# The nuclear self-energy and the strong equivalence principle 



ORCID: https://orcid.org/0000-0002-0985-1982
Contacts: LENNYUS@,ALICE.IT
Independent researcher
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## Abstract

In the present work I discuss whether the gravito-electric self-energy is a valid approach to study the nuclear structure and the nuclear forces.

In particular I investigate the validity of the strong equivalence principle (SEP) in the atomic nucleus, by assuming that in the nucleus the gravito-electric force $\left(F_{g e}=\frac{G K M m}{R^{2}}\right)$ to be operating and that the potential "self-energy" related to this force to be inversely proportional to the circumference $(2 \pi R)$, with $R$ equal to the nuclear radius observed in the electron scattering experiments.

The new approach here proposed offers an occasion for discussing about the physics and chemistry foundations, in particular about the nature of the nucleus of the atom, which perhaps should be reconsidered in deterministic terms, rather than probabilistic ones.

## - The nuclear radius and the gravito-electric force

The nuclear force operating within the atomic nucleus as responsible for the nuclear stability and for the cohesion of nucleons is, according to the nuclear standard physics, the strong nuclear force.

Though, there are some evidences that appears to deny the existence of this force.

First of all, if the strong nuclear force were responsible for the cohesion of the nucleons, it should be able to explain the nuclear size, which instead remains still without a theoretical motivation.

Moreover, in the reference [1] it has been remarked that the strong nuclear force is denied by the radius 7 fm of halo neutron in Be 11 .

In the mentioned reference we can read that the inexistence of the strong
nuclear force, inferred from an impartial analysis of what really happens in the alpha decay of U238, seems to have been confirmed in 2009 by an experiment, reported in the reference [2], because, while the range of maximum actuation of the strong force is less than 3 fm , the experiment has detected that a halo neutron in Be11 is separated from the rest of the nucleus by a distance of 7 fm .

In the present work we show the proofs of the existence of the force of gravity even within the atomic nucleus, as responsible for the nuclear stability and of the cohesion of nucleons (in fact our theoretical formula of the nuclear radius is perfectly in accordance with the data).

We know from Einstein's theory of relativity that the energy contained in the atomic nucleus is equal to $E=M c^{2}$, where $M$ is the mass of the nucleus.

The mass, in this formula, is understood as the inertial mass, namely it is considered as the inertial resistance to acceleration.

Now, one of the cornerstones of the theory of relativity is the strong equivalence principle (SEP), namely the equivalence between inertial mass and gravitational mass.

One way to theoretically demonstrate this equivalence is to hypothesize that the gravitational mass gives rise to a self-energy, namely a potential energy which depends on the mass of the body squared $\left(M^{2}\right)$.

In the reference [3] the author tries to demonstrate the existence of the selfenergy in the celestial body, by resorting to the PNN formalism, namely a modification of Newtonian potential energy, and the result is that, for the Sun, the ratio $\frac{E}{M c^{2}}$ is equal to $3.52 \times 10^{-6}$, where $E$ is the self-energy of the Sun, obtained by means of the PNN parameter.

In this paper we propose a different way to demonstrate the existence of the self-energy within the atomic nucleus.

As it's known, the gravitational potential energy of a mass body $m$ subjected to the attractive force of gravity exerted by a mass body $M$ is:

$$
\begin{equation*}
U=F_{g} * R \tag{1}
\end{equation*}
$$

where $F_{g}$ is the force of gravity $\frac{G M m}{R^{2}}$ :
Therefore the eq. (1) becomes (the value is considered as positive for the
reasons explained below):

$$
\begin{align*}
& U=\frac{G M m}{R^{2}} * R \\
& U=\frac{G M m}{R} \tag{1a}
\end{align*}
$$

This is the gravitational potential energy of the mass body $m$.
The reason of the direct proportionality between the potential energy and the distance - which we have seen in the equation (1) - rather than the inverse proportionality - which instead we have in the equation of force of gravity - is explained by the fact that in the first case we observe the phenomenon of gravitational attraction in terms of potentiality of the body subjected to a given gravitational force, located at a certain height and free to fall, to affect the surrounding reality, in particular by impacting the ground.

It is a logical consequence of this new vision of the gravitational potential energy that it is mathematically expressed as positive, unlike the traditional definition of the gravitational potential energy in which it is negative, because we have considered the potential energy in a different sense as the traditional way, by we having understood the energy in terms of potentiality of the a body, located at a certain height and free to fall, of affecting the surrounding reality, in particular by impacting the ground, so in this vision the motion of the body has the same sense as that of the force of gravity, whereas, in the traditional way of conceiving the gravitational potential energy, this latter is understood as the work needed to move a body, for instance, from the Earth up to infinite, in which case the sense of the motion of the body is opposite to that of the force, and consequently the potential energy is negative.

