Analyzing some Ramanujan's differential equations: new possible mathematical connections with ϕ, $\zeta(2)$, and various parameters of Particle Physics

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Abstract

In this paper we have described some Ramanujan's differential equations: new possible mathematical connections with ϕ, $\zeta(2)$, and various parameters of Particle Physics

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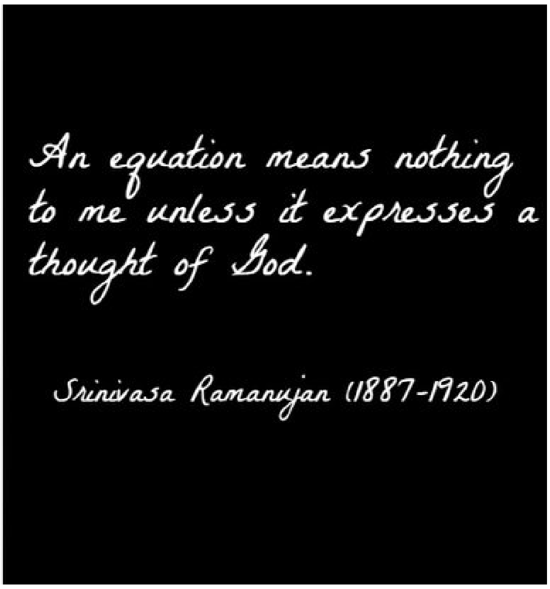


https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

From:

AUTOMORPHISM OF SOLUTIONS TO RAMANUJAN'S DIFFERENTIAL EQUATIONS AND OTHER RESULTS
MATTHEW RANDALL - arXiv:1806.07544v1 [math.CA] 20 Jun 2018

We have the following Ramanujan's differential equations:

We say that the triple of functions $(p(x), q(x), r(x))$ of the variable x satisfies Ramanujan's differential equations if the following set of equations are satisfied for the functions $p(x)$, $q(x)$ and $r(x)$ in the triple:

$$\frac{dp}{dx} = \frac{1}{6}(p^2 - q),$$
$$\frac{dq}{dx} = \frac{2}{3}(pq - r), \qquad (1.1)$$
$$\frac{dr}{dx} = pr - q^2.$$

Theorem 1.1. *Suppose $(P(x), Q(x), R(x))$ satisfies Ramanujan's differential equations, i.e. we have*

$$\frac{\mathrm{d}}{\mathrm{d}x}P = \frac{1}{6}(P^2 - Q),$$
$$\frac{\mathrm{d}}{\mathrm{d}x}Q = \frac{2}{3}(PQ - R), \tag{1.2}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}R = PR - Q^2.$$

Let $T = R + \sqrt{R^2 - Q^3}$ and consider the quantities

$$v = \frac{3}{2}T^{\frac{1}{3}} + \frac{3}{2}\frac{Q}{T^{\frac{1}{3}}}, \quad u = \pm\sqrt{3}\left(\frac{Q^2}{T^{\frac{2}{3}}} + Q + T^{\frac{2}{3}}\right)^{\frac{1}{2}}.$$

Then the following holds. The triples

$$(p_2, q_2, r_2)$$
$$= \left(P + \frac{u+v}{2}, \frac{8}{9}u(u+v) + \frac{1}{36}(v-u)^2, \frac{1}{54}(3u+v)(16u(u+v) - (\frac{v-u}{2})^2)\right),$$
$$(p_3, q_3, r_3)$$
$$= \left(P + \frac{v-u}{2}, \frac{8}{9}u(u-v) + \frac{1}{36}(v+u)^2, \frac{1}{54}(v-3u)(16u(u-v) - (\frac{u+v}{2})^2)\right),$$

also satisfy Ramanujan's differential equations (1.1), and furthermore so does the triple

$$(p_0, q_0, r_0) = \left(P + \frac{1}{2}T^{\frac{1}{3}} + \frac{1}{2}\frac{Q}{T^{\frac{1}{3}}}, \frac{3}{2}Q + \frac{5}{4}T^{\frac{2}{3}} + \frac{5}{4}\frac{Q^2}{T^{\frac{2}{3}}}, \frac{11}{4}R + \frac{21}{8}QT^{\frac{1}{3}} + \frac{21}{8}\frac{Q^2}{T^{\frac{1}{3}}}\right).$$

We now assume

that we know a solution of (1.2) given by

$$P = 1 - 24\sum_{n=1}^{\infty}\sigma_1(n)q^n = E_2,$$
$$Q = 1 + 240\sum_{n=1}^{\infty}\sigma_3(n)q^n = E_4,$$
$$R = 1 - 504\sum_{n=1}^{\infty}\sigma_5(n)q^n = E_6.$$

Here E_2 is a quasi-modular form given by the Eisenstein series of weight 2, while E_4 and E_6 are modular forms given by the Eisenstein series of weight 4 and 6 respectively. The functions here involve $\sigma_1(n)$ the sum of divisor function, $\sigma_3(n)$ the sum of cube of divisor function and $\sigma_5(n)$ the sum of fifth powers of divisor function. Also $q = e^{2\pi i x}$ is the nome, and with $\mathrm{d}q = 2\pi i \mathrm{d}x$, this gives

$$\frac{\mathrm{d}}{\mathrm{d}x} = 2\pi i q\frac{\mathrm{d}}{\mathrm{d}q}$$

as a change of variable, so that the Ramanujan system (1.2) can be rewritten as

$$\pi i q\frac{\mathrm{d}}{\mathrm{d}q}P = \frac{1}{12}(P^2 - Q),$$
$$\pi i q\frac{\mathrm{d}}{\mathrm{d}q}Q = \frac{1}{3}(PQ - R),$$
$$\pi i q\frac{\mathrm{d}}{\mathrm{d}q}R = \frac{1}{2}(PR - Q^2).$$

Now given $(p_1, q_1, r_1) - (P, Q, R)$, we compute and find that

$$p_2 = 4 \quad 96q^4 \quad 288q^8 \quad 384q^{12} \quad 672q^{16} \quad \ldots = 4P(q^4),$$
$$p_3 = 1 + 24q - 72q^2 + 96q^3 - 168q^4 + 144q^5 - \ldots = P(-q),$$

with $p_0 = \frac{1}{3}P(q) + \frac{4}{3}P(q^4) + \frac{1}{3}P(-q) = 2P(q^2)$. In the case for p_2, we see that identifying $\tilde{q} - q^4$ gives back the solution (P, Q, R) to the differential equations (1.2) with an appropriate constant rescaling and likewise for the case p_3, the variable to be identified is $\tilde{q} - -q$. Similarly, in the case of p_0, identifying $\tilde{q} - q^2$ gives us back a constant rescaling of (P, Q, R). Taking the triple

$$(p_0, q_0, r_0) = (2P(q^2), 4Q(q^2), 8R(q^2))$$

satisfying (1.1), we can apply Theorem 1.1 to iterate the process and get the triples

$$(p_4, q_4, r_4) = (8P(q^8), 64Q(q^8), 512R(q^8)),$$
$$(p_5, q_5, r_5) = (2P(-q^2), 4Q(-q^2), 8R(-q^2)),$$

Now, we want to analyze

$$p_2 = 4 - 96q^4 - 288q^8 - 384q^{12} - 672q^{16} - \ldots = 4P(q^4),$$
$$p_3 = 1 + 24q - 72q^2 + 96q^3 - 168q^4 + 144q^5 - \ldots = P(-q),$$

From

$$p_2 = 4 - 96q^4 - 288q^8 - 384q^{12} - 672q^{16} - \ldots = 4P(q^4),$$

For q = exp(2Pi) and $ix > 0$; $ix = 1$, we obtain:

4-96(exp(2Pi))^4-288(exp(2Pi))^8-384(exp(2Pi))^12-672(exp(2Pi))^16

Input:

$4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)$

Exact result:

$4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}$

Decimal approximation:

$-3.0719390594725462498724658169788893459280939318188818\ldots \times 10^{46}$

$-3.07193905947\ldots * 10^{46}$

Property:

$4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}$ is a transcendental number

Alternate forms:

$4 - 96 \left(e^{8\pi} + 3 e^{16\pi} + 4 e^{24\pi} + 7 e^{32\pi}\right)$

$-4\left(-1 + 24 e^{8\pi} + 72 e^{16\pi} + 96 e^{24\pi} + 168 e^{32\pi}\right)$

Series representations:

$$4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi) = 4 - 96 e^{32 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 288 e^{64 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 384 e^{96 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 672 e^{128 \sum_{k=0}^{\infty} (-1)^k/(1+2k)}$$

$$4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi) = -4\left(-1 + 24\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8\pi} + 72\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16\pi} + 96\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24\pi} + 168\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32\pi}\right)$$

$$4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi) = -4\left(-1 + 24\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi} + 72\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{16\pi} + 96\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{24\pi} + 168\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{32\pi}\right)$$

Integral representations:

$$4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi) = -4\left(-1 + 24 e^{16\int_0^\infty 1/(1+t^2)\,dt} + 72 e^{32\int_0^\infty 1/(1+t^2)\,dt} + 96 e^{48\int_0^\infty 1/(1+t^2)\,dt} + 168 e^{64\int_0^\infty 1/(1+t^2)\,dt}\right)$$

$$4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi) = 4 - 96 e^{32\int_0^1 \sqrt{1-t^2}\,dt} - 288 e^{64\int_0^1 \sqrt{1-t^2}\,dt} - 384 e^{96\int_0^1 \sqrt{1-t^2}\,dt} - 672 e^{128\int_0^1 \sqrt{1-t^2}\,dt}$$

$$4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi) = -4\left(-1 + 24 e^{16\int_0^1 1/\sqrt{1-t^2}\,dt} + 72 e^{32\int_0^1 1/\sqrt{1-t^2}\,dt} + 96 e^{48\int_0^1 1/\sqrt{1-t^2}\,dt} + 168 e^{64\int_0^1 1/\sqrt{1-t^2}\,dt}\right)$$

From

$$p_3 = 1 + 24q - 72q^2 + 96q^3 - 168q^4 + 144q^5 - \ldots = P(-q),$$

We obtain:

1+24(exp(2Pi))-72(exp(2Pi))^2+96(exp(2Pi))^3-168(exp(2Pi))^4+144(exp(2Pi))^5

Input:

$$1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)$$

Exact result:

$$1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}$$

Decimal approximation:

$6.326737543361188435211657321413990345743487287575434\ldots \times 10^{15}$

$6.32673754336\ldots * 10^{15}$

Property:

$1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}$ is a transcendental number

Alternate form:

$1 + 24 e^{2\pi} \left(1 - 3 e^{2\pi} + 4 e^{4\pi} - 7 e^{6\pi} + 6 e^{8\pi}\right)$

Series representations:

$$1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi) = 1 + 24 e^{8 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 72 e^{16 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 96 e^{24 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 168 e^{32 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 144 e^{40 \sum_{k=0}^{\infty} (-1)^k/(1+2k)}$$

$$1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi) = 1 + 24 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi} - 72 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4\pi} + 96 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6\pi} - 168 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8\pi} + 144 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{10\pi}$$

$$1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi) = 1 + 24 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi} - 72 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{4\pi} + 96 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{6\pi} - 168 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{8\pi} + 144 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{10\pi}$$

Integral representations:

$$1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi) = 1 + 24 e^{4 \int_0^{\infty} 1/(1+t^2)\,dt} - 72 e^{8 \int_0^{\infty} 1/(1+t^2)\,dt} + 96 e^{12 \int_0^{\infty} 1/(1+t^2)\,dt} - 168 e^{16 \int_0^{\infty} 1/(1+t^2)\,dt} + 144 e^{20 \int_0^{\infty} 1/(1+t^2)\,dt}$$

$$1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi) = 1 + 24 e^{4 \int_0^{\infty} \sin(t)/t\,dt} - 72 e^{8 \int_0^{\infty} \sin(t)/t\,dt} + 96 e^{12 \int_0^{\infty} \sin(t)/t\,dt} - 168 e^{16 \int_0^{\infty} \sin(t)/t\,dt} + 144 e^{20 \int_0^{\infty} \sin(t)/t\,dt}$$

$$1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi) = 1 + 24 e^{8 \int_0^1 \sqrt{1-t^2}\,dt} - 72 e^{16 \int_0^1 \sqrt{1-t^2}\,dt} + 96 e^{24 \int_0^1 \sqrt{1-t^2}\,dt} - 168 e^{32 \int_0^1 \sqrt{1-t^2}\,dt} + 144 e^{40 \int_0^1 \sqrt{1-t^2}\,dt}$$

From the ratio of the two expressions, we obtain:

-(((4-96(exp(2Pi))^4-288(exp(2Pi))^8-384(exp(2Pi))^12-672(exp(2Pi))^16)))/(((1+24(exp(2Pi))-72(exp(2Pi))^2+96(exp(2Pi))^3-168(exp(2Pi))^4+144(exp(2Pi))^5)))

Input:

$$-\frac{4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)}{1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)}$$

Exact result:

$$\frac{-4+96e^{8\pi}+288e^{16\pi}+384e^{24\pi}+672e^{32\pi}}{1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}}$$

Decimal approximation:

$4.855486794605558497274653064385872428736186147614140 2\ldots\times 10^{30}$

4.8554867946…*10^{30}

Property:

$$\frac{-4+96e^{8\pi}+288e^{16\pi}+384e^{24\pi}+672e^{32\pi}}{1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}}$$ is a transcendental number

Alternate forms:

$$\frac{96\left(e^{8\pi}+3e^{16\pi}+4e^{24\pi}+7e^{32\pi}\right)-4}{1+24e^{2\pi}\left(1-3e^{2\pi}+4e^{4\pi}-7e^{6\pi}+6e^{8\pi}\right)}$$

$$-\frac{4}{1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}}+\frac{96e^{8\pi}}{1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}}+\frac{288e^{16\pi}}{1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}}+\frac{384e^{24\pi}}{1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}}+\frac{672e^{32\pi}}{1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}}$$

Series representations:

$$-\frac{4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)}{1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)}=$$
$$\left(4\left(-1+24e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+72e^{64\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+96e^{96\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+168e^{128\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right)\Big/$$
$$\left(1+24e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}-72e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+96e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}-168e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+144e^{40\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)$$

$$-\frac{4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)}{1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)}=$$
$$\left(4\left(-1+24\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}+72\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\pi}+96\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{24\pi}+168\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{32\pi}\right)\right)\Big/$$
$$\left(1+24\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}-72\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}+96\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}-168\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}+144\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10\pi}\right)$$

$$-\frac{4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)}{1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)}=$$
$$\left(4\left(-1+24\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi}+72\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{16\pi}+\right.\right.$$
$$\left.\left.96\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{24\pi}+168\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{32\pi}\right)\right)\Big/$$
$$\left(1+24\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}-72\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi}+96\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{6\pi}-\right.$$
$$\left.168\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi}+144\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{10\pi}\right)$$

Integral representations:

$$-\frac{4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)}{1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)}=$$
$$\left(4\left(-1+24e^{16\int_0^\infty \sin(t)/t\,dt}+72e^{32\int_0^\infty \sin(t)/t\,dt}+96e^{48\int_0^\infty \sin(t)/t\,dt}+168e^{64\int_0^\infty \sin(t)/t\,dt}\right)\right)\Big/$$
$$\left(1+24e^{4\int_0^\infty \sin(t)/t\,dt}-72e^{8\int_0^\infty \sin(t)/t\,dt}+\right.$$
$$\left.96e^{12\int_0^\infty \sin(t)/t\,dt}-168e^{16\int_0^\infty \sin(t)/t\,dt}+144e^{20\int_0^\infty \sin(t)/t\,dt}\right)$$

$$-\frac{4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)}{1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)}=$$
$$\left(4\left(-1+24e^{16\int_0^\infty 1/(1+t^2)\,dt}+72e^{32\int_0^\infty 1/(1+t^2)\,dt}+\right.\right.$$
$$\left.\left.96e^{48\int_0^\infty 1/(1+t^2)\,dt}+168e^{64\int_0^\infty 1/(1+t^2)\,dt}\right)\right)\Big/$$
$$\left(1+24e^{4\int_0^\infty 1/(1+t^2)\,dt}-72e^{8\int_0^\infty 1/(1+t^2)\,dt}+96e^{12\int_0^\infty 1/(1+t^2)\,dt}-\right.$$
$$\left.168e^{16\int_0^\infty 1/(1+t^2)\,dt}+144e^{20\int_0^\infty 1/(1+t^2)\,dt}\right)$$

$$-\frac{4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)}{1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)}=$$

$$\left(4\left(-1+24 e^{16\int_0^\infty \sin^2(t)/t^2\,dt}+72 e^{32\int_0^\infty \sin^2(t)/t^2\,dt}+96 e^{48\int_0^\infty \sin^2(t)/t^2\,dt}+168 e^{64\int_0^\infty \sin^2(t)/t^2\,dt}\right)\right)\Big/$$
$$\left(1+24 e^{4\int_0^\infty \sin^2(t)/t^2\,dt}-72 e^{8\int_0^\infty \sin^2(t)/t^2\,dt}+96 e^{12\int_0^\infty \sin^2(t)/t^2\,dt}-168 e^{16\int_0^\infty \sin^2(t)/t^2\,dt}+144 e^{20\int_0^\infty \sin^2(t)/t^2\,dt}\right)$$

From which:

1/3 * 1/10^30 [-(((4-96(exp(2Pi))^4-288(exp(2Pi))^8-384(exp(2Pi))^12-672(exp(2Pi))^16)))/(((1+24(exp(2Pi))-72(exp(2Pi))^2+96(exp(2Pi))^3-168(exp(2Pi))^4+144(exp(2Pi))^5)))]

Input:

$$\frac{1}{3}\times\frac{1}{10^{30}}\left(-\frac{4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)}{1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)}\right)$$

Exact result:

$$\left(-4+96 e^{8\pi}+288 e^{16\pi}+384 e^{24\pi}+672 e^{32\pi}\right)\Big/\left(3\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24 e^{2\pi}-72 e^{4\pi}+96 e^{6\pi}-168 e^{8\pi}+144 e^{10\pi}\right)\right)$$

Decimal approximation:

1.618495598201852832424884354795290809578728715871380076005...

