Analyzing various Ramanujan equations: mathematical connections with some Prime Numbers linked to the Supersingular Elliptic Curves, ϕ, $\zeta(2)$ and to the mass of candidate glueball f$_0$(1710) scalar meson.

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Abstract

In this paper we have described and analyzed various Ramanujan equations. We have obtained several mathematical connections between some Prime Numbers linked to the Supersingular Elliptic Curves, ϕ, $\zeta(2)$ and to the mass of candidate glueball $f_0(1710)$ scalar meson.

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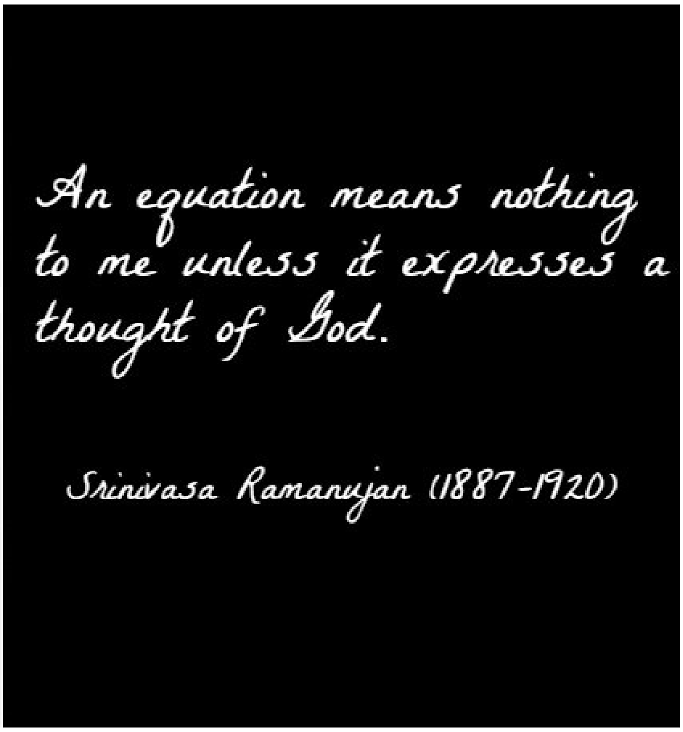
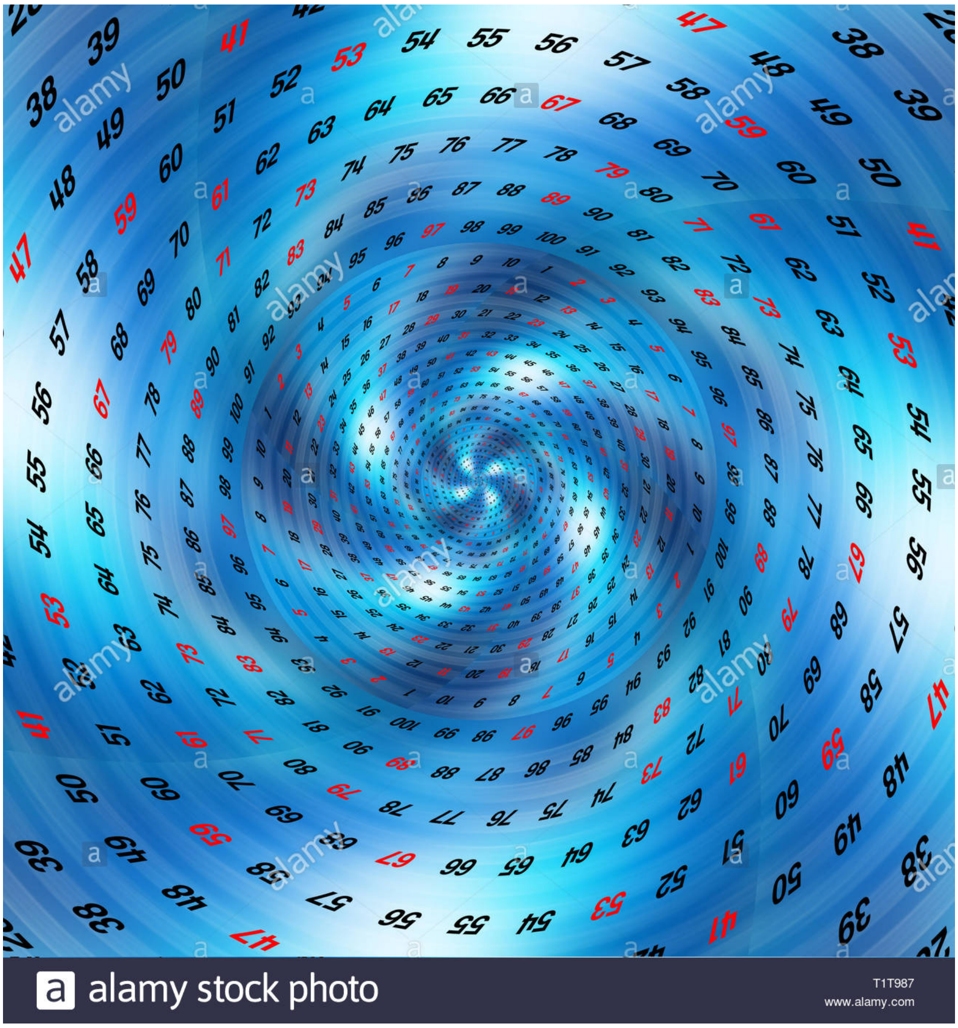


https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

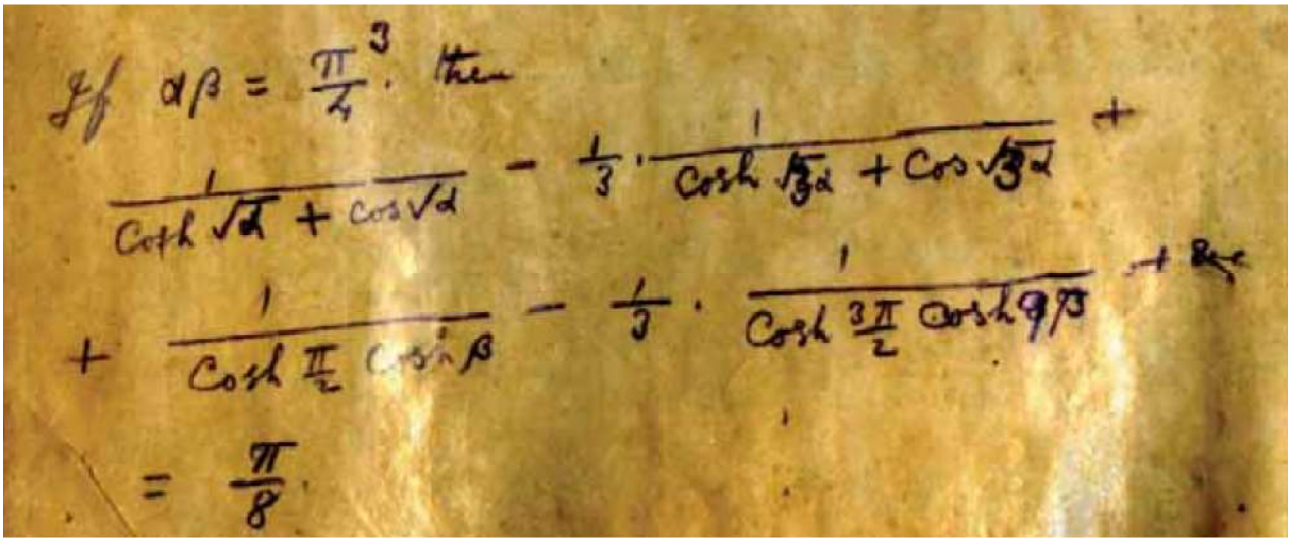


We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

Now, we take the following infinite series:

$$\sum_{n=2}^{\infty} \frac{\zeta(n)-1}{n} \operatorname{Im}\left((1+i)^n-(1+i^n)\right)=\frac{\pi}{4}$$

And the following Ramanujan expression (page 87 - **Manuscript Book I of Srinivasa Ramanujan**):



If $\alpha\beta = \frac{\pi^3}{4}$. then

$$\frac{1}{\cosh\sqrt{\alpha} + \cos\sqrt{\alpha}} - \frac{1}{3}\cdot\frac{1}{\cosh\sqrt{3}\alpha + \cos\sqrt{3}\alpha} + $$

$$+ \frac{1}{\cosh\frac{\pi}{2}\cosh\beta} - \frac{1}{3}\cdot\frac{1}{\cosh\frac{3\pi}{2}\cosh 9\beta} + \&c$$

$$= \frac{\pi}{8}.$$

We have:

((1/((cosh((sqrtPi)) + cos((sqrtPi)))))) – 1/3* ((1/((cosh(sqrt(3Pi))+cos(sqrt(3Pi)))))) + ((1/(cosh(Pi/2)*cosh((Pi^2)/4))) – ((1/3 * 1/(cosh((3Pi)/2) cosh(9*(Pi^2)/4)))))

Input:

$$\frac{1}{\cosh(\sqrt{\pi}) + \cos(\sqrt{\pi})} - \frac{1}{3}\times\frac{1}{\cosh(\sqrt{3\pi}) + \cos(\sqrt{3\pi})} + \left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)} - \frac{1}{3}\times\frac{1}{\cosh\left(\frac{3\pi}{2}\right)\cosh\left(9\times\frac{\pi^2}{4}\right)}\right)$$

$\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\operatorname{sech}\left(\frac{\pi}{2}\right)\operatorname{sech}\left(\frac{\pi^2}{4}\right) - \frac{1}{3}\operatorname{sech}\left(\frac{3\pi}{2}\right)\operatorname{sech}\left(\frac{9\pi^2}{4}\right) + \frac{1}{\cos(\sqrt{\pi}) + \cosh(\sqrt{\pi})} - \frac{1}{3(\cos(\sqrt{3\pi}) + \cosh(\sqrt{3\pi}))}$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

0.3867814004498545449072008765787832762853178989386392476 52...

0.38678140044985…. ≈ 0.392699081…≈ π /8

Alternate forms:

$$\frac{2\,e^{\pi/2}\,\mathrm{sech}\left(\frac{\pi^2}{4}\right)}{1+e^{\pi}} - \frac{1}{3}\,\mathrm{sech}\left(\frac{3\,\pi}{2}\right)\mathrm{sech}\left(\frac{9\,\pi^2}{4}\right) + \frac{2}{2\cos\left(\sqrt{\pi}\right)+2\cosh\left(\sqrt{\pi}\right)} - \frac{2}{3\left(2\cos\left(\sqrt{3\,\pi}\right)+2\cosh\left(\sqrt{3\,\pi}\right)\right)}$$

$$\frac{4\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}{(1+\cosh(\pi))\left(1+\cosh\left(\frac{\pi^2}{2}\right)\right)} - \frac{4\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)}{3\,(1+\cosh(3\,\pi))\left(1+\cosh\left(\frac{9\pi^2}{2}\right)\right)} + \frac{1}{\cos\left(\sqrt{\pi}\right)+\cosh\left(\sqrt{\pi}\right)} - \frac{1}{3\left(\cos\left(\sqrt{3\,\pi}\right)+\cosh\left(\sqrt{3\,\pi}\right)\right)}$$

$$\frac{4}{\left(e^{-\pi/2}+e^{\pi/2}\right)\left(e^{-\pi^2/4}+e^{\pi^2/4}\right)} - \frac{4}{3\left(e^{-(3\,\pi)/2}+e^{(3\,\pi)/2}\right)\left(e^{-(9\,\pi^2)/4}+e^{(9\,\pi^2)/4}\right)} + \frac{1}{\frac{1}{2}\left(e^{-\sqrt{\pi}}+e^{\sqrt{\pi}}\right)+\frac{1}{2}\left(e^{-i\sqrt{\pi}}+e^{i\sqrt{\pi}}\right)} - \frac{1}{3\left(\frac{1}{2}\left(e^{-\sqrt{3\,\pi}}+e^{\sqrt{3\,\pi}}\right)+\frac{1}{2}\left(e^{-i\sqrt{3\,\pi}}+e^{i\sqrt{3\,\pi}}\right)\right)}$$

Alternative representations:

$$\frac{1}{\cosh\left(\sqrt{\pi}\right)+\cos\left(\sqrt{\pi}\right)} - \frac{1}{\left(\cosh\left(\sqrt{3\,\pi}\right)+\cos\left(\sqrt{3\,\pi}\right)\right)3} + \left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)} - \frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right) = \frac{1}{\cos\left(\frac{i\pi}{2}\right)\cos\left(\frac{i\pi^2}{4}\right)} - \frac{1}{3\left(\cos\left(\frac{3i\pi}{2}\right)\cos\left(\frac{9i\pi^2}{4}\right)\right)} + \frac{1}{\cosh\left(-i\sqrt{\pi}\right)+\cos\left(i\sqrt{\pi}\right)} - \frac{1}{3\left(\cosh\left(-i\sqrt{3\,\pi}\right)+\cos\left(i\sqrt{3\,\pi}\right)\right)}$$

$$\frac{1}{\cosh\left(\sqrt{\pi}\right)+\cos\left(\sqrt{\pi}\right)} - \frac{1}{\left(\cosh\left(\sqrt{3\,\pi}\right)+\cos\left(\sqrt{3\,\pi}\right)\right)3} + \left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)} - \frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right) = \frac{1}{\cos\left(-\frac{i\pi}{2}\right)\cos\left(-\frac{i\pi^2}{4}\right)} - \frac{1}{3\left(\cos\left(-\frac{3i\pi}{2}\right)\cos\left(-\frac{9i\pi^2}{4}\right)\right)} + \frac{1}{\cosh\left(-i\sqrt{\pi}\right)+\cos\left(-i\sqrt{\pi}\right)} - \frac{1}{3\left(\cosh\left(-i\sqrt{3\,\pi}\right)+\cos\left(-i\sqrt{3\,\pi}\right)\right)}$$

$$\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{(\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi}))3}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right)=$$
$$\frac{1}{\cos\left(-\frac{i\pi}{2}\right)\cos\left(-\frac{i\pi^2}{4}\right)}-\frac{1}{3\left(\cos\left(-\frac{3i\pi}{2}\right)\cos\left(-\frac{9i\pi^2}{4}\right)\right)}+$$
$$\frac{1}{\cosh(i\sqrt{\pi})+\cos(-i\sqrt{\pi})}-\frac{1}{3(\cosh(i\sqrt{3\pi})+\cos(-i\sqrt{3\pi}))}$$

Series representations:

$$\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{(\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi}))3}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right)=\operatorname{sech}\left(\frac{\pi}{2}\right)\operatorname{sech}\left(\frac{\pi^2}{4}\right)-$$
$$\frac{1}{3}\operatorname{sech}\left(\frac{3\pi}{2}\right)\operatorname{sech}\left(\frac{9\pi^2}{4}\right)-\frac{1}{3\left(\sum_{k=0}^{\infty}\frac{(-3\pi)^k}{(2k)!}+\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{\left(-\frac{3\pi}{4}\right)^{-s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}+$$
$$\frac{1}{\sum_{k=0}^{\infty}\frac{(-\pi)^k}{(2k)!}+\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{\left(-\frac{\pi}{4}\right)^{-s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}}$$

$$\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{(\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi}))3}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right)=$$
$$\left(-\sum_{k=0}^{\infty}\frac{(-\pi)^k+\pi^k}{(2k)!}+3\sum_{k=0}^{\infty}\frac{(-3\pi)^k+(3\pi)^k}{(2k)!}+6\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\sum_{k_4=0}^{\infty}\right.$$
$$\frac{(-1)^{k_1+k_2}\left(\frac{(-\pi)^{k_3}}{(2k_3)!}+\frac{\pi^{k_3}}{(2k_3)!}\right)\left(\frac{(-3\pi)^{k_4}}{(2k_4)!}+\frac{(3\pi)^{k_4}}{(2k_4)!}\right)(1+2k_1)(1+2k_2)}{(1+2k_1+2k_1^2)\left(\frac{\pi^4}{16}+\pi^2\left(\frac{1}{2}+k_2\right)^2\right)}-$$
$$2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\sum_{k_4=0}^{\infty}$$
$$\left.\frac{(-1)^{k_1+k_2}\left(\frac{(-\pi)^{k_3}}{(2k_3)!}+\frac{\pi^{k_3}}{(2k_3)!}\right)\left(\frac{(-3\pi)^{k_4}}{(2k_4)!}+\frac{(3\pi)^{k_4}}{(2k_4)!}\right)(1+2k_1)(1+2k_2)}{(5+2k_1+2k_1^2)\left(\frac{81\pi^4}{16}+\pi^2\left(\frac{1}{2}+k_2\right)^2\right)}\right)/$$
$$\left(3\left(\sum_{k=0}^{\infty}\frac{(-\pi)^k+\pi^k}{(2k)!}\right)\sum_{k=0}^{\infty}\frac{(-3\pi)^k+(3\pi)^k}{(2k)!}\right)$$

$$\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{(\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi}))\,3}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right)=\operatorname{sech}\left(\frac{\pi}{2}\right)\operatorname{sech}\left(\frac{\pi^2}{4}\right)-\frac{1}{3}\operatorname{sech}\left(\frac{3\pi}{2}\right)\operatorname{sech}\left(\frac{9\pi^2}{4}\right)-\frac{1}{3\sqrt{\pi}\sum_{j=0}^{\infty}\left(\operatorname{Res}_{s=-j}\frac{\left(\frac{4}{3\pi}\right)^s\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}+\operatorname{Res}_{s=-j}\frac{\left(-\frac{3\pi}{4}\right)^{-s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}+\frac{1}{\sqrt{\pi}\sum_{j=0}^{\infty}\left(\operatorname{Res}_{s=-j}\frac{\left(\frac{4}{\pi}\right)^s\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}+\operatorname{Res}_{s=-j}\frac{\left(-\frac{\pi}{4}\right)^{-s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}$$

