On some Ramanujan Partition Congruences: mathematical connections with ϕ, $\zeta(2)$ and various Fractal Hausdorff Dimensions values. II

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Abstract

In this paper we have described some Ramanujan Partition Congruences, and obtained several mathematical connections with ϕ, $\zeta(2)$ and various Fractal Hausdorff Dimensions values

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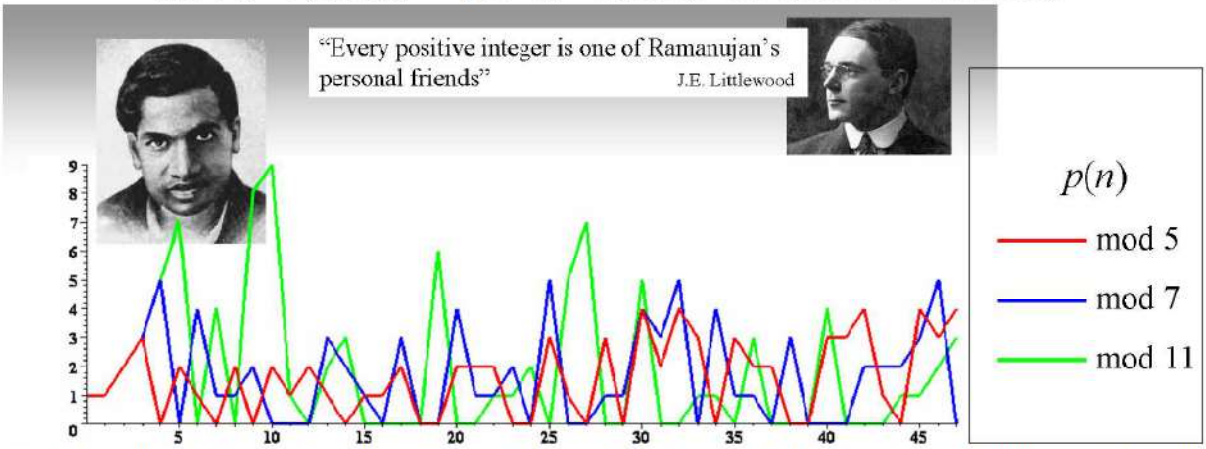
The Ramanujan Partition Congruences *Let n be a non-negative integer and let p(n) denote the number of partitions of n (that is, the number of ways to write n as a sum of positive integers). Then p(n) satisfies the congruence relations:*

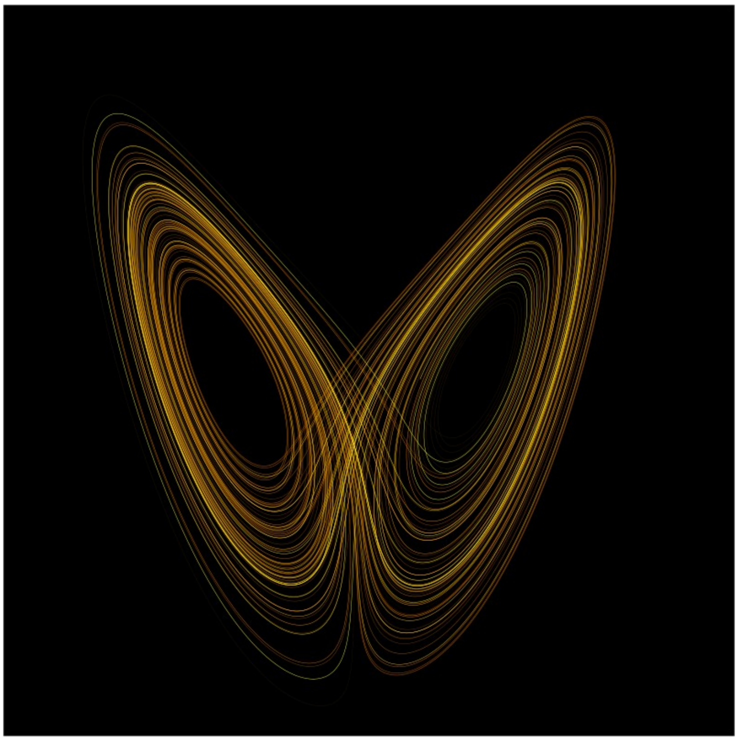
$$p(5t + 4) \equiv 0 \pmod 5, \qquad p(7t + 5) \equiv 0 \pmod 7, \quad \textit{and} \quad p(11t + 6) \equiv 0 \pmod{11}.$$



Ramanujan's congruences tell us that, in the set of values of n for which $p(n) \bmod q = 0$, when q is 5, 7 or 11, there is an infinite arithmetic progression of common difference q. Thus we see that, in the above plot, the three graphs touch the horizontal axis at intervals which appear quite irregular but are certainly constrained by this arithmetic progression property. The property extends to all primes $q > 5$, a deep result published in 2000 by Ken Ono, but the common differences will not generally be q: the set of values of n for which $p(n) \bmod 31 = 0$, for instance, contains an infinite arithmetic progression whose common difference is not 31 but 31×107^4, and which starts at $n = 30064597$. For $q = 3$, the situation is very different—it is not even known if the values of n for which $p(n) \bmod 3 = 0$ form an infinite set!

Ramanujan published proofs of the congruences for 5 and 7 in 1919. His proof for mod 11 remained unpublished at the time of his death in 1920 and was written up by G.H. Hardy.

https://www.theoremoftheday.org/NumberTheory/Ramanujan/TotDRamanujan.pdf



https://en.wikipedia.org/wiki/Chaos_theory#/media/File:Lorenz_attractor_yb.svg

From:

ℓ-ADIC PROPERTIES OF THE PARTITION FUNCTION
AMANDA FOLSOM, ZACHARY A. KENT, AND KEN ONO
(Appendix by Nick Ramsey) - Celebrating the life of A. O. L. Atkin

Ramanujan's famous partition congruences modulo powers of 5; 7; and 11 imply that certain sequences of partition generating functions tend ℓ-adically to 0. little is known about the ℓ-adic behaviour of these sequences for primes ℓ ≥13. Using the classical theory of "modular forms mod p", as developed by Serre in the 1970s, we show that these sequences are governed by "fractal" behavior.

Modulo any power of a prime $\ell \geq 5$, these sequences of generating functions ℓ-adically converge to linear combinations of at most $\lfloor \frac{\ell-1}{12} \rfloor - \lfloor \frac{\ell^2-1}{24\ell} \rfloor$ many special q-series. For $\ell \in \{5, 7, 11\}$ we have $\lfloor \frac{\ell-1}{12} \rfloor - \lfloor \frac{\ell^2-1}{24\ell} \rfloor = 0$, thereby giving a conceptual explanation of Ramanujan's congruences.

We have that:

$$P_\ell(b; z) := \sum_{n=0}^{\infty} p\left(\frac{\ell^b n + 1}{24}\right) q^{\frac{n}{24}} \tag{1.1}$$

(note that $q := e^{2\pi i z}$ throughout, $p(0) = 1$, and $p(\alpha) = 0$ if $\alpha < 0$ or $\alpha \notin \mathbb{Z}$).

Little is known about the ℓ-adic properties of the $P_\ell(b; z)$, as $b \to +\infty$, for primes $\ell \geq 13$. We address this topic, and we show, despite the absence of modular equations, that these functions are nicely constrained ℓ-adically. They are "self-similar", with resolution that improves as one "zooms in" appropriately. Throughout, if $\ell \geq 5$ is prime and $m \geq 1$, then we let

$$b_\ell(m) := \begin{cases} m^2 & \text{if } \ell \geq 5 \text{ and } \ell \neq 7, \\ m^2 & \text{if } \ell = 7 \text{ and } m \text{ is even}, \\ m(m+1) & \text{if } \ell = 7 \text{ and } m \text{ is odd}. \end{cases} \tag{1.2}$$

To illustrate the general theorem (see Theorem 1.2), we first highlight the phenomenon for powers of the primes $5 \leq \ell \leq 31$.

Theorem 1.2. *If $\ell \geq 5$ is prime and $m \geq 1$, then $\Omega_\ell(m)$ is a $\mathbb{Z}/\ell^m\mathbb{Z}$-module with rank $\leq \lfloor \frac{\ell-1}{12} \rfloor$. Moreover, if $b \geq b_\ell(m)$, then we have that*

$$P_\ell(b; z) \equiv \begin{cases} \frac{F_\ell(b;z)}{\eta(z)} \pmod{\ell^m} & \text{if } b \text{ is even}, \\ \frac{F_\ell(b;z)}{\eta(\ell z)} \pmod{\ell^m} & \text{if } b \text{ is odd}, \end{cases}$$

where $F_\ell(b; z) \in \Omega_\ell(m)$.

(4) Theorem 1.2 shows that the partition numbers are self-similar ℓ-adically with resolutions that improve as one zooms in properly using the stochastic process which defines the $P_\ell(b;z)$. Indeed, the $P_\ell(b;z) \pmod{\ell^m}$, for $b \geq b_\ell(m)$, form periodic orbits. Theorem 1.2 bounds the corresponding "Hausdorff dimensions", and these dimensions only depend on ℓ. For $\ell \in \{5,7,11\}$, the dimension is 0, a fact that is beautifully illustrated by Ramanujan's congruences, and for $13 \leq \ell \leq 23$, the dimension is 1. Theorem 1.1 summarizes these observations for $5 \leq \ell \leq 23$ and the proof will show how to include the primes $\ell = 29$ and 31.

Theorem 1.3. *If $5 \leq \ell \leq 31$ and $m \geq 1$, then for $b \geq b_\ell(m)$ we have that $P_\ell(b;24z) \pmod{\ell^m}$ is an eigenform of all of the weight $k_\ell(m) - \frac{1}{2}$ Hecke operators on $\Gamma_0(576)$.*

As an immediate corollary, we have the following congruences for $p(n)$.

Corollary 1.4. *Suppose that $5 \leq \ell \leq 31$ and that $m \geq 1$. If $b \geq b_\ell(m)$, then for every prime $c \geq 5$ there is an integer $\lambda_\ell(m,c)$ such that for all n coprime to c we have*

$$p\left(\frac{\ell^b nc^3 + 1}{24}\right) \equiv \lambda_\ell(m,c)p\left(\frac{\ell^b nc + 1}{24}\right) \pmod{\ell^m}.$$

Remark. Atkin [7] found such congruences modulo $13^2, 17^3, 19^2, 23^6$, 29, and 31.