1.6184955982...

Property:

$$\left(-4+96 e^{8\pi}+288 e^{16\pi}+384 e^{24\pi}+672 e^{32\pi}\right)\Big/\left(3\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24 e^{2\pi}-72 e^{4\pi}+96 e^{6\pi}-168 e^{8\pi}+144 e^{10\pi}\right)\right)$$
is a transcendental number

Alternate forms:

$$\left(-1+24 e^{8\pi}+72 e^{16\pi}+96 e^{24\pi}+168 e^{32\pi}\right)\Big/\left(750\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24 e^{2\pi}-72 e^{4\pi}+96 e^{6\pi}-168 e^{8\pi}+144 e^{10\pi}\right)\right)$$

$$\left(24\left(e^{8\pi}+3\,e^{16\pi}+4\,e^{24\pi}+7\,e^{32\pi}\right)-1\right)\Big/\left(750\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24\,e^{2\pi}\left(1-3\,e^{2\pi}+4\,e^{4\pi}-7\,e^{6\pi}+6\,e^{8\pi}\right)\right)\right)$$

$$-\left(1\Big/\left(750\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24\,e^{2\pi}-72\,e^{4\pi}+96\,e^{6\pi}-168\,e^{8\pi}+144\,e^{10\pi}\right)\right)\right)+$$
$$e^{8\pi}\Big/\left(31\,250\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24\,e^{2\pi}-72\,e^{4\pi}+96\,e^{6\pi}-168\,e^{8\pi}+144\,e^{10\pi}\right)\right)+$$
$$\left(3\,e^{16\pi}\right)\Big/\left(31\,250\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24\,e^{2\pi}-72\,e^{4\pi}+96\,e^{6\pi}-168\,e^{8\pi}+144\,e^{10\pi}\right)\right)+$$
$$e^{24\pi}\Big/\left(7\,812\,500\,000\,000\,000\,000\,000\,000\,000\left(1+24\,e^{2\pi}-72\,e^{4\pi}+96\,e^{6\pi}-168\,e^{8\pi}+144\,e^{10\pi}\right)\right)+$$
$$\left(7\,e^{32\pi}\right)\Big/\left(31\,250\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24\,e^{2\pi}-72\,e^{4\pi}+96\,e^{6\pi}-168\,e^{8\pi}+144\,e^{10\pi}\right)\right)$$

Series representations:

$$-\left(\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\Big/\left(\left(10^{30}\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)3\right)\right)=$$
$$\left(-1+24\,e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+72\,e^{64\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+96\,e^{96\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+168\,e^{128\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\Big/$$
$$\left(750\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24\,e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}-72\,e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+96\,e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}-168\,e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+144\,e^{40\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right)$$

$$-\left(\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\Big/\left(\left(10^{30}\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)3\right)\right)=$$
$$\left(-1+24\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}+72\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\pi}+96\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{24\pi}+168\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{32\pi}\right)\Big/$$
$$\left(750\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}-72\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}+96\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}-168\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}+144\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10\pi}\right)\right)$$

$$-\left(\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)/\left(\left(10^{30}\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)3\right)\right)=$$

$$\left(-1+24\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi}+72\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{16\pi}+96\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{24\pi}+168\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{32\pi}\right)/$$
$$\left(750\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}-72\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi}+96\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{6\pi}-168\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi}+144\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{10\pi}\right)\right)$$

Integral representations:

$$-\left(\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)/\left(\left(10^{30}\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)3\right)\right)=$$
$$\left(-1+24e^{16\int_0^\infty \sin(t)/t\,dt}+72e^{32\int_0^\infty \sin(t)/t\,dt}+96e^{48\int_0^\infty \sin(t)/t\,dt}+168e^{64\int_0^\infty \sin(t)/t\,dt}\right)/\left(750\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24e^{4\int_0^\infty \sin(t)/t\,dt}-72e^{8\int_0^\infty \sin(t)/t\,dt}+96e^{12\int_0^\infty \sin(t)/t\,dt}-168e^{16\int_0^\infty \sin(t)/t\,dt}+144e^{20\int_0^\infty \sin(t)/t\,dt}\right)\right)$$

$$-\left(\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)/\left(\left(10^{30}\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)3\right)\right)=$$
$$\left(-1+24e^{16\int_0^\infty 1/(1+t^2)\,dt}+72e^{32\int_0^\infty 1/(1+t^2)\,dt}+96e^{48\int_0^\infty 1/(1+t^2)\,dt}+168e^{64\int_0^\infty 1/(1+t^2)\,dt}\right)/$$
$$\left(750\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24e^{4\int_0^\infty 1/(1+t^2)\,dt}-72e^{8\int_0^\infty 1/(1+t^2)\,dt}+96e^{12\int_0^\infty 1/(1+t^2)\,dt}-168e^{16\int_0^\infty 1/(1+t^2)\,dt}+144e^{20\int_0^\infty 1/(1+t^2)\,dt}\right)\right)$$

$$-\left(\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\Big/\left(\left(10^{30}\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)3\right)\right)=$$

$$\left(-1+24\,e^{16\int_0^\infty \sin^2(t)/t^2\,dt}+72\,e^{32\int_0^\infty \sin^2(t)/t^2\,dt}+96\,e^{48\int_0^\infty \sin^2(t)/t^2\,dt}+168\,e^{64\int_0^\infty \sin^2(t)/t^2\,dt}\right)\Big/\left(750\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+24\,e^{4\int_0^\infty \sin^2(t)/t^2\,dt}-72\,e^{8\int_0^\infty \sin^2(t)/t^2\,dt}+96\,e^{12\int_0^\infty \sin^2(t)/t^2\,dt}-168\,e^{16\int_0^\infty \sin^2(t)/t^2\,dt}+144\,e^{20\int_0^\infty \sin^2(t)/t^2\,dt}\right)\right)$$

Multiplying the two expressions, we obtain:

[-(((4-96(exp(2Pi))^4-288(exp(2Pi))^8-384(exp(2Pi))^12-672(exp(2Pi))^16)))*(((1+24(exp(2Pi))-72(exp(2Pi))^2+96(exp(2Pi))^3-168(exp(2Pi))^4+144(exp(2Pi))^5)))]

Input:

$$-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)$$

Exact result:

$$-\left(1+24\,e^{2\pi}-72\,e^{4\pi}+96\,e^{6\pi}-168\,e^{8\pi}+144\,e^{10\pi}\right)\left(4-96\,e^{8\pi}-288\,e^{16\pi}-384\,e^{24\pi}-672\,e^{32\pi}\right)$$

Decimal approximation:

$1.943535217848261699882844774459603286698215281318580 6\ldots\times 10^{62}$

1.9435352178…*10^{62}

Property:

$-\left(1+24\,e^{2\pi}-72\,e^{4\pi}+96\,e^{6\pi}-168\,e^{8\pi}+144\,e^{10\pi}\right)\left(4-96\,e^{8\pi}-288\,e^{16\pi}-384\,e^{24\pi}-672\,e^{32\pi}\right)$ is a transcendental number

Alternate forms:

$$4\left(1+24\,e^{2\pi}\left(1-3\,e^{2\pi}+4\,e^{4\pi}-7\,e^{6\pi}+6\,e^{8\pi}\right)\right)\left(24\left(e^{8\pi}+3\,e^{16\pi}+4\,e^{24\pi}+7\,e^{32\pi}\right)-1\right)$$

$$-4 - 96\, e^{2\pi} + 288\, e^{4\pi} - 384\, e^{6\pi} + 768\, e^{8\pi} + 1728\, e^{10\pi} - 6912\, e^{12\pi} + 9216\, e^{14\pi} - 15\,840\, e^{16\pi} + 20\,736\, e^{18\pi} - 20\,736\, e^{20\pi} + 27\,648\, e^{22\pi} - 48\,000\, e^{24\pi} + 50\,688\, e^{26\pi} - 27\,648\, e^{28\pi} + 36\,864\, e^{30\pi} - 63\,840\, e^{32\pi} + 71\,424\, e^{34\pi} - 48\,384\, e^{36\pi} + 64\,512\, e^{38\pi} - 112\,896\, e^{40\pi} + 96\,768\, e^{42\pi}$$

$$4\left(-1 - 24\, e^{2\pi} + 72\, e^{4\pi} - 96\, e^{6\pi} + 192\, e^{8\pi} + 432\, e^{10\pi} - 1728\, e^{12\pi} + 2304\, e^{14\pi} - 3960\, e^{16\pi} + 5184\, e^{18\pi} - 5184\, e^{20\pi} + 6912\, e^{22\pi} - 12\,000\, e^{24\pi} + 12\,672\, e^{26\pi} - 6912\, e^{28\pi} + 9216\, e^{30\pi} - 15\,960\, e^{32\pi} + 17\,856\, e^{34\pi} - 12\,096\, e^{36\pi} + 16\,128\, e^{38\pi} - 28\,224\, e^{40\pi} + 24\,192\, e^{42\pi}\right)$$

Series representations:

$$-\left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right) \left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right) = 4\left(1 + 24\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi} - 72\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4\pi} + 96\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6\pi} - 168\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8\pi} + 144\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{10\pi}\right) \left(-1 + 24\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8\pi} + 72\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16\pi} + 96\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24\pi} + 168\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32\pi}\right)$$

$$-\left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right) \left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right) = 4\left(1 + 24\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi} - 72\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{4\pi} + 96\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{6\pi} - 168\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{8\pi} + 144\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{10\pi}\right) \left(-1 + 24\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{8\pi} + 72\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{16\pi} + 96\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{24\pi} + 168\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{32\pi}\right)$$

$$-\left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right) \left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right) = -4 - 96\, e^{8 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 288\, e^{16 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 384\, e^{24 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 768\, e^{32 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 1728\, e^{40 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 6912\, e^{48 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 9216\, e^{56 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 15\,840\, e^{64 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 20\,736\, e^{72 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 20\,736\, e^{80 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 27\,648\, e^{88 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 48\,000\, e^{96 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 50\,688\, e^{104 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 27\,648\, e^{112 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 36\,864\, e^{120 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 63\,840\, e^{128 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 71\,424\, e^{136 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 48\,384\, e^{144 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 64\,512\, e^{152 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} - 112\,896\, e^{160 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 96\,768\, e^{168 \sum_{k=0}^{\infty} (-1)^k/(1+2k)}$$

Integral representations:

$$-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)=-4-96e^{4\int_0^\infty \sin(t)/t\,dt}+288e^{8\int_0^\infty \sin(t)/t\,dt}-384e^{12\int_0^\infty \sin(t)/t\,dt}+768e^{16\int_0^\infty \sin(t)/t\,dt}+1728e^{20\int_0^\infty \sin(t)/t\,dt}-6912e^{24\int_0^\infty \sin(t)/t\,dt}+9216e^{28\int_0^\infty \sin(t)/t\,dt}-15840e^{32\int_0^\infty \sin(t)/t\,dt}+20736e^{36\int_0^\infty \sin(t)/t\,dt}-20736e^{40\int_0^\infty \sin(t)/t\,dt}+27648e^{44\int_0^\infty \sin(t)/t\,dt}-48000e^{48\int_0^\infty \sin(t)/t\,dt}+50688e^{52\int_0^\infty \sin(t)/t\,dt}-27648e^{56\int_0^\infty \sin(t)/t\,dt}+36864e^{60\int_0^\infty \sin(t)/t\,dt}-63840e^{64\int_0^\infty \sin(t)/t\,dt}+71424e^{68\int_0^\infty \sin(t)/t\,dt}-48384e^{72\int_0^\infty \sin(t)/t\,dt}+64512e^{76\int_0^\infty \sin(t)/t\,dt}-112896e^{80\int_0^\infty \sin(t)/t\,dt}+96768e^{84\int_0^\infty \sin(t)/t\,dt}$$

$$-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)=-4-96e^{4\int_0^\infty \sin^2(t)/t^2\,dt}+288e^{8\int_0^\infty \sin^2(t)/t^2\,dt}-384e^{12\int_0^\infty \sin^2(t)/t^2\,dt}+768e^{16\int_0^\infty \sin^2(t)/t^2\,dt}+1728e^{20\int_0^\infty \sin^2(t)/t^2\,dt}-6912e^{24\int_0^\infty \sin^2(t)/t^2\,dt}+9216e^{28\int_0^\infty \sin^2(t)/t^2\,dt}-15840e^{32\int_0^\infty \sin^2(t)/t^2\,dt}+20736e^{36\int_0^\infty \sin^2(t)/t^2\,dt}-20736e^{40\int_0^\infty \sin^2(t)/t^2\,dt}+27648e^{44\int_0^\infty \sin^2(t)/t^2\,dt}-48000e^{48\int_0^\infty \sin^2(t)/t^2\,dt}+50688e^{52\int_0^\infty \sin^2(t)/t^2\,dt}-27648e^{56\int_0^\infty \sin^2(t)/t^2\,dt}+36864e^{60\int_0^\infty \sin^2(t)/t^2\,dt}-63840e^{64\int_0^\infty \sin^2(t)/t^2\,dt}+71424e^{68\int_0^\infty \sin^2(t)/t^2\,dt}-48384e^{72\int_0^\infty \sin^2(t)/t^2\,dt}+64512e^{76\int_0^\infty \sin^2(t)/t^2\,dt}-112896e^{80\int_0^\infty \sin^2(t)/t^2\,dt}+96768e^{84\int_0^\infty \sin^2(t)/t^2\,dt}$$

$$-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)=-4-96e^{4\int_0^\infty 1/(1+t^2)\,dt}+288e^{8\int_0^\infty 1/(1+t^2)\,dt}-384e^{12\int_0^\infty 1/(1+t^2)\,dt}+768e^{16\int_0^\infty 1/(1+t^2)\,dt}+1728e^{20\int_0^\infty 1/(1+t^2)\,dt}-6912e^{24\int_0^\infty 1/(1+t^2)\,dt}+9216e^{28\int_0^\infty 1/(1+t^2)\,dt}-15840e^{32\int_0^\infty 1/(1+t^2)\,dt}+20736e^{36\int_0^\infty 1/(1+t^2)\,dt}-20736e^{40\int_0^\infty 1/(1+t^2)\,dt}+27648e^{44\int_0^\infty 1/(1+t^2)\,dt}-48000e^{48\int_0^\infty 1/(1+t^2)\,dt}+50688e^{52\int_0^\infty 1/(1+t^2)\,dt}-27648e^{56\int_0^\infty 1/(1+t^2)\,dt}+36864e^{60\int_0^\infty 1/(1+t^2)\,dt}-63840e^{64\int_0^\infty 1/(1+t^2)\,dt}+71424e^{68\int_0^\infty 1/(1+t^2)\,dt}-48384e^{72\int_0^\infty 1/(1+t^2)\,dt}+64512e^{76\int_0^\infty 1/(1+t^2)\,dt}-112896e^{80\int_0^\infty 1/(1+t^2)\,dt}+96768e^{84\int_0^\infty 1/(1+t^2)\,dt}$$

From which:

ln[-(((4-96(exp(2Pi))^4-288(exp(2Pi))^8-384(exp(2Pi))^12-672(exp(2Pi))^16)))*(((1+24(exp(2Pi))-72(exp(2Pi))^2+96(exp(2Pi))^3-168(exp(2Pi))^4+144(exp(2Pi))^5)))]-4

Input:

$$\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)-4$$

$\log(x)$ is the natural logarithm

Exact result:

$$\log\left(-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)-4$$

Decimal approximation:

139.42478435761459147483026871348292532572011970868293256 14...

