The nuclear self-energy and the strong equivalence principle

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Abstract

In the present work I discuss whether the gravito-electric self-energy is a valid approach to study the nuclear structure and the nuclear forces.

In particular I investigate the validity of the strong equivalence principle (SEP) in the atomic nucleus, by assuming that in the nucleus the gravito-electric force $\left(F_{ge} = \frac{GKMm}{R^2}\right)$ to be operating and that the potential "self-energy" related to this force to be inversely proportional to the circumference $(2\pi R)$, with R equal to the nuclear radius observed in the electron scattering experiments.

The new approach here proposed offers an occasion for discussing about the physics foundations, in particular about the nature of the nucleus of the atom, which perhaps should have to be reconsidered in deterministic terms, rather than probabilistic ones.

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- **The nuclear radius and the gravito-electric force**

We know from Einstein's theory of relativity that the energy contained in the atomic nucleus is equal to $E = Mc^2$, where M is the mass of the nucleus.

The mass, in this formula, is understood as the inertial mass, namely it is considered as the inertial resistance to acceleration.

Now, one of the cornerstones of the theory of relativity is the strong equivalence principle (SEP), namely the equivalence between inertial mass and gravitational mass.

One way to theoretically demonstrate this equivalence is to hypothesize that the gravitational mass gives rise to a self-energy, namely a potential energy which depends on the mass of the body squared (M^2).

In the reference [1] the author tries to demonstrate the existence of the self-energy in the celestial body, by resorting to the PNN formalism, namely a modification of Newtonian potential energy, and the result is that, for the Sun, the ratio $\frac{E}{Mc^2}$ is equal to 3.52×10^{-6}, where E is the self-energy of the Sun, obtained by means of the PNN parameter.

In this paper we propose a different way to demonstrate the existence of the self-energy within the atomic nucleus.

As it's known, the gravitational potential energy of a body subjected to the attractive force of gravity is:

$$U = F_g * R \tag{1}$$

where F_g is the force of gravity $\frac{GMm}{R^2}$:

Therefore the eq. (1) becomes:

$$U = \frac{GMm}{R^2} * R$$

$$U = \frac{GMm}{R}$$

If we consider the mass m as negligible with respect to the mass M, we have that the potential energy of a massless point orbitating about a greater body with mass M, will be:

$$U = \frac{GM}{R}$$

The reason of the direct proportionality between the potential energy and the distance — which we have seen in the equation (1) — rather than the inverse proportionality — which instead we have in the equation of force of gravity — is explained by the fact that in the first case we observe the phenomenon of gravitational attraction in terms of the potentiality of the body subjected to a given gravitational force, located at a certain height and free to fall, to affect the surrounding reality, in particular by impacting the ground.

It is obvious that the higher up the body is located, the greater its gravitational potential will be, because the damage it will cause to the Earth's soil is the greater, the greater the height from which it begins to fall is (in this case, in fact, a body would reach the Earth's soil with the greater speed, the greater the distance from the Earth).

But if we suppose that in the atomic nucleus there exists an attractive-repulsive field generated by the nucleus itself, and that this field gives rises to a pendulum, in particular to a peculiar harmonic oscillator which implies the revolution around the fixed point, rather than the oscillation, and in which:

1) the center of the nucleus would be the fixed point (*fulcrum*) of the pendulum;
2) the attractive force would play the same role as that played by the tension of the wire in the Galilean pendulum;
3) the repulsive force — equal in strength to the attractive force, but not aligned to it — would play the same role as that played by the force of gravity exerted on the Galilean pendulum by the Earth;
4) and in which $g = \frac{GM}{l^2}$ would be the repulsive gravity acceleration, which would play the same role as that played by the Earth's gravity acceleration on the pendulum, where l is the length of the wire;

it would follow that, by increasing the distance from the center of the nucleus, the repulsive gravity acceleration g decreases, and consequently the formula of potential energy has to change.

If we admit, indeed, that the effect of the attractive-repulsive field is not to make the bodies fall towards the central attractor-repulsor, but to make them move around it at decreasing speed as the distance from the central body increases, according to the formulae of a pendulum in which g is inversely proportional to the square of the length of the wire (l^2), then it would follow that the formula of the gravitational potential energy (E) would be as follows:

$$E = \frac{F_g}{2\,\pi R} \tag{2}$$

This time, differently from the eq. (1), the distance R is in the denominator, because, the greater is the distance, the lower will be the linear velocity produced by the attractive-repulsive field, then, in the final analysis, the lower will be the energy of the orbitating mass body m.