6. Examples

Here we give examples of Theorem 1.2 for the prime $\ell = 13$. We have that

$$\Phi_{13}(z) := \eta(169z)/\eta(z) = q^7 + q^8 + 2q^9 + \dots.$$

For $m = 1$, we have that $k_{13}(1) = 12$, and so if $b \geq 1$, then by Theorem 4.3 we have that $L_{13}(b; z)$ is congruent modulo 13 to a weight 12 cusp form of level 1, which of course must be a multiple of Ramanujan's $\Delta(z) = \eta(z)^{24}$. The first few terms of $L_{13}(1; z)$ are

$$L_{13}(1; z) = \Phi_{13}(z) \mid U(13) = 11q + 490q^2 + 8349q^3 + 89134q^4 + 715220q^5 + \dots$$
$$\equiv 11q + 9q^2 + 3q^3 + 6q^4 + 12q^5 + 6q^6 + \dots \pmod{13}.$$

On the other hand, we have that

$$11\Delta(z) = 11q - 264q^2 + 2772q^3 - 16192q^4 + \dots$$
$$\equiv 11q + 9q^2 + 3q^3 + 6q^4 + 12q^5 + 6q^6 + \dots \pmod{13}.$$

Therefore we have that $L_{13}(1; z) \equiv 11\Delta(z) \pmod{13}$, which, by Lemma 2.1, implies that

$$P_{13}(1; z) \equiv 11 \cdot \eta(z)^{11} \pmod{13}.$$

More generally (for example, see §4 of [27]), for every non-negative integer k we have that

$$P_{13}(2k + 1; z) \equiv 11 \cdot 6^k \eta(z)^{11} \pmod{13},$$
$$P_{13}(2k + 2; z) \equiv 10 \cdot 6^k \eta(z)^{23} \pmod{13}.$$

These congruences illustrate Theorem 1.1 for $\ell^m = 13$.

For $m = 2$, we have, for $b \geq 4$, that $L_{13}(b; z)$ is congruent modulo 169 to a form in $S_{156} \cap \mathbb{Z}[[q]]$. If $E_4(z)$ is the usual weight 4 Eisenstein series, then one directly computes and finds that

$$L_{13}(2; z) \equiv 36q + 150q^2 + 154q^3 + 100q^4 + 122q^5 + 22q^6 + 26q^7 + 60q^8 + \dots \pmod{169}$$
$$\equiv 36\Delta E_4^{36} + 89\Delta^2 E_4^{33} + 94\Delta^3 E_4^{30} + 16\Delta^4 E_4^{27} + 36\Delta^5 E_4^{24} + 102\Delta^6 E_4^{21} + 3\Delta^7 E_4^{18}$$
$$+ 80\Delta^8 E_4^{15} + 166\Delta^9 E_4^{12} + 115\Delta^{10} E_4^{9} + 3\Delta^{11} E_4^{6} + 145\Delta^{12} E_4^{3} + 88\Delta^{13} \pmod{169}.$$

Using Lemma 2.1, we find that

$$P_{13}(2; z) \equiv \frac{1}{\eta(z)} \cdot (36\Delta E_4^{36} + 89\Delta^2 E_4^{33} + 94\Delta^3 E_4^{30} + 16\Delta^4 E_4^{27} + 36\Delta^5 E_4^{24} + 102\Delta^6 E_4^{21} + 3\Delta^7 E_4^{18}$$
$$+ 80\Delta^8 E_4^{15} + 166\Delta^9 E_4^{12} + 115\Delta^{10} E_4^{9} + 3\Delta^{11} E_4^{6} + 145\Delta^{12} E_4^{3} + 88\Delta^{13}) \pmod{13^2}.$$

Theorem 1.3 is illustrated by the fact that this is a Hecke eigenform modulo 169.

Concerning Theorem 1.1, we have that

$$P_{13}(2; 24z) = 129913904637q^{23} + 78801255302666615q^{47} + \dots$$
$$\equiv 36q^{23} + 17q^{47} + 38q^{71} + 155q^{95} + \dots \pmod{13^2},$$

while we have that

$$154P_{13}(4; 24z) \equiv 36q^{23} + 17q^{47} + 38q^{71} + 155q^{95} + \dots \pmod{13^2}.$$

Indeed, it turns out that $P_{13}(4; 24z) \equiv 45P_{13}(2; 24z) \pmod{13^2}$, which in turn implies that

$$p(13^4 n + 27371) \equiv 45p(13^2 n + 162) \pmod{13^2}.$$

From

$$P_\ell(b; z) := \sum_{n=0}^{\infty} p\left(\frac{\ell^b n + 1}{24}\right) q^{\frac{n}{24}} \qquad (1.1)$$

For q = exp(2Pi), ℓ = 13, 17, 19, 21, 23, 29, 31 b = 1 and p = 55

We obtain the following expression:

sum 55((((13*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..7

Sum:

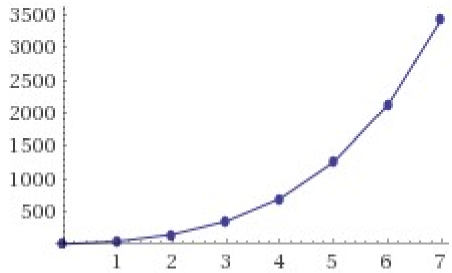
$$\sum_{n=0}^{7} \frac{55}{24}(13n+1)\exp^{\frac{n}{24}}(2\pi) \approx 3444.21274765595133934544471332612978380 5$$

Decimal approximation:

3444.212747655951339345444713326129783805480861636727680017...

3444.212747655…

Partial sums:



Alternate form:

$$\frac{55}{24}+\frac{385 e^{\pi/12}}{12}+\frac{495 e^{\pi/6}}{8}+\frac{275 e^{\pi/4}}{3}+\frac{2915 e^{\pi/3}}{24}+\frac{605}{4}e^{(5\pi)/12}+\frac{4345 e^{\pi/2}}{24}+\frac{1265}{6}e^{(7\pi)/12}$$

From which:

7 + 1/2 * sum 55((((13*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..7

Input interpretation:

$$7+\frac{1}{2}\sum_{n=0}^{7} 55\left(\frac{1}{24}(13n+1)\right)\exp^{\frac{n}{24}}(2\pi)$$

Result:

$$7+\frac{1}{2}\left(\frac{55}{24}+\frac{385 e^{\pi/12}}{12}+\frac{495 e^{\pi/6}}{8}+\frac{275 e^{\pi/4}}{3}+\frac{2915 e^{\pi/3}}{24}+\frac{605}{4} e^{(5\pi)/12}+\frac{4345 e^{\pi/2}}{24}+\frac{1265}{6} e^{(7\pi)/12}\right)\approx 1729.11$$

1729.11

Alternate forms:

$$\frac{1}{48}\left(391+770 e^{\pi/12}+1485 e^{\pi/6}+2200 e^{\pi/4}+2915 e^{\pi/3}+3630 e^{(5\pi)/12}+4345 e^{\pi/2}+5060 e^{(7\pi)/12}\right)$$

$$\frac{1}{48}\left(391+55 e^{\pi/12}\left(14+27 e^{\pi/12}+40 e^{\pi/6}+53 e^{\pi/4}+66 e^{\pi/3}+79 e^{(5\pi)/12}+92 e^{\pi/2}\right)\right)$$

$$\frac{391}{48}+\frac{385 e^{\pi/12}}{24}+\frac{495 e^{\pi/6}}{16}+\frac{275 e^{\pi/4}}{6}+\frac{2915 e^{\pi/3}}{48}+\frac{605}{8} e^{(5\pi)/12}+\frac{4345 e^{\pi/2}}{48}+\frac{1265}{12} e^{(7\pi)/12}$$

And:

((((7 + 1/2 * sum 55((((13*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..7))))^1/15

Input interpretation:

$$\sqrt[15]{7+\frac{1}{2}\sum_{n=0}^{7} 55\left(\frac{1}{24}(13n+1)\right)\exp^{\frac{n}{24}}(2\pi)}$$

Result:

$$\sqrt[15]{7+\frac{55}{48}\left(1+14 e^{\pi/12}+27 e^{\pi/6}+40 e^{\pi/4}+53 e^{\pi/3}+66 e^{(5\pi)/12}+79 e^{\pi/2}+92 e^{(7\pi)/12}\right)}\approx 1.64382$$

1.64382

Alternate form:

$$\frac{1}{2^{4/15}\sqrt[15]{\frac{3}{391+770 e^{\pi/12}+1485 e^{\pi/6}+2200 e^{\pi/4}+2915 e^{\pi/3}+3630 e^{(5\pi)/12}+4345 e^{\pi/2}+5060 e^{(7\pi)/12}}}}$$

((((7 + 1/2 * sum 55((((13*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..7))))^1/15 - (21+5)1/10^3

Input interpretation:

$$\sqrt[15]{7+\frac{1}{2}\sum_{n=0}^{7} 55\left(\frac{1}{24}(13n+1)\right)\exp^{\frac{n}{24}}(2\pi)} - (21+5)\times\frac{1}{10^3}$$

Result:

$$\left(7+\frac{1}{2}\left(\frac{55}{24}+\frac{385 e^{\pi/12}}{12}+\frac{495 e^{\pi/6}}{8}+\frac{275 e^{\pi/4}}{3}+\frac{2915 e^{\pi/3}}{24}+\frac{605}{4} e^{(5\pi)/12}+\frac{4345 e^{\pi/2}}{24}+\frac{1265}{6} e^{(7\pi)/12}\right)\right)^\wedge(1/15)-\frac{13}{500}\approx 1.61782$$

1.61782

Alternate forms:

$$\left(\frac{391}{48}+\frac{385 e^{\pi/12}}{24}+\frac{495 e^{\pi/6}}{16}+\frac{275 e^{\pi/4}}{6}+\frac{2915 e^{\pi/3}}{48}+\frac{605}{8} e^{(5\pi)/12}+\frac{4345 e^{\pi/2}}{48}+\frac{1265}{12} e^{(7\pi)/12}\right)^\wedge(1/15)-\frac{13}{500}$$

$$\frac{1}{2^{4/15}\sqrt[15]{\frac{3}{391+770 e^{\pi/12}+1485 e^{\pi/6}+2200 e^{\pi/4}+2915 e^{\pi/3}+3630 e^{(5\pi)/12}+4345 e^{\pi/2}+5060 e^{(7\pi)/12}}}}-\frac{13}{500}$$

$$\frac{1}{1500}\left(250\times 2^{11/15}\times 3^{14/15}\left(391+770 e^{\pi/12}+1485 e^{\pi/6}+2200 e^{\pi/4}+2915 e^{\pi/3}+3630 e^{(5\pi)/12}+4345 e^{\pi/2}+5060 e^{(7\pi)/12}\right)^\wedge(1/15)-39\right)$$

sum 55((((17*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5

Sum:

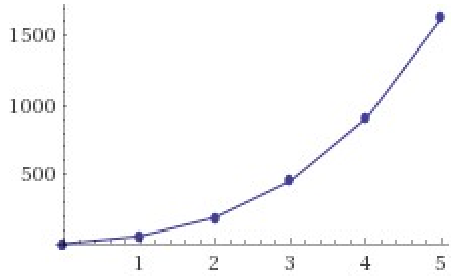
$$\sum_{n=0}^{5}\frac{55}{24}(17n+1)\exp^{\frac{n}{24}}(2\pi)\approx 1632.945585588580490688127199306358228747$$

Decimal approximation:

1632.945585588580490688127199306358228746981506388561191028...