139.4247843576…

Alternate forms:

$$\log\left(\left(-1-24e^{2\pi}+72e^{4\pi}-96e^{6\pi}+168e^{8\pi}-144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)-4$$

$$-4+\log\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)+\log\left(-4+96e^{8\pi}+288e^{16\pi}+384e^{24\pi}+672e^{32\pi}\right)$$

$$\log\left(4\left(1+24e^{2\pi}\left(1-3e^{2\pi}+4e^{4\pi}-7e^{6\pi}+6e^{8\pi}\right)\right)\left(24\left(e^{8\pi}+3e^{16\pi}+4e^{24\pi}+7e^{32\pi}\right)-1\right)\right)-4$$

Alternative representations:

$$\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)-4=-4+\log_e\left(-\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\right)$$

$$\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)-4=-4+\log(a)\log_a\left(-\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\right)$$

Series representations:

$$\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)-4=-4+2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)-z_0\right)^k z_0^{-k}$$

$$\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)-4 = -4+2i\pi\left\lfloor\frac{1}{2\pi}\arg\left(-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)-x\right)\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)-x\right)^k x^{-k} \quad \text{for } x<0$$

$$\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)-4 = -4+\log\left(-1-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)}\right)^k}{k}$$

Integral representations:

$$\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)-4 = -4+\int_1^{4\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(-1+24e^{8\pi}+72e^{16\pi}+96e^{24\pi}+168e^{32\pi}\right)}\frac{1}{t}\,dt$$

$$\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)-4 = -4-\frac{i}{2\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{1}{\Gamma(1-s)}\left(-1-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)\,ds \quad \text{for } -1<\gamma<0$$

ln[-(((4-96(exp(2Pi))^4-288(exp(2Pi))^8-384(exp(2Pi))^12-672(exp(2Pi))^16)))*(((1+24(exp(2Pi))-72(exp(2Pi))^2+96(exp(2Pi))^3-168(exp(2Pi))^4+144(exp(2Pi))^5)))]-18

Input:

$$\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)-18$$

$\log(x)$ is the natural logarithm

Exact result:

$$\log\left(-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)-18$$

Decimal approximation:

125.4247843576145914748302687134829253257201197086829325614...

125.4247843576...

Alternate forms:

$$\log\left(\left(-1-24e^{2\pi}+72e^{4\pi}-96e^{6\pi}+168e^{8\pi}-144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)-18$$

$$-18+\log\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)+\log\left(-4+96e^{8\pi}+288e^{16\pi}+384e^{24\pi}+672e^{32\pi}\right)$$

$$\log\left(4\left(1+24e^{2\pi}\left(1-3e^{2\pi}+4e^{4\pi}-7e^{6\pi}+6e^{8\pi}\right)\right)\left(24\left(e^{8\pi}+3e^{16\pi}+4e^{24\pi}+7e^{32\pi}\right)-1\right)\right)-18$$

Alternative representations:

$$\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)-18=-18+\log_e\left(-\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\right)$$

$$\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)-18=-18+\log(a)\log_a\left(-\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\right)$$

Series representations:

$$\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)-18=-18+2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)-z_0\right)^k z_0^{-k}$$

$$\log\Big(-\big(4 - 96\exp^4(2\pi) - 288\exp^8(2\pi) - 384\exp^{12}(2\pi) - 672\exp^{16}(2\pi)\big)\big(1 + 24\exp(2\pi) - 72\exp^2(2\pi) + 96\exp^3(2\pi) - 168\exp^4(2\pi) + 144\exp^5(2\pi)\big)\Big) - 18 =$$
$$-18 + 2 i \pi \left\lfloor \frac{1}{2\pi} \arg\Big(-\big(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\big)\big(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\big) - x\Big)\right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{1}{k}(-1)^k \Big(-\big(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\big)\big(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\big) - x\Big)^k x^{-k} \quad \text{for } x < 0$$

$$\log\Big(-\big(4 - 96\exp^4(2\pi) - 288\exp^8(2\pi) - 384\exp^{12}(2\pi) - 672\exp^{16}(2\pi)\big)\big(1 + 24\exp(2\pi) - 72\exp^2(2\pi) + 96\exp^3(2\pi) - 168\exp^4(2\pi) + 144\exp^5(2\pi)\big)\Big) - 18 = -18 + \log\Big(-1 - \big(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\big)\big(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\big)\Big) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{-1-\left(1+24 e^{2\pi}-72 e^{4\pi}+96 e^{6\pi}-168 e^{8\pi}+144 e^{10\pi}\right)\left(4-96 e^{8\pi}-288 e^{16\pi}-384 e^{24\pi}-672 e^{32\pi}\right)}\right)^k}{k}$$

Integral representations:

$$\log\Big(-\big(4 - 96\exp^4(2\pi) - 288\exp^8(2\pi) - 384\exp^{12}(2\pi) - 672\exp^{16}(2\pi)\big)\big(1 + 24\exp(2\pi) - 72\exp^2(2\pi) + 96\exp^3(2\pi) - 168\exp^4(2\pi) + 144\exp^5(2\pi)\big)\Big) - 18 =$$
$$-18 + \int_1^{4\left(1+24 e^{2\pi}-72 e^{4\pi}+96 e^{6\pi}-168 e^{8\pi}+144 e^{10\pi}\right)\left(-1+24 e^{8\pi}+72 e^{16\pi}+96 e^{24\pi}+168 e^{32\pi}\right)} \frac{1}{t}\, dt$$

$$\log\Big(-\big(4 - 96\exp^4(2\pi) - 288\exp^8(2\pi) - 384\exp^{12}(2\pi) - 672\exp^{16}(2\pi)\big)\big(1 + 24\exp(2\pi) - 72\exp^2(2\pi) + 96\exp^3(2\pi) - 168\exp^4(2\pi) + 144\exp^5(2\pi)\big)\Big) - 18 =$$
$$-18 - \frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\Gamma(1-s)} \Big(-1 - \big(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\big)\big(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\big)\Big)^{-s} \Gamma(-s)^2\, \Gamma(1+s)\, ds \quad \text{for } -1 < \gamma < 0$$

12ln[-(((4-96(exp(2Pi))^4-288(exp(2Pi))^8-384(exp(2Pi))^12-672(exp(2Pi))^16)))*(((1+24(exp(2Pi))-72(exp(2Pi))^2+96(exp(2Pi))^3-168(exp(2Pi))^4+144(exp(2Pi))^5)))]+8

Input:

$$12 \log\left(-\left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right)\left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right)\right) + 8$$

$\log(x)$ is the natural logarithm

Exact result:

$$8 + 12 \log\left(-\left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right)\left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right)\right)$$

Decimal approximation:

1729.0974122913750976979632245617951039086414365041951907 36...

1729.0974122913…

Alternate forms:

$$8 + 12 \log\left(\left(-1 - 24 e^{2\pi} + 72 e^{4\pi} - 96 e^{6\pi} + 168 e^{8\pi} - 144 e^{10\pi}\right)\left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right)\right)$$

$$8 + 12\left(\log\left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right) + \log\left(-4 + 96 e^{8\pi} + 288 e^{16\pi} + 384 e^{24\pi} + 672 e^{32\pi}\right)\right)$$

$$4\left(2 + 3 \log\left(-\left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right)\left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right)\right)\right)$$

Alternative representations:

$$12 \log\left(-\left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right)\left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right)\right) + 8 = 8 + 12 \log_e\left(-\left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right)\left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right)\right)$$

$$12 \log\left(-\left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right)\left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right)\right) + 8 = 8 + 12 \log(a) \log_a\left(-\left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right)\left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right)\right)$$

Series representations:

$$12 \log\Big(-\big(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\big) \big(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\big)\Big) + 8 = 8 + 24 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + 12 \log(z_0) - 12 \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \left(-\left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right) \left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right) - z_0\right)^k z_0^{-k}$$

$$12 \log\Big(-\big(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\big) \big(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\big)\Big) + 8 = 8 + 24 i \pi \left\lfloor \frac{1}{2\pi} \arg\left(-\left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right) \left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right) - x\right) \right\rfloor + 12 \log(x) - 12 \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \left(-\left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right) \left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right) - x\right)^k x^{-k} \quad \text{for } x < 0$$

$$12 \log\Big(-\big(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\big) \big(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\big)\Big) + 8 = 8 + 12 \log\left(-1 - \left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right) \left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right)\right) - 12 \sum_{k=1}^{\infty} \frac{1}{k} \left(-\left(1 / \left(-1 - \left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right) \left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right)\right)\right)\right)^k$$

Integral representations:

$$12 \log\Big(-\big(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\big) \big(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\big)\Big) + 8 = 8 + 12 \int_1^{4\left(1+24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right)\left(-1 + 24 e^{8\pi} + 72 e^{16\pi} + 96 e^{24\pi} + 168 e^{32\pi}\right)} \frac{1}{t} \, dt$$

$$12\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)+8 =$$

$$8-\frac{6i}{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{1}{\Gamma(1-s)}\left(-1-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)\,ds \quad \text{for } -1<\gamma<0$$

and:

Input:

$$\left(12\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)+8\right)\hat{}\,(1/15)$$

log(x) is the natural logarithm

Exact result:

$$\left(8+12\log\left(-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)\right)\hat{}\,(1/15)$$

Decimal approximation:

1.643821402783741956534674412152921362427171943366526058309...

1.64382140278…

Alternate forms:

$$\left(8+12\log\left(\left(-1-24e^{2\pi}+72e^{4\pi}-96e^{6\pi}+168e^{8\pi}-144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)\right)\hat{}\,(1/15)$$

$$\left(8+12\left(\log\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)+\log\left(-4+96e^{8\pi}+288e^{16\pi}+384e^{24\pi}+672e^{32\pi}\right)\right)\right)\hat{}\,(1/15)$$

$$\left(8+12\log\left(4\left(1+24e^{2\pi}\left(1-3e^{2\pi}+4e^{4\pi}-7e^{6\pi}+6e^{8\pi}\right)\right)\left(24\left(e^{8\pi}+3e^{16\pi}+4e^{24\pi}+7e^{32\pi}\right)-1\right)\right)\right)\hat{}\,(1/15)$$

All 15th roots of 8 + 12 log(-(1 + 24 e^(2 π) - 72 e^(4 π) + 96 e^(6 π) - 168 e^(8 π) + 144 e^(10 π)) (4 - 96 e^(8 π) - 288 e^(16 π) - 384 e^(24 π) - 672 e^(32 π))):

$$e^0\left(8+12\log\left(-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)\right)\hat{}\,(1/15)\approx 1.64382$$ (real, principal root)

$$e^{(2 i \pi)/15} \left(8 + 12 \log\left(-\left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right)\left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right)\right)\right) \text{^} (1/15) \approx 1.50171 + 0.6686 i$$

$$e^{(4 i \pi)/15} \left(8 + 12 \log\left(-\left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right)\left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right)\right)\right) \text{^} (1/15) \approx 1.0999 + 1.2216 i$$

$$e^{(2 i \pi)/5} \left(8 + 12 \log\left(-\left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right)\left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right)\right)\right) \text{^} (1/15) \approx 0.5080 + 1.5634 i$$

$$e^{(8 i \pi)/15} \left(8 + 12 \log\left(-\left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right)\left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right)\right)\right) \text{^} (1/15) \approx -0.17183 + 1.63482 i$$

Alternative representations:

$$\left(12 \log\left(-\left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right) \left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right)\right) + 8\right) \text{^} (1/15) = \left(8 + 12 \log_e\left(-\left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right) \left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right)\right)\right) \text{^} (1/15)$$

$$\left(12 \log\left(-\left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right) \left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right)\right) + 8\right) \text{^} (1/15) = \left(8 + 12 \log(a) \log_a\left(-\left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right) \left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right)\right)\right) \text{^} (1/15)$$

Series representations:

$$\left(12 \log\left(-\left(4 - 96 \exp^4(2\pi) - 288 \exp^8(2\pi) - 384 \exp^{12}(2\pi) - 672 \exp^{16}(2\pi)\right) \left(1 + 24 \exp(2\pi) - 72 \exp^2(2\pi) + 96 \exp^3(2\pi) - 168 \exp^4(2\pi) + 144 \exp^5(2\pi)\right)\right) + 8\right) \text{^} (1/15) = \left(8 + 12 \left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \left(-\left(1 + 24 e^{2\pi} - 72 e^{4\pi} + 96 e^{6\pi} - 168 e^{8\pi} + 144 e^{10\pi}\right)\left(4 - 96 e^{8\pi} - 288 e^{16\pi} - 384 e^{24\pi} - 672 e^{32\pi}\right) - z_0\right)^k z_0^{-k}\right)\right) \text{^} (1/15)$$

$$\left(12\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)+8\right)^\wedge(1/15) =$$

$$\left(8+12\left(\log\left(-1-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)-\sum_{k=1}^{\infty}\frac{1}{k}\left(-\left(1/\left(-1-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)\right)\right)^k\right)\right)^\wedge(1/15)$$

$$\left(12\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)+8\right)^\wedge(1/15) =$$

$$\left(8+12\left(2i\pi\left\lfloor\frac{1}{2\pi}\arg\left(-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)-x\right)\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)-x\right)^k x^{-k}\right)\right)^\wedge(1/15) \quad \text{for } x<0$$

Integral representations:

$$\left(12\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)+8\right)^\wedge(1/15) =$$

$$\left(8+12\int_1^{4\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(-1+24e^{8\pi}+72e^{16\pi}+96e^{24\pi}+168e^{32\pi}\right)}\frac{1}{t}\,dt\right)^\wedge(1/15)$$

$$\left(12\log\left(-\left(4-96\exp^4(2\pi)-288\exp^8(2\pi)-384\exp^{12}(2\pi)-672\exp^{16}(2\pi)\right)\left(1+24\exp(2\pi)-72\exp^2(2\pi)+96\exp^3(2\pi)-168\exp^4(2\pi)+144\exp^5(2\pi)\right)\right)+8\right)^\wedge(1/15) =$$

$$\left(8-\frac{6i}{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{1}{\Gamma(1-s)}\left(-1-\left(1+24e^{2\pi}-72e^{4\pi}+96e^{6\pi}-168e^{8\pi}+144e^{10\pi}\right)\left(4-96e^{8\pi}-288e^{16\pi}-384e^{24\pi}-672e^{32\pi}\right)\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)\,ds\right)^\wedge(1/15) \quad \text{for } -1<\gamma<0$$

From:

$$p_0 = \tfrac{1}{3}P(q) + \tfrac{4}{3}P(q^4) + \tfrac{1}{3}P(-q) = 2P(q^2).$$

for

$$p_2 = 4 - 96q^4 - 288q^8 - 384q^{12} - 672q^{16} - \ldots = 4P(q^4),$$

$-3.071939059472546249872465816978889345928093931818818\ldots \times 10^{46}$

and:

$$p_3 - 1 + 24q - 72q^2 + 96q^3 - 168q^4 + 144q^5 - \ldots - P(-q),$$

$6.3267375433611884352116573214139903457434872875754344\ldots \times 10^{15}$

we obtain:

$$p_0 - \tfrac{1}{3}P(q) + \tfrac{4}{3}P(q^4) + \tfrac{1}{3}P(-q) - 2P(q^2).$$

1/3(-6.3267375e+15)+4/3*1/4(-3.071939e+46)+1/3(6.3267375e+15)

Input interpretation:

$$\frac{1}{3}\left(-6.3267375 \times 10^{15}\right) + \frac{4}{3} \times \frac{1}{4}\left(-3.071939 \times 10^{46}\right) + \frac{1}{3} \times 6.3267375 \times 10^{15}$$

Result:

$-1.0239796666666666666666666666666666666666666666666\ldots \times 10^{46}$

$-1.023979666\ldots*10^{46} = p_0 = 2P(q^2)$

From

$$y = p_0 = \tfrac{1}{3}(p_1 + p_2 + p_3)$$

we obtain:

-1.023979666e+46 = 1/3(x-(3.071939e+46)+(6.3267375e+15))

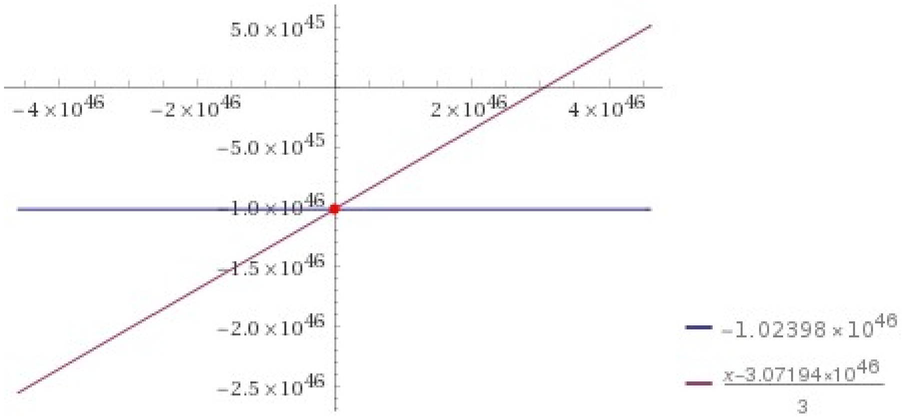
Input interpretation:

$$-1.023979666\times 10^{46} = \frac{1}{3}\left(x - 3.071939\times 10^{46} + 6.3267375\times 10^{15}\right)$$