In fact, the period T of the pendulum harmonic oscillator is directly proportional to the length (l) of the wire $\left(T = 2\,\pi * \sqrt{\frac{l}{g}}\right)$, so that it increases if the length increases, and in this case not only the angular velocity of the pendulum, but also its linear velocity (more precisely the tangential velocity) decreases, because above we have assumed that in such a particular type of pendulum, the gravity acceleration g decreases with the increase of the square of the wire's length ($g = \frac{GM}{l^2}$).

In fact, the formula of the tangential maximum velocity of pendulum is $v = \omega * l$, and, by knowing that the angular velocity of harmonic oscillator is $\omega = \sqrt{\frac{g}{l}}$, its tangential velocity will be $v = \sqrt{\frac{g}{l} * l^2} = \sqrt{\frac{GM}{l^3} * l^2} = \sqrt{\frac{GM}{l}}$ which demonstrates that, in such a particular pendulum, the increase of the wire implies the decrease of the tangential velocity of the pendulum.

In essence, if the linear velocity of pendulum decreases with the distance from the center of the nucleus, it means that its energy, in particular the kinetic energy, decreases, therefore, by assuming that the attractive-repulsive field generates a pendulum, in particular a harmonic oscillator, we can infer that the potential energy of a body inserted in such a field decreases as the distance from the central body increases, so that this energy can be mathematically expressed as inversely proportional to the circumference ($2\pi R$) described by the orbitating body.

The term π is extremely important because from it one can deduce that it's not the case of an exclusively repulsive field, in which the potential energy should be inversely proportional to the distance, not to the circumference.

But the equation (2) must still be modified if to be applied to the atomic nucleus.

Here, in fact, even if we admit that gravity operates, it would not be the only operating force, because it is not possible to neglect the electrostatic one.

Therefore I have supposed that in the atom the force of gravity and the electrostatic force were merged, giving rise to the *gravito-electric* force F_{ge} (or, if one prefers, *electro-gravitational* force) having this magnitude:

$$F_{ge} = \frac{GKMm}{R^2} \tag{3}$$

where K is the Coulomb's constant and G is the gravitational constant, so the eq. (2) becomes:

$$E = \frac{GKMm}{R^2} * \frac{1}{2\,\pi R} \tag{4}$$

Let's assume that in the nucleus there exists the gravito-electric **self**-energy, so we have to replace in eq. (4) m with M, i.e. with the mass of the nucleus itself, so that the eq. (4) becomes:

$$E = \frac{GKM^2}{2\,\pi R^3} \qquad (5)$$

where R is the nuclear radius detected in the electron scattering experiments: for medium and heavy atoms, $R = 1.21 * \sqrt[3]{A}\ fm$ (see references [2])

Now, in order to demonstrate the respect of the strong equivalence principle within the nucleus, we have to verify if the energy expressed in eq. (5) is equal to Mc^2, i.e. the total mass-energy, so we can write:

$$\frac{GKM^2}{2\,\pi R^3} = Mc^2 \qquad (6)$$

Let's test now the eq. (6), considering the nucleus of bromum atom (^{79}Br), which contains 35 protons and 44 neutrons, whose radius — according to the empirical formula $R = 1.21151 * \sqrt[3]{A}\ fm$ — is 5.1983 $femtometers$:

$$\frac{(6.6743*10^{-11})*(8.9875*10^{9})*\{[(35*1.6726)+(44*1.6749)]*10^{-27}\}^2}{2*3.1415*(5.1983*10^{-15})^3} = [(35*1.6726)+(44*1.6749)]*10^{-27}*c^2$$

where c is the speed of light in vacuum: $299{,}792{,}458\ m/sec$

$$1.1884 * 10^{-8}\ joule = 1.1884 * 10^{-8}\ joule$$

$$\frac{E}{mc^2} = \frac{1.1884 * 10^{-8}}{1.1884 * 10^{-8}} = 1$$

- **Nuclear self-energy or self-orbitating particles?**

The result achieved above gives rise to a philosophical question.

How to interpret the eq. (5)?

Does it contain the mathematic expression of the potential self-energy, or does it contain the potential energy of self-orbitating particles (i.e. the nucleons)?

In other words the fact that the energy expressed by the eq. (5) depends on the mass of nucleons squared, could also mean that they stay both in the center of the nucleus and, at *same time*, in orbit around it, because we have replaced in the eq. (5) the mass m — which denotes the orbiting body, having a very small mass with respect to the central one — with the mass M, that is the total mass of nucleons.

If we accept the second hypothesis (self-orbitating particles), there would be non-irrelevant consequences on the foundations of physics, to be understood as the philosophical bases of this particular science, because this would mean that the nucleons would have precise trajectory and velocity in while they are orbitating about the center of the nucleus (occupied by their at-rest alter ego).