1632.945585588…

Partial sums:



Alternate form:

$$\frac{55}{24}+\frac{165\,e^{\pi/12}}{4}+\frac{1925\,e^{\pi/6}}{24}+\frac{715\,e^{\pi/4}}{6}+\frac{1265\,e^{\pi/3}}{8}+\frac{2365}{12}\,e^{(5\pi)/12}$$

From which:

89+5+2 + sum 55((((17*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5

Input interpretation:

$$89+5+2+\sum_{n=0}^{5} 55\left(\frac{1}{24}(17n+1)\right)\exp^{\frac{n}{24}}(2\pi)$$

Result:

$$96+\frac{55}{24}\left(1+18\,e^{\pi/12}+35\,e^{\pi/6}+52\,e^{\pi/4}+69\,e^{\pi/3}+86\,e^{(5\pi)/12}\right)\approx 1728.95$$

1728.95 ≈ 1729

Alternate forms:

$$\frac{1}{24}\left(2359+990\,e^{\pi/12}+1925\,e^{\pi/6}+2860\,e^{\pi/4}+3795\,e^{\pi/3}+4730\,e^{(5\pi)/12}\right)$$

$$\frac{2359}{24}+\frac{165\,e^{\pi/12}}{4}+\frac{1925\,e^{\pi/6}}{24}+\frac{715\,e^{\pi/4}}{6}+\frac{1265\,e^{\pi/3}}{8}+\frac{2365}{12}\,e^{(5\pi)/12}$$

And:

((((89+5+2 + sum 55((((17*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5))))^1/15

Input interpretation:

$$\sqrt[15]{89+5+2+\sum_{n=0}^{5} 55\left(\frac{1}{24}(17n+1)\right)\exp^{\frac{n}{24}}(2\pi)}$$

Result:

$$\sqrt[15]{96+\frac{55}{24}\left(1+18e^{\pi/12}+35e^{\pi/6}+52e^{\pi/4}+69e^{\pi/3}+86e^{(5\pi)/12}\right)} \approx 1.64381$$

1.64381

Alternate form:

$$\frac{1}{\sqrt[5]{2}\sqrt[15]{\frac{3}{2359+990e^{\pi/12}+1925e^{\pi/6}+2860e^{\pi/4}+3795e^{\pi/3}+4730e^{(5\pi)/12}}}}$$

((((89+5+2 + sum 55((((17*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5))))^1/15-(21+5)1/10^3

Input interpretation:

$$\sqrt[15]{89+5+2+\sum_{n=0}^{5} 55\left(\frac{1}{24}(17n+1)\right)\exp^{\frac{n}{24}}(2\pi)} - (21+5)\times\frac{1}{10^3}$$

Result:

$$\sqrt[15]{96+\frac{55}{24}\left(1+18e^{\pi/12}+35e^{\pi/6}+52e^{\pi/4}+69e^{\pi/3}+86e^{(5\pi)/12}\right)} - \frac{13}{500} \approx 1.61781$$

1.61781

Alternate forms:

$$\frac{1}{1500}\left(250\times 2^{4/5}\times 3^{14/15}\sqrt[15]{2359+990e^{\pi/12}+1925e^{\pi/6}+2860e^{\pi/4}+3795e^{\pi/3}+4730e^{(5\pi)/12}}-39\right)$$

$$\frac{1}{500\sqrt[5]{2}\sqrt[15]{3}}\left(500\sqrt[15]{2359+990e^{\pi/12}+1925e^{\pi/6}+2860e^{\pi/4}+3795e^{\pi/3}+4730e^{(5\pi)/12}}-13\sqrt[5]{2}\sqrt[15]{3}\right)$$

sum 55((((19*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5

Sum:

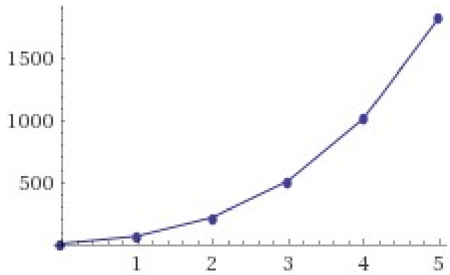
$$\sum_{n=0}^{5}\frac{55}{24}(19n+1)\exp^{\frac{n}{24}}(2\pi)\approx 1821.6239817736903531782797582192322628\,19$$

Decimal approximation:

1821.623981773690353178279758219232262818946591435141719084...

1821.623981773…

Partial sums:



Alternate form:

$$\frac{55}{24}+\frac{275e^{\pi/12}}{6}+\frac{715e^{\pi/6}}{8}+\frac{1595e^{\pi/4}}{12}+\frac{4235e^{\pi/3}}{24}+220e^{(5\pi)/12}$$

From which:

-(89+3)+sum 55((((19*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5

Input interpretation:

$$-(89+3)+\sum_{n=0}^{5}55\left(\frac{1}{24}(19n+1)\right)\exp^{\frac{n}{24}}(2\pi)$$

Result:

$$\frac{55}{24}\left(1+20e^{\pi/12}+39e^{\pi/6}+58e^{\pi/4}+77e^{\pi/3}+96e^{(5\pi)/12}\right)-92\approx 1729.62$$

1729.62

Alternate forms:

$$\frac{1}{24}\left(-2153+1100\,e^{\pi/12}+2145\,e^{\pi/6}+3190\,e^{\pi/4}+4235\,e^{\pi/3}+5280\,e^{(5\pi)/12}\right)$$

$$-\frac{2153}{24}+\frac{275\,e^{\pi/12}}{6}+\frac{715\,e^{\pi/6}}{8}+\frac{1595\,e^{\pi/4}}{12}+\frac{4235\,e^{\pi/3}}{24}+220\,e^{(5\pi)/12}$$

And:

((((-(89+3)+sum 55((((19*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5))))^1/15

Input interpretation:

$$\sqrt[15]{-(89+3)+\sum_{n=0}^{5}55\left(\frac{1}{24}(19\,n+1)\right)\exp^{\frac{n}{24}}(2\,\pi)}$$

Result:

$$\sqrt[15]{\frac{55}{24}\left(1+20\,e^{\pi/12}+39\,e^{\pi/6}+58\,e^{\pi/4}+77\,e^{\pi/3}+96\,e^{(5\pi)/12}\right)-92}\approx 1.64385$$

1.64385

Alternate form:

$$\frac{1}{\sqrt[5]{2}\,\sqrt[15]{\frac{3}{-2153+1100\,e^{\pi/12}+2145\,e^{\pi/6}+3190\,e^{\pi/4}+4235\,e^{\pi/3}+5280\,e^{(5\pi)/12}}}}$$

((((-(89+3)+sum 55((((19*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5))))^1/15-(21+5)1/10^3

Input interpretation:

$$\sqrt[15]{-(89+3)+\sum_{n=0}^{5}55\left(\frac{1}{24}(19\,n+1)\right)\exp^{\frac{n}{24}}(2\,\pi)}-(21+5)\times\frac{1}{10^3}$$

Result:

$$\sqrt[15]{\frac{55}{24}\left(1+20\,e^{\pi/12}+39\,e^{\pi/6}+58\,e^{\pi/4}+77\,e^{\pi/3}+96\,e^{(5\pi)/12}\right)-92}-\frac{13}{500}\approx 1.61785$$

1.61785

Alternate forms:

$$\frac{1}{1500}\left(250\times 2^{4/5}\times 3^{14/15}\sqrt[15]{-2153+1100\,e^{\pi/12}+2145\,e^{\pi/6}+3190\,e^{\pi/4}+4235\,e^{\pi/3}+5280\,e^{(5\pi)/12}}-39\right)$$

$$\frac{1}{500\sqrt[5]{2}\sqrt[15]{3}}\left(500\sqrt[15]{-2153+1100\,e^{\pi/12}+2145\,e^{\pi/6}+3190\,e^{\pi/4}+4235\,e^{\pi/3}+5280\,e^{(5\pi)/12}}-13\sqrt[5]{2}\sqrt[15]{3}\right)$$

sum 55((((21*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5

Sum:

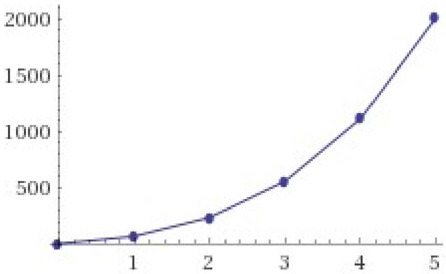
$$\sum_{n=0}^{5}\frac{55}{24}(21\,n+1)\exp^{\frac{n}{24}}(2\,\pi)\approx 2010.302377958800215668432317132106296891$$

Decimal approximation:

2010.302377958800215668432317132106296890911676481722247139...

2010.30237…

Partial sums:



Alternate form:

$$\frac{55}{24}+\frac{605\,e^{\pi/12}}{12}+\frac{2365\,e^{\pi/6}}{24}+\frac{440\,e^{\pi/4}}{3}+\frac{4675\,e^{\pi/3}}{24}+\frac{2915}{12}\,e^{(5\pi)/12}$$

From which:

(-199-76-4-2) + sum 55((((21*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5

Input interpretation:

$$(-199-76-4-2)+\sum_{n=0}^{5} 55\left(\frac{1}{24}(21n+1)\right)\exp^{\frac{n}{24}}(2\pi)$$

Result:

$$\frac{55}{24}\left(1+22e^{\pi/12}+43e^{\pi/6}+64e^{\pi/4}+85e^{\pi/3}+106e^{(5\pi)/12}\right)-281\approx 1729.3$$

1729.3

Alternate forms:

$$\frac{1}{24}\left(-6689+1210e^{\pi/12}+2365e^{\pi/6}+3520e^{\pi/4}+4675e^{\pi/3}+5830e^{(5\pi)/12}\right)$$

$$-\frac{6689}{24}+\frac{605e^{\pi/12}}{12}+\frac{2365e^{\pi/6}}{24}+\frac{440e^{\pi/4}}{3}+\frac{4675e^{\pi/3}}{24}+\frac{2915}{12}e^{(5\pi)/12}$$

And:

(((((-199-76-4-2) + sum 55((((21*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5))))^1/15

Input interpretation:

$$\sqrt[15]{(-199-76-4-2)+\sum_{n=0}^{5} 55\left(\frac{1}{24}(21n+1)\right)\exp^{\frac{n}{24}}(2\pi)}$$

Result:

$$\sqrt[15]{\frac{55}{24}\left(1+22e^{\pi/12}+43e^{\pi/6}+64e^{\pi/4}+85e^{\pi/3}+106e^{(5\pi)/12}\right)-281}\approx 1.64383$$

1.64383

Alternate form:

$$\frac{1}{\sqrt[5]{2}\sqrt[15]{\frac{3}{-6689+1210e^{\pi/12}+2365e^{\pi/6}+3520e^{\pi/4}+4675e^{\pi/3}+5830e^{(5\pi)/12}}}}$$