Result:

$$-1.02398\times 10^{46} = \frac{x - 3.07194\times 10^{46}}{3}$$

Plot:



Alternate forms:

$$-1.02398\times 10^{46} = 0.333333\left(x - 3.07194\times 10^{46}\right)$$

$$6.66667\times 10^{36} - \frac{x}{3} = 0$$

Expanded form:

$$-10\,239\,796\,659\,999\,999\,687\,048\,012\,579\,044\,993\,181\,646\,061\,568 = \frac{x}{3} - 10\,239\,796\,666\,666\,665\,464\,491\,487\,434\,033\,063\,101\,349\,232\,640$$

Solution:

$$x = 19\,999\,997\,332\,330\,424\,564\,964\,209\,759\,109\,513\,216$$

Integer solution:

$$x = 19\,999\,997\,332\,330\,424\,564\,964\,209\,759\,109\,513\,216$$

Scientific notation:

$$1.9999997332330424564964209759109513216\times 10^{37}$$

$1.999999733233...*10^{37} = p_1$

Thence, from:

From

$$y = p_0 = \frac{1}{3}(p_1 + p_2 + p_3)$$

We obtain:

1/3(1.999999733233e+37-(3.071939e+46)+(6.3267375e+15))

Input interpretation:

$\frac{1}{3}\left(1.999999733233\times10^{37} - 3.071939\times10^{46} + 6.3267375\times10^{15}\right)$

Result:

$-1.0239796660000000889223333333331224420833333333333... \times 10^{46}$

-1.023979666…*10^{46} result equal to the previous obtained with the following expression:

$$p_0 = \frac{1}{3}P(q) - \frac{4}{3}P(q^4) + \frac{1}{3}P(-q) = 2P(q^2).$$

Now, we have:

ln(((-1/3(1.999999733233e+37-(3.071939e+46)+(6.3267375e+15)))))+21-golden ratio

Input interpretation:

$\log\left(-\frac{1}{3}\left(1.999999733233\times10^{37} - 3.071939\times10^{46} + 6.3267375\times10^{15}\right)\right) + 21 - \phi$

log(x) is the natural logarithm

ϕ is the golden ratio

Result:

125.324577…

125.324577…

ln(((-1/3(1.999999733233e+37-(3.071939e+46)+(6.3267375e+15)))))+34-1/golden ratio

Input interpretation:

$\log\left(-\frac{1}{3}\left(1.999999733233\times10^{37} - 3.071939\times10^{46} + 6.3267375\times10^{15}\right)\right) + 34 - \frac{1}{\phi}$

log(x) is the natural logarithm

ϕ is the golden ratio

Result:

139.324577...

139.324577…

27*1/2[ln(((-1/3(1.999999733233e+37-(3.071939e+46)+(6.3267375e+15)))))+21+1/golden ratio]+7

Input interpretation:

$$27\times\frac{1}{2}\left(\log\left(-\frac{1}{3}\left(1.999999733233\times10^{37}-3.071939\times10^{46}+6.3267375\times10^{15}\right)\right)+21+\frac{1}{\phi}\right)+7$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Result:

1729.06871...

1729.06871…

((((27*1/2[ln(((-1/3(1.999999733233e+37-(3.071939e+46)+(6.3267375e+15)))))+21+1/golden ratio]+7))))^1/15

Input interpretation:

$$\left(27\times\frac{1}{2}\left(\log\left(-\frac{1}{3}\left(1.999999733233\times10^{37}-3.071939\times10^{46}+6.3267375\times10^{15}\right)\right)+21+\frac{1}{\phi}\right)+7\right)\text{^}(1/15)$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Result:

1.643819583...

1.643819583…

((((27*1/2[ln(((-1/3(1.999999733233e+37-(3.071939e+46)+(6.3267375e+15)))))+21+1/golden ratio]+7))))^1/15-26/10^3

Input interpretation:

$$\left(27\times\frac{1}{2}\left(\log\left(-\frac{1}{3}\left(1.999999733233\times10^{37}-3.071939\times10^{46}+6.3267375\times10^{15}\right)\right)+21+\frac{1}{\phi}\right)+7\right)^{\wedge}(1/15)-\frac{26}{10^3}$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Result:

1.617819583...

1.617819583…

From

On certain arithmetical functions – *Srinivasa Ramanujan*
Transactions of the Cambridge Philosophical Society, XXII, No.9, 1916, 159 – 184

We have that:

$$\left.\begin{aligned}\Phi_{0,s}(x) &= \frac{1^s x}{1-x}+\frac{2^s x^2}{1-x^2}+\frac{3^s x^3}{1-x^3}+\cdots = S_s - \tfrac{1}{2}\zeta(-s),\\ \Phi_{1,s}(x) &= \frac{1^s x}{(1-x)^2}+\frac{2^s x^2}{(1-x^2)^2}+\frac{3^s x^3}{(1-x^3)^2}+\cdots\end{aligned}\right\}. \qquad (24)$$

Further let

$$\left.\begin{aligned}P &= -24S_1 = 1-24\left(\frac{x}{1-x}+\frac{2x^2}{1-x^2}+\frac{3x^3}{1-x^3}+\cdots\right)^{*},\\ Q &= 240S_3 = 1+240\left(\frac{1^3x}{1-x}+\frac{2^3x^2}{1-x^2}+\frac{3^3x^3}{1-x^3}+\cdots\right),\\ R &= -540S_5 = 1-504\left(\frac{1^5x}{1-x}+\frac{2^5x^2}{1-x^2}+\frac{3^5x^3}{1-x^3}+\cdots\right)\end{aligned}\right\}. \qquad (25)$$

For x = q^2; q = $e^{\pi it} = e^{\pi}$; q = (exp(Pi))^2; s = 3, we obtain:

((exp(Pi)))^2 / ((1-((exp(Pi))^2))) + 8*((exp(Pi))^2)^2 / ((1-((exp(Pi))^2)^2)) + 27*((exp(Pi))^2)^3 / ((1-((exp(Pi))^2)^3))

Input:

$$\frac{\exp^2(\pi)}{1-\exp^2(\pi)} + 8\times\frac{\exp^2(\pi)^2}{1-\exp^2(\pi)^2} + 27\times\frac{\exp^2(\pi)^3}{1-\exp^2(\pi)^3}$$

Exact result:

$$\frac{e^{2\pi}}{1-e^{2\pi}} + \frac{8\,e^{4\pi}}{1-e^{4\pi}} + \frac{27\,e^{6\pi}}{1-e^{6\pi}}$$

Decimal approximation:

−36.00189901126993192830140942151506597976022088237629817 29...

-36.00189901126…

Property:

$$\frac{e^{2\pi}}{1-e^{2\pi}} + \frac{8\,e^{4\pi}}{1-e^{4\pi}} + \frac{27\,e^{6\pi}}{1-e^{6\pi}}$$ is a transcendental number

Alternate forms:

$$-\frac{10+e^{-2\pi}+37\,e^{2\pi}+36\,e^{4\pi}}{2\,(\sinh(2\,\pi)+\sinh(4\,\pi))}$$

$$-\frac{e^{2\pi}}{e^{2\pi}-1} - \frac{8\,e^{4\pi}}{e^{4\pi}-1} - \frac{27\,e^{6\pi}}{e^{6\pi}-1}$$

$$-36 - \frac{7}{e^{\pi}-1} + \frac{7}{1+e^{\pi}} + \frac{4}{1+e^{2\pi}} - \frac{9\,(e^{\pi}-2)}{2\,(1-e^{\pi}+e^{2\pi})} + \frac{9\,(2+e^{\pi})}{2\,(1+e^{\pi}+e^{2\pi})}$$

$\sinh(x)$ is the hyperbolic sine function

Series representations:

$$\frac{\exp^2(\pi)}{1-\exp^2(\pi)} + \frac{8\exp^2(\pi)^2}{1-\exp^2(\pi)^2} + \frac{27\exp^2(\pi)^3}{1-\exp^2(\pi)^3} =$$
$$-\left(\left(\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\left(1+10\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}+37\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}+36\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}\right)\right)\Big/\right.$$
$$\left(\left(-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\right.$$
$$\left.\left.\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\right)\right)$$

$$\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{8\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{27\exp^2(\pi)^3}{1-\exp^2(\pi)^3}=$$
$$-\left(\left(\left(1+10\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}+37\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi}+36\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{6\pi}\right)\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)\Big/\right.$$
$$\left(\left(-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)\right.$$
$$\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)\left(1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)$$
$$\left.\left.\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)\right)\right)$$

$$\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{8\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{27\exp^2(\pi)^3}{1-\exp^2(\pi)^3}=-\left(\left(e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right.\right.$$
$$\left.\left(1+10\,e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+37\,e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+36\,e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right)\Big/$$
$$\left(\left(-1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right.$$
$$\left(1-e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)$$
$$\left(1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)$$
$$\left(1-e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)$$
$$\left.\left(1-e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right)\Big)$$

Integral representations:

$$\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{8\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{27\exp^2(\pi)^3}{1-\exp^2(\pi)^3}=$$
$$\frac{e^{16/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}{1-e^{16/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}+\frac{8\,e^{32/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}{1-e^{32/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}+\frac{27\,e^{16\int_0^{\infty}\sin^3(t)/t^3\,dt}}{1-e^{16\int_0^{\infty}\sin^3(t)/t^3\,dt}}$$

$$\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{8\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{27\exp^2(\pi)^3}{1-\exp^2(\pi)^3}=$$
$$-\left(\left(e^{4\int_0^{\infty}\sin(t)/t\,dt}\left(1+10\,e^{4\int_0^{\infty}\sin(t)/t\,dt}+37\,e^{8\int_0^{\infty}\sin(t)/t\,dt}+36\,e^{12\int_0^{\infty}\sin(t)/t\,dt}\right)\right)\Big/\right.$$
$$\left(\left(-1+e^{\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1+e^{\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1+e^{2\int_0^{\infty}\sin(t)/t\,dt}\right)\right.$$
$$\left(1-e^{\int_0^{\infty}\sin(t)/t\,dt}+e^{2\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1+e^{\int_0^{\infty}\sin(t)/t\,dt}+e^{2\int_0^{\infty}\sin(t)/t\,dt}\right)$$
$$\left.\left.\left(1+e^{4\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1-e^{2\int_0^{\infty}\sin(t)/t\,dt}+e^{4\int_0^{\infty}\sin(t)/t\,dt}\right)\right)\right)$$

$$\frac{\exp^2(\pi)}{1-\exp^2(\pi)} + \frac{8\exp^2(\pi)^2}{1-\exp^2(\pi)^2} + \frac{27\exp^2(\pi)^3}{1-\exp^2(\pi)^3} =$$

$$-\left(\left(e^{4\int_0^\infty 1/(1+t^2)\,dt}\left(1+10\,e^{4\int_0^\infty 1/(1+t^2)\,dt}+37\,e^{8\int_0^\infty 1/(1+t^2)\,dt}+36\,e^{12\int_0^\infty 1/(1+t^2)\,dt}\right)\right)\Big/\right.$$
$$\left(\left(-1+e^{\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1+e^{\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1+e^{2\int_0^\infty 1/(1+t^2)\,dt}\right)\right.$$
$$\left(1-e^{\int_0^\infty 1/(1+t^2)\,dt}+e^{2\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1+e^{\int_0^\infty 1/(1+t^2)\,dt}+e^{2\int_0^\infty 1/(1+t^2)\,dt}\right)$$
$$\left.\left.\left(1+e^{4\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1-e^{2\int_0^\infty 1/(1+t^2)\,dt}+e^{4\int_0^\infty 1/(1+t^2)\,dt}\right)\right)\right)$$

((exp(Pi)))^2 / (1-((exp(Pi))^2))^2 + 8*((exp(Pi))^2)^2 / (1-((exp(Pi))^2)^2)^2 + 27*((exp(Pi))^2)^3 / (1-((exp(Pi))^2)^3)^2

Input:

$$\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2} + 8\times\frac{\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2} + 27\times\frac{\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}$$

Exact result:

$$\frac{e^{2\pi}}{(1-e^{2\pi})^2} + \frac{8\,e^{4\pi}}{(1-e^{4\pi})^2} + \frac{27\,e^{6\pi}}{(1-e^{6\pi})^2}$$

Decimal approximation:

0.001902511770982413535314427039665099167434985249917553048...

0.0019025117709...

Property:

$$\frac{e^{2\pi}}{(1-e^{2\pi})^2} + \frac{8\,e^{4\pi}}{(1-e^{4\pi})^2} + \frac{27\,e^{6\pi}}{(1-e^{6\pi})^2}$$ is a transcendental number

Alternate forms:

$$\frac{2\,(2+\cosh(2\pi))^3}{(\sinh(2\pi)+\sinh(4\pi))^2}$$

$$\frac{e^{2\pi}}{(e^{2\pi}-1)^2} + \frac{8\,e^{4\pi}}{(e^{4\pi}-1)^2} + \frac{27\,e^{6\pi}}{(e^{6\pi}-1)^2}$$

$$\frac{e^{2\pi}\,(1+4\,e^{2\pi}+e^{4\pi})^3}{(e^{\pi}-1)^2\,(1+e^{\pi})^2\,(1+e^{2\pi})^2\,(1-e^{\pi}+e^{2\pi})^2\,(1+e^{\pi}+e^{2\pi})^2}$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

Series representations:

$$\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+\frac{8\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+\frac{27\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}= \left(\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\left(1+4\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}\right)^3\right)\Bigg/\left(\left(-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)^2\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)^2 \left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)^2\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)^2\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)^2\right)$$

$$\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+\frac{8\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+\frac{27\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}= \left(\left(1+4\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi}\right)^3\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)\Bigg/ \left(\left(-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)^2\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)^2\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)^2 \left(1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)^2\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)^2\right)$$

$$\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+\frac{8\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+\frac{27\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}= \left(e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\left(1+4e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^3\right)\Big/ \left(\left(-1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2\left(1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2 \left(1+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2\left(1-e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2 \left(1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2\left(1+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2 \left(1-e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2\left(1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2 \left(1-e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2\right)$$

We note that:

1+ ((exp(Pi)))^2 / (1-((exp(Pi))^2))^2 + 8*((exp(Pi))^2)^2 / (1-((exp(Pi))^2)^2)^2 + 27*((exp(Pi))^2)^3 / (1-((exp(Pi))^2)^3)^2

Input:

$$1+\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+8\times\frac{\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+27\times\frac{\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}$$

Exact result:

$$1+\frac{e^{2\pi}}{(1-e^{2\pi})^2}+\frac{8\,e^{4\pi}}{(1-e^{4\pi})^2}+\frac{27\,e^{6\pi}}{(1-e^{6\pi})^2}$$

Decimal approximation:

1.001902511770982413535314427039665099167434985249917553048...

1.00190251177….. result that is very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}}-\varphi}=1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-4\pi}}{1+\cfrac{e^{-6\pi}}{1+\cfrac{e^{-8\pi}}{1+\dots}}}}\approx 1.0018674362$$

Property:

$$1+\frac{e^{2\pi}}{(1-e^{2\pi})^2}+\frac{8\,e^{4\pi}}{(1-e^{4\pi})^2}+\frac{27\,e^{6\pi}}{(1-e^{6\pi})^2}$$ is a transcendental number

Alternate forms:

$$\frac{42+49\cosh(2\pi)+13\cosh(4\pi)+3\cosh(6\pi)+\cosh(8\pi)}{2\,(\sinh(2\pi)+\sinh(4\pi))^2}$$

$$1+\frac{e^{2\pi}}{(e^{2\pi}-1)^2}+\frac{8\,e^{4\pi}}{(e^{4\pi}-1)^2}+\frac{27\,e^{6\pi}}{(e^{6\pi}-1)^2}$$

$$\frac{1+3\,e^{2\pi}+13\,e^{4\pi}+49\,e^{6\pi}+84\,e^{8\pi}+49\,e^{10\pi}+13\,e^{12\pi}+3\,e^{14\pi}+e^{16\pi}}{(e^{\pi}-1)^2\,(1+e^{\pi})^2\,(1+e^{2\pi})^2\,(1-e^{\pi}+e^{2\pi})^2\,(1+e^{\pi}+e^{2\pi})^2}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Series representations:

$$1+\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+\frac{8\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+\frac{27\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}=$$
$$1+\frac{e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2}+\frac{8e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2}+\frac{27e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2}$$

$$1+\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+\frac{8\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+\frac{27\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}=$$
$$1+\frac{\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2}+$$
$$\frac{8\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2}+\frac{27\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2}$$

$$1+\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+\frac{8\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+\frac{27\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}=$$
$$1+\frac{\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2}+$$
$$\frac{8\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2}+\frac{27\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2}$$

From the ratio of the two previous results, performing the square root, we obtain:

(((-36.001899011269931928 / -[((exp(Pi)))^2 / (1-((exp(Pi))^2))^2 + 8*((exp(Pi))^2)^2 / (1-((exp(Pi))^2)^2)^2 + 27*((exp(Pi))^2)^3 / (1-((exp(Pi))^2)^3)^2])))^1/2

Input interpretation:

$$\sqrt{\frac{-36.001899011269931928}{-\left(\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+8\times\frac{\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+27\times\frac{\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}\right)}}$$

Result:

137.56217328526607012...