So, in this new perspective, we can mathematically express the potential energy of the mass body $m$ as positive, like we have done in the eq. (1a).

It is obvious that the higher up the body is located, the greater its gravitational potential will be, because the damage it will cause to the Earth's soil is the greater, the greater the height from which it begins to fall is (in this case, in fact, a body would reach the Earth's soil with the greater speed, the greater the distance from the Earth).

In this regards it suffices to notice that the ratio $\frac{E}{F_{g}}$ increases as the distance $R$ increases, because $E$ is inversely proportional to $R$, whereas $F_{g}$ is inversely proportional to $R^{2}$.

But if we suppose that in the atomic nucleus there exists an attractiverepulsive field generated by the nucleus itself, and that this field gives rise to a pendulum, in particular to a peculiar harmonic oscillator which implies the revolution around the fixed point, rather than the oscillation, and in which:

1) the center of the nucleus would be the fixed point (fulcrum) of the pendulum;
2) the attractive force would play the same role as that played by the tension of the wire in the Galilean pendulum;
3) the repulsive force - equal in strength to the attractive force, but not aligned to it - would play the same role as that played by the force of gravity exerted on the Galilean pendulum by the Earth;
4) and in which $g=\frac{G M}{l^{2}}$ would be the repulsive gravity acceleration, which would play the same role as that played by the Earth's gravity acceleration on the pendulum, where $l$ is the length of the wire;
it would follow that, by increasing the distance from the center of the nucleus, the repulsive gravity acceleration $g$ decreases, and consequently the formula of potential energy has to change.

If we admit, indeed, that the effect of the attractive-repulsive field is not to make the bodies fall towards the central attractor-repulsor, but to make them move around it at decreasing speed as the distance from the central body increases, according to the formulae of a pendulum in which $g$ is inversely proportional to the square of the length of the wire $\left(l^{2}\right)$, then it would follow that the formula of the gravitational potential energy $(E)$ would be as follows:

$$
\begin{equation*}
E=\frac{F_{g}}{2 \pi R} \tag{2}
\end{equation*}
$$

This time, differently from the eq. (1), the distance $R$ is in the denominator, because, the greater is the distance, the lower will be the linear velocity produced by the attractive-repulsive field, then, in the final analysis, the lower will be the energy of
the orbitating mass body $m$.
In fact, the period $T$ of the pendulum harmonic oscillator is directly proportional to the length $(l)$ of the wire $\left(T=2 \pi * \sqrt{\frac{l}{g}}\right)$, so that it increases if the length increases, and in this case not only the angular velocity of the pendulum, but also its linear velocity (more precisely the tangential velocity) decreases, because above we have assumed that in such a particular type of pendulum, the gravity acceleration $g$ decreases with the increase of the square of the wire's length $\left(g=\frac{G M}{l^{2}}\right)$.

In fact, the formula of the tangential maximum velocity of the pendulum harmonic oscillator is $v=\omega * l$, and, by knowing that the angular velocity of harmonic oscillator is $\omega=\sqrt{\frac{g}{l}}$, its tangential velocity will be $v=\sqrt{\frac{g}{l} * l^{2}}=\sqrt{\frac{G M}{l^{3}} * l^{2}}$ $=\sqrt{\frac{G M}{l}}$ which demonstrates that, in such a particular pendulum, the increase of the wire implies the decrease of the tangential velocity of the pendulum.

In essence, if the linear velocity of pendulum decreases as the distance from the center of the nucleus increases, it means that its energy, in particular the kinetic energy, decreases, therefore, by assuming that the attractive-repulsive field generates a pendulum, in particular a harmonic oscillator, we can infer that the potential energy of a body inserted in such a field decreases as the distance from the central body increases, so that this energy can be mathematically expressed as inversely proportional to the circumference $(2 \pi R)$ described by the orbitating body.

The term $\pi$ is extremely important because from it one can deduce that it's not the case of an exclusively repulsive field, in which the potential energy should be inversely proportional to the distance, not to the circumference.

But the equation (2) must still be modified if to be applied to the atomic nucleus.

Here, in fact, even if we admit that gravity operates, it would not be the only operating force, because it is not possible to neglect the electrostatic one.