(((((-199-76-4-2) + sum 55((((21*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5))))^1/15-(21+5)1/10^3

Input interpretation:

$$\sqrt[15]{(-199-76-4-2)+\sum_{n=0}^{5} 55\left(\frac{1}{24}(21n+1)\right)\exp^{\frac{n}{24}}(2\pi)} - (21+5)\times\frac{1}{10^3}$$

Result:

$$\sqrt[15]{\frac{55}{24}\left(1+22e^{\pi/12}+43e^{\pi/6}+64e^{\pi/4}+85e^{\pi/3}+106e^{(5\pi)/12}\right)-281} - \frac{13}{500} \approx 1.61783$$

1.61783

Alternate forms:

$$\frac{1}{1500}\left(250\times 2^{4/5}\times 3^{14/15}\sqrt[15]{-6689+1210e^{\pi/12}+2365e^{\pi/6}+3520e^{\pi/4}+4675e^{\pi/3}+5830e^{(5\pi)/12}}-39\right)$$

$$\frac{1}{500\sqrt[5]{2}\sqrt[15]{3}}\left(500\sqrt[15]{-6689+1210e^{\pi/12}+2365e^{\pi/6}+3520e^{\pi/4}+4675e^{\pi/3}+5830e^{(5\pi)/12}}-13\sqrt[5]{2}\sqrt[15]{3}\right)$$

sum 55((((23*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5

Sum:

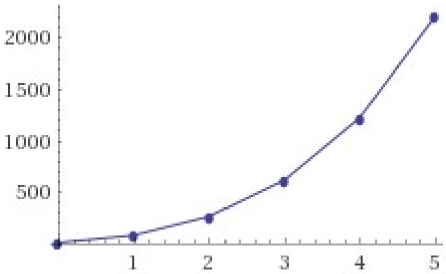
$$\sum_{n=0}^{5}\frac{55}{24}(23n+1)\exp^{\frac{n}{24}}(2\pi) \approx 2198.980774143910078158584876044980330963$$

Decimal approximation:

2198.9807741439100781585848760449803309628767615283027751 94...

2198.98077414…

Partial sums:



Alternate form:

$$\frac{55}{24} + 55\,e^{\pi/12} + \frac{2585\,e^{\pi/6}}{24} + \frac{1925\,e^{\pi/4}}{12} + \frac{1705\,e^{\pi/3}}{8} + \frac{1595}{6}\,e^{(5\pi)/12}$$

From which:

(-322-123-18-7) + sum 55((((23*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5

Input interpretation:

$$(-322 - 123 - 18 - 7) + \sum_{n=0}^{5} 55\left(\frac{1}{24}\,(23\,n + 1)\right)\exp^{\frac{n}{24}}(2\,\pi)$$

Result:

$$\frac{55}{24}\left(1 + 24\,e^{\pi/12} + 47\,e^{\pi/6} + 70\,e^{\pi/4} + 93\,e^{\pi/3} + 116\,e^{(5\pi)/12}\right) - 470 \approx 1728.98$$

1728.98 ≈ 1729

Alternate forms:

$$\frac{5}{24}\left(-2245 + 264\,e^{\pi/12} + 517\,e^{\pi/6} + 770\,e^{\pi/4} + 1023\,e^{\pi/3} + 1276\,e^{(5\pi)/12}\right)$$

$$-\frac{11\,225}{24} + 55\,e^{\pi/12} + \frac{2585\,e^{\pi/6}}{24} + \frac{1925\,e^{\pi/4}}{12} + \frac{1705\,e^{\pi/3}}{8} + \frac{1595}{6}\,e^{(5\pi)/12}$$

And:

(((((-322-123-18-7) + sum 55((((23*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5))))^1/15

Input interpretation:

$$\sqrt[15]{(-322-123-18-7)+\sum_{n=0}^{5} 55\left(\frac{1}{24}(23n+1)\right)\exp^{\frac{n}{24}}(2\pi)}$$

Result:

$$\sqrt[15]{\frac{55}{24}\left(1+24e^{\pi/12}+47e^{\pi/6}+70e^{\pi/4}+93e^{\pi/3}+116e^{(5\pi)/12}\right)-470} \approx 1.64381$$

1.64381

Alternate form:

$$\frac{\sqrt[15]{\frac{5}{3}\left(-2245+264e^{\pi/12}+517e^{\pi/6}+770e^{\pi/4}+1023e^{\pi/3}+1276e^{(5\pi)/12}\right)}}{\sqrt[5]{2}}$$

(((((-322-123-18-7) + sum 55((((23*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..5))))^1/15-(21+5)1/10^3

Input interpretation:

$$\sqrt[15]{(-322-123-18-7)+\sum_{n=0}^{5} 55\left(\frac{1}{24}(23n+1)\right)\exp^{\frac{n}{24}}(2\pi)}-(21+5)\times\frac{1}{10^3}$$

Result:

$$\sqrt[15]{\frac{55}{24}\left(1+24e^{\pi/12}+47e^{\pi/6}+70e^{\pi/4}+93e^{\pi/3}+116e^{(5\pi)/12}\right)-470}-\frac{13}{500} \approx 1.61781$$

1.61781

Alternate forms:

$$\frac{1}{1500}\left(250\times 2^{4/5}\times 3^{14/15}\sqrt[15]{5\left(-2245+264e^{\pi/12}+517e^{\pi/6}+770e^{\pi/4}+1023e^{\pi/3}+1276e^{(5\pi)/12}\right)}-39\right)$$

$$\frac{500\sqrt[15]{\frac{5}{3}\left(-2245+264e^{\pi/12}+517e^{\pi/6}+770e^{\pi/4}+1023e^{\pi/3}+1276e^{(5\pi)/12}\right)}-13\sqrt[5]{2}}{500\sqrt[5]{2}}$$

sum 55((((29*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..4

Sum:

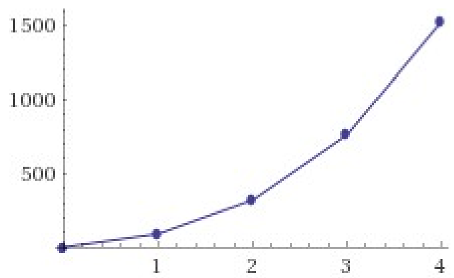
$$\sum_{n=0}^{4} \frac{55}{24}(29\,n+1)\exp^{\frac{n}{24}}(2\,\pi) = \frac{55}{24}\left(1+30\,e^{\pi/12}+59\,e^{\pi/6}+88\,e^{\pi/4}+117\,e^{\pi/3}\right)$$

Decimal approximation:

1526.2352042237362849123917487208301371068983068030059725 11...

1526.23520422…

Partial sums:



Alternate form:

$$\frac{55}{24}+\frac{275\,e^{\pi/12}}{4}+\frac{3245\,e^{\pi/6}}{24}+\frac{605\,e^{\pi/4}}{3}+\frac{2145\,e^{\pi/3}}{8}$$

(123+76+4) + sum 55((((29*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..4

Input interpretation:

$$(123+76+4)+\sum_{n=0}^{4} 55\left(\frac{1}{24}(29\,n+1)\right)\exp^{\frac{n}{24}}(2\,\pi)$$

Result:

$$203+\frac{55}{24}\left(1+30\,e^{\pi/12}+59\,e^{\pi/6}+88\,e^{\pi/4}+117\,e^{\pi/3}\right)\approx 1729.24$$

1729.24

Alternate forms:

$$\frac{1}{24}\left(4927+1650\,e^{\pi/12}+3245\,e^{\pi/6}+4840\,e^{\pi/4}+6435\,e^{\pi/3}\right)$$

$$\frac{4927}{24}+\frac{275\,e^{\pi/12}}{4}+\frac{3245\,e^{\pi/6}}{24}+\frac{605\,e^{\pi/4}}{3}+\frac{2145\,e^{\pi/3}}{8}$$

And:

(((((123+76+4) + sum 55((((29*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..4))))^1/15

Input interpretation:

$$\sqrt[15]{(123+76+4)+\sum_{n=0}^{4} 55\left(\frac{1}{24}\,(29\,n+1)\right)\exp^{\frac{n}{24}}(2\,\pi)}$$

Result:

$$\sqrt[15]{203+\frac{55}{24}\left(1+30\,e^{\pi/12}+59\,e^{\pi/6}+88\,e^{\pi/4}+117\,e^{\pi/3}\right)}\approx 1.64383$$

1.64383

Alternate form:

$$\frac{1}{\sqrt[5]{2}\ \sqrt[15]{\frac{3}{4927+1650\,e^{\pi/12}+3245\,e^{\pi/6}+4840\,e^{\pi/4}+6435\,e^{\pi/3}}}}$$

And:

(((((123+76+4) + sum 55((((29*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..4))))^1/15-(21+5)1/10^3

Input interpretation:

$$\sqrt[15]{(123+76+4)+\sum_{n=0}^{4} 55\left(\frac{1}{24}\,(29\,n+1)\right)\exp^{\frac{n}{24}}(2\,\pi)}-(21+5)\times\frac{1}{10^3}$$

Result:

$$\sqrt[15]{203+\frac{55}{24}\left(1+30\,e^{\pi/12}+59\,e^{\pi/6}+88\,e^{\pi/4}+117\,e^{\pi/3}\right)}-\frac{13}{500}\approx 1.61783$$

1.61783

Alternate forms:

$$\frac{250\times 2^{4/5}\times 3^{14/15}\sqrt[15]{4927+1650\,e^{\pi/12}+3245\,e^{\pi/6}+4840\,e^{\pi/4}+6435\,e^{\pi/3}}-39}{1500}$$

$$\frac{500\sqrt[15]{4927+1650\,e^{\pi/12}+3245\,e^{\pi/6}+4840\,e^{\pi/4}+6435\,e^{\pi/3}}-13\sqrt[5]{2}\sqrt[15]{3}}{500\sqrt[5]{2}\sqrt[15]{3}}$$

sum 55((((31*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..4

Sum:

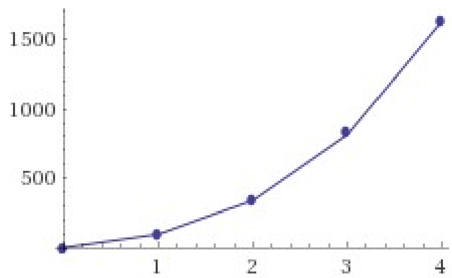
$$\sum_{n=0}^{4}\frac{55}{24}(31\,n+1)\exp^{\frac{n}{24}}(2\,\pi)=\frac{55}{24}\left(1+32\,e^{\pi/12}+63\,e^{\pi/6}+94\,e^{\pi/4}+125\,e^{\pi/3}\right)$$

Decimal approximation:

1630.065603252989751463047677218445794735584370625953734344...