137.56217328... result practically equal to the golden angle value 137.5 and very near to the inverse of fine-structure constant 137.035

(((-36.001899011269931928 / -[((exp(Pi)))^2 / (1-((exp(Pi))^2))^2 + 8*((exp(Pi))^2)^2 / (1-((exp(Pi))^2)^2)^2 + 27*((exp(Pi))^2)^3 / (1-((exp(Pi))^2)^3)^2])))^1/2 - 13+1/golden ratio

Input interpretation:

$$\sqrt{\frac{-36.001899011269931928}{-\left(\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+8\times\frac{\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+27\times\frac{\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}\right)}}-13+\frac{1}{\phi}$$

ϕ is the golden ratio

Result:

125.18020727401596497...

125.18020727…

27*1/2((((((-36.001899011269931928 / -[((exp(Pi)))^2 / (1-((exp(Pi))^2))^2 + 8*((exp(Pi))^2)^2 / (1-((exp(Pi))^2)^2)^2 + 27*((exp(Pi))^2)^3 / (1-((exp(Pi))^2)^3)^2])))^1/2 - 11+golden ratio)))-sqrt2

Input interpretation:

$$27\times\frac{1}{2}\left(\sqrt{\frac{-36.001899011269931928}{-\left(\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+8\times\frac{\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+27\times\frac{\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}\right)}}-11+\phi\right)-\sqrt{2}$$

ϕ is the golden ratio

Result:

1729.0185846368424320...

1729.0185846…

Series representations:

$$\frac{27}{2}\left(\sqrt{\frac{-36.0018990112699319280000}{-\left(\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+\frac{8\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+\frac{27\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}\right)}}-11+\phi\right)-\sqrt{2}=-\frac{297}{2}+\frac{27\phi}{2}+81.0021363595056367688483\sqrt{-\frac{1}{-\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}-\frac{8\exp^4(\pi)}{(1-\exp^4(\pi))^2}-\frac{27\exp^6(\pi)}{(1-\exp^6(\pi))^2}}}-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}$$ for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{27}{2}\left(\sqrt{\frac{-36.0018990112699319280000}{-\left(\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+\frac{8\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+\frac{27\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}\right)}}-11+\phi\right)-\sqrt{2}=-\frac{297}{2}+\frac{27\phi}{2}+81.0021363595056367688483\sqrt{-\frac{1}{-\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}-\frac{8\exp^4(\pi)}{(1-\exp^4(\pi))^2}-\frac{27\exp^6(\pi)}{(1-\exp^6(\pi))^2}}}-\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$ for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{27}{2}\left(\sqrt{\frac{-36.0018990112699319280000}{-\left(\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}+\frac{8\exp^2(\pi)^2}{(1-\exp^2(\pi)^2)^2}+\frac{27\exp^2(\pi)^3}{(1-\exp^2(\pi)^3)^2}\right)}}-11+\phi\right)-\sqrt{2}=-\frac{297}{2}+\frac{27\phi}{2}+81.0021363595056367688483\sqrt{-\frac{1}{-\frac{\exp^2(\pi)}{(1-\exp^2(\pi))^2}-\frac{8\exp^4(\pi)}{(1-\exp^4(\pi))^2}-\frac{27\exp^6(\pi)}{(1-\exp^6(\pi))^2}}}-\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}$$

Now, we have that:

$$P = -24S_1 = 1-24\left(\frac{x}{1-x}+\frac{2x^2}{1-x^2}+\frac{3x^3}{1-x^3}+\cdots\right)$$

$$Q = 240S_3 = 1+240\left(\frac{1^3x}{1-x}+\frac{2^3x^2}{1-x^2}+\frac{3^3x^3}{1-x^3}+\cdots\right),$$

$$R = -540S_5 = 1-504\left(\frac{1^5x}{1-x}+\frac{2^5x^2}{1-x^2}+\frac{3^5x^3}{1-x^3}+\cdots\right)$$

For x = q^2; q = $e^{\pi i t} = e^{\pi}$; q^2 = x = (exp(Pi))^2; s = 3, we obtain:

1-24[((exp(Pi))^2)/(1-(((exp(Pi))^2))) + 2(((exp(Pi))^2)^2)/(1-(((exp(Pi))^2)^2)) + 3(((exp(Pi))^2)^3)/(1-(((exp(Pi))^2)^3))]

Input:

$$1-24\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+2\times\frac{\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+3\times\frac{\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)$$

Exact result:

$$1-24\left(\frac{e^{2\pi}}{1-e^{2\pi}}+\frac{2e^{4\pi}}{1-e^{4\pi}}+\frac{3e^{6\pi}}{1-e^{6\pi}}\right)$$

Decimal approximation:

145.04507034027838709939527563816695432802242562362977872 26...

P = 145.04507034…..

Property:

$1-24\left(\frac{e^{2\pi}}{1-e^{2\pi}}+\frac{2e^{4\pi}}{1-e^{4\pi}}+\frac{3e^{6\pi}}{1-e^{6\pi}}\right)$ is a transcendental number

Alternate forms:

$$73+12\tanh(\pi)+36\coth(\pi)+\frac{48\sinh(2\pi)}{1+2\cosh(2\pi)}$$

$$1+\frac{24e^{2\pi}}{e^{2\pi}-1}+\frac{48e^{4\pi}}{e^{4\pi}-1}+\frac{72e^{6\pi}}{e^{6\pi}-1}$$

$$1-\frac{24e^{2\pi}}{1-e^{2\pi}}-\frac{48e^{4\pi}}{1-e^{4\pi}}-\frac{72e^{6\pi}}{1-e^{6\pi}}$$

coth(x) is the hyperbolic cotangent function

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

tanh(x) is the hyperbolic tangent function

Series representations:

$$1-24\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{2\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{3\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$\left(-1+23\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}+96\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}+169\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}+145\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}\right)\Big/$$
$$\left(\left(-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\right.$$
$$\left.\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\right)$$

$$1-24\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{2\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{3\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$\left(-1+23\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}+96\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi}+\right.$$
$$\left.169\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{6\pi}+145\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi}\right)\Big/$$
$$\left(\left(-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)\right.$$
$$\left.\left(1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)\right)$$

$$1-24\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{2\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{3\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$\left(-1+23\,e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+96\,e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+169\,e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+\right.$$
$$\left.145\,e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\Big/\left(\left(-1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right.$$
$$\left(1+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)$$
$$\left(1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)$$
$$\left(1-e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)$$
$$\left.\left(1-e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right)$$

Integral representations:

$$1-24\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{2\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{3\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$1-\frac{24\,e^{16/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}{1-e^{16/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}-\frac{48\,e^{32/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}{1-e^{32/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}-\frac{72\,e^{16\int_0^{\infty}\sin^3(t)/t^3\,dt}}{1-e^{16\int_0^{\infty}\sin^3(t)/t^3\,dt}}$$

$$1-24\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{2\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{3\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$\left(-1+23\,e^{4\int_0^\infty \sin(t)/t\,dt}+96\,e^{8\int_0^\infty \sin(t)/t\,dt}+169\,e^{12\int_0^\infty \sin(t)/t\,dt}+145\,e^{16\int_0^\infty \sin(t)/t\,dt}\right)/$$
$$\left(\left(-1+e^{\int_0^\infty \sin(t)/t\,dt}\right)\left(1+e^{\int_0^\infty \sin(t)/t\,dt}\right)\left(1+e^{2\int_0^\infty \sin(t)/t\,dt}\right)\right.$$
$$\left(1-e^{\int_0^\infty \sin(t)/t\,dt}+e^{2\int_0^\infty \sin(t)/t\,dt}\right)\left(1+e^{\int_0^\infty \sin(t)/t\,dt}+e^{2\int_0^\infty \sin(t)/t\,dt}\right)$$
$$\left.\left(1+e^{4\int_0^\infty \sin(t)/t\,dt}\right)\left(1-e^{2\int_0^\infty \sin(t)/t\,dt}+e^{4\int_0^\infty \sin(t)/t\,dt}\right)\right)$$

$$1-24\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{2\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{3\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$\left(-1+23\,e^{4\int_0^\infty 1/(1+t^2)\,dt}+96\,e^{8\int_0^\infty 1/(1+t^2)\,dt}+\right.$$
$$\left.169\,e^{12\int_0^\infty 1/(1+t^2)\,dt}+145\,e^{16\int_0^\infty 1/(1+t^2)\,dt}\right)/$$
$$\left(\left(-1+e^{\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1+e^{\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1+e^{2\int_0^\infty 1/(1+t^2)\,dt}\right)\right.$$
$$\left(1-e^{\int_0^\infty 1/(1+t^2)\,dt}+e^{2\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1+e^{\int_0^\infty 1/(1+t^2)\,dt}+e^{2\int_0^\infty 1/(1+t^2)\,dt}\right)$$
$$\left.\left(1+e^{4\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1-e^{2\int_0^\infty 1/(1+t^2)\,dt}+e^{4\int_0^\infty 1/(1+t^2)\,dt}\right)\right)$$

1+240[((exp(Pi))^2)/(1-(((exp(Pi))^2))) + 8(((exp(Pi))^2)^2)/(1-(((exp(Pi))^2)^2)) + 27(((exp(Pi))^2)^3)/(1-(((exp(Pi))^2)^3))]

Input:

$$1+240\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+8\times\frac{\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+27\times\frac{\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)$$

Exact result:

$$1+240\left(\frac{e^{2\pi}}{1-e^{2\pi}}+\frac{8\,e^{4\pi}}{1-e^{4\pi}}+\frac{27\,e^{6\pi}}{1-e^{6\pi}}\right)$$

Decimal approximation:

-8639.455762704783662792338261163615835142453011770311561 51...

Q = -8639.4557627….

Property:

$$1+240\left(\frac{e^{2\pi}}{1-e^{2\pi}}+\frac{8\,e^{4\pi}}{1-e^{4\pi}}+\frac{27\,e^{6\pi}}{1-e^{6\pi}}\right)$$ is a transcendental number

Alternate forms:

$$-4319 - \frac{240 (5 + 19 \cosh(2\pi) + 18 \cosh(4\pi))}{\sinh(2\pi) + \sinh(4\pi)}$$

$$1 - \frac{240 e^{2\pi}}{e^{2\pi} - 1} - \frac{1920 e^{4\pi}}{e^{4\pi} - 1} - \frac{6480 e^{6\pi}}{e^{6\pi} - 1}$$

$$1 - \frac{240 e^{2\pi} (1 + 10 e^{2\pi} + 37 e^{4\pi} + 36 e^{6\pi})}{-1 - e^{2\pi} + e^{6\pi} + e^{8\pi}}$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

Series representations:

$$1 + 240 \left(\frac{\exp^2(\pi)}{1 - \exp^2(\pi)} + \frac{8 \exp^2(\pi)^2}{1 - \exp^2(\pi)^2} + \frac{27 \exp^2(\pi)^3}{1 - \exp^2(\pi)^3}\right) = -\left(\left(1 + 241 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi} + 2400 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4\pi} + 8879 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6\pi} + 8639 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8\pi}\right) \Big/ \left(\left(-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{\pi}\right) \left(1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{\pi}\right) \left(1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi}\right) \left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{\pi} + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi}\right) \left(1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{\pi} + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi}\right)\right)\right)$$

$$1 + 240 \left(\frac{\exp^2(\pi)}{1 - \exp^2(\pi)} + \frac{8 \exp^2(\pi)^2}{1 - \exp^2(\pi)^2} + \frac{27 \exp^2(\pi)^3}{1 - \exp^2(\pi)^3}\right) = -\left(\left(1 + 241 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi} + 2400 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{4\pi} + 8879 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{6\pi} + 8639 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{8\pi}\right) \Big/ \left(\left(-1 + \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{\pi}\right) \left(1 + \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{\pi}\right) \left(1 + \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi}\right) \left(1 - \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{\pi} + \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi}\right) \left(1 + \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{\pi} + \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi}\right)\right)\right)$$

$$1+240\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{8\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{27\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$-\left(\left(1+241\,e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+2400\,e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+8879\,e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+8639\,e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\Big/\left(\left(-1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right)\right)$$

Integral representations:

$$1+240\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{8\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{27\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$1+\frac{240\,e^{16/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}{1-e^{16/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}+\frac{1920\,e^{32/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}{1-e^{32/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}+\frac{6480\,e^{16\int_0^{\infty}\sin^3(t)/t^3\,dt}}{1-e^{16\int_0^{\infty}\sin^3(t)/t^3\,dt}}$$

$$1+240\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{8\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{27\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$-\left(\left(1+241\,e^{4\int_0^{\infty}\sin(t)/t\,dt}+2400\,e^{8\int_0^{\infty}\sin(t)/t\,dt}+8879\,e^{12\int_0^{\infty}\sin(t)/t\,dt}+8639\,e^{16\int_0^{\infty}\sin(t)/t\,dt}\right)\Big/\left(\left(-1+e^{\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1+e^{\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1+e^{2\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1-e^{\int_0^{\infty}\sin(t)/t\,dt}+e^{2\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1+e^{\int_0^{\infty}\sin(t)/t\,dt}+e^{2\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1+e^{4\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1-e^{2\int_0^{\infty}\sin(t)/t\,dt}+e^{4\int_0^{\infty}\sin(t)/t\,dt}\right)\right)\right)$$

$$1+240\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{8\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{27\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$-\left(\left(1+241\,e^{4\int_0^{\infty}1/(1+t^2)\,dt}+2400\,e^{8\int_0^{\infty}1/(1+t^2)\,dt}+8879\,e^{12\int_0^{\infty}1/(1+t^2)\,dt}+8639\,e^{16\int_0^{\infty}1/(1+t^2)\,dt}\right)\Big/\left(\left(-1+e^{\int_0^{\infty}1/(1+t^2)\,dt}\right)\left(1+e^{\int_0^{\infty}1/(1+t^2)\,dt}\right)\left(1+e^{2\int_0^{\infty}1/(1+t^2)\,dt}\right)\left(1-e^{\int_0^{\infty}1/(1+t^2)\,dt}+e^{2\int_0^{\infty}1/(1+t^2)\,dt}\right)\left(1+e^{\int_0^{\infty}1/(1+t^2)\,dt}+e^{2\int_0^{\infty}1/(1+t^2)\,dt}\right)\left(1+e^{4\int_0^{\infty}1/(1+t^2)\,dt}\right)\left(1-e^{2\int_0^{\infty}1/(1+t^2)\,dt}+e^{4\int_0^{\infty}1/(1+t^2)\,dt}\right)\right)\right)$$

1-504[((exp(Pi))^2)/(1-(((exp(Pi))^2))) + 32(((exp(Pi))^2)^2)/(1-(((exp(Pi))^2)^2)) + 243(((exp(Pi))^2)^3)/(1-(((exp(Pi))^2)^3))]

Input:

$$1-504\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+32\times\frac{\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+243\times\frac{\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)$$

Exact result:

$$1-504\left(\frac{e^{2\pi}}{1-e^{2\pi}}+\frac{32\,e^{4\pi}}{1-e^{4\pi}}+\frac{243\,e^{6\pi}}{1-e^{6\pi}}\right)$$

Decimal approximation:

139105.99999368753249813099682365864533301692570414464246 31...