Therefore I have supposed that in the atom the force of gravity and the electrostatic force were merged, giving rise to the gravito-electric force $F_{g e}$ (or, if one prefers, electro-gravitational force) having this magnitude:

$$
\begin{equation*}
F_{g e}=\frac{G K M m}{R^{2}} \tag{3}
\end{equation*}
$$

where $K$ is the Coulomb's constant and $G$ is the gravitational constant, so the eq. (2) becomes:

$$
\begin{equation*}
E=\frac{G K M m}{R^{2}} * \frac{1}{2 \pi R} \tag{4}
\end{equation*}
$$

Let's assume that in the nucleus there exists the gravito-electric self-energy, so we have to replace in eq. (4) $m$ with $M$, i.e. with the mass of the nucleus itself, so that the eq. (4) becomes:

$$
\begin{equation*}
E=\frac{G K M^{2}}{2 \pi R^{3}} \tag{5}
\end{equation*}
$$

where $R$ is the nuclear radius detected in the electron scattering experiments: for medium and heavy atoms, $R=1.21 * \sqrt[3]{A} \mathrm{fm}$ (see references [4])

Now, in order to demonstrate the respect of the strong equivalence principle within the nucleus, we have to verify if the energy expressed in eq. (5) is equal to $M c^{2}$, i.e. the total mass-energy, so we can write:

$$
\begin{equation*}
\frac{G K M^{2}}{2 \pi R^{3}}=M c^{2} \tag{6}
\end{equation*}
$$

It's important to specify that $M$ is taken as the mass of the nucleus, intended as the sum of the masses of the protons and of neutrons, without taking into account the binding energy (mass defect), that therefore will not be subtracted from the mentioned sum.

Let's test now the eq. (6), considering the nucleus of bromum atom $\left({ }^{79} \mathrm{Br}\right)$, which contains 35 protons and 44 neutrons, whose radius - according to the empirical formula $R=1.21151 * \sqrt[3]{A} f m$ - is 5.1983 femtometers:
$\frac{\left(6.6743 * 10^{-11}\right) *\left(8.9875 * 10^{9}\right) *\left\{[(35 * 1.6726)+(44 * 1.6749)] * 10^{-27}\right\}^{2}}{2 * 3.1415 *\left(5.1983 * 10^{-15}\right)^{3}}=[(35 * 1.6726)+(44 * 1.6749)] * 10^{-27} * c^{2}$
where $c$ is the speed of light in vacuum: $299,792,458 \mathrm{~m} / \mathrm{sec}$

$$
\begin{gathered}
1.1884 * 10^{-8} \text { joule }=1.1884 * 10^{-8} \text { joule } \\
\frac{E}{M c^{2}}=\frac{1.1884 * 10^{-8}}{1.1884 * 10^{-8}}=1
\end{gathered}
$$

For summary reasons it's not worth repeating here the above calculation for all the atoms, since the empirical formula of the nuclear radius seen above $(R=1.21151 * \sqrt[3]{A} \mathrm{fm})$ is applicable to every medium and heavy atom.

The only further atom that we can consider as a demonstration of the validity of the eq. (6) is the lead atom, the heaviest among the stable atoms.

The nucleus of the lead atom contains 82 protons and 126 neutrons, and its radius, according to the mentioned empirical formula $R=1.21151 * \sqrt[3]{A} \mathrm{fm}$, is 7.1781 fm , hence, applying the eq. (6), we obtain the following values:

$$
\begin{gathered}
\frac{\left(6.6743 * 10^{-11}\right) *\left(8.9875 * 10^{9}\right) *\left\{[(82 * 1.6726)+(126 * 1.6749)] * 10^{-27}\right\}^{2}}{2 * 3.1415 *\left(7.1781 * 10^{-15}\right)^{3}}=[(82 * 1.6726)+(126 * 1.6749)] * 10^{-27} * c^{2} \\
3.1295 * 10^{-8} \text { joule }=3.1293 * 10^{-8} \text { joule } \\
\frac{E}{M c^{2}}=\frac{3.1295 * 10^{-8}}{3.1293 * 10^{-8}}=1,00005
\end{gathered}
$$

The eq. (6) holds again.
We reiterate that $M$ is taken as the mass of the nucleus, intended as the sum of the masses of the protons and of neutrons, without taking into account the mass-defect detected in the nuclear reaction experiments and ascribed, by the nuclear standard physics, to the binding energy of nucleons, which therefore here was not subtracted from the mentioned sum of the nucleonic masses, and, despite this, the equation (6) perfectly holds, and this seems to demonstrate that the mass-defect detected in the nuclear reactions is not the consequence of the mass-energy equivalence principle stated by Einstein's special theory of relativity, but most likely is the effect of the increase of the nuclear radius, which in turn implies the decrease of the nuclear potential energy - which in fact depends, as indicated in the eq. (5), on the variations of the nuclear radius - occurring very probably in the nuclear reactions.