1630.06560325…

Partial sums:



Alternate form:

$$\frac{55}{24}+\frac{220\,e^{\pi/12}}{3}+\frac{1155\,e^{\pi/6}}{8}+\frac{2585\,e^{\pi/4}}{12}+\frac{6875\,e^{\pi/3}}{24}$$

(76 +18+3+2) + sum 55((((31*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..4

Input interpretation:

$$(76+18+3+2)+\sum_{n=0}^{4}55\left(\frac{1}{24}(31\,n+1)\right)\exp^{\frac{n}{24}}(2\,\pi)$$

Result:

$$99+\frac{55}{24}\left(1+32\,e^{\pi/12}+63\,e^{\pi/6}+94\,e^{\pi/4}+125\,e^{\pi/3}\right)\approx 1729.07$$

1729.07

Alternate forms:

$$\frac{11}{24}\left(221+160\,e^{\pi/12}+315\,e^{\pi/6}+470\,e^{\pi/4}+625\,e^{\pi/3}\right)$$

$$\frac{2431}{24}+\frac{220\,e^{\pi/12}}{3}+\frac{1155\,e^{\pi/6}}{8}+\frac{2585\,e^{\pi/4}}{12}+\frac{6875\,e^{\pi/3}}{24}$$

And:

(((((76 +18+3+2) + sum 55((((31*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..4))))^1/15

Input interpretation:

$$\sqrt[15]{(76+18+3+2)+\sum_{n=0}^{4}55\left(\frac{1}{24}\,(31\,n+1)\right)\exp^{\frac{n}{24}}(2\,\pi)}$$

Result:

$$\sqrt[15]{99+\frac{55}{24}\left(1+32\,e^{\pi/12}+63\,e^{\pi/6}+94\,e^{\pi/4}+125\,e^{\pi/3}\right)}\approx 1.64382$$

1.64382

Alternate form:

$$\frac{\sqrt[15]{\frac{11}{3}\left(221+160\,e^{\pi/12}+315\,e^{\pi/6}+470\,e^{\pi/4}+625\,e^{\pi/3}\right)}}{\sqrt[5]{2}}$$

(((((76 +18+3+2) + sum 55((((31*n+1)/24)))*(exp(2Pi))^(n/24), n = 0..4))))^1/15-(21+5)1/10^3

Input interpretation:

$$\sqrt[15]{(76+18+3+2)+\sum_{n=0}^{4} 55\left(\frac{1}{24}(31n+1)\right)\exp^{\frac{n}{24}}(2\pi)}-(21+5)\times\frac{1}{10^3}$$

Result:

$$\sqrt[15]{99+\frac{55}{24}\left(1+32e^{\pi/12}+63e^{\pi/6}+94e^{\pi/4}+125e^{\pi/3}\right)}-\frac{13}{500}\approx 1.61782$$

1.61782

Alternate forms:

$$\frac{250\times 2^{4/5}\times 3^{14/15}\sqrt[15]{11\left(221+160e^{\pi/12}+315e^{\pi/6}+470e^{\pi/4}+625e^{\pi/3}\right)}-39}{1500}$$

$$\frac{500\sqrt[15]{\frac{11}{3}\left(221+160e^{\pi/12}+315e^{\pi/6}+470e^{\pi/4}+625e^{\pi/3}\right)}-13\sqrt[5]{2}}{500\sqrt[5]{2}}$$

We have:

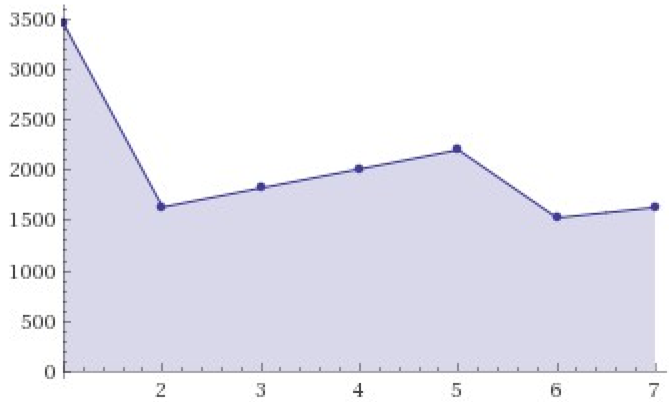
3444.212747655, 1632.945585588, 1821.623981773, 2010.30237, 2198.98077414, 1526.23520422, 1630.06560325

plot(3444.212747655, 1632.945585588, 1821.623981773, 2010.30237, 2198.98077414, 1526.23520422, 1630.06560325)

Input interpretation:

plot	{3444.212747655, 1632.945585588, 1821.623981773, 2010.30237, 2198.98077414, 1526.23520422, 1630.06560325}

Plot:



Dividing the sum of the results by 6912,

$$6P^2Q - 8PR + 3Q^2 - P^4 = 6912\Phi_{3,4}(x),$$

We obtain:

(3444.212747655+1632.945585588+1821.623981773+2010.30237+2198.98077414+1526.23520422+1630.06560325)/6912

Input interpretation:

$$\frac{1}{6912}(3444.212747655 + 1632.945585588 + 1821.623981773 + 2010.30237 + 2198.98077414 + 1526.23520422 + 1630.06560325)$$

Result:

2.063710397370659722222222222222222222222222222222222222...

Repeating decimal:

$2.06371039737065972\overline{2}$ (period 1)

2.063710397....... result very near to the value of the Hausdorff dimension of Lorenz attractor

From Wikipedia:

"Almost all initial points will tend to an invariant set – the Lorenz attractor – a strange attractor, a fractal, and a self-excited attractor with respect to all three equilibria. Its Hausdorff dimension is estimated to be 2.06 ± 0.01"

Now, we have that:

$$L_{13}(1; z) = \Phi_{13}(z) \mid U(13) = 11q + 490q^2 + 8349q^3 + 89134q^4 + 715220q^5 + \ldots$$
$$\equiv 11q + 9q^2 + 3q^3 + 6q^4 + 12q^5 + 6q^6 + \ldots \pmod{13}.$$

$$11\Delta(z) = 11q - 264q^2 + 2772q^3 - 16192q^4 + \ldots$$
$$\equiv 11q + 9q^2 + 3q^3 + 6q^4 + 12q^5 + 6q^6 + \ldots \pmod{13}.$$

We obtain:

11*e^(2Pi)+9*(e^(2Pi))^2+3*(e^(2Pi))^3+6*(e^(2Pi))^4+12*(e^(2Pi))^5+6*(e^(2Pi))^6

Input:

$$11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6$$

Exact result:

$$11 e^{2\pi} + 9 e^{4\pi} + 3 e^{6\pi} + 6 e^{8\pi} + 12 e^{10\pi} + 6 e^{12\pi}$$

Decimal approximation:

$$1.41999895702815913123548773088738027834183508646897 53\ldots \times 10^{17}$$

$1.419998957028\ldots * 10^{17}$

Property:

$11 e^{2\pi} + 9 e^{4\pi} + 3 e^{6\pi} + 6 e^{8\pi} + 12 e^{10\pi} + 6 e^{12\pi}$ is a transcendental number

Alternate forms:

$$e^{2\pi} \left(11 + 9 e^{2\pi} + 3 e^{4\pi} + 6 e^{6\pi} + 12 e^{8\pi} + 6 e^{10\pi}\right)$$

$$e^{2\pi} \left(11 + 3 e^{2\pi} \left(3 + e^{2\pi} + 2 e^{4\pi} + 4 e^{6\pi} + 2 e^{8\pi}\right)\right)$$

Alternative representations:

$$11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6 = 11 \exp^{2\pi}(z) + 9 \exp^{2\pi}(z)^2 + 3 \exp^{2\pi}(z)^3 + 6 \exp^{2\pi}(z)^4 + 12 \exp^{2\pi}(z)^5 + 6 \exp^{2\pi}(z)^6 \quad \text{for } z = 1$$

$$11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6 = 11 \exp^{2\cos^{-1}(-1)}(z) + 9 \exp^{2\cos^{-1}(-1)}(z)^2 + 3 \exp^{2\cos^{-1}(-1)}(z)^3 + 6 \exp^{2\cos^{-1}(-1)}(z)^4 + 12 \exp^{2\cos^{-1}(-1)}(z)^5 + 6 \exp^{2\cos^{-1}(-1)}(z)^6 \quad \text{for } z = 1$$

Series representations:

$$11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6 = e^{8 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} \left(11 + 9 e^{8 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 3 e^{16 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 6 e^{24 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 12 e^{32 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 6 e^{40 \sum_{k=0}^{\infty} (-1)^k/(1+2k)}\right)$$

$$11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6 = \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi} \left(11 + 9 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi} + 3 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4\pi} + 6 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6\pi} + 12 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8\pi} + 6 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{10\pi}\right)$$

$$11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6 = \left(11 + 9 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi} + 3 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{4\pi} + 6 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{6\pi} + 12 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{8\pi} + 6 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{10\pi}\right) \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi}$$

Integral representations:

$$11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6 = e^{4 \int_0^{\infty} 1/(1+t^2)\,dt} \left(11 + 9 e^{4 \int_0^{\infty} 1/(1+t^2)\,dt} + 3 e^{8 \int_0^{\infty} 1/(1+t^2)\,dt} + 6 e^{12 \int_0^{\infty} 1/(1+t^2)\,dt} + 12 e^{16 \int_0^{\infty} 1/(1+t^2)\,dt} + 6 e^{20 \int_0^{\infty} 1/(1+t^2)\,dt}\right)$$

$$11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6 = e^{4 \int_0^{\infty} \sin(t)/t\,dt} \left(11 + 9 e^{4 \int_0^{\infty} \sin(t)/t\,dt} + 3 e^{8 \int_0^{\infty} \sin(t)/t\,dt} + 6 e^{12 \int_0^{\infty} \sin(t)/t\,dt} + 12 e^{16 \int_0^{\infty} \sin(t)/t\,dt} + 6 e^{20 \int_0^{\infty} \sin(t)/t\,dt}\right)$$

$$11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6 = e^{8 \int_0^1 \sqrt{1-t^2}\,dt} \left(11 + 9 e^{8 \int_0^1 \sqrt{1-t^2}\,dt} + 3 e^{16 \int_0^1 \sqrt{1-t^2}\,dt} + 6 e^{24 \int_0^1 \sqrt{1-t^2}\,dt} + 12 e^{32 \int_0^1 \sqrt{1-t^2}\,dt} + 6 e^{40 \int_0^1 \sqrt{1-t^2}\,dt}\right)$$

From

$$L_{13}(2;z) \equiv 36q + 150q^2 + 154q^3 + 100q^4 + 122q^5 + 22q^6 + 26q^7 + 60q^8 + \dots \pmod{169}$$
$$\equiv 36\Delta E_4^{36} + 89\Delta^2 E_4^{33} + 94\Delta^3 E_4^{30} + 16\Delta^4 E_4^{27} + 36\Delta^5 E_4^{24} + 102\Delta^6 E_4^{21} + 3\Delta^7 E_4^{18}$$
$$+ 80\Delta^8 E_4^{15} + 166\Delta^9 E_4^{12} + 115\Delta^{10} E_4^{9} + 3\Delta^{11} E_4^{6} + 145\Delta^{12} E_4^{3} + 88\Delta^{13} \pmod{169}.$$