R = 139105.9999…

Property:

$$1-504\left(\frac{e^{2\pi}}{1-e^{2\pi}}+\frac{32\,e^{4\pi}}{1-e^{4\pi}}+\frac{243\,e^{6\pi}}{1-e^{6\pi}}\right)$$ is a transcendental number

Alternate forms:

$$69553+\frac{504\,(17+139\cosh(2\pi)+138\cosh(4\pi))}{\sinh(2\pi)+\sinh(4\pi)}$$

$$1+\frac{504\,e^{2\pi}}{e^{2\pi}-1}+\frac{16128\,e^{4\pi}}{e^{4\pi}-1}+\frac{122472\,e^{6\pi}}{e^{6\pi}-1}$$

$$1+\frac{504\,e^{2\pi}\left(1+34\,e^{2\pi}+277\,e^{4\pi}+276\,e^{6\pi}\right)}{-1-e^{2\pi}+e^{6\pi}+e^{8\pi}}$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

Series representations:

$$1-504\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{32\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{243\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$

$$\left(-1+503\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}+17136\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}+139609\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}+139105\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}\right)\Bigg/$$
$$\left(\left(-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\right.$$
$$\left.\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)\right)$$

$$1-504\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{32\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{243\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$\left(-1+503\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}+17136\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi}+139609\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{6\pi}+139105\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi}\right)\Big/$$
$$\left(\left(-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)\left(1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)\right)$$

$$1-504\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{32\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{243\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$\left(-1+503\,e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+17136\,e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+139609\,e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+139105\,e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\Big/$$
$$\left(\left(-1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right)$$

Integral representations:

$$1-504\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{32\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{243\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$1-\frac{504\,e^{16/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}{1-e^{16/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}-\frac{16128\,e^{32/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}{1-e^{32/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}-\frac{122472\,e^{16\int_0^{\infty}\sin^3(t)/t^3\,dt}}{1-e^{16\int_0^{\infty}\sin^3(t)/t^3\,dt}}$$

$$1-504\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{32\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{243\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$\left(-1+503\,e^{4\int_0^{\infty}\sin(t)/t\,dt}+17136\,e^{8\int_0^{\infty}\sin(t)/t\,dt}+139609\,e^{12\int_0^{\infty}\sin(t)/t\,dt}+139105\,e^{16\int_0^{\infty}\sin(t)/t\,dt}\right)\Big/\left(\left(-1+e^{\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1+e^{\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1+e^{2\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1-e^{\int_0^{\infty}\sin(t)/t\,dt}+e^{2\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1+e^{\int_0^{\infty}\sin(t)/t\,dt}+e^{2\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1+e^{4\int_0^{\infty}\sin(t)/t\,dt}\right)\left(1-e^{2\int_0^{\infty}\sin(t)/t\,dt}+e^{4\int_0^{\infty}\sin(t)/t\,dt}\right)\right)$$

$$1-504\left(\frac{\exp^2(\pi)}{1-\exp^2(\pi)}+\frac{32\exp^2(\pi)^2}{1-\exp^2(\pi)^2}+\frac{243\exp^2(\pi)^3}{1-\exp^2(\pi)^3}\right)=$$
$$\left(-1+503\,e^{4\int_0^\infty 1/(1+t^2)\,dt}+17136\,e^{8\int_0^\infty 1/(1+t^2)\,dt}+139609\,e^{12\int_0^\infty 1/(1+t^2)\,dt}+139105\,e^{16\int_0^\infty 1/(1+t^2)\,dt}\right)\Big/$$
$$\left(\left(-1+e^{\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1+e^{\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1+e^{2\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1-e^{\int_0^\infty 1/(1+t^2)\,dt}+e^{2\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1+e^{\int_0^\infty 1/(1+t^2)\,dt}+e^{2\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1+e^{4\int_0^\infty 1/(1+t^2)\,dt}\right)\left(1-e^{2\int_0^\infty 1/(1+t^2)\,dt}+e^{4\int_0^\infty 1/(1+t^2)\,dt}\right)\right)$$

Thence, we have obtained:

P = 145.04507034….. Q = -8639.4557627…. R = 139105.9999…
x = (exp(Pi))^2

We have that:

Suppose now that m and n are any two positive integers including zero, and that $m+n$ is not zero. Then

$$\begin{aligned} Q^m R^n(Q^3-R^2) &= Q(Q^3-R^2)Q^{m-1}R^n \\ &= \sum O(\nu^{10})x^\nu\{\sum O(\nu^3)x^\nu\}^{m-1}\{\sum O(\nu^5)x^\nu\}^n \\ &= \sum O(\nu^{10})x^\nu \sum O(\nu^{4m-5})x^\nu \sum O(\nu^{6n-1})x^\nu \\ &= \sum O(\nu^{4m+6n+6})x^\nu, \end{aligned}$$

If m is not zero, Similarly we can shew that

$$\begin{aligned} Q^m R^n(Q^3-R^2) &= R(Q^3-R^2)Q^m R^{n-1} \\ &= \sum O(\nu^{4m+6n+6})x^\nu, \end{aligned}$$

if n is not zero. Therefore in any case

$$(Q^3-R^2)Q^m R^n = \sum O(\nu^{4m+6n+6})x^\nu. \quad (52)$$

Thence:

Q = -8639.4557627…. R = 139105.9999 m = 2 and n = 3, we obtain:

(-8639.4557627^3 - 139105.9999^2)*(-8639.4557627^2 - 139105.9999^3)

Input interpretation:

$(-8639.4557627^3 - 139\,105.9999^2)(-8639.4557627^2 - 139\,105.9999^3)$

Result:

$1.787875289412735788824426522807006824137251487022439 7... \times 10^{27}$

1.7878752894…*10^27

From which:

ln[(-8639.4557627^3 - 139105.9999^2)*(-8639.4557627^2 - 139105.9999^3)]+sqrt5-1

Input interpretation:

$\log((-8639.4557627^3 - 139\,105.9999^2)(-8639.4557627^2 - 139\,105.9999^3)) + \sqrt{5} - 1$

log(x) is the natural logarithm

Result:

63.986893414...

63.986893414…≈ 64

Alternative representations:

$$\log((-8639.45576270000^3 - 139\,106.^2)(-8639.45576270000^2 - 139\,106.^3)) + \sqrt{5} - 1 = -1 + \log_e((-8639.45576270000^3 - 139\,106.^2)(-8639.45576270000^2 - 139\,106.^3)) + \sqrt{5}$$

$$\log((-8639.45576270000^3 - 139\,106.^2)(-8639.45576270000^2 - 139\,106.^3)) + \sqrt{5} - 1 = -1 + \log(a)\log_a((-8639.45576270000^3 - 139\,106.^2)(-8639.45576270000^2 - 139\,106.^3)) + \sqrt{5}$$

$$\log((-8639.45576270000^3 - 139\,106.^2)(-8639.45576270000^2 - 139\,106.^3)) + \sqrt{5} - 1 = -1 - \text{Li}_1(1 - (-8639.45576270000^3 - 139\,106.^2)(-8639.45576270000^2 - 139\,106.^3)) + \sqrt{5}$$

Series representations:

$$\log((-8639.45576270000^3 - 139\,106.^2)(-8639.45576270000^2 - 139\,106.^3)) + \sqrt{5} - 1 = -1 + \log(1.78788 \times 10^{27}) + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}$$

$$\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1 = -1 + \log(1.78788\times10^{27}) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-62.7508 k}}{k} + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}$$

$$\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1 = -1 + \log(1.78788\times10^{27}) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-62.7508 k}}{k} + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

Integral representations:

$$\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1 = -1 + \int_1^{1.78788\times10^{27}} \frac{1}{t}\, dt + \sqrt{5}$$

$$\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1 = -1 + \frac{1}{2 i \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-62.7508 s}\, \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds + \sqrt{5} \quad \text{for } -1 < \gamma < 0$$

and:

27*(((ln[(-8639.4557627^3 - 139105.9999^2)*(-8639.4557627^2 - 139105.9999^3)]+sqrt5-1)))+sqrt2

Input interpretation:

$$27\left(\log((-8639.4557627^3 - 139105.9999^2)(-8639.4557627^2 - 139105.9999^3)) + \sqrt{5} - 1\right) + \sqrt{2}$$

log(x) is the natural logarithm

Result:

1729.0603357…

1729.0603357…

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Alternative representations:

$$27\left(\log((-8639.45576270000^3-139106.^2)(-8639.45576270000^2-139106.^3))+\sqrt{5}-1\right)+\sqrt{2}=\sqrt{2}+27\left(-1+\log_e((-8639.45576270000^3-139106.^2)(-8639.45576270000^2-139106.^3))+\sqrt{5}\right)$$

$$27\left(\log((-8639.45576270000^3-139106.^2)(-8639.45576270000^2-139106.^3))+\sqrt{5}-1\right)+\sqrt{2}=\sqrt{2}+27\left(-1+\log(a)\log_a((-8639.45576270000^3-139106.^2)(-8639.45576270000^2-139106.^3))+\sqrt{5}\right)$$

$$27\left(\log((-8639.45576270000^3-139106.^2)(-8639.45576270000^2-139106.^3))+\sqrt{5}-1\right)+\sqrt{2}=\sqrt{2}+27\left(-1-\text{Li}_1(1-(-8639.45576270000^3-139106.^2)(-8639.45576270000^2-139106.^3))+\sqrt{5}\right)$$

Series representations:

$$27\left(\log((-8639.45576270000^3-139106.^2)(-8639.45576270000^2-139106.^3))+\sqrt{5}-1\right)+\sqrt{2}=-27+27\log(1.78788\times10^{27})+\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left((2-x)^k\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)+27(5-x)^k\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\right)\left(-\frac{1}{2}\right)_k\sqrt{x}}{k!}$$

for $(x\in\mathbb{R}$ and $x<0)$

$$27\left(\log((-8639.45576270000^3-139106.^2)(-8639.45576270000^2-139106.^3))+\sqrt{5}-1\right)+\sqrt{2}=-27+27\log(1.78788\times10^{27})-27\sum_{k=1}^{\infty}\frac{(-5.59323\times10^{-28})^k}{k}+\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left((2-x)^k\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)+27(5-x)^k\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\right)\left(-\frac{1}{2}\right)_k\sqrt{x}}{k!}$$

for $(x\in\mathbb{R}$ and $x<0)$

$$27\left(\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1\right) + \sqrt{2} = -27 + 54 i \pi \left\lfloor \frac{\arg(1.78788\times 10^{27} - x)}{2\pi} \right\rfloor + 27\log(x) - 27 \sum_{k=1}^{\infty} \frac{(-1)^k (1.78788\times 10^{27} - x)^k x^{-k}}{k} + \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left((2-x)^k \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) + 27 (5-x)^k \exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right)\right)\left(-\frac{1}{2}\right)_k \sqrt{x}}{k!}$$

for ($x \in \mathbb{R}$ and $x < 0$)

Integral representations:

$$27\left(\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1\right) + \sqrt{2} = -27 + 27 \int_1^{1.78788\times 10^{27}} \frac{1}{t}\, dt + \sqrt{2} + 27\sqrt{5}$$

$$27\left(\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1\right) + \sqrt{2} = -27 + \frac{27}{2 i \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-62.7508 s}\, \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds + \sqrt{2} + 27\sqrt{5}$$ for $-1 < \gamma < 0$

or:

Input interpretation:

$$\phi \sqrt[9]{(-8639.4557^3 - 139105.9999^2)(-8639.4557^2 - 139105.9999^3)} + \pi$$

ϕ is the golden ratio

Result:

1729.0793...

1729.0793…

Alternative representations:

$$\phi \sqrt[9]{(-8639.46^3 - 139106.^2)(-8639.46^2 - 139106.^3)} + \pi = \pi - 2\cos(216°) \sqrt[9]{(-8639.46^3 - 139106.^2)(-8639.46^2 - 139106.^3)}$$

$$\phi \sqrt[9]{(-8639.46^3 - 139\,106.^2)(-8639.46^2 - 139\,106.^3)} + \pi =$$
$$\pi + 2\cos\left(\frac{\pi}{5}\right) \sqrt[9]{(-8639.46^3 - 139\,106.^2)(-8639.46^2 - 139\,106.^3)}$$

$$\phi \sqrt[9]{(-8639.46^3 - 139\,106.^2)(-8639.46^2 - 139\,106.^3)} + \pi =$$
$$180\,° - 2\cos(216\,°) \sqrt[9]{(-8639.46^3 - 139\,106.^2)(-8639.46^2 - 139\,106.^3)}$$

Series representations:

$$\phi \sqrt[9]{(-8639.46^3 - 139\,106.^2)(-8639.46^2 - 139\,106.^3)} + \pi = 1066.69\,\phi + 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

$$\phi \sqrt[9]{(-8639.46^3 - 139\,106.^2)(-8639.46^2 - 139\,106.^3)} + \pi =$$
$$-2 + 1066.69\,\phi + 2 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\phi \sqrt[9]{(-8639.46^3 - 139\,106.^2)(-8639.46^2 - 139\,106.^3)} + \pi =$$
$$1066.69\,\phi + \sum_{k=0}^{\infty} \frac{2^{-k}(-6 + 50k)}{\binom{3k}{k}}$$

Integral representations:

$$\phi \sqrt[9]{(-8639.46^3 - 139\,106.^2)(-8639.46^2 - 139\,106.^3)} + \pi = 1066.69\,\phi + 2 \int_0^{\infty} \frac{1}{1 + t^2}\,dt$$

$$\phi \sqrt[9]{(-8639.46^3 - 139\,106.^2)(-8639.46^2 - 139\,106.^3)} + \pi =$$
$$1066.69\,\phi + 4 \int_0^1 \sqrt{1 - t^2}\,dt$$

$$\phi \sqrt[9]{(-8639.46^3 - 139\,106.^2)(-8639.46^2 - 139\,106.^3)} + \pi = 1066.69\,\phi + 2 \int_0^{\infty} \frac{\sin(t)}{t}\,dt$$

For the other expression, we have:

((((27*(((ln[(-8639.4557627^3 - 139105.9999^2)*(-8639.4557627^2 - 139105.9999^3)]+sqrt5-1)))+sqrt2))))^1/15

Input interpretation:

$$\left(27\left(\log\left((-8639.4557627^3 - 139\,105.9999^2)(-8639.4557627^2 - 139\,105.9999^3)\right) + \sqrt{5} - 1\right) + \sqrt{2}\right) \wedge (1/15)$$

log(x) is the natural logarithm

Result:

1.64381905289...

1.64381905289…

((((27*(((ln[(-8639.4557627^3 - 139105.9999^2)*(-8639.4557627^2 - 139105.9999^3)]+sqrt5-1)))+sqrt2))))^1/15 -(21+5)1/10^3

Input interpretation:

$$\left(27\left(\log\left((-8639.4557627^3 - 139\,105.9999^2)(-8639.4557627^2 - 139\,105.9999^3)\right) + \sqrt{5} - 1\right) + \sqrt{2}\right) \wedge (1/15) - (21+5) \times \frac{1}{10^3}$$

log(x) is the natural logarithm

Result:

1.61781905289...