In summary we can state that, according to the relevant results achieved in the eq. (6), the mass of an unbound nucleon (proton or neutron) is not greater than that of a bound nucleon, but is exactly the same, and that the discrepancy detected in the nuclear reactions is not due to a mass defect of a bound nucleons with respect the mass of a free nucleons, but is the consequence of the very probable increase of the nuclear radius occurring during the nuclear reaction which in turn implies, according to the eq. (5), the decrease of the nuclear potential energy, which is very likely responsible for what is detected in the nuclear reactions and interpreted, perhaps mistakenly, as the mass-defect by the nuclear standard physics.

- The gravito-electric force seen in the light of the dimensional analysis
According to the dimensional analysis, the force that in the eq. (3) we have supposed to be existing in the atomic nucleus should not be a force, because it scales as the square of a force over a charge or, in units of the International System, as $\left(\frac{N}{C}\right)^{2}$.

Firstly we can say that the dimensional analysis has empirical bases, hence it cannot reasonably represent the unique obstacle to the validity of a new theoretical equation, especially when, as in our case, the theoretical mathematical result is perfectly equal to the empirical one.

Anyway, even though we consider the dimensional analysis as a substantial issue, it is possible to overcome it simply by to considering $K$ as a dimensionless quantity precisely equal to the numerical value of the Coulomb's constant (8.9875* $10^{9}$ ), so that the new value of the gravitational constant, only operating at microscopic scale, would be:

$$
G_{k}=6.6743 * 10^{-11} * 8.9875 * 10^{9} \mathrm{Nm}^{2} \mathrm{~kg}^{-2}
$$

Consequently the mathematical expression of the gravito-electric force, seen in the eq. (3), would become:

$$
F_{g e}=\frac{G_{k} M m}{R^{2}}
$$

We reiterate that $G_{k}$ is the value of the gravitational constant that is only operating at microscopic scale, in particular at nuclear scale (at least until further studies will not confirm it even at atomic and molecular scale).

It's important to emphasize that the $8.9875 * 10^{9}$ parameter is a number with a precise physical meaning, because it is the numerical value of the Coulomb's constant, therefore it would not be possible to argue that the mentioned number be an ad hoc parameter arbitrarily chosen to fit the data.

The mathematical identity demonstrated in the eq. (6) is so perfect that an underlying physical meaning subsists beyond any reasonable doubt.

But what is this underlying physical meaning of the eq. (6)?
In the next paragraph we'll try to discuss this issue.

## - Nuclear self-energy or self-orbitating particles?

The result achieved above gives rise to a philosophical question.
How to interpret the eq. (5)?
Does it contain the mathematic expression of the nuclear potential selfenergy, or does it contain the potential energy of self-orbitating particles (i.e. the nucleons)?

In other words the fact that the energy expressed by the eq. (5) depends on the mass of nucleons squared, could also mean that they stay both in the center of the nucleus and, at same time, in orbit around it, because we have replaced in the eq. (5) the mass $m$ - which denotes the orbiting body, having a very small mass with respect to the central one - with the mass $M$, that is the total mass of nucleons.

If we accept the second hypothesis (self-orbitating particles), there would be non-irrelevant consequences on the foundations of physics, to be understood as the philosophical bases of this particular science, because this would mean that the nucleons would have precise trajectory and velocity in while they are orbitating about the center of the nucleus (occupied by their at-rest alter ego).

In this weird scenario, one would have to accept not only the idea that the nucleons stay in two places at the same time, but also the fact that they are both at rest, in the center of nucleus, and revolving at same time around this point, with the specification that, when they are moving, they would do at the speed of light at a distance equal to the nuclear radius.

In this framework, in fact, the right-hand side of the eq. (6) would be twice the kinetic energy of the nucleons $\left(2 * \frac{1}{2} M c^{2}=M c^{2}\right)$.

From the planetary orbits, indeed, we know that the orbit will be as stable as possible whether the gravitational potential energy will be equal to twice the kinetic energy of the planet.

In our solar system we have in particular that, for each planet, the following relation is operating:

$$
\begin{equation*}
U=2 E_{k} \tag{7}
\end{equation*}
$$

where $U$ is the gravitational potential energy and $E_{k}$ is the kinetic energy of the planet, which is equal to $E_{k}=\frac{1}{2} m v^{2}$, where $m$ is the mass of the planet.