Integral representations:

$$\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{(\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi}))\,3}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right)=\frac{4\left(\int_0^\infty\frac{t^i}{1+t^2}\,dt\right)\int_0^\infty\frac{t^{(i\pi)/2}}{1+t^2}\,dt}{\pi^2}-\frac{4\left(\int_0^\infty\frac{t^{3i}}{1+t^2}\,dt\right)\int_0^\infty\frac{t^{(9i\pi)/2}}{1+t^2}\,dt}{3\pi^2}+\frac{1}{-\frac{i}{2\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi/(4s)+s}}{\sqrt{s}}\,ds-\int_{\frac{\pi}{2}}^{\sqrt{\pi}}\sin(t)\,dt}-\frac{1}{3\left(-\frac{i}{2\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{(3\pi)/(4s)+s}}{\sqrt{s}}\,ds-\int_{\frac{\pi}{2}}^{\sqrt{3\pi}}\sin(t)\,dt\right)}\quad\text{for }\gamma>0$$

$$\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{(\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi}))\,3}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right)=\frac{4\left(\int_0^\infty\frac{t^i}{1+t^2}\,dt\right)\int_0^\infty\frac{t^{(i\pi)/2}}{1+t^2}\,dt}{\pi^2}-\frac{4\left(\int_0^\infty\frac{t^{3i}}{1+t^2}\,dt\right)\int_0^\infty\frac{t^{(9i\pi)/2}}{1+t^2}\,dt}{3\pi^2}+\frac{1}{-\frac{i}{2\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi/(4s)+s}}{\sqrt{s}}\,ds+\int_{\frac{i\pi}{2}}^{\sqrt{\pi}}\sinh(t)\,dt}-\frac{1}{3\left(-\frac{i}{2\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(3\pi)/(4s)+s}}{\sqrt{s}}\,ds+\int_{\frac{i\pi}{2}}^{\sqrt{3\pi}}\sinh(t)\,dt\right)}\quad\text{for }\gamma>0$$

$$\frac{1}{\cosh(\sqrt{\pi}) + \cos(\sqrt{\pi})} - \frac{1}{(\cosh(\sqrt{3\pi}) + \cos(\sqrt{3\pi}))\,3} + \left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)} - \frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right) =$$

$$\frac{4\left(\int_0^\infty \frac{t^i}{1+t^2}\,dt\right)\int_0^\infty \frac{t^{(i\pi)/2}}{1+t^2}\,dt}{\pi^2} - \frac{4\left(\int_0^\infty \frac{t^{3i}}{1+t^2}\,dt\right)\int_0^\infty \frac{t^{(9i\pi)/2}}{1+t^2}\,dt}{3\pi^2} +$$
$$\frac{1}{1 - \frac{i}{2\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi/(4s)+s}}{\sqrt{s}}\,ds + \sqrt{\pi}\int_0^1 \sinh(\sqrt{\pi}\,t)\,dt} -$$
$$\frac{1}{3\left(1 - \frac{i}{2\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(3\pi)/(4s)+s}}{\sqrt{s}}\,ds + \sqrt{3\pi}\int_0^1 \sinh(\sqrt{3\pi}\,t)\,dt\right)} \quad \text{for } \gamma > 0$$

from

$$\sum_{n=2}^{\infty} \frac{\zeta(n)-1}{n}\,\mathrm{Im}\left((1+i)^n - (1+i^n)\right) = \frac{\pi}{4}$$

Sum((zeta(n)-1)/n) Im((1+i)^n-(1+i^n)), n=2..6

Sum:

$$\sum_{n=2}^{6} \frac{\mathrm{Im}((1+i)^n - (1+i^n))\,(\zeta(n)-1)}{n} = \zeta(3) - \zeta(5) + \frac{1}{3} + \frac{\pi^2}{6} - \frac{4\pi^6}{2835}$$

$\zeta(s)$ is the Riemann zeta function

$\mathrm{Im}(z)$ is the imaginary part of z

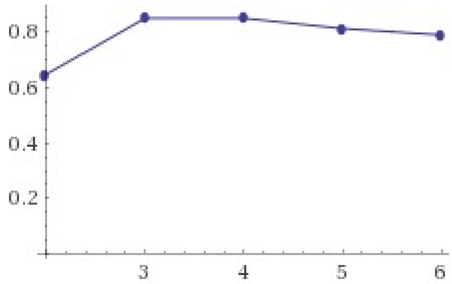
i is the imaginary unit

Decimal approximation:

0.7869391322185186092547639353125469747244319540015797584 30...

0.7869391322185…..... ≈ 0.78539816… = π/4

Partial sums:



Alternate forms:

$$\frac{5670\,\zeta(3) - 5670\,\zeta(5) + 1890 + 945\,\pi^2 - 8\,\pi^6}{5670}$$

$$\frac{5670\,\zeta(3) + 1890 + 945\,\pi^2 - 8\,\pi^6}{5670} - \zeta(5)$$

$$\frac{1890\,(3\,\zeta(3) - 3\,\zeta(5) + 1) + 945\,\pi^2 - 8\,\pi^6}{5670}$$

From which:

1/2((Sum((zeta(n)-1)/n) Im((1+i)^n-(1+i^n)), n=2..6))

Input interpretation:

$$\frac{1}{2}\sum_{n=2}^{6}\frac{\zeta(n)-1}{n}\,\mathrm{Im}\left((1+i)^n - (1+i^n)\right)$$

$\zeta(s)$ is the Riemann zeta function

$\mathrm{Im}(z)$ is the imaginary part of z

i is the imaginary unit

Result:

$$\frac{1}{2}\left(\zeta(3) - \zeta(5) + \frac{1}{3} + \frac{\pi^2}{6} - \frac{4\,\pi^6}{2835}\right) \approx 0.39347$$

0.39347… ≈ 0.392699081 = π/8

Alternate forms:

$$\frac{5670\,\zeta(3) - 5670\,\zeta(5) + 1890 + 945\,\pi^2 - 8\,\pi^6}{11\,340}$$

$$\frac{\zeta(3)}{2} - \frac{\zeta(5)}{2} + \frac{1}{6} + \frac{\pi^2}{12} - \frac{2\,\pi^6}{2835}$$

$$\frac{5670\,\zeta(3)+1890+945\,\pi^2-8\,\pi^6}{11\,340}-\frac{\zeta(5)}{2}$$

Further:

2[(((((1/((cosh((sqrtPi)) + cos((sqrtPi)))))) – 1/3* ((1/((cosh(sqrt(3Pi))+cos(sqrt(3Pi)))))) + ((1/(cosh(Pi/2)*cosh((Pi^2)/4))) – ((1/3 * 1/(cosh((3Pi)/2) cosh(9*(Pi^2)/4))))))))]

Input:

$$2\left(\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{3}\times\frac{1}{\cosh(\sqrt{3\,\pi})+\cos(\sqrt{3\,\pi})}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3}\times\frac{1}{\cosh\left(\frac{3\,\pi}{2}\right)\cosh\left(9\times\frac{\pi^2}{4}\right)}\right)\right)$$

$\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$2\left(\operatorname{sech}\left(\frac{\pi}{2}\right)\operatorname{sech}\left(\frac{\pi^2}{4}\right)-\frac{1}{3}\operatorname{sech}\left(\frac{3\,\pi}{2}\right)\operatorname{sech}\left(\frac{9\,\pi^2}{4}\right)+\frac{1}{\cos(\sqrt{\pi})+\cosh(\sqrt{\pi})}-\frac{1}{3\,(\cos(\sqrt{3\,\pi})+\cosh(\sqrt{3\,\pi}))}\right)$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

0.7735628008997090898144017531575665525706357978772784953005...

0.7735628008... ≈ 0.78539816… = π/4

Alternate forms:

$$\frac{4\,e^{\pi/2}\operatorname{sech}\left(\frac{\pi^2}{4}\right)}{1+e^{\pi}}-\frac{2}{3}\operatorname{sech}\left(\frac{3\,\pi}{2}\right)\operatorname{sech}\left(\frac{9\,\pi^2}{4}\right)+\frac{4}{2\cos(\sqrt{\pi})+2\cosh(\sqrt{\pi})}-\frac{4}{3\,(2\cos(\sqrt{3\,\pi})+2\cosh(\sqrt{3\,\pi}))}$$

$$\frac{8\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}{(1+\cosh(\pi))\left(1+\cosh\left(\frac{\pi^2}{2}\right)\right)}-\frac{8\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)}{3\,(1+\cosh(3\pi))\left(1+\cosh\left(\frac{9\pi^2}{2}\right)\right)}+\frac{2}{\cos(\sqrt{\pi})+\cosh(\sqrt{\pi})}-\frac{2}{3\,(\cos(\sqrt{3\pi})+\cosh(\sqrt{3\pi}))}$$

$$\frac{8}{\left(e^{-\pi/2}+e^{\pi/2}\right)\left(e^{-\pi^2/4}+e^{\pi^2/4}\right)}-\frac{8}{3\left(e^{-(3\pi)/2}+e^{(3\pi)/2}\right)\left(e^{-(9\pi^2)/4}+e^{(9\pi^2)/4}\right)}+\frac{2}{\frac{1}{2}\left(e^{-\sqrt{\pi}}+e^{\sqrt{\pi}}\right)+\frac{1}{2}\left(e^{-i\sqrt{\pi}}+e^{i\sqrt{\pi}}\right)}-\frac{2}{3\left(\frac{1}{2}\left(e^{-\sqrt{3\pi}}+e^{\sqrt{3\pi}}\right)+\frac{1}{2}\left(e^{-i\sqrt{3\pi}}+e^{i\sqrt{3\pi}}\right)\right)}$$

Alternative representations:

$$2\left(\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{(\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi}))\,3}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right)\right)=2\left(\frac{1}{\cos\left(\frac{i\pi}{2}\right)\cos\left(\frac{i\pi^2}{4}\right)}-\frac{1}{3\left(\cos\left(\frac{3i\pi}{2}\right)\cos\left(\frac{9i\pi^2}{4}\right)\right)}+\frac{1}{\cosh(-i\sqrt{\pi})+\cos(i\sqrt{\pi})}-\frac{1}{3\,(\cosh(-i\sqrt{3\pi})+\cos(i\sqrt{3\pi}))}\right)$$

$$2\left(\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{(\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi}))\,3}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right)\right)=2\left(\frac{1}{\cos\left(-\frac{i\pi}{2}\right)\cos\left(-\frac{i\pi^2}{4}\right)}-\frac{1}{3\left(\cos\left(-\frac{3i\pi}{2}\right)\cos\left(-\frac{9i\pi^2}{4}\right)\right)}+\frac{1}{\cosh(-i\sqrt{\pi})+\cos(-i\sqrt{\pi})}-\frac{1}{3\,(\cosh(-i\sqrt{3\pi})+\cos(-i\sqrt{3\pi}))}\right)$$

$$2\left(\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{(\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi}))\,3}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right)\right)=$$
$$2\left(\frac{1}{\cos\left(-\frac{i\pi}{2}\right)\cos\left(-\frac{i\pi^2}{4}\right)}-\frac{1}{3\left(\cos\left(-\frac{3i\pi}{2}\right)\cos\left(-\frac{9i\pi^2}{4}\right)\right)}+\frac{1}{\cosh(i\sqrt{\pi})+\cos(-i\sqrt{\pi})}-\frac{1}{3\left(\cosh(i\sqrt{3\pi})+\cos(-i\sqrt{3\pi})\right)}\right)$$

Series representations:

$$2\left(\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{(\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi}))\,3}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right)\right)=$$
$$2\,\mathrm{sech}\left(\frac{\pi}{2}\right)\mathrm{sech}\left(\frac{\pi^2}{4}\right)-\frac{2}{3}\,\mathrm{sech}\left(\frac{3\pi}{2}\right)\mathrm{sech}\left(\frac{9\pi^2}{4}\right)-\frac{2}{3\left(\sum_{k=0}^{\infty}\frac{(-3\pi)^k}{(2k)!}+\sqrt{\pi}\sum_{j=0}^{\infty}\mathrm{Res}_{s=-j}\frac{\left(-\frac{3\pi}{4}\right)^{-s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}+\frac{2}{\sum_{k=0}^{\infty}\frac{(-\pi)^k}{(2k)!}+\sqrt{\pi}\sum_{j=0}^{\infty}\mathrm{Res}_{s=-j}\frac{\left(-\frac{\pi}{4}\right)^{-s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}}$$

$$2\left(\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{(\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi}))\,3}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right)\right)=$$
$$2\,\mathrm{sech}\left(\frac{\pi}{2}\right)\mathrm{sech}\left(\frac{\pi^2}{4}\right)-\frac{2}{3}\,\mathrm{sech}\left(\frac{3\pi}{2}\right)\mathrm{sech}\left(\frac{9\pi^2}{4}\right)-\frac{2}{3\sqrt{\pi}\sum_{j=0}^{\infty}\left(\mathrm{Res}_{s=-j}\frac{\left(\frac{4}{3\pi}\right)^{s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}+\mathrm{Res}_{s=-j}\frac{\left(-\frac{3\pi}{4}\right)^{-s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}+\frac{2}{\sqrt{\pi}\sum_{j=0}^{\infty}\left(\mathrm{Res}_{s=-j}\frac{\left(\frac{4}{\pi}\right)^{s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}+\mathrm{Res}_{s=-j}\frac{\left(-\frac{\pi}{4}\right)^{-s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}$$

$$2\left(\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{(\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi}))\,3}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3\left(\cosh\left(\frac{3\pi}{2}\right)\cosh\left(\frac{9\pi^2}{4}\right)\right)}\right)\right)=$$

$$\left(2\left(-\sum_{k=0}^{\infty}\frac{(-\pi)^k+\pi^k}{(2k)!}+3\sum_{k=0}^{\infty}\frac{(-3\pi)^k+(3\pi)^k}{(2k)!}+3\pi^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\sum_{k_4=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(\frac{(-\pi)^{k_3}}{(2k_3)!}+\frac{\pi^{k_3}}{(2k_3)!}\right)\left(\frac{(-3\pi)^{k_4}}{(2k_4)!}+\frac{(3\pi)^{k_4}}{(2k_4)!}\right)(1+2k_1)(1+2k_2)}{\left(\frac{\pi^2}{4}+\pi^2\left(\frac{1}{2}+k_1\right)^2\right)\left(\frac{\pi^4}{16}+\pi^2\left(\frac{1}{2}+k_2\right)^2\right)}-\pi^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\sum_{k_4=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(\frac{(-\pi)^{k_3}}{(2k_3)!}+\frac{\pi^{k_3}}{(2k_3)!}\right)\left(\frac{(-3\pi)^{k_4}}{(2k_4)!}+\frac{(3\pi)^{k_4}}{(2k_4)!}\right)(1+2k_1)(1+2k_2)}{\left(\frac{9\pi^2}{4}+\pi^2\left(\frac{1}{2}+k_1\right)^2\right)\left(\frac{81\pi^4}{16}+\pi^2\left(\frac{1}{2}+k_2\right)^2\right)}\right)\right)\Bigg/\left(3\left(\sum_{k=0}^{\infty}\frac{(-\pi)^k+\pi^k}{(2k)!}\right)\sum_{k=0}^{\infty}\frac{(-3\pi)^k+(3\pi)^k}{(2k)!}\right)$$

$n!$ is the factorial function

We have also:

1+1/ (((4[((((((1/((cosh((sqrtPi)) + cos((sqrtPi)))))) -1/3 ((1/((cosh(sqrt(3Pi))+cos(sqrt(3Pi))))))+((1/(cosh(Pi/2)*cosh((Pi^2)/4)))-((1/3 1/(cosh((3Pi)/2) cosh(9*(Pi^2)/4))))))))])))

Input:

$$1+\frac{1}{4\left(\frac{1}{\cosh(\sqrt{\pi})+\cos(\sqrt{\pi})}-\frac{1}{3}\times\frac{1}{\cosh(\sqrt{3\pi})+\cos(\sqrt{3\pi})}+\left(\frac{1}{\cosh\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi^2}{4}\right)}-\frac{1}{3}\times\frac{1}{\cosh\left(\frac{3\pi}{2}\right)\cosh\left(9\times\frac{\pi^2}{4}\right)}\right)\right)}$$

$\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$1+1\Bigg/\left(4\left(\operatorname{sech}\left(\frac{\pi}{2}\right)\operatorname{sech}\left(\frac{\pi^2}{4}\right)-\frac{1}{3}\operatorname{sech}\left(\frac{3\pi}{2}\right)\operatorname{sech}\left(\frac{9\pi^2}{4}\right)+\frac{1}{\cos(\sqrt{\pi})+\cosh(\sqrt{\pi})}-\frac{1}{3(\cos(\sqrt{3\pi})+\cosh(\sqrt{3\pi}))}\right)\right)$$

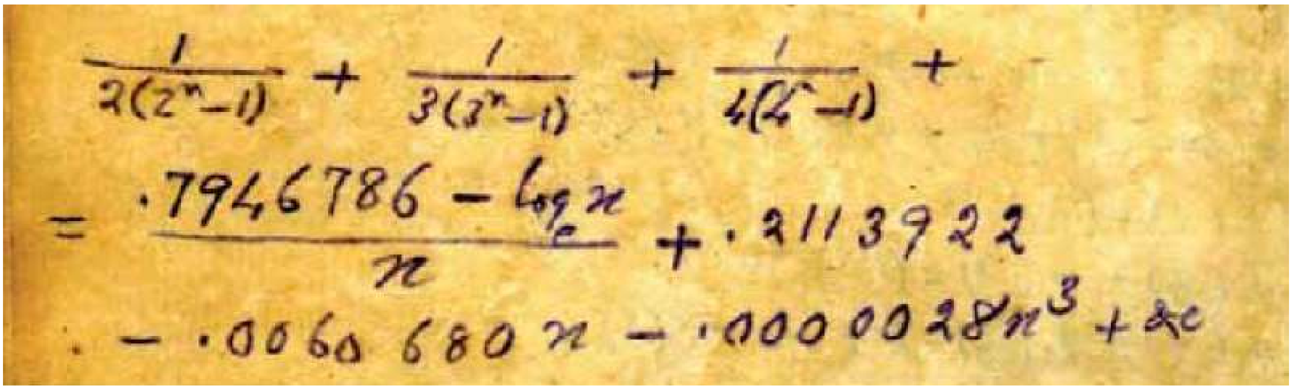
sech(x) is the hyperbolic secant function

Decimal approximation:

1.6463599327920940525774082868470066049842047980573776615284...