1.61781905289…

Alternative representations:

$$\left(27\left(\log\left((-8639.45576270000^3 - 139\,106.^2)(-8639.45576270000^2 - 139\,106.^3)\right) + \sqrt{5} - 1\right) + \sqrt{2}\right) \wedge (1/15) - \frac{21+5}{10^3} = -\frac{26}{10^3} + \left(\sqrt{2} + 27\left(-1 + \log_e\left((-8639.45576270000^3 - 139\,106.^2)(-8639.45576270000^2 - 139\,106.^3)\right) + \sqrt{5}\right)\right) \wedge (1/15)$$

$$\left(27\left(\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1\right) + \sqrt{2}\right)^\wedge(1/15) - \frac{21+5}{10^3} = -\frac{26}{10^3} + \left(\sqrt{2} + 27\left(-1 + \log(a)\log_a((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5}\right)\right)^\wedge(1/15)$$

$$\left(27\left(\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1\right) + \sqrt{2}\right)^\wedge(1/15) - \frac{21+5}{10^3} = -\frac{26}{10^3} + \left(\sqrt{2} + 27\left(-1 - \text{Li}_1(1 - (-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5}\right)\right)^\wedge(1/15)$$

Series representations:

$$\left(27\left(\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1\right) + \sqrt{2}\right)^\wedge(1/15) - \frac{21+5}{10^3} = \frac{1}{500}\left(-13 + 500\left(-27 + 27\log(1.78788\times10^{27}) + \exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + 27\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^\wedge(1/15)\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\left(27\left(\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1\right) + \sqrt{2}\right)^\wedge(1/15) - \frac{21+5}{10^3} = \frac{1}{500}\left(-13 + 500\left(-27 + 27\log(1.78788\times10^{27}) - 27\sum_{k=1}^{\infty}\frac{(-5.59323\times10^{-28})^k}{k} + \exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + 27\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^\wedge(1/15)\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\left(27\left(\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1\right) + \sqrt{2}\right) \wedge (1/15) - \frac{21+5}{10^3} =$$

$$\frac{1}{500}\left(-13 + 500\left(-27 + 54 i\pi \left\lfloor \frac{\arg(1.78788\times 10^{27} - x)}{2\pi} \right\rfloor + 27\log(x) - 27\sum_{k=1}^{\infty} \frac{(-1)^k (1.78788\times 10^{27} - x)^k x^{-k}}{k} + \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + 27\exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) \wedge (1/15)\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

Integral representations:

$$\left(27\left(\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1\right) + \sqrt{2}\right) \wedge (1/15) - \frac{21+5}{10^3} =$$

$$-\frac{13}{500} + \sqrt[15]{\sqrt{2} + 27\left(-1 + \int_1^{1.78788\times 10^{27}} \frac{1}{t}\, dt + \sqrt{5}\right)}$$

$$\left(27\left(\log((-8639.45576270000^3 - 139106.^2)(-8639.45576270000^2 - 139106.^3)) + \sqrt{5} - 1\right) + \sqrt{2}\right) \wedge (1/15) - \frac{21+5}{10^3} =$$

$$-\frac{13}{500} + \sqrt[15]{\sqrt{2} + 27\left(-1 + \frac{1}{2 i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1.78788\times 10^{27-s}\, \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds + \sqrt{5}\right)}$$

for $-1 < \gamma < 0$

Now, we have that:

Finally I find that

$$\sum_{1}^{\infty} \frac{e_{16}(n)}{n^s} = \frac{e_{16}(1)}{1+2^{3-s}} \prod_p \left(\frac{1}{1+2c_p \cdot p^{-s} + p^{7-2s}} \right), \tag{162}$$

p being an odd prime and $c_p^2 \leq p^7$. From this it would follow that

$$\left| \frac{e_{16}(n)}{e_{16}(1)} \right| \leq n^{\frac{7}{2}} d(n) \tag{163}$$

for all values of n, and

$$\left| \frac{e_{16}(n)}{e_{16}(1)} \right| \geq n^{\frac{7}{2}} \tag{164}$$

for an infinity of values of n.
In the case in which $2s = 24$ we have

$$\tfrac{691}{64} e_{24}(n) = (-1)^{n-1} 259\tau(n) - 512\tau(\tfrac{1}{2}n).$$

I have already stated the reasons for supposing that

$$|\tau(n)| \leq n^{\frac{11}{2}} d(n)$$

for all values of n, and

$$|\tau(n)| \geq n^{\frac{11}{2}}$$

for an infinity of values of n.

For n = 3, we obtain from

$$\tfrac{691}{64} e_{24}(n) = (-1)^{n-1} 259\tau(n) - 512\tau(\tfrac{1}{2}n).$$

(-1)^2 * 259*3^(11/2) – 512*1/2*3^(11/2)

Input:

$$(-1)^2 \times 259 \times 3^{11/2} - 512 \times \frac{1}{2} \times 3^{11/2}$$

Result:

$$729\sqrt{3}$$

Decimal approximation:

1262.665038717711546981508382957780955501305030027767477852...

1262.665….

From

Modular equations and approximations to π – *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

$$64g_{22}^{24} = e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots,$$

exp(Pi*sqrt22) – 24 + 276*exp(-Pi*sqrt22)

Input:

$$\exp\left(\pi\sqrt{22}\right) - 24 + 276\exp\left(-\pi\sqrt{22}\right)$$

Exact result:

$$-24 + 276\,e^{-\sqrt{22}\,\pi} + e^{\sqrt{22}\,\pi}$$

Decimal approximation:

2.50892799836743055743417280363072461379337839941915425... × 10^6

2.50892799836….*10^6

Property:

$-24 + 276\,e^{-\sqrt{22}\,\pi} + e^{\sqrt{22}\,\pi}$ is a transcendental number

Alternate form:

$$e^{-\sqrt{22}\,\pi}\left(276 - 24\,e^{\sqrt{22}\,\pi} + e^{2\sqrt{22}\,\pi}\right)$$

Series representations:

$$\exp\left(\pi\sqrt{22}\right) - 24 + 276\exp\left(-\pi\sqrt{22}\right) = -24 + 276\exp\left(-\pi\sqrt{21}\sum_{k=0}^{\infty} 21^{-k}\binom{\frac{1}{2}}{k}\right) + \exp\left(\pi\sqrt{21}\sum_{k=0}^{\infty} 21^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\exp\left(\pi\sqrt{22}\right) - 24 + 276\exp\left(-\pi\sqrt{22}\right) = -24 + 276\exp\left(-\pi\sqrt{21}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{21}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right) + \exp\left(\pi\sqrt{21}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{21}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\exp\left(\pi\sqrt{22}\right) - 24 + 276\exp\left(-\pi\sqrt{22}\right) = -24 + 276\exp\left(-\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (22-z_0)^k z_0^{-k}}{k!}\right) + \exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (22-z_0)^k z_0^{-k}}{k!}\right) \text{ for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

(((-(exp(Pi*sqrt22) – 24)+ 2.50892799836743e+6)))/(((exp(-Pi*sqrt22))))

Input interpretation:

$$\frac{-\left(\exp\left(\pi\sqrt{22}\right) - 24\right) + 2.50892799836743 \times 10^6}{\exp\left(-\pi\sqrt{22}\right)}$$

Result:

275.999...

275.999…. = 276

For n = 8, from the above expression

$$\frac{691}{64} e_{24}(n) = (-1)^{n-1} 259\tau(n) - 512\tau(\tfrac{1}{2}n).$$

we obtain:

(-1)^7 * 259*8^(11/2) – 512*1/2*8^(11/2)

Input:

$$(-1)^7 \times 259 \times 8^{11/2} - 512 \times \frac{1}{2} \times 8^{11/2}$$

Exact result:

$$-33\,751\,040\sqrt{2}$$

Decimal approximation:

$$-4.7731178512196825915907748198350488237728138252722389\ldots \times 10^7$$

$-4.7731178512196....*10^7$

from which, changing the sign, we obtain:

-(-1)^7 * 259*8^(11/2) – 512*1/2*8^(11/2)

Input:

$$-(-1)^7 \left(259\times 8^{11/2}\right)-512\times\frac{1}{2}\times 8^{11/2}$$

Result:

$$196\,608\sqrt{2}$$

Decimal approximation:

278045.70007104947135480241668942031983142604807411100677 11...

278045.7......

and again, by the expression above for to obtain 276:

1/sqrt2 ((((-(-1)^7 * 259*8^(11/2) – 512*1/2*8^(11/2) + (((-(exp(Pi*sqrt22) – 24)+ 2.50892799836743e+6)))/(((exp(-Pi*sqrt22))))sqrt2))))

Input interpretation:

$$\frac{1}{\sqrt{2}}\left(-(-1)^7\left(259\times 8^{11/2}\right)-512\times\frac{1}{2}\times 8^{11/2}+\frac{-\left(\exp\left(\pi\sqrt{22}\right)-24\right)+2.50892799836743\times 10^6}{\exp\left(-\pi\sqrt{22}\right)}\sqrt{2}\right)$$

Result:

196884.0...

196884

196884 is a fundamental number of the following j-invariant

$$j(\tau)=q^{-1}+744+196884q+21493760q^2+864299970q^3+20245856256q^4+\cdots$$

(In mathematics, Felix Klein's j-invariant or j function, regarded as a function of a complex variable τ, is a modular function of weight zero for SL(2, Z) defined on the upper half plane of complex numbers. Several remarkable properties of j have to do with its q expansion (Fourier series expansion), written as a Laurent series in terms of $q = e^{2\pi i\tau}$ (the square of the nome), which begins:

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

Note that *j* has a simple pole at the cusp, so its *q*-expansion has no terms below q^{-1}.

All the Fourier coefficients are integers, which results in several almost integers, notably Ramanujan's constant:

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

The asymptotic formula for the coefficient of q^n is given by

$$\frac{e^{4\pi\sqrt{n}}}{\sqrt{2}\,n^{3/4}},$$

as can be proved by the Hardy–Littlewood circle method)

From

ON NON-LINEAR DIFFERENTIAL EQUATIONS OF THE SECOND ORDER: III. THE EQUATION $\ddot{y} - k(1-y^2)\dot{y} + y = b\mu k\cos(\mu t + \alpha)$ FOR LARGE k, AND ITS GENERALIZATIONS

BY *J. E. LITTLEWOOD* in Cambridge

1. We are concerned with equations in real variables of the form

$$\ddot{y} + f(y)\dot{y} + g(y) = p(t),$$

where f, g, p are smooth functions of their arguments, and p has period $\lambda = 2\pi/\mu$ in t. About f we suppose that $\underline{\lim}\, f > 0$ as $y \to \pm\infty$; that is to say, we suppose the "damping" to be positive for large $|y|$. About g we suppose that it has a "restoring" effect, i.e. has the sign of y. The simplest case, and a specially important one, to be covered in any generalization, is $g = ay$ for positive a. We do in fact assume always that $g(0) = 0$, and that g' exists and has a positive lower bound.

Now, from:

JOURNAL OF MATHEMATICAL ANALYSIS AND APPLICATIONS 8, 423-444 (1964) **A Study of Second Order Nonlinear Systems** –
Y. S. LIM AND L. F. KAZDA

In the last two decades, a considerable amount of work has been given to the study of second-order nonlinear differential systems [1-4]. This paper is to study a class of second-order differential systems of the form

$$\ddot{x} + f(\dot{x}) + g(x) = e(t) . \qquad (1)$$

We note that the equation (1) is very similar to the above equation of the J. E. Littlewood paper.

Consider the Duffing equation

$$\ddot{x} + b\dot{x} + x + ax^3 = E \sin \omega t$$

with $0 < b < 2$, $a > 0$. The solutions are bounded as

$$|x(t)| \leqslant I = E\left[1 + \sqrt{\frac{4}{b^2} - 1}\, \exp\left(-\frac{\pi}{2\sqrt{(4/b^2) - 1}}\right)\right] \qquad (45)$$

$$|\dot{x}(t)| \leqslant J = \frac{2E}{b} . \qquad (46)$$

Since

$$g'(x) = 1 + 3ax^2 > 1 , \qquad g''(x) = 6ax ,$$

then $C_2 = 6aI$, and

$$\frac{2EC_2}{q_1} = 12aEI.$$

If a or E is sufficiently small such that the inequality

$$b^2 > \frac{2EC_2}{q_1} = 12aE^2\left[1 + \sqrt{\frac{4}{b^2} - 1}\, \exp\left(-\frac{\pi}{2\sqrt{(4/b^2) - 1}}\right)\right] \qquad (47)$$

holds, then all solutions will converge to unique periodic solution having the same period as that of the forcing function. Otherwise it may have three periodic solutions or subharmonics.

For a = 1.526939, E = b = 1, we obtain from (47)

12*(1+0.526939) * [1+sqrt3 * exp(-Pi/(2sqrt3))]

Where 0.526939 is the value of the following Rogers-Ramanujan continued fraction:

$$2\int_0^\infty \frac{t^2\,dt}{e^{\sqrt{3}t}\sinh t} = \cfrac{1}{1+\cfrac{1^3}{1+\cfrac{1^3}{3+\cfrac{2^3}{1+\cfrac{2^3}{5+\cfrac{3^3}{1+\cfrac{3^3}{7+\dots}}}}}}} \approx 0.5269391135$$

Input interpretation:

$$12\,(1+0.526939)\left(1+\sqrt{3}\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)$$

Result:

31.1378...

31.1378...

Series representations:

$$12\,(1+0.526939)\left(1+\sqrt{3}\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right) = 18.3233\left(1+\exp\left(-\frac{\pi}{2\sqrt{2}\sum_{k=0}^\infty 2^{-k}\binom{\frac{1}{2}}{k}}\right)\sqrt{2}\sum_{k=0}^\infty 2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$12\,(1+0.526939)\left(1+\sqrt{3}\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right) = 18.3233\left(1+\exp\left(-\frac{\pi}{2\sqrt{2}\sum_{k=0}^\infty \frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\sqrt{2}\sum_{k=0}^\infty \frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$12\,(1+0.526939)\left(1+\sqrt{3}\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right) = \frac{9.16163\left(2\sqrt{\pi}+\exp\left(-\frac{\pi\sqrt{\pi}}{\sum_{j=0}^\infty \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\sum_{j=0}^\infty \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)}{\sqrt{\pi}}$$

For a = 1.65578, that is the value of the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 , we obtain:

12*(1.65578) * [1+sqrt3 * exp(-Pi/(2sqrt3))]

Input interpretation:

$$12\times 1.65578\left(1+\sqrt{3}\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)$$

Result:

33.7651...

33.7651…

Series representations:

$$12\times 1.65578\left(1+\sqrt{3}\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right) = 19.8694\left(1+\exp\left(-\frac{\pi}{2\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}\right)\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$12\times 1.65578\left(1+\sqrt{3}\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right) = 19.8694\left(1+\exp\left(-\frac{\pi}{2\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$12\times 1.65578\left(1+\sqrt{3}\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right) = \frac{9.93468\left(2\sqrt{\pi}+\exp\left(-\frac{\pi\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)}{\sqrt{\pi}}$$

Multiplying by 4, we obtain:

48*(1.65578) * [1+sqrt3 * exp(-Pi/(2sqrt3))]

Input interpretation:

$$48\times 1.65578\left(1+\sqrt{3}\,\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)$$

Result:

135.061...

135.061…

Series representations:

$$48\times 1.65578\left(1+\sqrt{3}\,\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)=79.4774\left(1+\exp\left(-\frac{\pi}{2\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}\right)\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$48\times 1.65578\left(1+\sqrt{3}\,\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)=79.4774\left(1+\exp\left(-\frac{\pi}{2\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$48\times 1.65578\left(1+\sqrt{3}\,\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)=\frac{39.7387\left(2\sqrt{\pi}+\exp\left(-\frac{\pi\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)}{\sqrt{\pi}}$$

48*(1.65578) * [1+sqrt3 * exp(-Pi/(2sqrt3))]+3

Input interpretation:

$$48\times 1.65578\left(1+\sqrt{3}\,\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)+3$$

Result:

138.061...

138.061…

Series representations:

$$48\times 1.65578\left(1+\sqrt{3}\,\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)+3=$$
$$79.4774\left(1.03775+\exp\left(-\frac{\pi}{2\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}\right)\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$48\times 1.65578\left(1+\sqrt{3}\,\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)+3=$$
$$79.4774\left(1.03775+\exp\left(-\frac{\pi}{2\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$48\times 1.65578\left(1+\sqrt{3}\,\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)+3=\frac{1}{\sqrt{\pi}}\,39.7387\left(2.07549\sqrt{\pi}+\right.$$
$$\left.\exp\left(-\frac{\pi\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)$$

(((48*(1.65578) * [1+sqrt3 * exp(-Pi/(2sqrt3))] - 11+sqrt2)))

Input interpretation:

$$48\times 1.65578\left(1+\sqrt{3}\,\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)-11+\sqrt{2}$$

Result:

125.475...

125.475...

Series representations:

$$48\times 1.65578\left(1+\sqrt{3}\,\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)-11+\sqrt{2} = 68.4774+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}+79.4774\exp\left(-\frac{\pi}{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\le 0$))

$$48\times 1.65578\left(1+\sqrt{3}\,\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)-11+\sqrt{2} = 68.4774+\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+79.4774\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\exp\left(-\frac{\pi}{2\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for ($x\in\mathbb{R}$ and $x<0$)

$$48\times 1.65578\left(1+\sqrt{3}\,\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)-11+\sqrt{2} = 68.4774+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}+79.4774\exp\left(-\frac{\pi\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}}{2\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}}\right)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}$$

$$48 \times 1.65578 \left(1 + \sqrt{3} \exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right) - 11 + \sqrt{2} =$$

$$68.4774 + \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2+1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} +$$

$$79.4774 \exp\left(-\frac{\pi \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} z_0^{1/2 (-1-\lfloor \arg(3-z_0)/(2\pi) \rfloor)}}{2 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}}\right)$$

$$\left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} z_0^{1/2+1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}$$

27*1/2(((48*(1.65578) * [1+sqrt3 * exp(-Pi/(2sqrt3))] - 7)))+1/(2Pi)

Input interpretation:

$$27 \times \frac{1}{2} \left(48 \times 1.65578 \left(1 + \sqrt{3} \exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right) - 7\right) + \frac{1}{2\pi}$$

Result:

1728.98…

1728.98… ≈ 1729

Series representations:

$$\frac{27}{2} \left(48 \times 1.65578 \left(1 + \sqrt{3} \exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right) - 7\right) + \frac{1}{2\pi} = \frac{1072.95 \left(0.000466007 + 0.911925\,\pi + \pi \exp\left(-\frac{\pi}{2\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}\right) \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}\right)}{\pi}$$

$$\frac{27}{2} \left(48 \times 1.65578 \left(1 + \sqrt{3} \exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right) - 7\right) + \frac{1}{2\pi} = \frac{1}{\pi} 1072.95 \left(0.000466007 + 0.911925\,\pi + \pi \exp\left(-\frac{\pi}{2\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}\right) \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{27}{2}\left(48\times 1.65578\left(1+\sqrt{3}\exp\left(-\frac{\pi}{2\sqrt{3}}\right)\right)-7\right)+\frac{1}{2\pi}=$$

$$\frac{1}{\pi\sqrt{\pi}}536.473\left(0.000932014\sqrt{\pi}+1.82385\pi\sqrt{\pi}+\pi\exp\left(-\frac{\pi\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)$$

Now, we have that:

$$x_2' \leqslant x_7 = \frac{E}{c}\left[1+\sqrt{\frac{4c}{b^2}-1}\cdot\exp\left(-\frac{\pi}{2\sqrt{(4c/b^2)-1}}\right)\right]. \qquad (14)$$

For E = b = 1 and c = 0.4 , we obtain:

1/(0.4) [1+sqrt(4*0.4-1) * exp(-Pi/(2*sqrt(4*0.4-1)))]

Input:

$$\frac{1}{0.4}\left(1+\sqrt{4\times 0.4-1}\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)$$

Result:

2.754867514964326226814762812390744783092640383903000245604...