By knowing that $U$ is equal to $m * g * R$, the eq. (7) becomes:

$$
\begin{gather*}
m * g * R=2\left(\frac{1}{2} m v^{2}\right) \\
\rightarrow m * g * R=z\left(\frac{1}{z} m v^{2}\right) \\
\rightarrow g * R=v^{2} \\
\rightarrow \frac{G M}{R^{2}} * R=v^{2} \\
\rightarrow \frac{G M}{R}=v^{2} \\
v=\sqrt{\frac{G M}{R}} \tag{8}
\end{gather*}
$$

which is the velocity necessary to have a circular orbit, namely the most stable orbit (where $M$ is the mass of the Sun).

After all, from the eq. (6) it is possible to derive the theoretical value of $c$ :

$$
c=\sqrt{\frac{G K M}{2 \pi R^{3}}}
$$

which is not very different from the planetary orbital velocity seen in the eq. (8).

Furthermore in a recent research [5] it has been experimentally shown that the missing momentum of a knockout proton, in some collisions, can be up to $1,000 \mathrm{Mev} / \mathrm{c}$, in contrast with the previous experiments, from which the value of the missing momentum turned out to be $250 \mathrm{Mev} / \mathrm{c}$.

The value of $1,000 \mathrm{Mev} / \mathrm{c}$ is very high and could be well-justified by assuming that the nucleons move within the nucleus at the speed of light, or at a speed which is approaching it.

Moreover, in the mentioned research it has been shown that in the nucleus not only an attractive force exists, but also a repulsive force, and it is very likely that these two opposed forces are not aligned and this consequently gives life to the particular pendulum descripted in this work, which guarantees the dynamical equilibrium of nucleonic orbits.

In this regard it is important to remark the similarity of the nuclear model here proposed with the interatomic and intermolecular chemical bond, by being the balance between attractive and repulsive forces a fundamental feature of all these systems (see reference [9]).

## - Is the virial theorem always valid?

The virial theorem (by R. Clausius, 1870) states, for a central potential $\langle\phi\rangle(\vec{R})=\phi(R) \propto \pm R^{ \pm b}$, that:

$$
\begin{equation*}
\left\langle E_{K}\right\rangle= \pm \frac{b}{2} \cdot\langle\phi\rangle \tag{9}
\end{equation*}
$$

where $\langle\phi\rangle$ is the average over time of the potential energy, $\left\langle E_{K}\right\rangle$ is the average over time of the kinetic energy and $b$ is the exponent of the radius as it appears in the formula of the potential energy.

Since the gravitational potential energy, according to its synthetical formula, is inversely proportional to the distance $\left(U=\frac{G M}{R}\right)$, then the exponent of the radius is $b=-1$ and the eq. (9) becomes:

$$
\left\langle E_{K}\right\rangle=-\frac{1}{2} \cdot\langle\phi\rangle
$$

Yet, in the light of the result reached in eq. (5), which denotes quite indisputably the nuclear potential energy, the virial theorem [eq. (9)] doesn't hold.

Indeed, applying the eq. (9) and considering that the nuclear gravitoelectric potential energy, as expressed in eq. (5) (even if considered as negative), is inversely proportional to $R^{3}$, the virial theorem would lead to:

$$
\begin{aligned}
\left\langle E_{K}\right\rangle & =-\frac{3}{2} \cdot\langle\phi\rangle \\
\rightarrow \frac{1}{2} M c^{2} & =-\frac{3}{2} \cdot\left(-\frac{G K M^{2}}{2 \pi R^{3}}\right)
\end{aligned}
$$

Multiplying both member by 2 :

$$
\rightarrow \quad M c^{2}=\frac{3 G K M^{2}}{2 \pi R^{3}}
$$

which is not true.
In fact, if we again apply the above equation to the bromum atom ${ }^{79} \mathrm{Br}$, it leads to:

$$
\rightarrow \frac{M c^{2}}{\frac{3 G K M^{2}}{2 \pi R^{3}}}=\frac{1.1884 * 10^{-8}}{3.5652 * 10^{-8}} \neq 1
$$

At this point, the fact that the virial theorem doesn't hold for the nuclear gravitoelectric potential energy can be explained in two different ways.

The first is to assert that the eq. (5) doesn't contain the nuclear potential
self-energy, and consequently that $M c^{2}$ wouldn't represent twice the kinetic energy of nucleons, but would be, as the theory of relativity states, the total mass-energy of nucleons, more precisely the energy that the nucleons contains for the very fact of having a mass, even if they are at rest.

This interpretation, yet, doesn't allow to explain which would be the physical meaning of the perfect mathematical identity given by the eq. (6), which, consequently, should be ascribed, we repeat, only to the fortuity, nothing short of unrealistically.