We obtain:

36(e^(2Pi))+150(e^(2Pi))^2+154(e^(2Pi))^3+100(e^(2Pi))^4+122(e^(2Pi))^5+22(e^(2Pi))^6+26(e^(2Pi))^7+60(e^(2Pi))^8

Input:

$$36 e^{2\pi} + 150 (e^{2\pi})^2 + 154 (e^{2\pi})^3 + 100 (e^{2\pi})^4 + 122 (e^{2\pi})^5 + 22 (e^{2\pi})^6 + 26 (e^{2\pi})^7 + 60 (e^{2\pi})^8$$

Exact result:

$$36 e^{2\pi} + 150 e^{4\pi} + 154 e^{6\pi} + 100 e^{8\pi} + 122 e^{10\pi} + 22 e^{12\pi} + 26 e^{14\pi} + 60 e^{16\pi}$$

Decimal approximation:

$$4.0599882098933702334164642774166500000584867499262261\ldots \times 10^{23}$$

$4.05998820989\ldots*10^{23}$

Property:

$36 e^{2\pi} + 150 e^{4\pi} + 154 e^{6\pi} + 100 e^{8\pi} + 122 e^{10\pi} + 22 e^{12\pi} + 26 e^{14\pi} + 60 e^{16\pi}$ is a transcendental number

Alternate forms:

$$2 e^{9\pi} (11 \sinh(\pi) - 66 \sinh(3\pi) - 62 \sinh(5\pi) + 12 \sinh(7\pi) + 111 \cosh(\pi) + 88 \cosh(3\pi) + 88 \cosh(5\pi) + 48 \cosh(7\pi))$$

$$2 e^{2\pi} \left(18 + 75 e^{2\pi} + 77 e^{4\pi} + 50 e^{6\pi} + 61 e^{8\pi} + 11 e^{10\pi} + 13 e^{12\pi} + 30 e^{14\pi}\right)$$

$$2 \left(18 e^{2\pi} + 75 e^{4\pi} + 77 e^{6\pi} + 50 e^{8\pi} + 61 e^{10\pi} + 11 e^{12\pi} + 13 e^{14\pi} + 30 e^{16\pi}\right)$$

Alternative representations:

$$36 e^{2\pi} + 150 (e^{2\pi})^2 + 154 (e^{2\pi})^3 + 100 (e^{2\pi})^4 + 122 (e^{2\pi})^5 + 22 (e^{2\pi})^6 + 26 (e^{2\pi})^7 + 60 (e^{2\pi})^8 = 36 \exp^{2\cos^{-1}(-1)}(z) + 150 \exp^{2\cos^{-1}(-1)}(z)^2 + 154 \exp^{2\cos^{-1}(-1)}(z)^3 + 100 \exp^{2\cos^{-1}(-1)}(z)^4 + 122 \exp^{2\cos^{-1}(-1)}(z)^5 + 22 \exp^{2\cos^{-1}(-1)}(z)^6 + 26 \exp^{2\cos^{-1}(-1)}(z)^7 + 60 \exp^{2\cos^{-1}(-1)}(z)^8 \quad \text{for } z = 1$$

$$36 e^{2\pi} + 150 (e^{2\pi})^2 + 154 (e^{2\pi})^3 + 100 (e^{2\pi})^4 + 122 (e^{2\pi})^5 + 22 (e^{2\pi})^6 + 26 (e^{2\pi})^7 + 60 (e^{2\pi})^8 = 36 \exp^{2\pi}(z) + 150 \exp^{2\pi}(z)^2 + 154 \exp^{2\pi}(z)^3 + 100 \exp^{2\pi}(z)^4 + 122 \exp^{2\pi}(z)^5 + 22 \exp^{2\pi}(z)^6 + 26 \exp^{2\pi}(z)^7 + 60 \exp^{2\pi}(z)^8 \quad \text{for } z = 1$$

Series representations:

$$36 e^{2\pi} + 150 (e^{2\pi})^2 + 154 (e^{2\pi})^3 + 100 (e^{2\pi})^4 + 122 (e^{2\pi})^5 + 22 (e^{2\pi})^6 + 26 (e^{2\pi})^7 + 60 (e^{2\pi})^8 = 2 e^{8 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} \left(18 + 75 e^{8 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 77 e^{16 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 50 e^{24 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 61 e^{32 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 11 e^{40 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 13 e^{48 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 30 e^{56 \sum_{k=0}^{\infty} (-1)^k/(1+2k)}\right)$$

$$36 e^{2\pi} + 150 (e^{2\pi})^2 + 154 (e^{2\pi})^3 + 100 (e^{2\pi})^4 + 122 (e^{2\pi})^5 + 22 (e^{2\pi})^6 + 26 (e^{2\pi})^7 + 60 (e^{2\pi})^8 = 2 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi} \left(18 + 75 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi} + 77 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4\pi} + 50 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6\pi} + 61 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8\pi} + 11 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{10\pi} + 13 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{12\pi} + 30 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{14\pi}\right)$$

$$36 e^{2\pi} + 150 (e^{2\pi})^2 + 154 (e^{2\pi})^3 + 100 (e^{2\pi})^4 + 122 (e^{2\pi})^5 + 22 (e^{2\pi})^6 + 26 (e^{2\pi})^7 + 60 (e^{2\pi})^8 = 2 \left(18 + 75 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi} + 77 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{4\pi} + 50 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{6\pi} + 61 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{8\pi} + 11 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{10\pi} + 13 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{12\pi} + 30 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{14\pi}\right) \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi}$$

Integral representations:

$$36 e^{2\pi} + 150 (e^{2\pi})^2 + 154 (e^{2\pi})^3 + 100 (e^{2\pi})^4 + 122 (e^{2\pi})^5 + 22 (e^{2\pi})^6 + 26 (e^{2\pi})^7 + 60 (e^{2\pi})^8 = 2 e^{4 \int_0^{\infty} \sin(t)/t \, dt} \left(18 + 75 e^{4 \int_0^{\infty} \sin(t)/t \, dt} + 77 e^{8 \int_0^{\infty} \sin(t)/t \, dt} + 50 e^{12 \int_0^{\infty} \sin(t)/t \, dt} + 61 e^{16 \int_0^{\infty} \sin(t)/t \, dt} + 11 e^{20 \int_0^{\infty} \sin(t)/t \, dt} + 13 e^{24 \int_0^{\infty} \sin(t)/t \, dt} + 30 e^{28 \int_0^{\infty} \sin(t)/t \, dt}\right)$$

$$36 e^{2\pi} + 150 (e^{2\pi})^2 + 154 (e^{2\pi})^3 + 100 (e^{2\pi})^4 + 122 (e^{2\pi})^5 + 22 (e^{2\pi})^6 + 26 (e^{2\pi})^7 + 60 (e^{2\pi})^8 = 2 e^{4 \int_0^\infty 1/(1+t^2) dt} \left(18 + 75 e^{4 \int_0^\infty 1/(1+t^2) dt} + 77 e^{8 \int_0^\infty 1/(1+t^2) dt} + 50 e^{12 \int_0^\infty 1/(1+t^2) dt} + 61 e^{16 \int_0^\infty 1/(1+t^2) dt} + 11 e^{20 \int_0^\infty 1/(1+t^2) dt} + 13 e^{24 \int_0^\infty 1/(1+t^2) dt} + 30 e^{28 \int_0^\infty 1/(1+t^2) dt}\right)$$

$$36 e^{2\pi} + 150 (e^{2\pi})^2 + 154 (e^{2\pi})^3 + 100 (e^{2\pi})^4 + 122 (e^{2\pi})^5 + 22 (e^{2\pi})^6 + 26 (e^{2\pi})^7 + 60 (e^{2\pi})^8 = 2 e^{4 \int_0^\infty \sin^2(t)/t^2 dt} \left(18 + 75 e^{4 \int_0^\infty \sin^2(t)/t^2 dt} + 77 e^{8 \int_0^\infty \sin^2(t)/t^2 dt} + 50 e^{12 \int_0^\infty \sin^2(t)/t^2 dt} + 61 e^{16 \int_0^\infty \sin^2(t)/t^2 dt} + 11 e^{20 \int_0^\infty \sin^2(t)/t^2 dt} + 13 e^{24 \int_0^\infty \sin^2(t)/t^2 dt} + 30 e^{28 \int_0^\infty \sin^2(t)/t^2 dt}\right)$$

From

$$\begin{aligned} P_{13}(2; 24z) &= 129913904637q^{23} + 78801255302666615q^{47} + \dots \\ &\equiv 36q^{23} + 17q^{47} + 38q^{71} + 155q^{95} + \dots \pmod{13^2}, \\ 154P_{13}(4; 24z) &\equiv 36q^{23} + 17q^{47} + 38q^{71} + 155q^{95} + \dots \pmod{13^2}. \end{aligned}$$

We obtain:

36*(e^(2Pi))^23+17*(e^(2Pi))^47+38*(e^(2Pi))^71+155*(e^(2Pi))^95

Input:

$$36 (e^{2\pi})^{23} + 17 (e^{2\pi})^{47} + 38 (e^{2\pi})^{71} + 155 (e^{2\pi})^{95}$$

Exact result:

$$36 e^{46\pi} + 17 e^{94\pi} + 38 e^{142\pi} + 155 e^{190\pi}$$

Decimal approximation:

$$2.6414289179584490160367546047919389851456044639605698\ldots \times 10^{261}$$

2.6414289179…*10^{261}

Property:

$36 e^{46\pi} + 17 e^{94\pi} + 38 e^{142\pi} + 155 e^{190\pi}$ is a transcendental number

Alternate form:

$$e^{46\pi} (36 + 17 e^{48\pi} + 38 e^{96\pi} + 155 e^{144\pi})$$

Alternative representations:

$$36\left(e^{2\pi}\right)^{23}+17\left(e^{2\pi}\right)^{47}+38\left(e^{2\pi}\right)^{71}+155\left(e^{2\pi}\right)^{95}= 36\exp^{2\pi}(z)^{23}+17\exp^{2\pi}(z)^{47}+38\exp^{2\pi}(z)^{71}+155\exp^{2\pi}(z)^{95} \quad \text{for } z=1$$

$$36\left(e^{2\pi}\right)^{23}+17\left(e^{2\pi}\right)^{47}+38\left(e^{2\pi}\right)^{71}+155\left(e^{2\pi}\right)^{95}=36\exp^{2\cos^{-1}(-1)}(z)^{23}+ 17\exp^{2\cos^{-1}(-1)}(z)^{47}+38\exp^{2\cos^{-1}(-1)}(z)^{71}+155\exp^{2\cos^{-1}(-1)}(z)^{95} \quad \text{for } z=1$$