1.646359932792094….. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

Now, we have that (page 99):



For n = 2, we obtain:

1/(2(4-1)) + 1/(3(9-1)) + 1/(4(16-1)) = 1/x((0.7946786 – ln(x)))+0.2113922-0.0060680x-0.0000028x^3

Input interpretation:

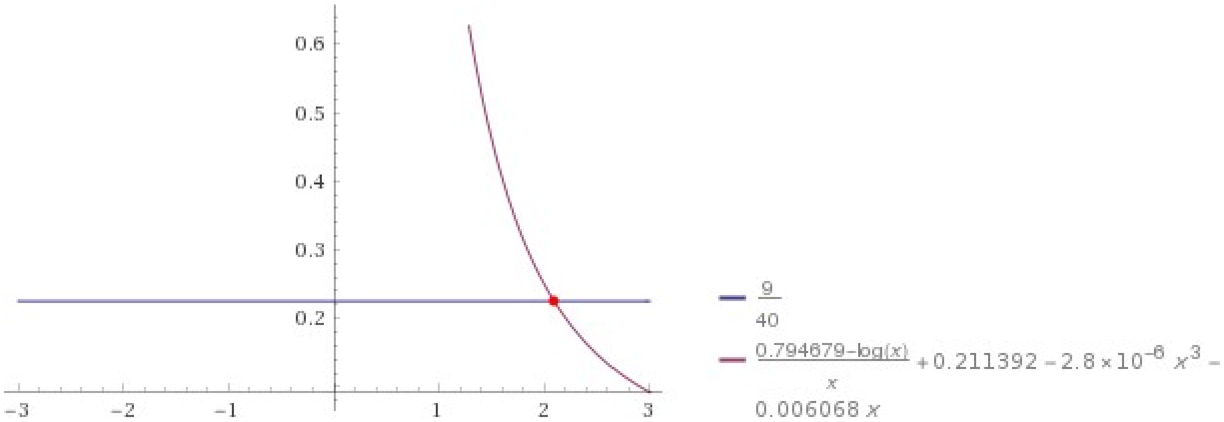
$$\frac{1}{2(4-1)} + \frac{1}{3(9-1)} + \frac{1}{4(16-1)} = \frac{1}{x}(0.7946786 - \log(x)) + 0.2113922 - 0.006068\,x - 2.8\times 10^{-6}\,x^3$$

log(x) is the natural logarithm

Result:

$$\frac{9}{40} = \frac{0.794679 - \log(x)}{x} + 0.211392 - 2.8\times 10^{-6}\,x^3 - 0.006068\,x$$

Plot:



Alternate form assuming x is real:

$$0.0136078 + 2.8\times10^{-6}\,x^3 + 0.006068\,x + \frac{\log(x) - 0.794679}{x} = 0$$

Alternate forms:

$$\frac{9}{40} = \frac{0.794679 - \log(x)}{x} + 0.211392 - 2.8\times10^{-6}\,x^3 - 0.006068\,x$$

$$\frac{9}{40} = -\frac{2.8\times10^{-6}\,(x^4 + 2167.14\,x^2 - 75\,497.2\,x + 357\,143.\,\log(x) - 283\,814.)}{x}$$

$$\frac{9}{40} = -\frac{2.8\times10^{-6}\,(x - 28.6094)\,(x + 3.42142)\,(x^2 + 25.188\,x + 2899.46)}{x} - \frac{\log(x)}{x}$$

Alternate form assuming x is positive:

$$x\,(x^3 + 2167.14\,x + 4859.93) + 357\,143.\,\log(x) = 283\,814.$$

Expanded form:

$$\frac{9}{40} = 0.211392 - 2.8\times10^{-6}\,x^3 - 0.006068\,x + \frac{0.794679}{x} - \frac{\log(x)}{x}$$

Solution:

$x = 2.09486$

2.09486

Numerical solution:

$x \approx 2.09486478400017\ldots$

Indeed:

1/(2(4-1)) + 1/(3(9-1)) + 1/(4(16-1))

Input:

$$\frac{1}{2(4-1)}+\frac{1}{3(9-1)}+\frac{1}{4(16-1)}$$

Exact result:

$$\frac{9}{40}$$

Decimal form:

0.225

0.225

1/(2.09486)((0.7946786 – ln(2.09486)))+0.2113922-0.0060680(2.09486)-0.0000028(2.09486)^3

Input interpretation:

$$\frac{1}{2.09486}(0.7946786-\log(2.09486))+ \\ 0.2113922+0.006068\times(-2.09486)-2.8\times10^{-6}\times2.09486^3$$

$\log(x)$ is the natural logarithm

Result:

0.225001...

0.225001…

Alternative representations:

$$\frac{0.794679-\log(2.09486)}{2.09486}+0.211392-0.006068\times2.09486-2.8\times10^{-6}\times2.09486^3 = \\ 0.198681+\frac{0.794679-\log_e(2.09486)}{2.09486}-2.8\times10^{-6}\times2.09486^3$$

$$\frac{0.794679 - \log(2.09486)}{2.09486} + 0.211392 - 0.006068 \times 2.09486 - 2.8 \times 10^{-6} \times 2.09486^3 = 0.198681 + \frac{0.794679 - \log(a)\log_a(2.09486)}{2.09486} - 2.8 \times 10^{-6} \times 2.09486^3$$

$$\frac{0.794679 - \log(2.09486)}{2.09486} + 0.211392 - 0.006068 \times 2.09486 - 2.8 \times 10^{-6} \times 2.09486^3 = 0.198681 + \frac{0.794679 + \mathrm{Li}_1(-1.09486)}{2.09486} - 2.8 \times 10^{-6} \times 2.09486^3$$

Series representations:

$$\frac{0.794679 - \log(2.09486)}{2.09486} + 0.211392 - 0.006068 \times 2.09486 - 2.8 \times 10^{-6} \times 2.09486^3 = 0.578002 - 0.477359 \log(1.09486) + 0.477359 \sum_{k=1}^{\infty} \frac{(-1)^k e^{-0.0906265 k}}{k}$$

$$\frac{0.794679 - \log(2.09486)}{2.09486} + 0.211392 - 0.006068 \times 2.09486 - 2.8 \times 10^{-6} \times 2.09486^3 = 0.578002 - 0.954718 i \pi \left\lfloor \frac{\arg(2.09486 - x)}{2\pi} \right\rfloor - 0.477359 \log(x) + 0.477359 \sum_{k=1}^{\infty} \frac{(-1)^k (2.09486 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\frac{0.794679 - \log(2.09486)}{2.09486} + 0.211392 - 0.006068 \times 2.09486 - 2.8 \times 10^{-6} \times 2.09486^3 = 0.578002 - 0.477359 \left\lfloor \frac{\arg(2.09486 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - 0.477359 \log(z_0) - 0.477359 \left\lfloor \frac{\arg(2.09486 - z_0)}{2\pi} \right\rfloor \log(z_0) + 0.477359 \sum_{k=1}^{\infty} \frac{(-1)^k (2.09486 - z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$\frac{0.794679 - \log(2.09486)}{2.09486} + 0.211392 - 0.006068 \times 2.09486 - 2.8 \times 10^{-6} \times 2.09486^3 = 0.578002 - 0.477359 \int_1^{2.09486} \frac{1}{t}\, dt$$

$$\frac{0.794679 - \log(2.09486)}{2.09486} + 0.211392 - 0.006068 \times 2.09486 - 2.8 \times 10^{-6} \times 2.09486^3 = 0.578002 - \frac{0.238679}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-0.0906265 s}\, \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds \quad \text{for } -1 < \gamma < 0$$

Where 0.225… ≈ 0.2243994… = π / 14

We have also:

1+1/ (((7(((1/(2.09486)((0.7946786 – ln(2.09486)))+0.2113922-0.0060680(2.09486)-0.0000028(2.09486)^3))))))

Input interpretation:

$$1+1/\left(7\left(\frac{1}{2.09486}(0.7946786-\log(2.09486))+0.2113922+0.006068\times(-2.09486)-2.8\times10^{-6}\times2.09486^3\right)\right)$$

log(x) is the natural logarithm

Result:

1.634917306526777776856967122540683784613892857828712931714...

1.63491730652… ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

Alternative representations:

$$1+\frac{1}{7\left(\frac{0.794679-\log(2.09486)}{2.09486}+0.211392-0.006068\times2.09486-2.8\times10^{-6}\times2.09486^3\right)}=1+\frac{1}{7\left(0.198681+\frac{0.794679-\log_e(2.09486)}{2.09486}-2.8\times10^{-6}\times2.09486^3\right)}$$

$$1+\frac{1}{7\left(\frac{0.794679-\log(2.09486)}{2.09486}+0.211392-0.006068\times2.09486-2.8\times10^{-6}\times2.09486^3\right)}=1+\frac{1}{7\left(0.198681+\frac{0.794679+\mathrm{Li}_1(-1.09486)}{2.09486}-2.8\times10^{-6}\times2.09486^3\right)}$$

$$1+\frac{1}{7\left(\frac{0.794679-\log(2.09486)}{2.09486}+0.211392-0.006068\times2.09486-2.8\times10^{-6}\times2.09486^3\right)}=1+\frac{1}{7\left(0.198681+\frac{0.794679-\log(a)\log_a(2.09486)}{2.09486}-2.8\times10^{-6}\times2.09486^3\right)}$$

Series representations:

$$1+\frac{1}{7\left(\frac{0.794679-\log(2.09486)}{2.09486}+0.211392-0.006068\times 2.09486-2.8\times 10^{-6}\times 2.09486^3\right)} = 1-\frac{0.299266}{-1.21083+\log(1.09486)-\sum_{k=1}^{\infty}\frac{(-1)^k e^{-0.0906265\,k}}{k}}$$

$$1+\frac{1}{7\left(\frac{0.794679-\log(2.09486)}{2.09486}+0.211392-0.006068\times 2.09486-2.8\times 10^{-6}\times 2.09486^3\right)} = 1-\frac{0.149633}{-0.605416+i\pi\left\lfloor\frac{\arg(2.09486-x)}{2\pi}\right\rfloor+0.5\log(x)-0.5\sum_{k=1}^{\infty}\frac{(-1)^k(2.09486-x)^k x^{-k}}{k}} \quad \text{for } x<0$$

$$1+\frac{1}{7\left(\frac{0.794679-\log(2.09486)}{2.09486}+0.211392-0.006068\times 2.09486-2.8\times 10^{-6}\times 2.09486^3\right)} = 1-\frac{0.299266}{-1.21083+\log(z_0)+\left\lfloor\frac{\arg(2.09486-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2.09486-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$1+\frac{1}{7\left(\frac{0.794679-\log(2.09486)}{2.09486}+0.211392-0.006068\times 2.09486-2.8\times 10^{-6}\times 2.09486^3\right)} = 1+\frac{0.299266}{1.21083-\int_1^{2.09486}\frac{1}{t}\,dt}$$

$$1+\frac{1}{7\left(\frac{0.794679-\log(2.09486)}{2.09486}+0.211392-0.006068\times 2.09486-2.8\times 10^{-6}\times 2.09486^3\right)} = 1+\frac{0.247157\,i\pi}{i\pi-0.412939\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-0.0906265\,s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

Now, we have that (page 99)

$$\frac{1}{2\log_e 2} + \frac{1}{3\log_e 3} + \frac{1}{4\log_e 4} + \cdots + \frac{1}{n\log_e n}.$$
$$= \cdot 7946786 + \log_e \log_e \left(n + \tfrac{1}{2}\right) \text{ nearly.}$$

1/(2ln2) + 1/(3ln3) + 1/(4ln4) + …+1/(nln(n))

Input interpretation:

$$\frac{1}{2\log(2)} + \frac{1}{3\log(3)} + \frac{1}{4\log(4)} + \cdots + \frac{1}{n\log(n)}$$

$\log(x)$ is the natural logarithm

Result:

$$\sum_{k=2}^{n} \frac{1}{k\log(k)}$$

Sum:

$$\sum_{k=2}^{n} \frac{1}{k\log(k)}$$

While:

1/(2ln2) + 1/(3ln3) + 1/(4ln4)

Input:

$$\frac{1}{2\log(2)} + \frac{1}{3\log(3)} + \frac{1}{4\log(4)}$$

$\log(x)$ is the natural logarithm

Decimal approximation:

1.205097476097881260804699647538218252762532407097353457076…

1.205097476…

Note that:

(1,205097476 / 2) + 1 = 1.602548738

Alternate forms:

$$\frac{1}{\log(27)} + \frac{5}{\log(256)}$$

$$\frac{8\log(2) + 15\log(3)}{24\log(2)\log(3)}$$

$$\frac{1}{12}\left(\frac{6}{\log(2)} + \frac{4}{\log(3)} + \frac{3}{\log(4)}\right)$$

Alternative representations:

$$\frac{1}{2\log(2)} + \frac{1}{3\log(3)} + \frac{1}{4\log(4)} = \frac{1}{2\log_e(2)} + \frac{1}{3\log_e(3)} + \frac{1}{4\log_e(4)}$$

$$\frac{1}{2\log(2)} + \frac{1}{3\log(3)} + \frac{1}{4\log(4)} = \frac{1}{2\log(a)\log_a(2)} + \frac{1}{3\log(a)\log_a(3)} + \frac{1}{4\log(a)\log_a(4)}$$

$$\frac{1}{2\log(2)} + \frac{1}{3\log(3)} + \frac{1}{4\log(4)} = -\frac{1}{4\,\mathrm{Li}_1(-3)} + -\frac{1}{3\,\mathrm{Li}_1(-2)} + -\frac{1}{2\,\mathrm{Li}_1(-1)}$$

Series representations:

$$\frac{1}{2\log(2)} + \frac{1}{3\log(3)} + \frac{1}{4\log(4)} = \frac{1}{2\left(2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)} + \frac{1}{3\left(2 i \pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k}\right)} + \frac{1}{4\left(2 i \pi \left\lfloor \frac{\arg(4-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (4-x)^k x^{-k}}{k}\right)} \quad \text{for } x < 0$$

$$\frac{1}{2\log(2)}+\frac{1}{3\log(3)}+\frac{1}{4\log(4)}=$$

$$\frac{1}{2\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)}+$$
$$\frac{1}{3\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(3-z_0)^k z_0^{-k}}{k}\right)}+$$
$$\frac{1}{4\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(4-z_0)^k z_0^{-k}}{k}\right)}$$

$$\frac{1}{2\log(2)}+\frac{1}{3\log(3)}+\frac{1}{4\log(4)}=$$

$$\frac{1}{2\left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)}+$$
$$\frac{1}{3\left(\log(z_0)+\left\lfloor\frac{\arg(3-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(3-z_0)^k z_0^{-k}}{k}\right)}+$$
$$\frac{1}{4\left(\log(z_0)+\left\lfloor\frac{\arg(4-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(4-z_0)^k z_0^{-k}}{k}\right)}$$

Integral representations:

$$\frac{1}{2\log(2)}+\frac{1}{3\log(3)}+\frac{1}{4\log(4)}=$$
$$\frac{\int_0^1\int_0^1\frac{1}{(1+t_1)(1+2t_2)}\,dt_2\,dt_1+\int_0^1\int_0^1\frac{1}{(1+t_1)(1+3t_2)}\,dt_2\,dt_1+\int_0^1\int_0^1\frac{1}{(1+2t_1)(1+3t_2)}\,dt_2\,dt_1}{12\left(\int_1^2\frac{1}{t}\,dt\right)\left(\int_1^3\frac{1}{t}\,dt\right)\int_1^4\frac{1}{t}\,dt}$$

$$\frac{1}{2\log(2)}+\frac{1}{3\log(3)}+\frac{1}{4\log(4)}=$$
$$\left(i\pi\left(3\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds+\right.\right.$$
$$4\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{3^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds+$$
$$\left.\left.6\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{3^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\right)\Big/$$
$$\left(6\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\right.$$
$$\left.\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{3^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\quad \text{for } -1<\gamma<0$$

0.7946786 + ln ln(4+1/2)

Input interpretation:

$$0.7946786+\log\left(\log\left(4+\frac{1}{2}\right)\right)$$

log(x) is the natural logarithm

Result:

1.202858...