The second possibility is to claim that the virial theorem, as formulated in eq. (9), is incorrect, and that the correct law would be:

$$
\begin{equation*}
\left\langle E_{K}\right\rangle=\frac{1}{2} \cdot\langle\phi\rangle \tag{10}
\end{equation*}
$$

This interpretation is based on the fact that the virial theorem is an ad hoc solution, valid only in the case that the force of gravity were inversely proportional to the square of the distance.

Though, this is a fact that has never been explained logically, mathematically or geometrically, in essence scientifically, in particular nobody has never demonstrated the reason why the force of gravity can't be other than inversely proportional to the distance squared.

Consequently one can argue, in abstract, that, if the gravitational force were, for instance, inversely proportional to the fourth power of the distance, the theorem would fail, as we'll show shortly.

In fact, in the case that the force of gravity were $F=\frac{G M m}{R^{4}}$, the kinetic energy needed to have a stable orbit, applying the virial theorem, would turn out to be greater than the potential energy.

In particular, supposing that in the mentioned hypothesis the force of gravity to be only attractive, then the gravitational potential energy, expressed as positive as we have explained above, would be:

$$
U=\frac{G M m}{R^{4}} \cdot R=\frac{G M m}{R^{3}}
$$

Consequently the exponent of the radius that would appear in the eq. (9) would be $b=-3$, so that the necessary condition to have a stable orbit would turn out to be:

$$
\begin{aligned}
&\left\langle E_{K}\right\rangle=-\frac{3}{2} \cdot\langle\phi\rangle \\
& \rightarrow \quad \frac{1}{2} m v^{2}=\frac{3}{2} \cdot \frac{G M m}{R^{3}}
\end{aligned}
$$

but this is impossible because the kinetic energy of the mass body $m$ would be greater than its potential energy $\left(E_{K}=1.5 \cdot U\right)$, and we know that in such a condition the orbit will be hyperbolic.

The same result would turn out in the case that the force of gravity were inversely proportional to the third power of the distance, in which case, applying the virial theorem, the most stable orbit would occur if the kinetic energy were equal to the potential energy, but it is well-known that in this case the orbiting body would reach the escape velocity, so the virial theorem would fail again.

The virial theorem, therefore, is implicitly based on a premise (namely the fact that the force of gravity can't be other than inversely proportional to the square of the distance) which is not logically demonstrable, and this implies that it cannot be considered a theorem in the proper sense of the term, because a theorem is, by definition, a proposition which can be scientifically demonstrated, and this also holds for its logical premises.

Consequently one should admit that the eq. (9) would be replaced by the eq. (10), and that this latter would apply in any case, both when the object (body or particle) is subjected to only one attractive gravitational force, and when it is subjected to two gravitational forces (attractive and repulsive) at same time, regardless of the mathematical configuration of the potential energy (namely, regardless of the exponent of radius, $b$, appearing in the formula of the potential energy).

In other words, in this scenario one should admit that the eq. (10) to be a fundamental principle of Nature, in the sense that it wouldn't have any mathematical derivation, but should be accepted as it is.

After all, there are some aspects of the force of gravity that are not entirely explainable, just think of the fact, we repeat, that it depends, without any apparent logical reason, on the inverse of the square - rather than on the inverse of the cube or of the fourth power - of the distance, or rather than simply on the inverse of the distance.

However the aim of this paper is not getting into the details of the debate between those who believe in the existence of the fundamental laws of Nature, and those who believe that the physical laws are created by humans to describe the reality and consequently that every natural law should be explainable in the light of the reason, but it's undeniable that the answer to the question here proposed depends on the way of solving this dispute.

The only thing that I can say in this regard is that the deductive method doesn't seem the best way of approaching the force of gravity, as it is shown by the paradoxical results of the virial theorem seen above.

The inductive method, on the contrary, by starting from the single cases in order to deduce, case by case, the existence of a general principle, seems to be more suitable to study the issues related to the force of gravity, which, as for every phenomenological entity, isn't a-priori knowable in its every single aspect.

Obviously, the latter considerations would fail if we believe, as Einstein teaches, that the force of gravity is a geometrical entity, which would find its logical primary cause in the spacetime, in particular in its curvature, but we have already said that this is not entirely true, at least until the force of gravity will continue to receive no geometrical, logical, mathematical, scientific explanation with regard to the fact that it can't be other than inversely proportional to the square of the distance.

## - Relative facts and absolute self-facts

In the reference [6] the authors distinguish relative facts from stable facts, and conclude that the stable facts are only a subset of the more general category of relative facts.