Series representations:

$$36\left(e^{2\pi}\right)^{23}+17\left(e^{2\pi}\right)^{47}+38\left(e^{2\pi}\right)^{71}+155\left(e^{2\pi}\right)^{95}=36\, e^{184\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+ 17\, e^{376\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+38\, e^{568\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+155\, e^{760\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

$$36\left(e^{2\pi}\right)^{23}+17\left(e^{2\pi}\right)^{47}+38\left(e^{2\pi}\right)^{71}+155\left(e^{2\pi}\right)^{95}= 36\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{46\pi}+17\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{94\pi}+38\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{142\pi}+155\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{190\pi}$$

$$36\left(e^{2\pi}\right)^{23}+17\left(e^{2\pi}\right)^{47}+38\left(e^{2\pi}\right)^{71}+155\left(e^{2\pi}\right)^{95}=36\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{46\pi}+ 17\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{94\pi}+38\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{142\pi}+155\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{190\pi}$$

Integral representations:

$$36\left(e^{2\pi}\right)^{23}+17\left(e^{2\pi}\right)^{47}+38\left(e^{2\pi}\right)^{71}+155\left(e^{2\pi}\right)^{95}= 36\, e^{92\int_0^{\infty}1/(1+t^2)\,dt}+17\, e^{188\int_0^{\infty}1/(1+t^2)\,dt}+38\, e^{284\int_0^{\infty}1/(1+t^2)\,dt}+155\, e^{380\int_0^{\infty}1/(1+t^2)\,dt}$$

$$36\left(e^{2\pi}\right)^{23}+17\left(e^{2\pi}\right)^{47}+38\left(e^{2\pi}\right)^{71}+155\left(e^{2\pi}\right)^{95}= 36\, e^{184\int_0^1\sqrt{1-t^2}\,dt}+17\, e^{376\int_0^1\sqrt{1-t^2}\,dt}+38\, e^{568\int_0^1\sqrt{1-t^2}\,dt}+155\, e^{760\int_0^1\sqrt{1-t^2}\,dt}$$

$$36\left(e^{2\pi}\right)^{23}+17\left(e^{2\pi}\right)^{47}+38\left(e^{2\pi}\right)^{71}+155\left(e^{2\pi}\right)^{95}=36\, e^{92\int_0^1 1/\sqrt{1-t^2}\,dt}+ 17\, e^{188\int_0^1 1/\sqrt{1-t^2}\,dt}+38\, e^{284\int_0^1 1/\sqrt{1-t^2}\,dt}+155\, e^{380\int_0^1 1/\sqrt{1-t^2}\,dt}$$

We have also:

(((11*e^(2Pi)+9*(e^(2Pi))^2+3*(e^(2Pi))^3+6*(e^(2Pi))^4+12*(e^(2Pi))^5+6*(e^(2Pi))^6)))^1/80-(18+2)1/10^3

Input:

$$\sqrt[80]{11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6} - (18+2) \times \frac{1}{10^3}$$

Exact result:

$$\sqrt[80]{11 e^{2\pi} + 9 e^{4\pi} + 3 e^{6\pi} + 6 e^{8\pi} + 12 e^{10\pi} + 6 e^{12\pi}} - \frac{1}{50}$$

Decimal approximation:

1.618338361458603708818579482840899021836783119186054277674...

1.6183383614586…

Property:

$$-\frac{1}{50} + \sqrt[80]{11 e^{2\pi} + 9 e^{4\pi} + 3 e^{6\pi} + 6 e^{8\pi} + 12 e^{10\pi} + 6 e^{12\pi}}$$
is a transcendental number

Alternate forms:

$$e^{(7\pi)/80} \sqrt[80]{-5 \sinh(5\pi) + 21 \cosh(3\pi) + 17 \cosh(5\pi) + (9 + 6 \sinh(2\pi)) \cosh(\pi)} - \frac{1}{50}$$

$$\frac{1}{50} \left(50 e^{\pi/40} \sqrt[80]{11 + 9 e^{2\pi} + 3 e^{4\pi} + 6 e^{6\pi} + 12 e^{8\pi} + 6 e^{10\pi}} - 1\right)$$

$$\frac{1}{50} \left(50 \sqrt[80]{11 e^{2\pi} + 9 e^{4\pi} + 3 e^{6\pi} + 6 e^{8\pi} + 12 e^{10\pi} + 6 e^{12\pi}} - 1\right)$$

Alternative representations:

$$\sqrt[80]{11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6} - \frac{18+2}{10^3} = -\frac{20}{10^3} + \sqrt[80]{11 e^{360^\circ} + 9 (e^{360^\circ})^2 + 3 (e^{360^\circ})^3 + 6 (e^{360^\circ})^4 + 12 (e^{360^\circ})^5 + 6 (e^{360^\circ})^6}$$

$$\sqrt[80]{11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6} - \frac{18+2}{10^3} = \left(11 \exp^{2\pi}(z) + 9 \exp^{2\pi}(z)^2 + 3 \exp^{2\pi}(z)^3 + 6 \exp^{2\pi}(z)^4 + 12 \exp^{2\pi}(z)^5 + 6 \exp^{2\pi}(z)^6\right) \wedge (1/80) - \frac{18+2}{10^3} \quad \text{for } z = 1$$

$$\sqrt[80]{11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6} - \frac{18+2}{10^3} =$$
$$\left(11 \exp^{2\cos^{-1}(-1)}(z) + 9 \exp^{2\cos^{-1}(-1)}(z)^2 + 3 \exp^{2\cos^{-1}(-1)}(z)^3 + 6 \exp^{2\cos^{-1}(-1)}(z)^4 + 12 \exp^{2\cos^{-1}(-1)}(z)^5 + 6 \exp^{2\cos^{-1}(-1)}(z)^6\right) \wedge (1/80) - \frac{18+2}{10^3} \quad \text{for } z = 1$$

Series representations:

$$\sqrt[80]{11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6} - \frac{18+2}{10^3} =$$
$$\frac{1}{50}\left(-1 + 50\left(e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\left(11 + 9 e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)} + 3 e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)} + 6 e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)} + 12 e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)} + 6 e^{40\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right) \wedge (1/80)\right)$$

$$\sqrt[80]{11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6} - \frac{18+2}{10^3} =$$
$$\frac{1}{50}\left(-1 + 50\left(\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\left(11 + 9\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi} + 3\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi} + 6\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi} + 12\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi} + 6\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10\pi}\right)\right) \wedge (1/80)\right)$$

$$\sqrt[80]{11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6} - \frac{18+2}{10^3} =$$
$$\frac{1}{50}\left(-1 + 50\left(\left(11 + 9\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi} + 3\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi} + 6\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{6\pi} + 12\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi} + 6\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{10\pi}\right)\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right) \wedge (1/80)\right)$$

Integral representations:

$$\sqrt[80]{11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6} - \frac{18+2}{10^3} =$$
$$\frac{1}{50}\left(-1 + 50\left(e^{4\int_0^{\infty}\sin(t)/t\,dt}\left(11 + 9 e^{4\int_0^{\infty}\sin(t)/t\,dt} + 3 e^{8\int_0^{\infty}\sin(t)/t\,dt} + 6 e^{12\int_0^{\infty}\sin(t)/t\,dt} + 12 e^{16\int_0^{\infty}\sin(t)/t\,dt} + 6 e^{20\int_0^{\infty}\sin(t)/t\,dt}\right)\right) \wedge (1/80)\right)$$

$$\sqrt[80]{11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6} - \frac{18+2}{10^3} = \frac{1}{50}$$
$$\left(-1 + 50 \left(e^{4 \int_0^\infty 1/(1+t^2) dt} \left(11 + 9 e^{4 \int_0^\infty 1/(1+t^2) dt} + 3 e^{8 \int_0^\infty 1/(1+t^2) dt} + 6 e^{12 \int_0^\infty 1/(1+t^2) dt} + 12 e^{16 \int_0^\infty 1/(1+t^2) dt} + 6 e^{20 \int_0^\infty 1/(1+t^2) dt}\right)\right) \hat{}\ (1/80)\right)$$

$$\sqrt[80]{11 e^{2\pi} + 9 (e^{2\pi})^2 + 3 (e^{2\pi})^3 + 6 (e^{2\pi})^4 + 12 (e^{2\pi})^5 + 6 (e^{2\pi})^6} - \frac{18+2}{10^3} =$$
$$\frac{1}{50} \left(-1 + 50 \left(e^{4 \int_0^\infty \sin^2(t)/t^2 dt} \left(11 + 9 e^{4 \int_0^\infty \sin^2(t)/t^2 dt} + 3 e^{8 \int_0^\infty \sin^2(t)/t^2 dt} + 6 e^{12 \int_0^\infty \sin^2(t)/t^2 dt} + 12 e^{16 \int_0^\infty \sin^2(t)/t^2 dt} + 6 e^{20 \int_0^\infty \sin^2(t)/t^2 dt}\right)\right) \hat{}\ (1/80)\right)$$

((((36(e^(2Pi))+150(e^(2Pi))^2+154(e^(2Pi))^3+100(e^(2Pi))^4+122(e^(2Pi))^5+22(e^(2Pi))^6+26(e^(2Pi))^7+60(e^(2Pi))^8))))^1/113

Input:

$$\left(36 e^{2\pi} + 150 (e^{2\pi})^2 + 154 (e^{2\pi})^3 + 100 (e^{2\pi})^4 + 122 (e^{2\pi})^5 + 22 (e^{2\pi})^6 + 26 (e^{2\pi})^7 + 60 (e^{2\pi})^8\right) \hat{}\ (1/113)$$

Exact result:

$$\sqrt[113]{36 e^{2\pi} + 150 e^{4\pi} + 154 e^{6\pi} + 100 e^{8\pi} + 122 e^{10\pi} + 22 e^{12\pi} + 26 e^{14\pi} + 60 e^{16\pi}}$$

Decimal approximation:

1.617800621743731153031214786920360496793586710298668327732...