1.202858…

Alternative representations:

$$0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)=0.794679+\log_e\left(\log\left(\frac{9}{2}\right)\right)$$

$$0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)=0.794679+\log(a)\log_a\left(\log\left(\frac{9}{2}\right)\right)$$

$$0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)=0.794679-\text{Li}_1\left(1-\log\left(\frac{9}{2}\right)\right)$$

Series representations:

$$0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)=0.794679-\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\log\left(\frac{9}{2}\right)\right)^k}{k}$$

$$0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)=$$
$$0.794679+2\,i\,\pi\left\lfloor\frac{\arg\left(-x+\log\left(\frac{9}{2}\right)\right)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,x^{-k}\left(-x+\log\left(\frac{9}{2}\right)\right)^k}{k}\quad\text{for } x<0$$

$$0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)=0.794679+\left\lfloor\frac{\arg\left(\log\left(\frac{9}{2}\right)-z_0\right)}{2\,\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+$$
$$\log(z_0)+\left\lfloor\frac{\arg\left(\log\left(\frac{9}{2}\right)-z_0\right)}{2\,\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\log\left(\frac{9}{2}\right)-z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)=0.794679+\int_1^{\log\left(\frac{9}{2}\right)}\frac{1}{t}\,dt$$

$$0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)=0.794679+\frac{1}{2\,i\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\log\left(\frac{9}{2}\right)\right)^{-s}}{\Gamma(1-s)}\,ds$$

for $-1<\gamma<0$

From which:

1+1/2((0.7946786 + ln ln(4+1/2)))

Input interpretation:

$$1+\frac{1}{2}\left(0.7946786+\log\left(\log\left(4+\frac{1}{2}\right)\right)\right)$$

$\log(x)$ is the natural logarithm

Result:

1.601429142413059162995674435042583407000034582070751332460...

1.601429142413…..

Alternative representations:

$$1+\frac{1}{2}\left(0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)\right)=1+\frac{1}{2}\left(0.794679+\log_e\left(\log\left(\frac{9}{2}\right)\right)\right)$$

$$1+\frac{1}{2}\left(0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)\right)=1+\frac{1}{2}\left(0.794679+\log(a)\log_a\left(\log\left(\frac{9}{2}\right)\right)\right)$$

$$1+\frac{1}{2}\left(0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)\right)=1+\frac{1}{2}\left(0.794679-\text{Li}_1\left(1-\log\left(\frac{9}{2}\right)\right)\right)$$

Series representations:

$$1+\frac{1}{2}\left(0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)\right)=1.39734-0.5\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\log\left(\frac{9}{2}\right)\right)^k}{k}$$

$$1+\frac{1}{2}\left(0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)\right)=1.39734+i\pi\left\lfloor\frac{\arg\left(-x+\log\left(\frac{9}{2}\right)\right)}{2\pi}\right\rfloor+0.5\log(x)-0.5\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\log\left(\frac{9}{2}\right)\right)^k}{k}\quad\text{for } x<0$$

$$1+\frac{1}{2}\left(0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)\right)=1.39734+\frac{1}{2}\left\lfloor\frac{\arg\left(\log\left(\frac{9}{2}\right)-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\frac{\log(z_0)}{2}+\frac{1}{2}\left\lfloor\frac{\arg\left(\log\left(\frac{9}{2}\right)-z_0\right)}{2\pi}\right\rfloor\log(z_0)-\frac{1}{2}\sum_{k=1}^{\infty}\frac{(-1)^k\left(\log\left(\frac{9}{2}\right)-z_0\right)^k z_0^{-k}}{k}$$

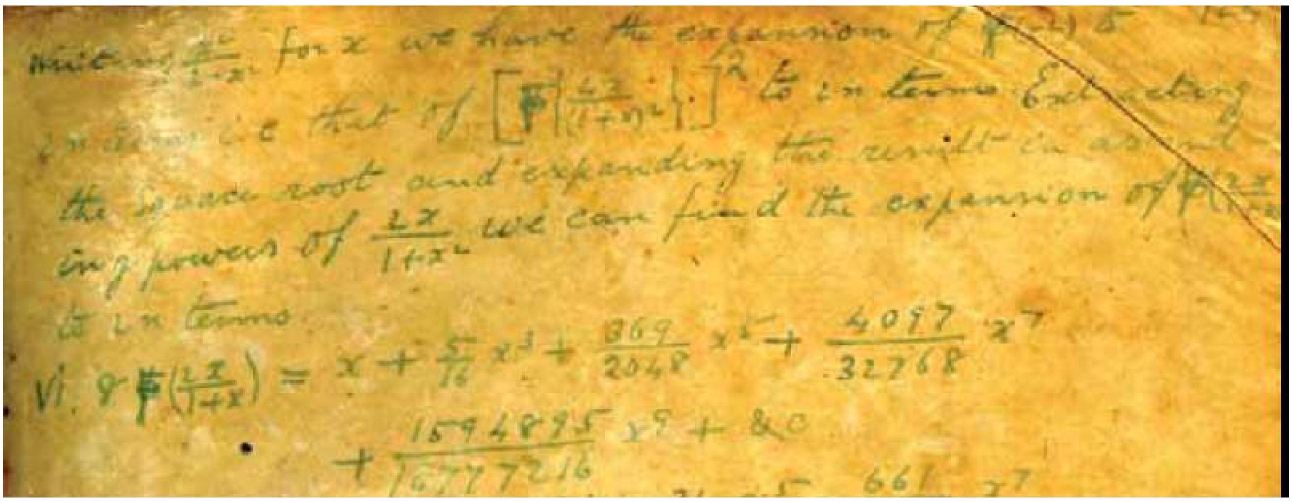
Integral representations:

$$1+\frac{1}{2}\left(0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)\right)=1.39734+0.5\int_1^{\log\left(\frac{9}{2}\right)}\frac{1}{t}\,dt$$

$$1+\frac{1}{2}\left(0.794679+\log\left(\log\left(4+\frac{1}{2}\right)\right)\right)=1.39734+\frac{0.25}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\log\left(\frac{9}{2}\right)\right)^{-s}}{\Gamma(1-s)}\,ds\quad\text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

Now, we have that (page 234):



writing $\frac{2x}{1+x^2}$ for x we have the expansion of F(x) in terms i.e. that of $\left[F\left\{\frac{2x}{(1+x)^2}\right\}\right]^2$ in terms. Extracting the square root and expanding the result in ascending powers of $\frac{2x}{1+x^2}$ we can find the expansion of $F\left(\frac{2x}{1+x^2}\right)$ in terms.

$$\text{Vi. } 8F\left(\frac{2x}{1+x}\right)=x+\frac{5}{16}x^3+\frac{369}{2048}x^5+\frac{4097}{32768}x^7+\frac{1594895}{16777216}x^9+\&c$$

For x = 2, we obtain:

5+(5/16)*5^3+(369/2048)*5^5+(4097/32768)*5^7+(1594895/16777216)*5^9

Input:

$$5+\frac{5}{16}\times 5^3+\frac{369}{2048}\times 5^5+\frac{4097}{32768}\times 5^7+\frac{1594895}{16777216}\times 5^9$$

Exact result:

$$\frac{3289094942955}{16777216}$$

Decimal form:

196045.33570736646652216796875

196045.3357…

From

A002410	Nearest integer to imaginary part of n-th zero of Riemann **zeta** function. (Formerly M4924 N2113)

14, 21, 25, 30, 33, 38, 41, 43, 48, 50, 53, 56, 59, 61, 65, 67, 70, 72, 76, 77, 79, 83, 85, 87, 89, 92, 95, 96, 99, 101, 104, 105, 107, 111, 112, 114, 116, 119, 121, 123, 124, 128, 130, 131, 133, 135, 138, 140, 141, 143, 146, 147, 150, 151, 153, 156, 158, 159, 161 (list; graph; refs; listen; history; text; internal format)

and the relative formula

a(n) ~ 2*Pi*(n - 11/8)/Product Log((n - 11/8)/exp(1))

for n = 521, where 521 is also a Lucas number, we obtain:

2*Pi*(521 - 11/8)/Product Log((521 - 11/8)/exp(1))

Input:

$$2\pi \times \frac{521 - \frac{11}{8}}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}$$

$W(z)$ is the product log function

Exact result:

$$\frac{4157\pi}{4\,W\left(\frac{4157}{8e}\right)}$$

Decimal approximation:

838.50025727537266563331424553425215707125810331283305444462...

838.5002572….

Alternate form:

$$2\pi e^{W(4157/(8e))+1}$$

Alternative representation:

$$\frac{(2\pi)\left(521 - \frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)} = \frac{2\pi\left(521 - \frac{11}{8}\right)}{W_0\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}$$

$W_k(z)$ is the analytic continuation of the product log function

Series representation:

$$\frac{(2\pi)\left(521 - \frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)} = (4157\pi) \Bigg/ \left(4\left(\log\left(\frac{4157}{8\exp(1)}\right) - \log\left(\log\left(\frac{4157}{8\exp(1)}\right)\right) - \sum_{k=0}^{\infty} (-1)^k \log^{-k}\left(\frac{4157}{8\exp(1)}\right) \sum_{j=1}^{k} \frac{\log^j\left(\log\left(\frac{4157}{8\exp(1)}\right)\right) S_k^{(1-j+k)}}{j!}\right)\right) \text{ for } \infty \to \frac{4157}{8e}$$

$S_n^{(m)}$ is the Stirling number of the first kind

Integral representations:

$$\frac{(2\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)} = -\frac{4157\pi^2}{4\int_{-\infty}^{-\frac{1}{e}} \operatorname{Im}\left(\frac{\partial W(x)}{\partial x}\right)\log\left(1-\frac{4157}{8\,e\,x}\right)dx}$$

$$\frac{(2\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)} = \frac{4157\pi}{4\left(1+\exp\left(-\frac{i}{2\pi}\int_0^\infty \frac{\log\left(\frac{-i\pi+t+\log\left(\frac{4157}{8e}\right)-\log(t)}{i\pi+t+\log\left(\frac{4157}{8e}\right)-\log(t)}\right)}{1+t}\,dt\right)\left(-1+\log\left(\frac{4157}{8e}\right)\right)\right)}$$

$\operatorname{Im}(z)$ is the imaginary part of z

From the two expressions, we obtain:

(((5+(5/16)*5^3+(369/2048)*5^5+(4097/32768)*5^7+(1594895/16777216)*5^9))) + ((2*Pi*(521 - 11/8)/Product Log((521 - 11/8)/exp(1))))

Input:

$$\left(5+\frac{5}{16}\times 5^3+\frac{369}{2048}\times 5^5+\frac{4097}{32768}\times 5^7+\frac{1594895}{16777216}\times 5^9\right)+2\pi\times\frac{521-\frac{11}{8}}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}$$

$W(z)$ is the product log function

Exact result:

$$\frac{4157\pi}{4\,W\left(\frac{4157}{8e}\right)}+\frac{3289094942955}{16777216}$$

Decimal approximation:

196883.8359646418391878501111205342521570712581033128330544...

196883.835964…. 196884 is a fundamental number of the following j-invariant

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

(In mathematics, Felix Klein's j-invariant or j function, regarded as a function of a complex variable τ, is a modular function of weight zero for SL(2, Z) defined on the upper half plane of complex numbers. Several remarkable properties of j have to do with its q expansion (Fourier series expansion), written as a Laurent series in terms of $q = e^{2\pi i\tau}$ (the square of the nome), which begins:

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

Note that j has a simple pole at the cusp, so its q-expansion has no terms below q^{-1}.

All the Fourier coefficients are integers, which results in several almost integers, notably Ramanujan's constant:

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

The asymptotic formula for the coefficient of q^n is given by

$$\frac{e^{4\pi\sqrt{n}}}{\sqrt{2}\,n^{3/4}},$$

as can be proved by the Hardy–Littlewood circle method)

Alternate forms:

$$2\pi e^{W(4157/(8e))+1} + \frac{3289094942955}{16777216}$$

$$\frac{3289094942955\, W\left(\frac{4157}{8e}\right) + 17435721728\,\pi}{16777216\, W\left(\frac{4157}{8e}\right)}$$

Alternative representation:

$$\left(5 + \frac{5^3 \times 5}{16} + \frac{5^5 \times 369}{2048} + \frac{5^7 \times 4097}{32768} + \frac{5^9 \times 1594895}{16777216}\right) + \frac{(2\pi)\left(521 - \frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)} =$$

$$5 + \frac{5 \times 5^3}{16} + \frac{369 \times 5^5}{2048} + \frac{4097 \times 5^7}{32768} + \frac{1594895 \times 5^9}{16777216} + \frac{2\pi\left(521 - \frac{11}{8}\right)}{W_0\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}$$

$W_k(z)$ is the analytic continuation of the product log function

Series representation:

$$\left(5+\frac{5^3\times 5}{16}+\frac{5^5\times 369}{2048}+\frac{5^7\times 4097}{32\,768}+\frac{5^9\times 1\,594\,895}{16\,777\,216}\right)+\frac{(2\,\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}=$$

$$\left(17\,435\,721\,728\,\pi+3\,289\,094\,942\,955\,\log\left(\frac{4157}{8\exp(1)}\right)-3\,289\,094\,942\,955\,\log\left(\log\left(\frac{4157}{8\exp(1)}\right)\right)-3\,289\,094\,942\,955\sum_{k=0}^{\infty}(-1)^k\log^{-k}\left(\frac{4157}{8\exp(1)}\right)\sum_{j=1}^{k}\frac{\log^j\left(\log\left(\frac{4157}{8\exp(1)}\right)\right)S_k^{(1-j+k)}}{j!}\right)\Big/\left(16\,777\,216\left(\log\left(\frac{4157}{8\exp(1)}\right)-\log\left(\log\left(\frac{4157}{8\exp(1)}\right)\right)-\sum_{k=0}^{\infty}(-1)^k\log^{-k}\left(\frac{4157}{8\exp(1)}\right)\sum_{j=1}^{k}\frac{\log^j\left(\log\left(\frac{4157}{8\exp(1)}\right)\right)S_k^{(1-j+k)}}{j!}\right)\right)\quad\text{for } \infty\to\frac{4157}{8\,e}$$

$S_n^{(m)}$ is the Stirling number of the first kind

Integral representations:

$$\left(5+\frac{5^3\times 5}{16}+\frac{5^5\times 369}{2048}+\frac{5^7\times 4097}{32\,768}+\frac{5^9\times 1\,594\,895}{16\,777\,216}\right)+\frac{(2\,\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}=$$

$$\frac{3\,289\,094\,942\,955}{16\,777\,216}-\frac{4157\,\pi^2}{4\int_{-\infty}^{-\frac{1}{e}}\operatorname{Im}\left(\frac{\partial W(x)}{\partial x}\right)\log\left(1-\frac{4157}{8\,e\,x}\right)dx}$$

$$\left(5+\frac{5^3\times 5}{16}+\frac{5^5\times 369}{2048}+\frac{5^7\times 4097}{32\,768}+\frac{5^9\times 1\,594\,895}{16\,777\,216}\right)+\frac{(2\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}=$$

$$\frac{3\,289\,094\,942\,955}{16\,777\,216}+\frac{4157\,\pi}{4\left(1+\exp\left(-\frac{i}{2\pi}\int_0^\infty\frac{\log\left(\frac{-i\pi+t+\log\left(\frac{4157}{8e}\right)-\log(t)}{i\pi+t+\log\left(\frac{4157}{8e}\right)-\log(t)}\right)}{1+t}\,dt\right)\right)\left(-1+\log\left(\frac{4157}{8e}\right)\right)}$$

Im(z) is the imaginary part of z

Performing the 24th root, we obtain:

((((((5+(5/16)*5^3+(369/2048)*5^5+(4097/32768)*5^7+(1594895/16777216)*5^9))) + ((2*Pi*(521 - 11/8)/Product Log((521 - 11/8)/exp(1)))))))^1/24

Input:

$$\sqrt[24]{\left(5+\frac{5}{16}\times 5^3+\frac{369}{2048}\times 5^5+\frac{4097}{32\,768}\times 5^7+\frac{1\,594\,895}{16\,777\,216}\times 5^9\right)+2\pi\times\frac{521-\frac{11}{8}}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}}$$

W(z) is the product log function

Exact result:

$$\sqrt[24]{\frac{4157\,\pi}{4\,W\left(\frac{4157}{8e}\right)}+\frac{3\,289\,094\,942\,955}{16\,777\,216}}$$

Decimal approximation:

1.661851012134917071061766720515441435654451011637318008710...

1.6618510121349…… result very near to the 14th root of the following Ramanujan's class invariant $Q=\left(G_{505}/G_{101/5}\right)^3=1164.2696$ i.e. 1.65578...