According to this theory, called relational quantum mechanics (RMQ), relative facts are also those concerning the particles that are in two superimposed states, or even the particles that are demonstrated to be ubiquitous, which instead are stable according to quantum mechanics because they are ubiquitous as ubiquitous the decoherence is.

In essence, according to RQM, "Schrodinger's cat has no reason to feel superimposed", because this situation is similar as the man in Einstein's elevator,
which doesn't feel that the elevator, in which he stays, is moving in the interagalactic space, where the absence of gravity is assumed, with uniform linear accelerated motion, but thinks that the elevator is coming up and that he, together with the lift, is subjected to the gravitational force.

No matter what the observer sees, the important thing is what the observed feels, what he perceives.

Consequently, if Schrodinger's cat doesn't feel any change after the measurement, then it means that, to cat, nothing has changed, in the sense that, after the measurement, it feels to be in a single state and doesn't perceive any difference with respect to the superimposition situation in which it was before the measurement.

If nothing has changed, it means that no wave function collapse has occurred.

A logical corollary of this fundamental conclusion is that a fact is absolute when the relationality is not possible, namely when observer and observed coincide.

In particular it is possible to arrive at the conclusion that no wave function collapse occurs even by assuming that the equation (5) expresses the potential energy of self-orbitating particles (nucleons), rather than the self-energy of the nucleus.

In this framework, in fact, we have assumed that the nucleons revolve around themselves, but this means that the nucleons are observers and observed at same time.

In particular, the orbiting nucleons are revolving particles with respect to their central alter ego, but these latter are not different and separated particles from the orbitating ones: are the nucleons themselves.

Analogously, the central nucleons are at-rest with respect to their orbitating alter-ego, but these latter are not different and separated particles from the central ones: are the nucleons themselves.

We can conclude, hence, that the nucleus constitutes a self-system, meaning that the nucleons are observers and observed at same time, and, in this case, the relationality isn't possible anymore.

In fact, claiming that every system is always relative to another one, and consequently that it cannot ever be absolute, holds until observer and observed are different and separated objects or systems, but obviously doesn't apply when observer and observed coincide.

In this particular case, we deal with systems (more precisely self-systems) originating absolute facts, because the relationality, as necessary requisite for a fact to be relative, lacks.

But this does not invalidate the aforementioned principle of relationality of quantum world stated by RQM, rather it is an exception to this principle that confirms its validity, since this exception is justified by the absence, in the nucleons, of a necessary requisite for the relationality to be operating, namely the material separation between observer and observed.

If the nucleons constitute a self-system originating only absolute facts, it means that their wave function cannot collapse, because absolute facts, by definition, cannot collapse, and this is the reason why we are able to see the proofs of this superimposition, as we'll see later.

Finding the proof of superimposed states is fundamental to demonstrate that this phenomenon really occurs before the measurement.

In other words, are we really sure that two entangled photons or electrons are really superimposed before measurement?

The question arises because, when we measure (namely observe) one photon entangled to another photon, both of them are never found superimposed, in the sense that the entangled photons manifest themselves in only one state (for instance only the spin "up" or only the spin "down"), even if opposed with respect to each other, but never in two states simultaneously.

But the fact that there is the absolute certainty that, when we measure a photon, the non-observed entangled photon has the opposite spin with respect to the observed photon doesn't necessary mean that the two photons were superimposed before measurement, and that, due to the measurement, they have collapsed in only one status, because we can also reasonably argue that the two photons were moving in that strange, entangled way even before the measurement, meaning that they were moving in such a way to have in every instant an opposite
spin with respect to each other, namely changing their spin continuously, instant by instant, hence it's obvious that they show always opposite spin after measurement.

From another conceptual point of view, having two spin simultaneously, for instance up and down, doesn't mean that the states of a particle are superimposed, because being superimposed means being and not being at same time in a certain situation, as we'll say better shortly, with the consequence that a particle would be superimposed only if it had a spin and simultaneously no spin, not also when it had two contrary spin at same time.

Moreover, to have the absolute certainty that the two photons were superimposed before the measurement we should observe them in this superimposed state.

Well, in this regard we can say that the nucleons represent a case in which this is possible.

Indeed it has been shown that the nuclear size is bigger than that resulting from the electron scattering experiments.

In particular it has been demonstrated, see reference [7], that a beam of incident particles hitting a target nucleus is both diffracted and absorbed, and, when the absorption is maximum, the scattering cross section and the absorption cross section are identical, so that the total cross section, given by the sum between the two cross sections, is twice the scattering cross section.