In this weird scenario, one would have to accept not only the idea that the nucleons stay in two places at the same time, but also the fact that they are both at rest, in the center of nucleus, and revolving at same time around this point, with the specification that, when they are moving, they would do at the speed of light at a distance equal to the nuclear radius.

In this framework, in fact, the right-hand side of the eq. (6) would be twice the kinetic energy of the nucleons ($2 * \frac{1}{2}\, mc^2 = mc^2$).

From the planetary orbits, indeed, we know that the orbit will be as stable as possible whether the gravitational potential energy will be equal to twice the kinetic energy of the planet.

In our solar system we have in particular that, for each planet, the following relation is operating:

$$U = 2\, E_k \qquad (7)$$

where U is the gravitational potential energy and E_k is the kinetic energy of the planet, which is equal to $E_k = \frac{1}{2}\, mv^2$

By knowing that U is equal to $m * g * R$, the eq. (7) becomes:

$$m * g * R = 2\left(\frac{1}{2}\, mv^2\right)$$

$$\rightarrow \cancel{m} * g * R = \cancel{2}\left(\frac{1}{\cancel{2}}\, \cancel{m}v^2\right)$$

$$\rightarrow g * R = v^2$$

$$\rightarrow \frac{GM}{R^{\cancel{2}}} * \cancel{R} = v^2$$

$$\rightarrow \frac{GM}{R} = v^2$$

$$v = \sqrt{\frac{GM}{R}} \qquad (8)$$

which is the velocity necessary to have a circular orbit, namely the most stable orbit.

After all, from the eq. (6) it is possible to derive the theoretical value of c:

$$c = \sqrt{\frac{GKM}{2\,\pi R^3}}$$

which is not very different from the planetary orbital velocity seen in the eq. (8).

Furthermore in a recent research [3] it has been experimentally shown that the missing momentum of a knockout proton, in some collisions, can be up to 1,000 Mev/c, in contrast with the previous experiments, from which the value of the missing momentum turned out to be 250 Mev/c.

The value of 1,000 Mev/c is very high and could be well-justified by assuming that the nucleons move within the nucleus at the speed of light, or at a speed which is approaching it.

Moreover, in the mentioned research it has been shown that in the nucleus not only an attractive force exists, but also a repulsive force, and it is very likely that these two opposed forces are not aligned and this consequently gives rise to the particular pendulum descripted in this work.

- **Is the virial theorem always valid?**

The virial theorem (by R. Clausius, 1870) states, for a central potential $\langle \phi \rangle \left(\vec{R}\right) = \phi\,(r) \propto \pm\, R^{\pm b}$, that:

$$\langle E_K \rangle = \pm \frac{b}{2} \cdot \langle \phi \rangle \tag{9}$$

where $\langle \phi \rangle$ is the average over time of the potential energy, $\langle E_K \rangle$ is the average over time of the kinetic energy and b is the exponent of the radius as it appears in the formula of the potential energy.

Since the gravitational potential energy, according to its synthetical formula, is inversely proportional to the distance ($U = \frac{GM}{r}$), then the exponent of the radius is $b = -1$ and the eq. (9) becomes:

$$\langle E_K \rangle = -\frac{1}{2} \cdot \langle \phi \rangle$$

Yet, in the light of the result reached in eq. (5), which denotes quite indisputably the nuclear potential energy, the virial theorem [eq. (9)] doesn't hold.

Indeed, applying the eq. (9) and considering that the nuclear gravitoelectric potential energy, as expressed in eq. (5), is inversely proportional to R^3, the virial theorem would lead to:

$$\langle E_K \rangle = -\frac{3}{2} \cdot \langle \phi \rangle$$

$$\rightarrow \frac{1}{2} Mc^2 = -\frac{3}{2} \cdot \left(-\frac{GKM^2}{2\,\pi R^3} \right)$$

Dividing both member by 2:

$$\rightarrow Mc^2 = \frac{3GKM^2}{2\,\pi R^3}$$

which is not true.

In fact, if we again apply the above equation to the bromum atom ^{79}Br, it leads to:

$$\rightarrow \frac{Mc^2}{\frac{3GKM^2}{2\,\pi R^3}} = \frac{1.1884 * 10^{-8}}{3{,}5652 * 10^{-8}} \neq 1$$

At this point, the fact that the virial theorem doesn’t hold for the nuclear gravitoelectric potential energy can be explained in two different ways.

The first is to assert that the eq. (5) doesn’t contain the nuclear potential self-energy, and consequently that mc^2 wouldn’t represent twice the kinetic energy of nucleons, but would be, as the theory of relativity states, the total mass-energy of nucleons, more precisely the energy that the nucleons contains for the very fact of having a mass, even if they are at rest.

This interpretation, yet, doesn’t allow to explain which would be the physical meaning of the perfect mathematical identity given by the eq. (6), which, consequently, should be ascribable, nothing short of unrealistically, only to the fortuity.