Alternate forms:

$$\frac{1}{2}\sqrt[24]{33\,554\,432\,\pi\, e^{W(4157/(8e))+1}+3\,289\,094\,942\,955}$$

$$\frac{1}{2}\sqrt[24]{\frac{3\,289\,094\,942\,955\,W\left(\frac{4157}{8e}\right)+17\,435\,721\,728\,\pi}{W\left(\frac{4157}{8e}\right)}}$$

All 24th roots of (4157 π)/(4 W(4157/(8 e))) + 3289094942955/16777216:

$$e^{0}\sqrt[24]{\frac{4157\,\pi}{4\,W\left(\frac{4157}{8e}\right)}+\frac{3\,289\,094\,942\,955}{16\,777\,216}}\approx 1.661851 \text{ (real, principal root)}$$

$$e^{(i\pi)/12}\sqrt[24]{\frac{4157\,\pi}{4\,W\left(\frac{4157}{8e}\right)}+\frac{3\,289\,094\,942\,955}{16\,777\,216}}\approx 1.60522+0.43012\,i$$

$$e^{(i\pi)/6}\sqrt[24]{\frac{4157\,\pi}{4\,W\left(\frac{4157}{8e}\right)}+\frac{3\,289\,094\,942\,955}{16\,777\,216}}\approx 1.4392+0.83093\,i$$

$$e^{(i\pi)/4}\sqrt[24]{\frac{4157\,\pi}{4\,W\left(\frac{4157}{8e}\right)}+\frac{3\,289\,094\,942\,955}{16\,777\,216}}\approx 1.17511+1.17511\,i$$

$$e^{(i\pi)/3}\sqrt[24]{\frac{4157\,\pi}{4\,W\left(\frac{4157}{8e}\right)}+\frac{3\,289\,094\,942\,955}{16\,777\,216}}\approx 0.83093+1.4392\,i$$

Alternative representation:

$$\sqrt[24]{\left(5+\frac{5^3\times 5}{16}+\frac{5^5\times 369}{2048}+\frac{5^7\times 4097}{32\,768}+\frac{5^9\times 1\,594\,895}{16\,777\,216}\right)+\frac{(2\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}}=$$

$$\sqrt[24]{5+\frac{5\times 5^3}{16}+\frac{369\times 5^5}{2048}+\frac{4097\times 5^7}{32\,768}+\frac{1\,594\,895\times 5^9}{16\,777\,216}+\frac{2\pi\left(521-\frac{11}{8}\right)}{W_0\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}}$$

$W_k(z)$ is the analytic continuation of the product log function

Series representation:

$$\sqrt[24]{\left(5+\frac{5^3\times5}{16}+\frac{5^5\times369}{2048}+\frac{5^7\times4097}{32\,768}+\frac{5^9\times1\,594\,895}{16\,777\,216}\right)+\frac{(2\,\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}} =$$

$$\left(\frac{3\,289\,094\,942\,955}{16\,777\,216}+\right.$$
$$(4157\,\pi)\Big/\left(4\left(\log\left(\frac{4157}{8\exp(1)}\right)-\log\left(\log\left(\frac{4157}{8\exp(1)}\right)\right)-\sum_{k=0}^{\infty}(-1)^k\log^{-k}\left(\frac{4157}{8\exp(1)}\right)\right.\right.$$
$$\left.\left.\left.\sum_{j=1}^{k}\frac{\log^j\left(\log\left(\frac{4157}{8\exp(1)}\right)\right)S_k^{(1-j+k)}}{j!}\right)\right)\right)\text{^}(1/24)\ \text{for}\ \infty\to\frac{4157}{8\,e}$$

$S_n^{(m)}$ is the Stirling number of the first kind

Integral representations:

$$\sqrt[24]{\left(5+\frac{5^3\times5}{16}+\frac{5^5\times369}{2048}+\frac{5^7\times4097}{32\,768}+\frac{5^9\times1\,594\,895}{16\,777\,216}\right)+\frac{(2\,\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}} =$$
$$\sqrt[24]{\frac{3\,289\,094\,942\,955}{16\,777\,216}-\frac{4157\,\pi^2}{4\int_{-\infty}^{-\frac{1}{e}}\operatorname{Im}\left(\frac{\partial W(x)}{\partial x}\right)\log\left(1-\frac{4157}{8\,e\,x}\right)dx}}$$

$$\sqrt[24]{\left(5+\frac{5^3\times 5}{16}+\frac{5^5\times 369}{2048}+\frac{5^7\times 4097}{32768}+\frac{5^9\times 1594895}{16777216}\right)+\frac{(2\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}} =$$

$$\left(\frac{3289094942955}{16777216}+\frac{4157\pi}{4\left(1+\exp\left(-\frac{i}{2\pi}\int_0^\infty \frac{\log\left(\frac{-i\pi+t+\log\left(\frac{4157}{8e}\right)-\log(t)}{i\pi+t+\log\left(\frac{4157}{8e}\right)-\log(t)}\right)}{1+t}\,dt\right)\left(-1+\log\left(\frac{4157}{8e}\right)\right)\right)}\right)\hat{}\,(1/24)$$

and also, performing the 25th root:

((((((5+(5/16)*5^3+(369/2048)*5^5+(4097/32768)*5^7+(1594895/16777216)*5^9))) + ((2*Pi*(521 - 11/8)/Product Log((521 - 11/8)/exp(1)))))))^1/25

Input:

$$\sqrt[25]{\left(5+\frac{5}{16}\times 5^3+\frac{369}{2048}\times 5^5+\frac{4097}{32768}\times 5^7+\frac{1594895}{16777216}\times 5^9\right)+2\pi\times\frac{521-\frac{11}{8}}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}}$$

$W(z)$ is the product log function

Exact result:

$$\sqrt[25]{\frac{4157\pi}{4\,W\left(\frac{4157}{8e}\right)}+\frac{3289094942955}{16777216}}$$

Decimal approximation:

1.628427404735603885971569085260099026759808219177003408687...

1.62842740473560

Alternate forms:

$$\sqrt[25]{2\,\pi\, e^{W(4157/(8\,e))+1} + \frac{3\,289\,094\,942\,955}{16\,777\,216}}$$

$$\frac{\sqrt[25]{\frac{3\,289\,094\,942\,955\; W\left(\frac{4157}{8\,e}\right)+17\,435\,721\,728\,\pi}{W\left(\frac{4157}{8\,e}\right)}}}{2^{24/25}}$$

All 25th roots of (4157 π)/(4 W(4157/(8 e))) + 3289094942955/16777216:

$$e^{0}\sqrt[25]{\frac{4157\,\pi}{4\,W\left(\frac{4157}{8\,e}\right)} + \frac{3\,289\,094\,942\,955}{16\,777\,216}} \approx 1.628427 \text{ (real, principal root)}$$

$$e^{(2\,i\,\pi)/25}\sqrt[25]{\frac{4157\,\pi}{4\,W\left(\frac{4157}{8\,e}\right)} + \frac{3\,289\,094\,942\,955}{16\,777\,216}} \approx 1.57727 + 0.40497\, i$$

$$e^{(4\,i\,\pi)/25}\sqrt[25]{\frac{4157\,\pi}{4\,W\left(\frac{4157}{8\,e}\right)} + \frac{3\,289\,094\,942\,955}{16\,777\,216}} \approx 1.42700 + 0.7845\, i$$

$$e^{(6\,i\,\pi)/25}\sqrt[25]{\frac{4157\,\pi}{4\,W\left(\frac{4157}{8\,e}\right)} + \frac{3\,289\,094\,942\,955}{16\,777\,216}} \approx 1.1871 + 1.1147\, i$$

$$e^{(8\,i\,\pi)/25}\sqrt[25]{\frac{4157\,\pi}{4\,W\left(\frac{4157}{8\,e}\right)} + \frac{3\,289\,094\,942\,955}{16\,777\,216}} \approx 0.8726 + 1.3749\, i$$

Alternative representation:

$$\sqrt[25]{\left(5 + \frac{5^3\times 5}{16} + \frac{5^5\times 369}{2048} + \frac{5^7\times 4097}{32\,768} + \frac{5^9\times 1\,594\,895}{16\,777\,216}\right) + \frac{(2\,\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}} =$$

$$\sqrt[25]{5 + \frac{5\times 5^3}{16} + \frac{369\times 5^5}{2048} + \frac{4097\times 5^7}{32\,768} + \frac{1\,594\,895\times 5^9}{16\,777\,216} + \frac{2\,\pi\left(521-\frac{11}{8}\right)}{W_0\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}}$$

$W_k(z)$ is the analytic continuation of the product log function

Series representation:

$$\sqrt[25]{\left(5+\frac{5^3\times 5}{16}+\frac{5^5\times 369}{2048}+\frac{5^7\times 4097}{32768}+\frac{5^9\times 1594895}{16777216}\right)+\frac{(2\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}} =$$

$$\left(\frac{3289094942955}{16777216}+\right.$$

$$(4157\pi)\Bigg/\left(4\left(\log\left(\frac{4157}{8\exp(1)}\right)-\log\left(\log\left(\frac{4157}{8\exp(1)}\right)\right)-\sum_{k=0}^{\infty}(-1)^k\log^{-k}\left(\frac{4157}{8\exp(1)}\right)\right.\right.$$

$$\left.\left.\left.\sum_{j=1}^{k}\frac{\log^j\left(\log\left(\frac{4157}{8\exp(1)}\right)\right)S_k^{(1-j+k)}}{j!}\right)\right)\right)\hat{}\,(1/25)\ \text{ for } \infty\to\frac{4157}{8e}$$

$S_n^{(m)}$ is the Stirling number of the first kind

Integral representations:

$$\sqrt[25]{\left(5+\frac{5^3\times 5}{16}+\frac{5^5\times 369}{2048}+\frac{5^7\times 4097}{32768}+\frac{5^9\times 1594895}{16777216}\right)+\frac{(2\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}} =$$

$$\sqrt[25]{\frac{3289094942955}{16777216}-\frac{4157\pi^2}{4\int_{-\infty}^{-\frac{1}{e}}\operatorname{Im}\left(\frac{\partial W(x)}{\partial x}\right)\log\left(1-\frac{4157}{8ex}\right)dx}}$$

$$\sqrt[25]{\left(5+\frac{5^3\times 5}{16}+\frac{5^5\times 369}{2048}+\frac{5^7\times 4097}{32768}+\frac{5^9\times 1594895}{16777216}\right)+\frac{(2\pi)\left(521-\frac{11}{8}\right)}{W\left(\frac{521-\frac{11}{8}}{\exp(1)}\right)}} =$$

$$\left(\frac{3289094942955}{16777216}+\frac{4157\pi}{4\left(1+\exp\left(-\frac{i}{2\pi}\int_0^\infty \frac{\log\left(\frac{-i\pi+t+\log\left(\frac{4157}{8e}\right)-\log(t)}{i\pi+t+\log\left(\frac{4157}{8e}\right)-\log(t)}\right)}{1+t}\,dt\right)\left(-1+\log\left(\frac{4157}{8e}\right)\right)\right)}\right)\text{^}(1/25)$$

$\mathrm{Im}(z)$ is the imaginary part of z

We highlight that (from Wikipedia):

In the mathematical branch of moonshine theory, a **supersingular prime** is a prime number that divides the order of the Monster group *M*, which is the largest sporadic simple group. There are precisely fifteen supersingular prime numbers: the first eleven primes (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, and 31), as well as 41, 47, 59, and 71. (sequence A002267 in the OEIS). Supersingular primes are related to the notion of supersingular elliptic curves

In algebraic geometry, **supersingular elliptic curves** form a certain class of elliptic curves over a field of characteristic $p > 0$ with unusually large endomorphism rings. Elliptic curves over such fields which are not supersingular are called *ordinary* and these two classes of elliptic curves behave fundamentally differently in many aspects. Hasse (1936) discovered supersingular elliptic curves during his work on the Riemann hypothesis for elliptic curves by observing that in positive characteristic elliptic curves could have endomorphism rings of unusually large rank **4**, and Deuring (1941) developed their basic theory.

The elliptic curve given by $y^2 = x(x-1)(x+2)$ is nonsingular over $\mathbb{F}_p$ for $p \neq 2, 3$. It is supersingular for p = 23 and ordinary for every other $p \leq 73$ (see Hartshorne1977, 4.23.6).

The modular curve $X_0(11)$ has j-invariant $-2^{12}11^{-5}31^3$, and is isomorphic to the curve $y^2 + y = x^3 - x^2 - 10x - 20$. The primes p for which it is supersingular are those for which the coefficient of q^p in $\eta(\tau)^2\eta(11\tau)^2$ vanishes mod p, and are given by the list

2, 19, 29, 199, 569, 809, 1289, 1439, 2539, 3319, 3559, 3919, 5519, 9419, 9539, 9929,...
OEIS: A006962

If an elliptic curve over the rationals has complex multiplication then the set of primes for which it is supersingular has density 1/2. If it does not have complex multiplication then Serre showed that the set of primes for which it is supersingular has density zero. Elkies (1987) showed that any elliptic curve defined over the rationals is supersingular for an infinite number of primes.

We note that 809 + 29 = 838, thence, we have also the following expression:

(((5+(5/16)*5^3+(369/2048)*5^5+(4097/32768)*5^7+(1594895/16777216)*5^9))) + 809 +29

Practically, to the Ramanujan expression we adding 29 and 809 that are supersingular primes of the elliptic curve $X_0(11)$, as showed below:

A006962	Supersingular primes of the elliptic curve X_0 (11). (Formerly M2115)

2, 19, 29, 199, 569, 809, 1289, 1439, 2539, 3319, 3559, 3919, 5519, 9419, 9539, 9929, 11279, 11549, 13229, 14489, 17239, 18149, 18959, 19319, 22279, 24359, 27529, 28789, 32999, 33029, 36559, 42899, 45259, 46219, 49529, 51169, 52999, 55259 (list; graph; refs; listen; history; text; internal format)

Indeed:

Input:

$$\left(5+\frac{5}{16}\times 5^3+\frac{369}{2048}\times 5^5+\frac{4097}{32768}\times 5^7+\frac{1594895}{16777216}\times 5^9\right)+809+29$$

Exact result:

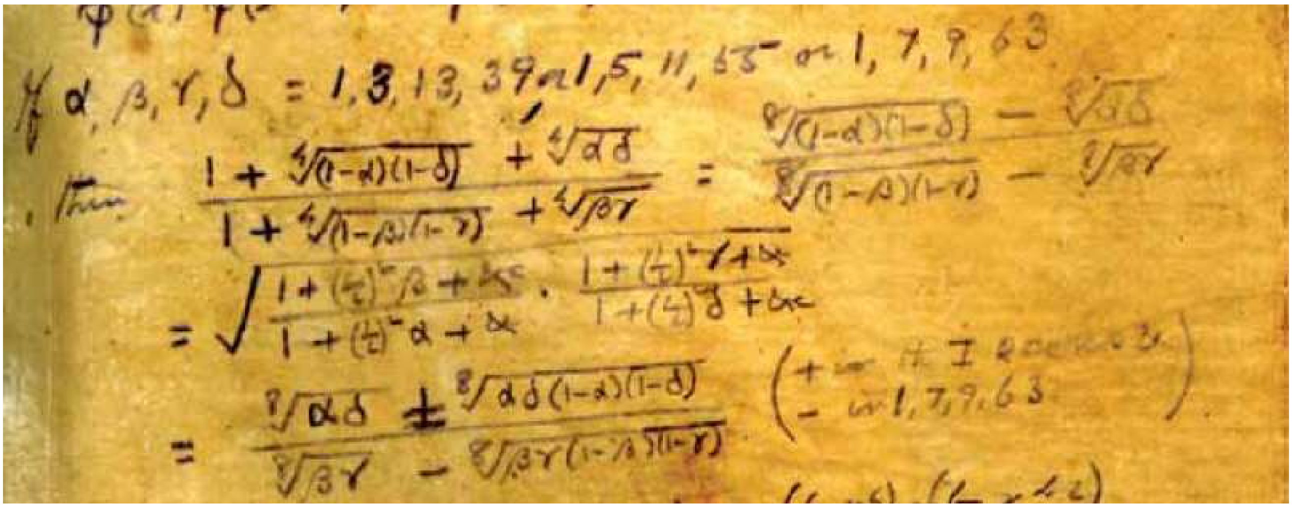
$$\frac{3303154249963}{16777216}$$

Decimal form:

196883.3357073664665222167968 75

196883.33570736….. as above

We have that (page 286)



For α = 1, β = 7,   γ = 9,   δ = 63   where $9 = 3^2$ and $63 = 3^2 * 7$ (we note that 3 and 7 are Lucas numbers), we obtain:

(((1+((((1-1)(1-63))))^(1/4)+(1*63)^(1/4))))/(((1+(((1-7)(1-9)))^(1/4)+(7*9)^(1/4))))

Input:

$$\frac{1+\sqrt[4]{(1-1)(1-63)}+\sqrt[4]{1\times 63}}{1+\sqrt[4]{(1-7)(1-9)}+\sqrt[4]{7\times 9}}$$

Result:

$$\frac{1+\sqrt{3}\,\sqrt[4]{7}}{1+2\sqrt[4]{3}+\sqrt{3}\,\sqrt[4]{7}}$$

Decimal approximation:

0.591880947195333566274469579006634378160734890206707866148...

