conclusions

For the sake of clarification of the phenomena introduced, the very large quantity of new concepts proposed in this article investing the whole numbers requires the fusion of discussions and conclusions. Also, always because of the need to instantly clarify their scope, some discussions are already present in several demonstrations.

13.1 Definition of ultimates

Until now, the definition of thus called *prime numbers* did not allow the numbers zero (0) and one (1) to be included in this set of primes. Thus, the set of whole numbers was scattered in four entities: prime numbers, non-prime numbers, but also ambiguous numbers *zero* and *one* at exotic arithmetic characteristics. The double definition of ultimate and non-ultimate numbers proposed here makes it possible to properly divide the set of whole numbers into two groups of numbers with well-defined and absolute characteristics: a number is either ultimate or non-ultimate. In addition to its non-triviality, the fact of specifying the numerically lower nature of a divisor to any envisaged number effectively allows that there is no difference in status between the ultimate numbers zero (0) and one (1) and any other number described as ultimate.

13.2 Concept of ultimate divisor and ultimate algebra

The attention paid to the ultimate divisors shows that these are organized not randomly, in particular within the matrix of the first hundred numbers where one counts 5*x* (15) different but also 5*x*' (210) in total which make up the non-ultimates in this matrix of 5*x* by 5*x* (10 by 10) entities. The arrangement of their first appearance is itself non-random within this matrix and is also organized into 3/2 value ratios in the same configurations as the set of all of the ultimate divisors of the first hundred numbers.

Also and so, the notion of ultimate divisor is inseparable from that of ultimate number. Although inferior to any other number, zero (0) and one (1) do not divide any: the division by zero is not defined and one (1) does not divide a number: it does not divide it into smaller dividers. In ultimate algebra, zero (0) and one (1) do not multiply any either. For example, the absolute composites 7×0 or 7×1 do not exist in ultimate algebra: 7×0 is reduced to 0 (ultimate number) and 7×1 is reduced to 7 (other ultimate number).

13.3 The four classes of numbers

The concept and definition of an ultimate number (but also those of ultimate divisor) allows the classification of the set of whole numbers into four classes of entities with properties that are simultaneously interactive, unique and of progressive degree (from the class of ultimates) of complexity. Also, any whole number can only belong to one of these four classes and, simultaneously, must belong to one of these four classes: ultimate, raised, composite or mixed numbers. This quantity of classes, of value four, makes it possible to form ten different combinations of two numbers. This links this number of classes to the decimal system because in fact, 5 versus 5 combinations (qualified as configurations *N* and *Z*) generate two sets of 30 versus 20 pairs in the matrix of the first 100 (so 10 by 10) numbers.

13.4 The 3/2 ratio and the decimal system

3/2 ratio, this term appears hundreds of times in this article! It is always involved between and in sets of entities of 5*x* sizes (so 3*x* + 2*x*) including, in most situations, various matrices of ten by ten entities. These arithmetic phenomena demonstrate the equality of importance of the different types of entities studied as the ultimates or non-ultimates, the primordials or non-primordials, the digit numbers or non-digit numbers among the fundamentals, the numbers of extreme classes and those of median classes, fertile or sterile numbers, etc. . Thus is revealed in this article quantity of dualities distinguishing whole numbers in always pairs of subsets opposing in various ratios of exact value 3/2 or, more incidentally, of exact value 1/1.

Also, many of the phenomena presented, in addition to involving this arithmetic ratio of 3/2, revolve around the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have the values 3 and 2. This generates many entanglement in the arithmetic arrangements operating between the different entities considered and therefore strengthens their credibility by the dimensional amplification of these arithmetic phenomena.

The first ten this, the first ten that, indeed, this is really at the source of different entities sequences that it is manifested these numerous arrangements of 3*x* versus 2*x* sizes and not further downstream of these sequences. All this demonstrates intimate links between the different natures, introduced in this study, of whole numbers and the decimal system.

13.5 Diversity of fields of investigation

The phenomena revealed in this study of whole numbers invest either separately or in a transcendent way different mathematical concepts. These demonstrations, describing oppositions of entities in always to 3/2 or 1/1 value ratios, for example simultaneously involve Fibonacci sequences and Pascal triangles. Also, many phenomena operate inside various matrices with exotic but geographically symmetrical configurations. Moreover, the fact of being able, with ease, to continue these investigations in the domain of notion of number of Sophie Germain, where the concepts of ultimity and non-ultimity can be enriched, confirms the veracity of these latter concepts which are the main subjects of this article.

13.6 Concept of Sophie Germain number

Concerning the concept of Sophie Germain number, with reference to genetics, it was introduced those of fertile numbers and sterile numbers generating safe or unsafe numbers. Also we propose, in this genetic logic, to replace the terms of safe and unsafe number by *pure number* and *hybrid number* which can apply as well to the ultimates as to the non-ultimates as explained in chapter 12. So we can say that a fertile number *generates* (by the function of Sophie Germain: $2x + 1$) a pure number and that a sterile number generates a hybrid number. By this same logic, and although any number can be fertile or sterile, some, by the irreversibility of the function (even numbers) can be neither pure nor hybrid but orphans.

13.7 The fundamentals

The highlighting of the first twenty numbers qualified as fundamental finds all its legitimacy by the almost quantum entanglement of the components of this particular group of numbers. Formed by the first ten ultimates and the first ten non-ultimates but also by the ten digits and the first ten non-digit numbers, this set is a mathematical space where and from which a large number of the phenomena introduced in this article since the simple matrix of addition of its components up to the intricacies linked to the concept of numbers of Sophie Germain.

13.8 To conclude

From the various arithmetic illustrations introduced in this article, we legitimately propose a double new mathematical concept classifying whole numbers into two sets of entities with distinct well defined properties:

- the ultimates, not admitting any divisor being inferior to them,
- the non-ultimates, admitting at least one divisor being inferior to them.

Also, from this double definition, we propose to subclass whole numbers into ultimates, raiseds, composites and mixes.

Finally, we propose to enrich notions of Sophie Germain numbers (prime, safe) with this double new concept of ultimity and non-ultimity.

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