In particular the particles beam is $50 \%$ diffracted and $50 \%$ absorbed, meaning that the nuclear dimension is twice that detected in the scattering experiments, and that the innermost part of nucleus is positively charged, whereas the outermost part is neutral.

This can be well-explained by assuming that the nucleons are self-orbitating particles which are globally charged in while they are at-rest and, at same time, electrically neutral in while they are in orbit.

In essence, the nucleons are in a double superimposed state, namely, they are both at rest and, at same time, in orbit, with the specification that, they are (positively) charged when they are at-rest, and uncharged when in orbit; obviously all these considerations holds under the assumption that the repulsive force among protons is neutralized by the neutrons.

And this two superimpositions are both of them detectable in the experiments, descripted in the mentioned reference [7].

But in order to justify the cited experiments in the light of the gravitoelectric force and gravitoelectric energy proposed in this paper, it's necessary to modify the eq. (5) as follows:

$$
\begin{equation*}
E=\frac{4 G K M^{2}}{\pi R^{3}} \tag{11}
\end{equation*}
$$

So the eq. (6) becomes:

$$
\frac{4 G K M^{2}}{\pi R^{3}}=M c^{2}
$$

In this way we obtain a nuclear radius $\left(R=\sqrt[3]{\frac{4 G K M}{\pi c^{2}}}\right)$ which is exactly twice the radius observed in the electron scattering experiment, and therefore we manage to explain the real, total size of the nucleus resulting from both the electron scattering phenomenon and the absorption phenomenon descripted in the reference [7], provided that we assume that the orbitating alter ego of nucleons to be electrically neutral and that the repulsive force among the central protons is neutralized by the neutrons, but accepting the eq. (11) implies to accept the containing-energy concept as defined in the reference [8], where it is clarified the reason of the adjunct of 4 in the numerator and the lack of 2 in the denominator of eq. (5).

But why can we detect only superimposed states concerning nucleons and not also those concerning photons, or in general, entangled particles?

This question has two possible answers.
The first is to think that the wave function of nucleons, as we have already said, cannot collapse because it involves objects which originate only absolute facts.

The second is to think that the wave function doesn't physically exist, in the sense that it is only a mathematical artifice and, consequently, the superimposed states which are not detected, but only supposed, have to be considered inexistent until they are experimentally demonstrated.

After all, "entangled" doesn't mean superimposed, but just means "united", "linked" to each other, in the sense that, by measuring only one particle, also the
other is immediately affected.
As regards the feature of particles' ubiquity, which is shown in the double slits experiment, again it doesn't mean that these particles are superimposed, because being everywhere doesn't mean being simultaneously in two superimposed, opposed states.

Being superimposed means being in two contrary states in the same instant, namely two states which contradict one another, for instance at rest and in movement, charged and uncharged, having a spin and not having a spin, dead and alive, but if a particle moves toward two slits, and passes simultaneously in these two slits, it doesn't mean that the particle was superimposed, but only that, in while it was moving toward the slits, it was not concentrated in only one point, but was everywhere, yet this is a different situation from the superimposition paradox, and can be also explained by resorting to the pilot wave concept of De Broglie-Bohm.

Anyway the aim of the present paper is seeking to give a response only to the superimposition paradox in microscopic mechanics, and how to understand when it occurs, so we don't go here in the details of the debate concerning the possible interpretations of double slits experiment, which, we repeat, denotes weirdness, but not paradoxicalness.

The only thing that we can say in concluding this study is that considering the nucleons as objects originating absolute facts can represent a useful tool to conceptually motivate not only the fact that they remain superimposed even after the measurement and to elucidate the experiments reported in reference [7], but even to justify some other absolute facts.

In particular, if we accept the existence of self-systems, then we should accept even that the facts they produce can't be other than absolute, for instance the constancy of the speed of light, which is independent from any observer.

The endorsement of the idea that the photons can produce absolute facts could be supported by arguing that they are in a certain way related to protons, in particular if we think about the possibility that their mass could be equal to the proton mass squared, as it is better shown again in the reference [8].

## - Conclusions

This study has revealed that the self-energy approach is a valid way to study the nuclear structure and the nuclear forces.

In particular the demonstration of the validity of strong equivalence principle even within the atomic nucleus confirms that the Einstein's theory of relativity can work even at this scale.

Anyway the self-energy approach is not the solely possible way to interpret our theoretical achievements, by being also possible to argue that the nucleons are self-orbitating particles which revolve around themselves at the speed of light, and, in this latter case, the foundations of chemical physics regarding the atomic nucleus, as well as those concerning the theory of relativity, could be questioned.

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