0.591880947…

Alternate forms:

$$1-\frac{2\sqrt[4]{3}}{1+2\sqrt[4]{3}+\sqrt{3}\,\sqrt[4]{7}}$$

$$\frac{1}{1+2\sqrt[4]{3}+\sqrt{3}\,\sqrt[4]{7}}+\frac{\sqrt{3}\,\sqrt[4]{7}}{1+2\sqrt[4]{3}+\sqrt{3}\,\sqrt[4]{7}}$$

$$\text{root of } 2709503 x^{16} - 27866096 x^{15} + 132281736 x^{14} - 350923216 x^{13} + 387338212 x^{12} + 598648080 x^{11} - 3301656904 x^{10} + 7019141296 x^9 - 9660282822 x^8 + 9574974640 x^7 - 7098524488 x^6 + 3979933968 x^5 - 1676307740 x^4 + 517171760 x^3 - 110822520 x^2 + 14776336 x - 923521 \text{ near } x = 0.591881$$

Minimal polynomial:

$$2709503 x^{16} - 27866096 x^{15} + 132281736 x^{14} - 350923216 x^{13} + 387338212 x^{12} + 598648080 x^{11} - 3301656904 x^{10} + 7019141296 x^9 - 9660282822 x^8 + 9574974640 x^7 - 7098524488 x^6 + 3979933968 x^5 - 1676307740 x^4 + 517171760 x^3 - 110822520 x^2 + 14776336 x - 923521$$

And multiplying by Euler number:

e * (((1+((((1-1)(1-63))))^(1/4)+(1*63)^(1/4))))/(((1+(((1-7)(1-9)))^(1/4)+(7*9)^(1/4))))

Input:

$$e \times \frac{1 + \sqrt[4]{(1-1)(1-63)} + \sqrt[4]{1 \times 63}}{1 + \sqrt[4]{(1-7)(1-9)} + \sqrt[4]{7 \times 9}}$$

Result:

$$\frac{\left(1 + \sqrt{3}\ \sqrt[4]{7}\right) e}{1 + 2\sqrt[4]{3} + \sqrt{3}\ \sqrt[4]{7}}$$

Decimal approximation:

1.608899223372202928312876094593221970797128405653392575573...

1.60889922337…

Property:

$$\frac{\left(1 + \sqrt{3}\ \sqrt[4]{7}\right) e}{1 + 2\sqrt[4]{3} + \sqrt{3}\ \sqrt[4]{7}} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{e}{1 + 2\sqrt[4]{3} + \sqrt{3}\ \sqrt[4]{7}} + \frac{\sqrt{3}\ \sqrt[4]{7}\ e}{1 + 2\sqrt[4]{3} + \sqrt{3}\ \sqrt[4]{7}}$$

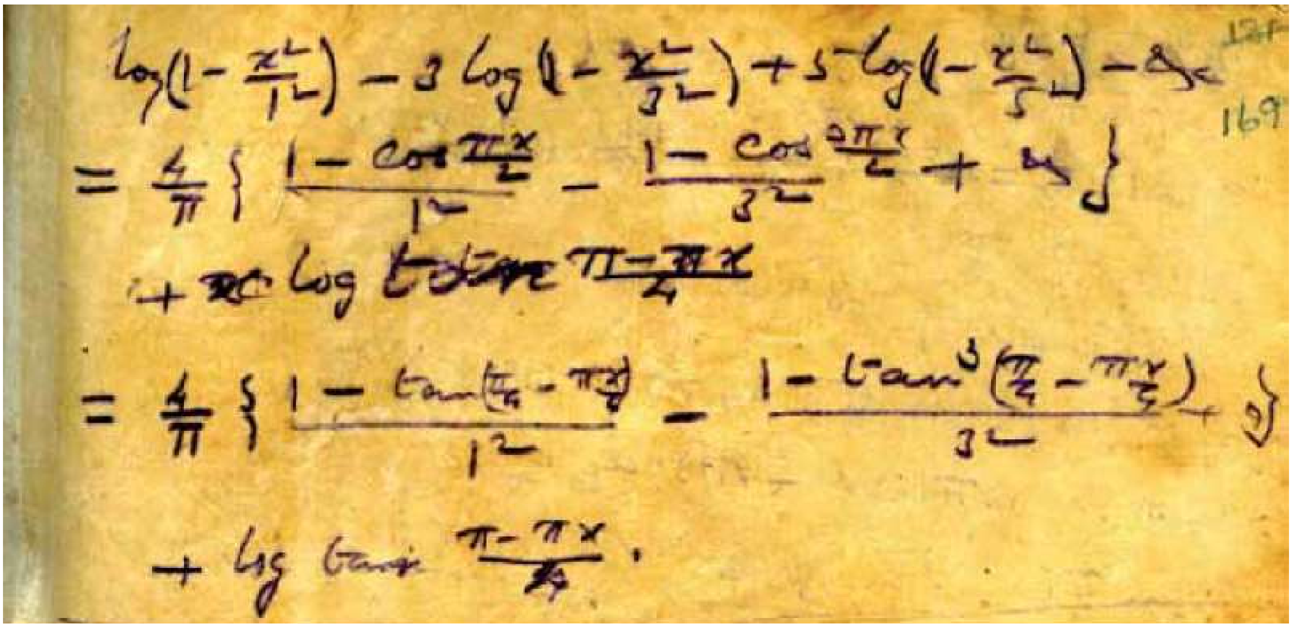
$$e \left(\text{root of } 2709503 x^{16} - 27866096 x^{15} + 132281736 x^{14} - 350923216 x^{13} + 387338212 x^{12} + 598648080 x^{11} - 3301656904 x^{10} + 7019141296 x^9 - 9660282822 x^8 + 9574974640 x^7 - 7098524488 x^6 + 3979933968 x^5 - 1676307740 x^4 + 517171760 x^3 - 110822520 x^2 + 14776336 x - 923521 \text{ near } x = 0.591881\right)$$

Alternative representation:

$$\frac{e\left(1+\sqrt[4]{(1-1)(1-63)}+\sqrt[4]{1\times 63}\right)}{1+\sqrt[4]{(1-7)(1-9)}+\sqrt[4]{7\times 9}}=$$

$$\frac{\exp(z)\left(1+\sqrt[4]{(1-1)(1-63)}+\sqrt[4]{1\times 63}\right)}{1+\sqrt[4]{(1-7)(1-9)}+\sqrt[4]{7\times 9}} \quad \text{for } z=1$$

We have that (page 324)



$$\log\left(1-\frac{x^2}{1^2}\right)-3\log\left(1-\frac{x^2}{3^2}\right)+5\log\left(1-\frac{x^2}{5^2}\right)-\&c$$

$$=\frac{4}{\pi}\left\{\frac{1-\cos\frac{\pi x}{2}}{1^2}-\frac{1-\cos\frac{3\pi x}{2}}{3^2}+\&c\right\}$$

$$+\cancel{x}\log \cancel{\tan}\ \frac{\pi-\pi x}{4}$$

$$=\frac{4}{\pi}\left\{\frac{1-\tan\left(\frac{\pi}{4}-\frac{\pi x}{4}\right)}{1^2}-\frac{1-\tan^3\left(\frac{\pi}{4}-\frac{\pi x}{4}\right)}{3^2}+\&c\right\}$$

$$+\log\tan\frac{\pi-\pi x}{4}.$$

For x = 2, we obtain:

4/Pi(1-tan((Pi/4-(2Pi)/4)))-1/9(1-tan^3((Pi/4-(2Pi)/4)))+ln tan((Pi-2Pi)/4)

Input:

$$\frac{4}{\pi}\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)$$

log(x) is the natural logarithm

Exact result:

$$-\frac{2}{9}+\frac{8}{\pi}+i\pi$$

Decimal approximation:

2.3242568672481031500799179917380075703291321096250809577 4... +
3.1415926535897932384626433832795028841971693993751058209 7... i

Property:

$-\frac{2}{9} + \frac{8}{\pi} + i\pi$ is a transcendental number

Polar coordinates:

$r \approx 3.90791$ (radius), $\theta \approx 53.5047°$ (angle)

3.90791

Alternate form:

$$\frac{72 - 2\pi + 9 i \pi^2}{9\pi}$$

Alternative representations:

$$\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi} - \frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right) + \log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right) = \log_e\left(\tan\left(-\frac{\pi}{4}\right)\right) + \frac{4\left(1+\cot\left(-\frac{\pi}{2}-\frac{\pi}{4}\right)\right)}{\pi} - \frac{1}{9}\left(1-\left(-\cot\left(-\frac{\pi}{2}-\frac{\pi}{4}\right)\right)^3\right)$$

$$\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi} - \frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right) + \log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right) = \log(a)\log_a\left(\tan\left(-\frac{\pi}{4}\right)\right) + \frac{4\left(1+\cot\left(-\frac{\pi}{2}-\frac{\pi}{4}\right)\right)}{\pi} - \frac{1}{9}\left(1-\left(-\cot\left(-\frac{\pi}{2}-\frac{\pi}{4}\right)\right)^3\right)$$

$$\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi} - \frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right) + \log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right) = \log\left(-i+\frac{2i}{1+e^{-(2i\pi)/4}}\right) + \frac{4\left(1+i-\frac{2i}{1+e^{-(2i\pi)/4}}\right)}{\pi} - \frac{1}{9}\left(1-\left(-i+\frac{2i}{1+e^{-(2i\pi)/4}}\right)^3\right)$$

Series representations:

$$\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi} - \frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right) + \log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right) = \frac{2\left(9-\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}+18i\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^2\right)}{9\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}$$

$$\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)=$$
$$\frac{72-2\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)+9i\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^2}{9\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}$$

$$\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)=$$
$$\left(72-2\times\sum_{k=0}^{\infty}16^{-k}\left(\frac{1}{-5-8k}-\frac{1}{2+4k}+\frac{4}{1+8k}-\frac{1}{6+8k}\right)+9i\left(\sum_{k=0}^{\infty}16^{-k}\left(\frac{1}{-5-8k}-\frac{1}{2+4k}+\frac{4}{1+8k}-\frac{1}{6+8k}\right)\right)^2\right)/$$
$$\left(9\times\sum_{k=0}^{\infty}16^{-k}\left(\frac{1}{-5-8k}-\frac{1}{2+4k}+\frac{4}{1+8k}-\frac{1}{6+8k}\right)\right)$$

Integral representations:

$$\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)=$$
$$\frac{2\left(18-\int_0^{\infty}\frac{1}{1+t^2}\,dt+9i\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2\right)}{9\int_0^{\infty}\frac{1}{1+t^2}\,dt}$$

$$\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)=$$
$$\frac{2\left(18-\int_0^{\infty}\frac{\sin(t)}{t}\,dt+9i\left(\int_0^{\infty}\frac{\sin(t)}{t}\,dt\right)^2\right)}{9\int_0^{\infty}\frac{\sin(t)}{t}\,dt}$$

$$\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)=$$
$$\frac{2\left(9-\int_0^{1}\sqrt{1-t^2}\,dt+18i\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^2\right)}{9\int_0^{1}\sqrt{1-t^2}\,dt}$$

Multiple-argument formulas:

$$\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)=$$
$$\log\left(-\frac{2\tan\left(-\frac{\pi}{8}\right)}{-1+\tan^2\left(-\frac{\pi}{8}\right)}\right)+\frac{1}{9}\left(-1-\frac{8\tan^3\left(-\frac{\pi}{8}\right)}{\left(-1+\tan^2\left(-\frac{\pi}{8}\right)\right)^3}\right)+\frac{4\left(1+\frac{2\tan\left(-\frac{\pi}{8}\right)}{-1+\tan^2\left(-\frac{\pi}{8}\right)}\right)}{\pi}$$

$$\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)=$$
$$\log\left(\frac{\tan\left(-\frac{\pi}{12}\right)\left(-3+\tan^2\left(-\frac{\pi}{12}\right)\right)}{-1+3\tan^2\left(-\frac{\pi}{12}\right)}\right)+$$
$$\frac{4\left(1-\frac{\tan\left(-\frac{\pi}{12}\right)\left(-3+\tan^2\left(-\frac{\pi}{12}\right)\right)}{-1+3\tan^2\left(-\frac{\pi}{12}\right)}\right)}{\pi}+\frac{1}{9}\left(-1+\frac{\left(-3\tan\left(-\frac{\pi}{12}\right)+\tan^3\left(-\frac{\pi}{12}\right)\right)^3}{\left(-1+3\tan^2\left(-\frac{\pi}{12}\right)\right)^3}\right)$$

$$\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)=$$
$$\log\left(\frac{3\tan\left(-\frac{\pi}{12}\right)-\tan^3\left(-\frac{\pi}{12}\right)}{1-3\tan^2\left(-\frac{\pi}{12}\right)}\right)+$$
$$\frac{4\left(1-\frac{3\tan\left(-\frac{\pi}{12}\right)-\tan^3\left(-\frac{\pi}{12}\right)}{1-3\tan^2\left(-\frac{\pi}{12}\right)}\right)}{\pi}+\frac{1}{9}\left(-1+\frac{\left(3\tan\left(-\frac{\pi}{12}\right)-\tan^3\left(-\frac{\pi}{12}\right)\right)^3}{\left(1-3\tan^2\left(-\frac{\pi}{12}\right)\right)^3}\right)$$

From which:

0.9991104684 + 1/(((4/Pi(1-tan((Pi/4-(2Pi)/4)))-1/9(1-tan^3((Pi/4-(2Pi)/4)))+ln tan((Pi-2Pi)/4))))^1/3

Where 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

Input interpretation:

$$0.9991104684+\frac{1}{\sqrt[3]{\frac{4}{\pi}\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)}}$$

$\log(x)$ is the natural logarithm

Result:

1.603470906... −
0.1944450923... i

Polar coordinates:

$r = 1.61522$ (radius), $\theta = -6.91422°$ (angle)

1.61522 result that is a good approximation to the value of the golden ratio 1.618033988749...

Alternative representations:

$$0.99911+\frac{1}{\sqrt[3]{\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)}}=$$
$$0.99911+\frac{1}{\sqrt[3]{\log_e\left(\tan\left(-\frac{\pi}{4}\right)\right)+\frac{4\left(1+\cot\left(-\frac{\pi}{2}-\frac{\pi}{4}\right)\right)}{\pi}-\frac{1}{9}\left(1-\left(-\cot\left(-\frac{\pi}{2}-\frac{\pi}{4}\right)\right)^3\right)}}$$

$$0.99911+\frac{1}{\sqrt[3]{\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)}}=$$
$$0.99911+\frac{1}{\sqrt[3]{\log(a)\log_a\left(\tan\left(-\frac{\pi}{4}\right)\right)+\frac{4\left(1+\cot\left(-\frac{\pi}{2}-\frac{\pi}{4}\right)\right)}{\pi}-\frac{1}{9}\left(1-\left(-\cot\left(-\frac{\pi}{2}-\frac{\pi}{4}\right)\right)^3\right)}}$$

$$0.99911+\frac{1}{\sqrt[3]{\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)}}=$$
$$0.99911+\frac{1}{\sqrt[3]{\log_e\left(\tan\left(-\frac{\pi}{4}\right)\right)+\frac{4\left(1+i-\frac{2i}{1+e^{-(2i\pi)/4}}\right)}{\pi}-\frac{1}{9}\left(1-\left(-i+\frac{2i}{1+e^{-(2i\pi)/4}}\right)^3\right)}}$$

Series representations:

$$0.99911+\frac{1}{\sqrt[3]{\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)}}=$$

$$0.99911+1\Big/\left(\left(\log\left(i\left(1+2\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)\right)+\frac{4\left(1-i\left(1+2\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)\right)}{\pi}+\frac{1}{9}\left(-1+i^3\left(1+2\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^3\right)\right)\hat{}\,(1/3)\right) \text{ for } (-1)^{3/4}+q=0$$

$$0.99911+\frac{1}{\sqrt[3]{\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)}}=$$

$$0.99911+1\Big/\left(\left(\log\left(-1+\tan\left(-\frac{\pi}{4}\right)\right)+\frac{4\left(1-i\left(1+2\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)\right)}{\pi}+\frac{1}{9}\left(-1+i^3\left(1+2\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^3\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\tan\left(-\frac{\pi}{4}\right)\right)^{-k}}{k}\right)\hat{}\,(1/3)\right) \text{ for } (-1)^{3/4}+q=0$$

$$0.99911+\frac{1}{\sqrt[3]{\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)}}=0.99911+$$

$$1\Big/\left(\left(\log\left(-1+\tan\left(-\frac{\pi}{4}\right)\right)+\frac{4\left(1-i\sum_{k=1}^{\infty}(-1)^k e^{-1/2\,i\,k\,\pi}+i\sum_{k=-\infty}^{-1}(-1)^k e^{-1/2\,i\,k\,\pi}\right)}{\pi}+\frac{1}{9}\left(-1+\left(i\sum_{k=1}^{\infty}(-1)^k e^{-1/2\,i\,k\,\pi}-i\sum_{k=-\infty}^{-1}(-1)^k e^{-1/2\,i\,k\,\pi}\right)^3\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\tan\left(-\frac{\pi}{4}\right)\right)^{-k}}{k}\right)\hat{}\,(1/3)\right)$$