2.75486751496…

Series representations:

$$\frac{1+\sqrt{4\times 0.4-1}\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)}{0.4}=$$

$$2.5\left(1+\exp\left(-\frac{\pi}{2\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{1+\sqrt{4\times 0.4-1}\ \exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)}{0.4} = -\frac{1}{\sqrt{\pi}}\ 1.25\left(-2\sqrt{\pi}+\exp\left(\frac{\pi\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)$$

$$\frac{1+\sqrt{4\times 0.4-1}\ \exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)}{0.4} = 2.5\left(1+\exp\left(-\frac{\pi}{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}\right) \text{ for (not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

(((1/(0.4) [1+sqrt(4*0.4-1) * exp(-Pi/(2*sqrt(4*0.4-1)))])))^1/2

Input:

$$\sqrt{\frac{1}{0.4}\left(1+\sqrt{4\times 0.4-1}\ \exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)}$$

Result:

1.659779357313593723565156314006164949731007559929984208905...

1.659779357313…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164{,}2696$ i.e. 1,65578...

48(((1/(0.4) [1+sqrt(4*0.4-1) * exp(-Pi/(2*sqrt(4*0.4-1)))])))+e

Input:

$$48\left(\frac{1}{0.4}\left(1+\sqrt{4\times 0.4-1}\ \exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)\right)+e$$

Result:

134.952...

134.952…

Series representations:

$$\frac{48\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)}{0.4}+e = 120+e+120\exp\left(-\frac{\pi}{2\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}$$

$$\frac{48\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)}{0.4}+e = \frac{1}{\sqrt{\pi}}\left(120\sqrt{\pi}+e\sqrt{\pi}-60\exp\left(\frac{\pi\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)$$

$$\frac{48\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)}{0.4}+e = 120+e+120\exp\left(-\frac{\pi}{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

48(((1/(0.4) [1+sqrt(4*0.4-1) * exp(-Pi/(2*sqrt(4*0.4-1)))])))+e+Pi

Input:

$$48\left(\frac{1}{0.4}\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)\right)+e+\pi$$

Result:

138.094...

138.094…

Series representations:

$$\frac{48\left(1+\sqrt{4\times0.4-1}\ \exp\left(-\frac{\pi}{2\sqrt{4\times0.4-1}}\right)\right)}{0.4}+e+\pi = 120+e+\pi+120\exp\left(-\frac{\pi}{2\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}$$

$$\frac{48\left(1+\sqrt{4\times0.4-1}\ \exp\left(-\frac{\pi}{2\sqrt{4\times0.4-1}}\right)\right)}{0.4}+e+\pi = \frac{1}{\sqrt{\pi}}\left(120\sqrt{\pi}+e\sqrt{\pi}+\pi\sqrt{\pi}-60\exp\left(\frac{\pi\sqrt{\pi}}{\sum_{j=0}^{\infty}\text{Res}_{s=-j}\,(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\sum_{j=0}^{\infty}\text{Res}_{s=-j}\,(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)$$

$$\frac{48\left(1+\sqrt{4\times0.4-1}\ \exp\left(-\frac{\pi}{2\sqrt{4\times0.4-1}}\right)\right)}{0.4}+e+\pi = 120+e+\pi+120\exp\left(-\frac{\pi}{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

48(((1/(0.4) [1+sqrt(4*0.4-1) * exp(-Pi/(2*sqrt(4*0.4-1)))])))+7

Input:

$$48\left(\frac{1}{0.4}\left(1+\sqrt{4\times0.4-1}\ \exp\left(-\frac{\pi}{2\sqrt{4\times0.4-1}}\right)\right)\right)+7$$

Result:

139.234...

139.234…

Series representations:

$$\frac{48\left(1+\sqrt{4\times0.4-1}\ \exp\left(-\frac{\pi}{2\sqrt{4\times0.4-1}}\right)\right)}{0.4}+7 = 120\left(1.05833+\exp\left(-\frac{\pi}{2\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{48\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)}{0.4}+7=-\frac{1}{\sqrt{\pi}}\,60\left(-2.11667\sqrt{\pi}+\exp\left(\frac{\pi\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)$$

$$\frac{48\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)}{0.4}+7=120\left(1.05833+\exp\left(-\frac{\pi}{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}\right)\text{ for (not }(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0))$$

48(((1/(0.4) [1+sqrt(4*0.4-1) * exp(-Pi/(2*sqrt(4*0.4-1)))])))-7

Input:

$$48\left(\frac{1}{0.4}\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)\right)-7$$

Result:

125.234...

125.234…

Series representations:

$$\frac{48\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)}{0.4}-7=120\left(0.941667+\exp\left(-\frac{\pi}{2\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{48\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)}{0.4}-7=-\frac{1}{\sqrt{\pi}}\,60.\left(-1.88333\sqrt{\pi}+\exp\left(\frac{\pi\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)$$

$$\frac{48\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)}{0.4}-7=120\left(0.941667+\exp\left(-\frac{\pi}{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}\right)\text{ for (not }(z_0\in\mathbb{R}\text{ and }-\infty<z_0\leq 0))$$

27*1/2[48(((1/(0.4) [1+sqrt(4*0.4-1) * exp(-Pi/(2*sqrt(4*0.4-1)))])))-5+2*0.4]+1/2

Input:

$$27\times\frac{1}{2}\left(48\left(\frac{1}{0.4}\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)\right)-5+2\times 0.4\right)+\frac{1}{2}$$

Result:

1728.95...

1728.95… ≈ 1729

Series representations:

$$\frac{27}{2}\left(\frac{48\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)}{0.4}-5+2\times 0.4\right)+\frac{1}{2}=1620\left(0.965309+\exp\left(-\frac{\pi}{2\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{27}{2}\left(\frac{48\left(1+\sqrt{4\times0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times0.4-1}}\right)\right)}{0.4}-5+2\times0.4\right)+\frac{1}{2}=$$

$$-\frac{1}{\sqrt{\pi}}\,810\left(-1.93062\sqrt{\pi}+\exp\left(\frac{\pi\sqrt{\pi}}{\sum_{j=0}^{\infty}\text{Res}_{s=-j}\,(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\sum_{j=0}^{\infty}\text{Res}_{s=-j}\,(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)$$

$$\frac{27}{2}\left(\frac{48\left(1+\sqrt{4\times0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times0.4-1}}\right)\right)}{0.4}-5+2\times0.4\right)+\frac{1}{2}=$$

$$1620\left(0.965309+\exp\left(-\frac{\pi}{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}\right)\quad\text{for (not }(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0))$$

((((27*1/2[48(((1/(0.4) [1+sqrt(4*0.4-1) * exp(-Pi/(2*sqrt(4*0.4-1)))])))-5+2*0.4]+1/2))))^1/15

Input:

$$\sqrt[15]{27\times\frac{1}{2}\left(48\left(\frac{1}{0.4}\left(1+\sqrt{4\times0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times0.4-1}}\right)\right)\right)-5+2\times0.4\right)+\frac{1}{2}}$$

Result:

1.643812322623444255738268430161892006732962911077374886043...

1.643812322623...

((((27*1/2[48(((1/(0.4) [1+sqrt(4*0.4-1) * exp(-Pi/(2*sqrt(4*0.4-1)))]))-5+2*0.4]+1/2))))^1/15-(21+5)1/10^3

Input:

$$\sqrt[15]{27\times\frac{1}{2}\left(48\left(\frac{1}{0.4}\left(1+\sqrt{4\times0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times0.4-1}}\right)\right)\right)-5+2\times0.4\right)+\frac{1}{2}}-(21+5)\times\frac{1}{10^3}$$

Result:

1.617812322623444255738268430161892006732962911077374886043...

1.617812322623…

Series representations:

$$\sqrt[15]{\frac{27}{2}\left(\frac{48\left(1+\sqrt{4\times0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times0.4-1}}\right)\right)}{0.4}-5+2\times0.4\right)+\frac{1}{2}}-\frac{21+5}{10^3}=\frac{1}{500}\left(-13+500\sqrt[15]{1563.8+1620\exp\left(-\frac{\pi}{2\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(-0.4)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

$$\sqrt[15]{\frac{27}{2}\left(\frac{48\left(1+\sqrt{4\times0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times0.4-1}}\right)\right)}{0.4}-5+2\times0.4\right)+\frac{1}{2}}-\frac{21+5}{10^3}=\frac{1}{500}\left(-13+500\left(1563.8-\frac{1}{\sqrt{\pi}}810\exp\left(\frac{\pi\sqrt{\pi}}{\sum_{j=0}^{\infty}\text{Res}_{s=-j}(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\sum_{j=0}^{\infty}\text{Res}_{s=-j}(-0.4)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)\hat{}\,(1/15)\right)$$

$$\sqrt[15]{\frac{27}{2}\left(\frac{48\left(1+\sqrt{4\times 0.4-1}\,\exp\left(-\frac{\pi}{2\sqrt{4\times 0.4-1}}\right)\right)}{0.4}-5+2\times 0.4\right)+\frac{1}{2}}-\frac{21+5}{10^3}=$$

$$\frac{1}{500}\left(-13+500\left(1563.8+1620\exp\left(-\frac{\pi}{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}}\right)\right.\right.$$
$$\left.\left.\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.6-z_0)^k z_0^{-k}}{k!}\right)\hat{}\,(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Now, we have:

$$p_2 = p_1 + \frac{8q_1}{p_1} + 4\sqrt{q_1}$$

For: $p_1 = 3$; $q_1 = 5$, we obtain:

3+(8*5)/3+4sqrt(5)

Input:

$$3+\frac{8\times 5}{3}+4\sqrt{5}$$

Result:

$$\frac{49}{3}+4\sqrt{5}$$

Decimal approximation:

25.27760524333249211897002800825843827509580677177943623041...

25.27760524…

Alternate form:

$$\frac{1}{3}\left(49+12\sqrt{5}\right)$$

Minimal polynomial:

$9x^2 - 294x + 1681$

From

$$\sqrt{q_2'} = \frac{2\sqrt{AB-1}}{B} - \sqrt{q_1} = \sqrt{q_1}\left[\frac{2(p_2-p_1)}{(p_2+p_1) \pm 2\sqrt{p_1p_2-4q_1}} - 1\right]. \quad (36)$$

We obtain:

sqrt5 [((2(25.277605-3)))/(((25.277605+3)+2sqrt(3*25.277605-4*5)))-1]

Input interpretation:

$$\sqrt{5}\left(\frac{2(25.277605-3)}{(25.277605+3)+2\sqrt{3\times 25.277605-4\times 5}}-1\right)$$

Result:

0.068979504460062226725871218113897631521454048952194431655...

0.06897950446…

From which:

((((sqrt5 [((2(25.277605-3)))/(((25.277605+3)+2sqrt(3*25.277605-4*5)))-1]))))^2

Input interpretation:

$$\left(\sqrt{5}\left(\frac{2(25.277605-3)}{(25.277605+3)+2\sqrt{3\times 25.277605-4\times 5}}-1\right)\right)^2$$

Result:

0.00475817203555574462902953364671188157476088384957565302...

0.00475817203…

and:

sqrt5 [((2(25.277605-3)))/(((25.277605+3)-2sqrt(3*25.277605-4*5)))-1]

Input interpretation:

$$\sqrt{5}\left(\frac{2(25.277605-3)}{(25.277605+3)-2\sqrt{3\times 25.277605-4\times 5}}-1\right)$$

Result:

5.236068...

5.236068…

From which:

(((sqrt5 [((2(25.277605-3)))/(((25.277605+3)-2sqrt(3*25.277605-4*5)))-1])))^2

Input interpretation:

$$\left(\sqrt{5}\left(\frac{2(25.277605-3)}{(25.277605+3)-2\sqrt{3\times 25.277605-4\times 5}}-1\right)\right)^2$$

Result:

27.41641...

27.41641...

From this last expression, we obtain:

5(((sqrt5 [((2(25.277605-3)))/(((25.277605+3)-2sqrt(3*25.277605-4*5)))-1])))^2-2

Input interpretation:

$$5\left(\sqrt{5}\left(\frac{2(25.277605-3)}{(25.277605+3)-2\sqrt{3\times 25.277605-4\times 5}}-1\right)\right)^2-2$$

Result:

135.0820...

135.082…

5(((sqrt5 [((2(25.277605-3)))/(((25.277605+3)-2sqrt(3*25.277605-4*5)))-1])))^2+1

Input interpretation:

$$5\left(\sqrt{5}\left(\frac{2(25.277605-3)}{(25.277605+3)-2\sqrt{3\times 25.277605-4\times 5}}-1\right)\right)^2+1$$

Result:

138.0820...

138.082…

27*1/2(((5(((sqrt5 [((2(25.277605-3)))/(((25.277605+3)-2sqrt(3*25.277605-4*5)))-1])))^2+1-2*5)))

Input interpretation:

$$27\times\frac{1}{2}\left(5\left(\sqrt{5}\left(\frac{2\,(25.277605-3)}{(25.277605+3)-2\sqrt{3\times25.277605-4\times5}}-1\right)\right)^2+1-2\times5\right)$$

Result:

1729.108...

1729.108…

We have also:

135.0820^3 + ((((5(((sqrt5 [((2(25.277605-3)))/(((25.277605+3)-2sqrt(3*25.277605-4*5)))-1])))^2+1))))^3

Input interpretation:

$$135.0820^3+\left(5\left(\sqrt{5}\left(\frac{2\,(25.277605-3)}{(25.277605+3)-2\sqrt{3\times25.277605-4\times5}}-1\right)\right)^2+1\right)^3$$

Result:

$5.097623\ldots\times10^6$

$5.097623\ldots*10^6$

((((135.0820^3 + ((((5(((sqrt5 [((2(25.277605-3)))/(((25.277605+3)-2sqrt(3*25.277605-4*5)))-1])))^2+1))))^3))))^1/3

Input interpretation:

$$\sqrt[3]{135.0820^3+\left(5\left(\sqrt{5}\left(\frac{2\,(25.277605-3)}{(25.277605+3)-2\sqrt{3\times25.277605-4\times5}}-1\right)\right)^2+1\right)^3}$$

Result:

172.1033...

172.1033…

We have:

Input interpretation:

$135.0820^3 + 138.0820^3$

Result:

$5.097620682058736 \times 10^6$

$5.0967620682058736*10^6$

and:

Input interpretation:

$172.1033^3 - 1$

Result:

$5.0976205888815 42937 \times 10^6$

$5.097620588881542937*10^6$

$135.0820^3 + 138.0820^3 \approx 172.1033^3 - 1$; $5097620.68 \approx 5097620.58$

very similar to the Ramanujan expression:

$$135^3 + 138^3 = 172^3 - 1$$

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

AUTOMORPHISM OF SOLUTIONS TO RAMANUJAN'S DIFFERENTIAL EQUATIONS AND OTHER RESULTS
MATTHEW RANDALL - arXiv:1806.07544v1 [math.CA] 20 Jun 2018

On certain arithmetical functions – *Srinivasa Ramanujan*
Transactions of the Cambridge Philosophical Society, XXII, No.9, 1916, 159 – 184

Modular equations and approximations to π – *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

ON NON-LINEAR DIFFERENTIAL EQUATIONS OF THE SECOND ORDER: III. THE EQUATION $\ddot{y} - k(1-y^2)\dot{y} + y = b\mu k \cos(\mu t + \alpha)$ **FOR LARGE k, AND ITS GENERALIZATIONS**

BY *J. E. LITTLEWOOD* in Cambridge

JOURNAL OF MATHEMATICAL ANALYSIS AND APPLICATIONS 8, 423-444 (1964) **A Study of Second Order Nonlinear Systems** –
Y. S. LIM AND L. F. KAZDA