The second possibility is to claim that the virial theorem, as formulated in eq. (9), is incorrect, and that the correct *law* would be:

$$\langle E_K \rangle = \frac{1}{2} \cdot \langle \phi \rangle \qquad (10)$$

This interpretation is based on the fact that the virial theorem is an *ad hoc* solution, valid only in the case that the force of gravity to be inversely proportional to the square of the distance.

Though, this is a fact that has never been explained logically, mathematically, or geometrically, in essence scientifically, in particular nobody has never demonstrated the reason why the force of gravity can't be other than inversely proportional to the distance squared.

Consequently one can argue, in abstract, that, if the gravitational force were inversely proportional to the fourth power of the distance, the theorem would fail, as we'll show shortly.

In fact, in the case that the force of gravity were $F = \frac{GMm}{R^4}$, the kinetic energy, applying the virial theorem, would turn out to be greater than the potential energy.

In particular, supposing that in the mentioned hypothesis the force of gravity to be only attractive, then the gravitational potential energy would be:

$$U = \frac{GMm}{R^4} \cdot R = \frac{GMm}{R^3}$$

Consequently the exponent of the radius that would appear in the eq. (9) would be $b = -3$, so that the necessary condition to have a stable orbit would turn out to be:

$$\langle E_K \rangle = -\frac{3}{2} \cdot \langle \phi \rangle$$

$$\rightarrow \quad \frac{1}{2} Mv^2 = \frac{3}{2} \cdot \frac{GMm}{R^3}$$

but this is impossible because the kinetic energy would be greater than the potential energy ($E_K = 1.5 \cdot U$), and we know that in such a condition the orbit will be hyperbolic.

The same result would turn out in the case that the force of gravity to be inversely proportional to the third power of the distance, in which case, applying the virial theorem, the most stable orbit would be obtained if the kinetic energy were equal to the potential energy, but it is well-known that in this case the orbiting body would reach the escape velocity, so the virial theorem would fail again.

The virial theorem, therefore, is implicitly based on a premise (namely the fact that the force of gravity can't be other than inversely proportional to the square of the distance) which is not logically demonstrable, and this implies that it cannot be considered a *theorem* in the proper sense of the term, because a theorem is, by definition, a proposition which can be scientifically demonstrated, and this also holds for its logical premises.

Consequently one should admit that the eq. (9) would be replaced by the eq. (10), and that this latter would apply in any case, both when the object (body or particle) is subjected to only one attractive gravitational force, and when it is subjected to two gravitational forces (attractive and repulsive) at same time, regardless of the mathematical configuration of the potential energy (namely, regardless of the exponent of radius, *b*, appearing in the formula of the potential energy).

In other words, in this scenario one should admit that the eq. (10) to be a fundamental principle of Nature, in the sense that it wouldn't have any mathematical derivation, but should be accepted as it is.

After all, there are some aspects of the force of gravity that are not entirely explainable, just think of the fact, we repeat, that it depends, without any apparent logical reason, on the inverse of the square — rather than on the inverse of the cube or of the fourth power — of the distance, or rather than simply on the inverse of the distance.

However the aim of this paper is not getting into the details of the debate between those who believe in the existence of the fundamental laws of Nature, and those who believe that the physical laws are created by humans to describe the reality and consequently that every natural law should be explainable in the light of the rationality, but it's undeniable that the answer to the question here proposed depends on the way of solving this dispute.

The only thing that I can say in this regard is that the deductive method doesn't seem the best way of approaching the force of gravity, as it is shown by the paradoxical results of the virial theorem seen above.

The inductive method, on the contrary, by starting from the single cases in order to deduce the existence of a general principle, seems to be more suitable to study the issues related to the force of gravity, which, as for every phenomenological entity, isn't a-priori knowable in its every single aspect.

Obviously, the latter considerations would fail if we believe, as Einstein teaches, that the force of gravity is a geometrical entity, which would find its logical primary cause in the spacetime, but we have already said that this is not entirely true, at least until the force of gravity will continue to receive no geometrical, logical, mathematical, scientific explanation with regard to the fact that it can't be other than inversely proportional to the square of the distance.

- **Conclusions**

This study has revealed that the self-energy approach is a valid way to study the nuclear structure and the nuclear forces.

In particular the demonstration of the validity of strong equivalence principle even within the atomic nucleus confirms that the Einstein's theory of relativity can work even at this scale.

Anyway the self-energy approach is not the solely possible way to interpret our theoretical achievements, by being also possible to argue that the nucleons are self-orbitating particles which revolve around themselves at the speed of light, and, in this latter case, the foundations of physics, included those concerning the theory of relativity, could be questioned.

References

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