Multiple-argument formulas:

$$0.99911+\frac{1}{\sqrt[3]{\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)}}=$$

$$0.99911+\frac{1}{\sqrt[3]{\log\left(\frac{2\tan\left(-\frac{\pi}{8}\right)}{1-\tan^2\left(-\frac{\pi}{8}\right)}\right)+\frac{1}{9}\left(-1+\frac{8\tan^3\left(-\frac{\pi}{8}\right)}{\left(1-\tan^2\left(-\frac{\pi}{8}\right)\right)^3}\right)+\frac{4\left(1-\frac{2\tan\left(-\frac{\pi}{8}\right)}{1-\tan^2\left(-\frac{\pi}{8}\right)}\right)}{\pi}}}$$

$$0.99911+\frac{1}{\sqrt[3]{\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)}}=$$

$$0.99911+\frac{1}{\sqrt[3]{\log\left(\frac{U_{-\frac{5}{4}}(\cos(\pi))\sin(\pi)}{T_{-\frac{1}{4}}(\cos(\pi))}\right)+\frac{4\left(1-\frac{U_{-\frac{5}{4}}(\cos(\pi))\sin(\pi)}{T_{-\frac{1}{4}}(\cos(\pi))}\right)}{\pi}+\frac{1}{9}\left(-1+\frac{U_{-\frac{5}{4}}(\cos(\pi))^3\sin^3(\pi)}{T_{-\frac{1}{4}}(\cos(\pi))^3}\right)}}$$

$$0.99911+\frac{1}{\sqrt[3]{\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)}}=$$

$$0.99911+\frac{1}{\sqrt[3]{\log\left(\frac{\tan\left(-\frac{5\pi}{4}\right)+\tan(\pi)}{1-\tan\left(-\frac{5\pi}{4}\right)\tan(\pi)}\right)+\frac{1}{9}\left(-1+\frac{\left(\tan\left(-\frac{5\pi}{4}\right)+\tan(\pi)\right)^3}{\left(1-\tan\left(-\frac{5\pi}{4}\right)\tan(\pi)\right)^3}\right)+\frac{4\left(1-\frac{\tan\left(-\frac{5\pi}{4}\right)+\tan(\pi)}{1-\tan\left(-\frac{5\pi}{4}\right)\tan(\pi)}\right)}{\pi}}}$$

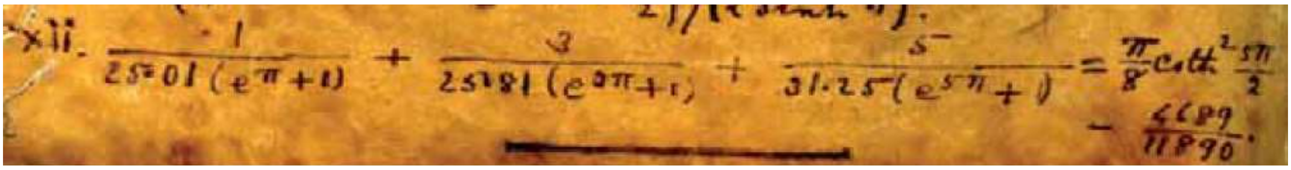
$$0.99911+\frac{1}{\sqrt[3]{\frac{\left(1-\tan\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)4}{\pi}-\frac{1}{9}\left(1-\tan^3\left(\frac{\pi}{4}-\frac{2\pi}{4}\right)\right)+\log\left(\tan\left(\frac{1}{4}(\pi-2\pi)\right)\right)}}=0.99911+$$

$$\frac{1}{\sqrt[3]{\log\left(\frac{3\tan\left(-\frac{\pi}{12}\right)-\tan^3\left(-\frac{\pi}{12}\right)}{1-3\tan^2\left(-\frac{\pi}{12}\right)}\right)+\frac{4\left(1-\frac{3\tan\left(-\frac{\pi}{12}\right)-\tan^3\left(-\frac{\pi}{12}\right)}{1-3\tan^2\left(-\frac{\pi}{12}\right)}\right)}{\pi}+\frac{1}{9}\left(-1+\frac{\left(3\tan\left(-\frac{\pi}{12}\right)-\tan^3\left(-\frac{\pi}{12}\right)\right)^3}{\left(1-3\tan^2\left(-\frac{\pi}{12}\right)\right)^3}\right)}}$$

$T_n(x)$ is the Chebyshev polynomial of the first kind

$U_n(x)$ is the Chebyshev polynomial of the second kind

Now, we have that (from **Manuscript Book II of Srinivasa Ramanujan**) page 183:



$$\text{xii. } \frac{1}{25\cdot 01\,(e^{\pi}+1)} + \frac{3}{25\cdot 81\,(e^{3\pi}+1)} + \frac{5}{31\cdot 25\,(e^{5\pi}+1)} = \frac{\pi}{8}\coth^2\frac{5\pi}{2} - \frac{6689}{11890}.$$

-1/((25*01)(e^Pi+1)) + 3/((25*81)(e^(3Pi)+1))+5/((31*25)(e^(5Pi)+1))

Input:

$$-\frac{1}{(25\times 1)(e^{\pi}+1)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}$$

Exact result:

$$-\frac{1}{25\,(1+e^{\pi})}+\frac{1}{675\left(1+e^{3\pi}\right)}+\frac{1}{155\left(1+e^{5\pi}\right)}$$

Decimal approximation:

-0.001656832769191844222197095230547924969145285774317127 15...

-0.0016568327…

Property:

$$-\frac{1}{25\,(1+e^{\pi})}+\frac{1}{675\left(1+e^{3\pi}\right)}+\frac{1}{155\left(1+e^{5\pi}\right)} \text{ is a transcendental number}$$

Alternate forms:

$$-\frac{2399}{62\,775\,(1+e^{\pi})}+\frac{2-e^{\pi}}{2025\left(1-e^{\pi}+e^{2\pi}\right)}+\frac{4-3\,e^{\pi}+2\,e^{2\pi}-e^{3\pi}}{775\left(1-e^{\pi}+e^{2\pi}-e^{3\pi}+e^{4\pi}\right)}$$

$$-\frac{671-1508\,e^{\pi}+2345\,e^{2\pi}-2480\,e^{3\pi}+2480\,e^{4\pi}-1674\,e^{5\pi}+837\,e^{6\pi}}{20\,925\,(1+e^{\pi})\left(1-e^{\pi}+e^{2\pi}\right)\left(1-e^{\pi}+e^{2\pi}-e^{3\pi}+e^{4\pi}\right)}$$

Alternative representations:

$$-\frac{1}{(e^{\pi}+1)\,25}+\frac{3}{\left(e^{3\pi}+1\right)25\times 81}+\frac{5}{\left(e^{5\pi}+1\right)31\times 25}=$$
$$-\frac{1}{25\left(1+e^{180^{\circ}}\right)}+\frac{3}{2025\left(1+e^{540^{\circ}}\right)}+\frac{5}{775\left(1+e^{900^{\circ}}\right)}$$

$$-\frac{1}{(e^\pi+1)25}+\frac{3}{(e^{3\pi}+1)25\times 81}+\frac{5}{(e^{5\pi}+1)31\times 25}=$$
$$\frac{5}{775\left(1+e^{-5 i \log(-1)}\right)}+\frac{3}{2025\left(1+e^{-3 i \log(-1)}\right)}-\frac{1}{25\left(1+e^{-i \log(-1)}\right)}$$

$$-\frac{1}{(e^\pi+1)25}+\frac{3}{(e^{3\pi}+1)25\times 81}+\frac{5}{(e^{5\pi}+1)31\times 25}=$$
$$-\frac{1}{(\exp^\pi(z)+1)25}+\frac{3}{(\exp^{3\pi}(z)+1)25\times 81}+\frac{5}{(\exp^{5\pi}(z)+1)31\times 25} \quad \text{for } z=1$$

Series representations:

$$-\frac{1}{(e^\pi+1)25}+\frac{3}{(e^{3\pi}+1)25\times 81}+\frac{5}{(e^{5\pi}+1)31\times 25}=-\frac{1}{25\left(1+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}+$$
$$\frac{1}{675\left(1+e^{12\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}+\frac{1}{155\left(1+e^{20\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}$$

$$-\frac{1}{(e^\pi+1)25}+\frac{3}{(e^{3\pi}+1)25\times 81}+\frac{5}{(e^{5\pi}+1)31\times 25}=$$
$$-\frac{1}{25\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)}+\frac{1}{675\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{3\pi}\right)}+\frac{1}{155\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{5\pi}\right)}$$

$$-\frac{1}{(e^\pi+1)25}+\frac{3}{(e^{3\pi}+1)25\times 81}+\frac{5}{(e^{5\pi}+1)31\times 25}=$$
$$-\frac{1}{25\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)}+\frac{1}{675\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{3\pi}\right)}+\frac{1}{155\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{5\pi}\right)}$$

Integral representations:

$$-\frac{1}{(e^\pi+1)25}+\frac{3}{(e^{3\pi}+1)25\times 81}+\frac{5}{(e^{5\pi}+1)31\times 25}=$$
$$-\frac{1}{25\left(1+e^{2\int_0^{\infty}1/(1+t^2)\,dt}\right)}+\frac{1}{675\left(1+e^{6\int_0^{\infty}1/(1+t^2)\,dt}\right)}+\frac{1}{155\left(1+e^{10\int_0^{\infty}1/(1+t^2)\,dt}\right)}$$

$$-\frac{1}{(e^\pi+1)25}+\frac{3}{(e^{3\pi}+1)25\times 81}+\frac{5}{(e^{5\pi}+1)31\times 25}=$$
$$-\frac{1}{25\left(1+e^{2\int_0^{\infty}\sin(t)/t\,dt}\right)}+\frac{1}{675\left(1+e^{6\int_0^{\infty}\sin(t)/t\,dt}\right)}+\frac{1}{155\left(1+e^{10\int_0^{\infty}\sin(t)/t\,dt}\right)}$$

$$-\frac{1}{(e^{\pi}+1)25}+\frac{3}{(e^{3\pi}+1)25\times 81}+\frac{5}{(e^{5\pi}+1)31\times 25}=$$

$$-\frac{1}{25\left(1+e^{4\int_0^1\sqrt{1-t^2}\,dt}\right)}+\frac{1}{675\left(1+e^{12\int_0^1\sqrt{1-t^2}\,dt}\right)}+\frac{1}{155\left(1+e^{20\int_0^1\sqrt{1-t^2}\,dt}\right)}$$

and:

Pi/8 coth^2((5Pi)/2) – 4689/11890

Input:

$$\frac{\pi}{8}\coth^2\left(\frac{5\pi}{2}\right)-\frac{4689}{11890}$$

coth(x) is the hyperbolic cotangent function

Exact result:

$$\frac{1}{8}\pi\coth^2\left(\frac{5\pi}{2}\right)-\frac{4689}{11890}$$

Decimal approximation:

−0.001665694195127834328346934326933649713197872426167604420...

-0.00166569419… (note that: 0,0016568327 × 10^3 = 1,6568327 ≈ 14th root of the following Ramanujan's class invariant $Q=\left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...)

Alternate forms:

$$\frac{5945\pi\coth^2\left(\frac{5\pi}{2}\right)-18756}{47560}$$

$$-\frac{4689}{11890}+\frac{\pi\cosh^2\left(\frac{5\pi}{2}\right)}{8\sinh^2\left(\frac{5\pi}{2}\right)}$$

$$\frac{\pi\sinh^2(5\pi)}{8(1-\cosh(5\pi))^2}-\frac{4689}{11890}$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

Alternative representations:

$$\frac{1}{8}\coth^2\left(\frac{5\pi}{2}\right)\pi - \frac{4689}{11\,890} = -\frac{4689}{11\,890} + \frac{1}{8}\pi\left(1 + \frac{2}{-1+e^{5\pi}}\right)^2$$

$$\frac{1}{8}\coth^2\left(\frac{5\pi}{2}\right)\pi - \frac{4689}{11\,890} = -\frac{4689}{11\,890} + \frac{1}{8}\pi\left(i\cot\left(\frac{5 i\pi}{2}\right)\right)^2$$

$$\frac{1}{8}\coth^2\left(\frac{5\pi}{2}\right)\pi - \frac{4689}{11\,890} = -\frac{4689}{11\,890} + \frac{1}{8}\pi\left(-i\cot\left(-\frac{5 i\pi}{2}\right)\right)^2$$

Series representations:

$$\frac{1}{8}\coth^2\left(\frac{5\pi}{2}\right)\pi - \frac{4689}{11\,890} = -\frac{4689}{11\,890} + \frac{1}{8}\pi\left(1 + 2\sum_{k=0}^{\infty} e^{-5(1+k)\pi}\right)^2$$

$$\frac{1}{8}\coth^2\left(\frac{5\pi}{2}\right)\pi - \frac{4689}{11\,890} = -\frac{4689}{11\,890} + \frac{1}{8}\pi\left(1 + 2\sum_{k=1}^{\infty} q^{2k}\right)^2 \quad \text{for } q = e^{(5\pi)/2}$$

$$\frac{1}{8}\coth^2\left(\frac{5\pi}{2}\right)\pi - \frac{4689}{11\,890} = -\frac{4689}{11\,890} + \frac{\left(2 + 100\sum_{k=1}^{\infty}\frac{1}{25+4k^2}\right)^2}{200\pi}$$

Integral representation:

$$\frac{1}{8}\coth^2\left(\frac{5\pi}{2}\right)\pi - \frac{4689}{11\,890} = -\frac{4689}{11\,890} + \frac{1}{8}\pi\left(\int_{\frac{i\pi}{2}}^{\frac{5\pi}{2}} \operatorname{csch}^2(t)\,dt\right)^2$$

From which, after some calculations, we obtain:

-e /((((-1/((25*01)(e^Pi+1)) + 3/((25*81)(e^(3Pi)+1))+5/((31*25)(e^(5Pi)+1))))))+89 -3/5

Input:

$$-\frac{e}{-\frac{1}{(25\times 1)(e^{\pi}+1)} + \frac{3}{(25\times 81)(e^{3\pi}+1)} + \frac{5}{(31\times 25)(e^{5\pi}+1)}} + 89 - \frac{3}{5}$$

Exact result:

$$\frac{442}{5}-\frac{e}{-\frac{1}{25\left(1+e^{\pi}\right)}+\frac{1}{675\left(1+e^{3\pi}\right)}+\frac{1}{155\left(1+e^{5\pi}\right)}}$$

Decimal approximation:

1729.049484368265842244210321378698935950972371542255589862...

1729.04948436…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms:

$$\frac{442}{5}+\frac{20925\, e\left(1+e^{\pi}\right)\left(1-e^{\pi}+e^{2\pi}\right)\left(1-e^{\pi}+e^{2\pi}-e^{3\pi}+e^{4\pi}\right)}{671-1508\, e^{\pi}+2345\, e^{2\pi}-2480\, e^{3\pi}+2480\, e^{4\pi}-1674\, e^{5\pi}+837\, e^{6\pi}}$$

$$\left(296582+104625\, e-666536\, e^{\pi}+1036490\, e^{2\pi}-1096160\, e^{3\pi}+1096160\, e^{4\pi}-739908\, e^{5\pi}+369954\, e^{6\pi}-104625\, e^{1+\pi}+104625\, e^{1+2\pi}+104625\, e^{1+5\pi}-104625\, e^{1+6\pi}+104625\, e^{1+7\pi}\right)/\left(5\left(671-1508\, e^{\pi}+2345\, e^{2\pi}-2480\, e^{3\pi}+2480\, e^{4\pi}-1674\, e^{5\pi}+837\, e^{6\pi}\right)\right)$$

Alternative representations:

$$-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}=89-\frac{3}{5}-\frac{e}{-\frac{1}{25\left(1+e^{180^{\circ}}\right)}+\frac{3}{2025\left(1+e^{540^{\circ}}\right)}+\frac{5}{775\left(1+e^{900^{\circ}}\right)}}$$

$$-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}=89-\frac{3}{5}-\frac{e}{\frac{5}{775\left(1+e^{-5 i \log(-1)}\right)}+\frac{3}{2025\left(1+e^{-3 i \log(-1)}\right)}-\frac{1}{25\left(1+e^{-i \log(-1)}\right)}}$$

$$-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}= -\frac{\exp(z)}{-\frac{1}{25\left(\exp^{\pi}(z)+1\right)}+\frac{3}{(25\times 81)\left(\exp^{3\pi}(z)+1\right)}+\frac{5}{(31\times 25)\left(\exp^{5\pi}(z)+1\right)}}+89-\frac{3}{5} \quad \text{for } z=1$$

Series representations:

$$-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}=\frac{442}{5}-\frac{e}{-\frac{1}{25\left(1+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}+\frac{1}{675\left(1+e^{12\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}+\frac{1}{155\left(1+e^{20\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}}$$

$$-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}=\frac{442}{5}-\frac{\sum_{k=0}^{\infty}\frac{1}{k!}}{-\frac{1}{25\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)}+\frac{1}{675\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{3\pi}\right)}+\frac{1}{155\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{5\pi}\right)}}$$

$$-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}=\frac{442}{5}-\frac{1}{\left(-\frac{1}{25\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)}+\frac{1}{675\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{3\pi}\right)}+\frac{1}{155\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{5\pi}\right)}\right)\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}$$

Integral representations:

$$-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}=\frac{442}{5}-\frac{e}{-\frac{1}{25\left(1+e^{2\int_0^{\infty}1/(1+t^2)\,dt}\right)}+\frac{1}{675\left(1+e^{6\int_0^{\infty}1/(1+t^2)\,dt}\right)}+\frac{1}{155\left(1+e^{10\int_0^{\infty}1/(1+t^2)\,dt}\right)}}$$

$$-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}=$$
$$\frac{442}{5}-\frac{e}{-\frac{1}{25\left(1+e^{2\int_0^{\infty}\sin(t)/t\,dt}\right)}+\frac{1}{675\left(1+e^{6\int_0^{\infty}\sin(t)/t\,dt}\right)}+\frac{1}{155\left(1+e^{10\int_0^{\infty}\sin(t)/t\,dt}\right)}}$$

$$-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}=$$
$$\frac{442}{5}-\frac{e}{-\frac{1}{25\left(1+e^{4\int_0^{1}\sqrt{1-t^2}\,dt}\right)}+\frac{1}{675\left(1+e^{12\int_0^{1}\sqrt{1-t^2}\,dt}\right)}+\frac{1}{155\left(1+e^{20\int_0^{1}\sqrt{1-t^2}\,dt}\right)}}$$

(((-e /((((-1/((25*01)(e^Pi+1)) + 3/((25*81)(e^(3Pi)+1))+5/((31*25)(e^(5Pi)+1))))))+89 -3/5)))^1/15

Input:

$$\sqrt[15]{-\frac{e}{-\frac{1}{(25\times 1)\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}}$$

Exact result:

$$\sqrt[15]{\frac{442}{5}-\frac{e}{-\frac{1}{25\left(1+e^{\pi}\right)}+\frac{1}{675\left(1+e^{3\pi}\right)}+\frac{1}{155\left(1+e^{5\pi}\right)}}}$$

Decimal approximation:

1.643818365130904058673177861364591471405610602132024450844...

$1.64381836513\ldots. \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

Alternate form:

$1/\left(\left(\left(5\left(671-1508\,e^{\pi}+2345\,e^{2\pi}-2480\,e^{3\pi}+2480\,e^{4\pi}-1674\,e^{5\pi}+837\,e^{6\pi}\right)\right)/\left(296582+104625\,e-666536\,e^{\pi}+1036490\,e^{2\pi}-1096160\,e^{3\pi}+1096160\,e^{4\pi}-739908\,e^{5\pi}+369954\,e^{6\pi}-104625\,e^{1+\pi}+104625\,e^{1+2\pi}+104625\,e^{1+5\pi}-104625\,e^{1+6\pi}+104625\,e^{1+7\pi}\right)\right)\text{^}(1/15)\right)$

All 15th roots of 442/5 - e/(-1/(25 (1 + e^π)) + 1/(675 (1 + e^(3 π))) + 1/(155 (1 + e^(5 π)))):

$$\sqrt[15]{\frac{442}{5} - \frac{e}{-\frac{1}{25\left(1+e^{\pi}\right)} + \frac{1}{675\left(1+e^{3\pi}\right)} + \frac{1}{155\left(1+e^{5\pi}\right)}}}\; e^{0} \approx 1.64382 \text{ (real, principal root)}$$

$$\sqrt[15]{\frac{442}{5} - \frac{e}{-\frac{1}{25\left(1+e^{\pi}\right)} + \frac{1}{675\left(1+e^{3\pi}\right)} + \frac{1}{155\left(1+e^{5\pi}\right)}}}\; e^{(2 i \pi)/15} \approx 1.5017 + 0.6686\, i$$

$$\sqrt[15]{\frac{442}{5} - \frac{e}{-\frac{1}{25\left(1+e^{\pi}\right)} + \frac{1}{675\left(1+e^{3\pi}\right)} + \frac{1}{155\left(1+e^{5\pi}\right)}}}\; e^{(4 i \pi)/15} \approx 1.0999 + 1.2216\, i$$

$$\sqrt[15]{\frac{442}{5} - \frac{e}{-\frac{1}{25\left(1+e^{\pi}\right)} + \frac{1}{675\left(1+e^{3\pi}\right)} + \frac{1}{155\left(1+e^{5\pi}\right)}}}\; e^{(2 i \pi)/5} \approx 0.5080 + 1.5634\, i$$

$$\sqrt[15]{\frac{442}{5} - \frac{e}{-\frac{1}{25\left(1+e^{\pi}\right)} + \frac{1}{675\left(1+e^{3\pi}\right)} + \frac{1}{155\left(1+e^{5\pi}\right)}}}\; e^{(8 i \pi)/15} \approx -0.17183 + 1.63481\, i$$

Alternative representations:

$$\sqrt[15]{-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)} + \frac{3}{(25\times 81)\left(e^{3\pi}+1\right)} + \frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}} + 89 - \frac{3}{5}} =$$
$$\sqrt[15]{89 - \frac{3}{5} - \frac{e}{-\frac{1}{25\left(1+e^{180\,^{\circ}}\right)} + \frac{3}{2025\left(1+e^{540\,^{\circ}}\right)} + \frac{5}{775\left(1+e^{900\,^{\circ}}\right)}}}$$

$$\sqrt[15]{-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)} + \frac{3}{(25\times 81)\left(e^{3\pi}+1\right)} + \frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}} + 89 - \frac{3}{5}} =$$
$$\sqrt[15]{89 - \frac{3}{5} - \frac{e}{\frac{5}{775\left(1+e^{-5 i \log(-1)}\right)} + \frac{3}{2025\left(1+e^{-3 i \log(-1)}\right)} - \frac{1}{25\left(1+e^{-i \log(-1)}\right)}}}$$

$$\sqrt[15]{-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)} + \frac{3}{(25\times 81)\left(e^{3\pi}+1\right)} + \frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}} + 89 - \frac{3}{5}} =$$
$$\sqrt[15]{-\frac{\exp(z)}{-\frac{1}{25\left(\exp^{\pi}(z)+1\right)} + \frac{3}{(25\times 81)\left(\exp^{3\pi}(z)+1\right)} + \frac{5}{(31\times 25)\left(\exp^{5\pi}(z)+1\right)}} + 89 - \frac{3}{5}} \quad \text{for } z = 1$$

Series representations:

$$\sqrt[15]{-\frac{e}{-\frac{1}{25(e^{\pi}+1)}+\frac{3}{(25\times 81)(e^{3\pi}+1)}+\frac{5}{(31\times 25)(e^{5\pi}+1)}}+89-\frac{3}{5}} =$$

$$\left(\frac{442}{5}-e\Big/\left(-\frac{1}{25\left(1+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}+\frac{1}{675\left(1+e^{12\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}+\frac{1}{155\left(1+e^{20\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}\right)\right)\hat{}\,(1/15)$$

$$\sqrt[15]{-\frac{e}{-\frac{1}{25(e^{\pi}+1)}+\frac{3}{(25\times 81)(e^{3\pi}+1)}+\frac{5}{(31\times 25)(e^{5\pi}+1)}}+89-\frac{3}{5}} =$$

$$\sqrt[15]{\frac{442}{5}-\frac{\sum_{k=0}^{\infty}\frac{1}{k!}}{-\frac{1}{25\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)}+\frac{1}{675\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{3\pi}\right)}+\frac{1}{155\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{5\pi}\right)}}}$$

$$\sqrt[15]{-\frac{e}{-\frac{1}{25(e^{\pi}+1)}+\frac{3}{(25\times 81)(e^{3\pi}+1)}+\frac{5}{(31\times 25)(e^{5\pi}+1)}}+89-\frac{3}{5}} = \left(\frac{442}{5}-\frac{1}{\left(-\frac{1}{25\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)}+\frac{1}{675\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{3\pi}\right)}+\frac{1}{155\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{5\pi}\right)}\right)\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)\hat{}\,(1/15)$$

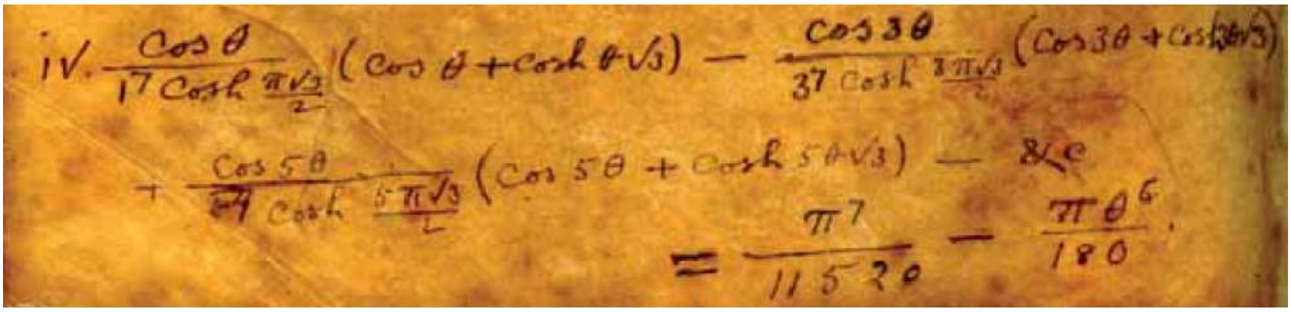
Integral representations:

$$\sqrt[15]{-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}} =$$
$$\sqrt[15]{\frac{442}{5}-\frac{e}{-\frac{1}{25\left(1+e^{2\int_0^\infty \sin(t)/t\,dt}\right)}+\frac{1}{675\left(1+e^{6\int_0^\infty \sin(t)/t\,dt}\right)}+\frac{1}{155\left(1+e^{10\int_0^\infty \sin(t)/t\,dt}\right)}}}$$

$$\sqrt[15]{-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}} =$$
$$\sqrt[15]{\frac{442}{5}-\frac{e}{-\frac{1}{25\left(1+e^{2\int_0^\infty 1/\left(1+t^2\right)dt}\right)}+\frac{1}{675\left(1+e^{6\int_0^\infty 1/\left(1+t^2\right)dt}\right)}+\frac{1}{155\left(1+e^{10\int_0^\infty 1/\left(1+t^2\right)dt}\right)}}}$$

$$\sqrt[15]{-\frac{e}{-\frac{1}{25\left(e^{\pi}+1\right)}+\frac{3}{(25\times 81)\left(e^{3\pi}+1\right)}+\frac{5}{(31\times 25)\left(e^{5\pi}+1\right)}}+89-\frac{3}{5}} =$$
$$\sqrt[15]{\frac{442}{5}-\frac{e}{-\frac{1}{25\left(1+e^{2\int_0^\infty \sin^2(t)/t^2\,dt}\right)}+\frac{1}{675\left(1+e^{6\int_0^\infty \sin^2(t)/t^2\,dt}\right)}+\frac{1}{155\left(1+e^{10\int_0^\infty \sin^2(t)/t^2\,dt}\right)}}}$$

From (page 221)



iv. $\frac{\cos\theta}{1^7\cosh\frac{\pi\sqrt{3}}{2}}(\cos\theta+\cosh\theta\sqrt{3})-\frac{\cos 3\theta}{3^7\cosh\frac{3\pi\sqrt{3}}{2}}(\cos 3\theta+\cosh 3\theta\sqrt{3})$
$+\frac{\cos 5\theta}{5^7\cosh\frac{5\pi\sqrt{3}}{2}}(\cos 5\theta+\cosh 5\theta\sqrt{3})-\&c$
$=\frac{\pi^7}{11520}-\frac{\pi\theta^6}{180}.$

We obtain, for θ = π:

((((Pi^7/11520 - Pi/180*Pi^6))))

Input:

$$\frac{\pi^7}{11520} - \frac{\pi}{180}\pi^6$$

Result:

$$-\frac{7\pi^7}{1280}$$

Decimal approximation:

-16.51722858940433161921831675898772876829845884927173112 44...

-16.517228589… result very near to the mass of the hypothetical light particle, the boson m_X = 16.84 MeV with minus sign

Property:

$-\frac{7\pi^7}{1280}$ is a transcendental number

Alternative representations:

$$\frac{\pi^7}{11520} - \frac{\pi^6\pi}{180} = -°(180°)^6 + \frac{(180°)^7}{11520}$$

$$\frac{\pi^7}{11520} - \frac{\pi^6\pi}{180} = -\frac{1}{180}\cos^{-1}(-1)\cos^{-1}(-1)^6 + \frac{\cos^{-1}(-1)^7}{11520}$$

$$\frac{\pi^7}{11520} - \frac{\pi^6\pi}{180} = \frac{1}{180}\,i\log(-1)(-i\log(-1))^6 + \frac{(-i\log(-1))^7}{11520}$$

Integral representations:

$$\frac{\pi^7}{11520} - \frac{\pi^6\pi}{180} = -\frac{7}{10}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^7$$

$$\frac{\pi^7}{11520} - \frac{\pi^6\pi}{180} = -\frac{7}{10}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^7$$

$$\frac{\pi^7}{11520} - \frac{\pi^6\pi}{180} = -\frac{448}{5}\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^7$$

108(((-(Pi^7/11520 - Pi/180*Pi^6))))-55

Input:

$$108\left(-\left(\frac{\pi^7}{11520}-\frac{\pi}{180}\pi^6\right)\right)-55$$

Result:

$$\frac{189\pi^7}{320}-55$$

Decimal approximation:

1728.86068765567814875578209970674706976233555721346961439...

1728.860687655… ≈ 1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Property:

$-55+\frac{189\pi^7}{320}$ is a transcendental number

Alternate form:

$$\frac{1}{320}\left(189\pi^7-17600\right)$$

Alternative representations:

$$108(-1)\left(\frac{\pi^7}{11520}-\frac{\pi\,\pi^6}{180}\right)-55=-55+108\left(°\,(180\,°)^6-\frac{(180\,°)^7}{11520}\right)$$

$$108(-1)\left(\frac{\pi^7}{11520}-\frac{\pi\,\pi^6}{180}\right)-55=-55+108\left(\frac{1}{180}\cos^{-1}(-1)\cos^{-1}(-1)^6-\frac{\cos^{-1}(-1)^7}{11520}\right)$$

$$108(-1)\left(\frac{\pi^7}{11520}-\frac{\pi\,\pi^6}{180}\right)-55=$$
$$-55+108\left(-\frac{1}{180}\,i\left(\log(-1)\,(-i\log(-1))^6\right)-\frac{(-i\log(-1))^7}{11520}\right)$$

Series representations:

$$108(-1)\left(\frac{\pi^7}{11520}-\frac{\pi\,\pi^6}{180}\right)-55=-55+\frac{48\,384}{5}\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^7$$

$$108(-1)\left(\frac{\pi^7}{11520}-\frac{\pi\,\pi^6}{180}\right)-55=$$
$$-55+\frac{48\,384}{5}\left(\sum_{k=0}^{\infty}\frac{(-1)^{1+k}\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^7$$

$$108(-1)\left(\frac{\pi^7}{11520}-\frac{\pi\,\pi^6}{180}\right)-55=-55+\frac{189}{320}\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^7$$

Integral representations:

$$108(-1)\left(\frac{\pi^7}{11520}-\frac{\pi\,\pi^6}{180}\right)-55=-55+\frac{378}{5}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^7$$

$$108(-1)\left(\frac{\pi^7}{11520}-\frac{\pi\,\pi^6}{180}\right)-55=-55+\frac{378}{5}\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^7$$

$$108(-1)\left(\frac{\pi^7}{11520}-\frac{\pi\,\pi^6}{180}\right)-55=-55+\frac{48\,384}{5}\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^7$$

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64 g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276 e^{-\pi\sqrt{22}} - \cdots, \\ 64 g_{22}^{-24} &= \qquad\qquad 4096 e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} \quad = \qquad 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that *sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

From Wikipedia:

For each positive characteristic there are only a finite number of possible j-invariants of supersingular elliptic curves. Over an algebraically closed field K an elliptic curve is determined by its j-invariant, so there are only a finite number of supersingular elliptic curves. If each such curve is weighted by $1/|\mathrm{Aut}(E)|$ then the total weight of the supersingular curves is $(p-1)/24$. Elliptic curves have automorphism groups of order 2 unless their j-invariant is 0 or 1728, so the supersingular elliptic curves are classified as follows. There are exactly $\lfloor p/12 \rfloor$ supersingular elliptic curves with automorphism groups of order 2. In addition if $p\equiv 3 \bmod 4$ there is a supersingular elliptic curve (with j-invariant 1728) whose automorphism group is cyclic or order 4 unless $p=3$ in which case it has order 12, and if $p\equiv 2 \bmod 3$ there is a supersingular elliptic curve (with j-invariant 0) whose automorphism group is cyclic of order 6 unless $p=2$ in which case it has order 24.

Birch & Kuyk (1975) give a table of all j-invariants of supersingular curves for primes up to 307. For the first few primes the supersingular elliptic curves are given as follows. The number of supersingular values of j other than 0 or 1728 is the integer part of $(p-1)/12$.

prime	supersingular j invariants
2	0
3	1728
5	0
7	1728
11	0, 1728
13	5
17	0,8
19	7, 1728
23	0,19, 1728
29	0,2, 25
31	2, 4, 1728
37	8, $3\pm\sqrt{15}$

References

Manuscript Book I of Srinivasa Ramanujan

Manuscript Book II of Srinivasa Ramanujan