1.617800621743…

Property:

$$\sqrt[113]{36 e^{2\pi} + 150 e^{4\pi} + 154 e^{6\pi} + 100 e^{8\pi} + 122 e^{10\pi} + 22 e^{12\pi} + 26 e^{14\pi} + 60 e^{16\pi}}$$
is a transcendental number

Alternate forms:

$$\sqrt[113]{2 \left(18 e^{2\pi} + 75 e^{4\pi} + 77 e^{6\pi} + 50 e^{8\pi} + 61 e^{10\pi} + 11 e^{12\pi} + 13 e^{14\pi} + 30 e^{16\pi}\right)}$$

$$e^{(2\pi)/113} \sqrt[113]{2 \left(18 + 75 e^{2\pi} + 77 e^{4\pi} + 50 e^{6\pi} + 61 e^{8\pi} + 11 e^{10\pi} + 13 e^{12\pi} + 30 e^{14\pi}\right)}$$

$$(2 (11 \sinh(\pi) - 66 \sinh(3\pi) - 62 \sinh(5\pi) + 12 \sinh(7\pi) + 111 \cosh(\pi) + 88 \cosh(3\pi) + 88 \cosh(5\pi) + 48 \cosh(7\pi)) (\sinh(9\pi) + \cosh(9\pi))) \hat{}\ (1/113)$$

All 113th roots of 36 e^(2 π) + 150 e^(4 π) + 154 e^(6 π) + 100 e^(8 π) + 122 e^(10 π) + 22 e^(12 π) + 26 e^(14 π) + 60 e^(16 π):

$$\sqrt[113]{36 e^{2\pi} + 150 e^{4\pi} + 154 e^{6\pi} + 100 e^{8\pi} + 122 e^{10\pi} + 22 e^{12\pi} + 26 e^{14\pi} + 60 e^{16\pi}}\ e^{0} \approx 1.6178 \text{ (real, principal root)}$$

$$\sqrt[113]{36 e^{2\pi} + 150 e^{4\pi} + 154 e^{6\pi} + 100 e^{8\pi} + 122 e^{10\pi} + 22 e^{12\pi} + 26 e^{14\pi} + 60 e^{16\pi}}\ e^{(2 i \pi)/113} \approx 1.6153 + 0.08991 i$$

$$\sqrt[113]{36 e^{2\pi} + 150 e^{4\pi} + 154 e^{6\pi} + 100 e^{8\pi} + 122 e^{10\pi} + 22 e^{12\pi} + 26 e^{14\pi} + 60 e^{16\pi}}\ e^{(4 i \pi)/113} \approx 1.6078 + 0.17954 i$$

$$\sqrt[113]{36 e^{2\pi} + 150 e^{4\pi} + 154 e^{6\pi} + 100 e^{8\pi} + 122 e^{10\pi} + 22 e^{12\pi} + 26 e^{14\pi} + 60 e^{16\pi}}\ e^{(6 i \pi)/113} \approx 1.5953 + 0.26862 i$$

$$\sqrt[113]{36 e^{2\pi} + 150 e^{4\pi} + 154 e^{6\pi} + 100 e^{8\pi} + 122 e^{10\pi} + 22 e^{12\pi} + 26 e^{14\pi} + 60 e^{16\pi}}\ e^{(8 i \pi)/113} \approx 1.5780 + 0.3569 i$$

Alternative representations:

$$\left(36 e^{2\pi} + 150 \left(e^{2\pi}\right)^2 + 154 \left(e^{2\pi}\right)^3 + 100 \left(e^{2\pi}\right)^4 + 122 \left(e^{2\pi}\right)^5 + 22 \left(e^{2\pi}\right)^6 + 26 \left(e^{2\pi}\right)^7 + 60 \left(e^{2\pi}\right)^8\right) \wedge (1/113) = \left(36 \exp^{2\pi}(z) + 150 \exp^{2\pi}(z)^2 + 154 \exp^{2\pi}(z)^3 + 100 \exp^{2\pi}(z)^4 + 122 \exp^{2\pi}(z)^5 + 22 \exp^{2\pi}(z)^6 + 26 \exp^{2\pi}(z)^7 + 60 \exp^{2\pi}(z)^8\right) \wedge (1/113) \text{ for } z = 1$$

$$\left(36 e^{2\pi} + 150 \left(e^{2\pi}\right)^2 + 154 \left(e^{2\pi}\right)^3 + 100 \left(e^{2\pi}\right)^4 + 122 \left(e^{2\pi}\right)^5 + 22 \left(e^{2\pi}\right)^6 + 26 \left(e^{2\pi}\right)^7 + 60 \left(e^{2\pi}\right)^8\right) \wedge (1/113) = \left(36 e^{360^\circ} + 150 \left(e^{360^\circ}\right)^2 + 154 \left(e^{360^\circ}\right)^3 + 100 \left(e^{360^\circ}\right)^4 + 122 \left(e^{360^\circ}\right)^5 + 22 \left(e^{360^\circ}\right)^6 + 26 \left(e^{360^\circ}\right)^7 + 60 \left(e^{360^\circ}\right)^8\right) \wedge (1/113)$$

$$\left(36 e^{2\pi} + 150 \left(e^{2\pi}\right)^2 + 154 \left(e^{2\pi}\right)^3 + 100 \left(e^{2\pi}\right)^4 + 122 \left(e^{2\pi}\right)^5 + 22 \left(e^{2\pi}\right)^6 + 26 \left(e^{2\pi}\right)^7 + 60 \left(e^{2\pi}\right)^8\right) \wedge (1/113) = \left(36 \exp^{2\cos^{-1}(-1)}(z) + 150 \exp^{2\cos^{-1}(-1)}(z)^2 + 154 \exp^{2\cos^{-1}(-1)}(z)^3 + 100 \exp^{2\cos^{-1}(-1)}(z)^4 + 122 \exp^{2\cos^{-1}(-1)}(z)^5 + 22 \exp^{2\cos^{-1}(-1)}(z)^6 + 26 \exp^{2\cos^{-1}(-1)}(z)^7 + 60 \exp^{2\cos^{-1}(-1)}(z)^8\right) \wedge (1/113) \text{ for } z = 1$$

Series representations:

$$\left(36 e^{2\pi} + 150 \left(e^{2\pi}\right)^2 + 154 \left(e^{2\pi}\right)^3 + 100 \left(e^{2\pi}\right)^4 + 122 \left(e^{2\pi}\right)^5 + 22 \left(e^{2\pi}\right)^6 + 26 \left(e^{2\pi}\right)^7 + 60 \left(e^{2\pi}\right)^8\right) \hat{}\ (1/113) = \sqrt[113]{2} \left(e^{8 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} \left(18 + 75 e^{8 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 77 e^{16 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 50 e^{24 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 61 e^{32 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 11 e^{40 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 13 e^{48 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 30 e^{56 \sum_{k=0}^{\infty} (-1)^k/(1+2k)}\right)\right) \hat{}\ (1/113)$$

$$\left(36 e^{2\pi} + 150 \left(e^{2\pi}\right)^2 + 154 \left(e^{2\pi}\right)^3 + 100 \left(e^{2\pi}\right)^4 + 122 \left(e^{2\pi}\right)^5 + 22 \left(e^{2\pi}\right)^6 + 26 \left(e^{2\pi}\right)^7 + 60 \left(e^{2\pi}\right)^8\right) \hat{}\ (1/113) = \sqrt[113]{2} \left(\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi} \left(18 + 75 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi} + 77 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4\pi} + 50 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6\pi} + 61 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8\pi} + 11 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{10\pi} + 13 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{12\pi} + 30 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{14\pi}\right)\right) \hat{}\ (1/113)$$

$$\left(36 e^{2\pi} + 150 \left(e^{2\pi}\right)^2 + 154 \left(e^{2\pi}\right)^3 + 100 \left(e^{2\pi}\right)^4 + 122 \left(e^{2\pi}\right)^5 + 22 \left(e^{2\pi}\right)^6 + 26 \left(e^{2\pi}\right)^7 + 60 \left(e^{2\pi}\right)^8\right) \hat{}\ (1/113) = \sqrt[113]{2} \left(\left(18 + 75 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi} + 77 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{4\pi} + 50 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{6\pi} + 61 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{8\pi} + 11 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{10\pi} + 13 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{12\pi} + 30 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{14\pi}\right) \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi}\right) \hat{}\ (1/113)$$

Integral representations:

$$\left(36 e^{2\pi} + 150 \left(e^{2\pi}\right)^2 + 154 \left(e^{2\pi}\right)^3 + 100 \left(e^{2\pi}\right)^4 + 122 \left(e^{2\pi}\right)^5 + 22 \left(e^{2\pi}\right)^6 + 26 \left(e^{2\pi}\right)^7 + 60 \left(e^{2\pi}\right)^8\right) \hat{}\ (1/113) = \sqrt[113]{2} \left(e^{4 \int_0^{\infty} \sin(t)/t\, dt} \left(18 + 75 e^{4 \int_0^{\infty} \sin(t)/t\, dt} + 77 e^{8 \int_0^{\infty} \sin(t)/t\, dt} + 50 e^{12 \int_0^{\infty} \sin(t)/t\, dt} + 61 e^{16 \int_0^{\infty} \sin(t)/t\, dt} + 11 e^{20 \int_0^{\infty} \sin(t)/t\, dt} + 13 e^{24 \int_0^{\infty} \sin(t)/t\, dt} + 30 e^{28 \int_0^{\infty} \sin(t)/t\, dt}\right)\right) \hat{}\ (1/113)$$

$$\left(36 e^{2\pi} + 150 (e^{2\pi})^2 + 154 (e^{2\pi})^3 + 100 (e^{2\pi})^4 + 122 (e^{2\pi})^5 + 22 (e^{2\pi})^6 + 26 (e^{2\pi})^7 + 60 (e^{2\pi})^8\right) \wedge (1/113) = \sqrt[113]{2} \left(e^{4 \int_0^\infty 1/(1+t^2)\,dt} \left(18 + 75 e^{4 \int_0^\infty 1/(1+t^2)\,dt} + 77 e^{8 \int_0^\infty 1/(1+t^2)\,dt} + 50 e^{12 \int_0^\infty 1/(1+t^2)\,dt} + 61 e^{16 \int_0^\infty 1/(1+t^2)\,dt} + 11 e^{20 \int_0^\infty 1/(1+t^2)\,dt} + 13 e^{24 \int_0^\infty 1/(1+t^2)\,dt} + 30 e^{28 \int_0^\infty 1/(1+t^2)\,dt}\right)\right) \wedge (1/113)$$

$$\left(36 e^{2\pi} + 150 (e^{2\pi})^2 + 154 (e^{2\pi})^3 + 100 (e^{2\pi})^4 + 122 (e^{2\pi})^5 + 22 (e^{2\pi})^6 + 26 (e^{2\pi})^7 + 60 (e^{2\pi})^8\right) \wedge (1/113) = \sqrt[113]{2} \left(e^{4 \int_0^\infty \sin^2(t)/t^2\,dt} \left(18 + 75 e^{4 \int_0^\infty \sin^2(t)/t^2\,dt} + 77 e^{8 \int_0^\infty \sin^2(t)/t^2\,dt} + 50 e^{12 \int_0^\infty \sin^2(t)/t^2\,dt} + 61 e^{16 \int_0^\infty \sin^2(t)/t^2\,dt} + 11 e^{20 \int_0^\infty \sin^2(t)/t^2\,dt} + 13 e^{24 \int_0^\infty \sin^2(t)/t^2\,dt} + 30 e^{28 \int_0^\infty \sin^2(t)/t^2\,dt}\right)\right) \wedge (1/113)$$

((((36*(e^(2Pi))^23+17*(e^(2Pi))^47+38*(e^(2Pi))^71+155*(e^(2Pi))^95))))^1/1250

Input:

$$\sqrt[1250]{36 (e^{2\pi})^{23} + 17 (e^{2\pi})^{47} + 38 (e^{2\pi})^{71} + 155 (e^{2\pi})^{95}}$$

Exact result:

$$\sqrt[1250]{36 e^{46\pi} + 17 e^{94\pi} + 38 e^{142\pi} + 155 e^{190\pi}}$$

Decimal approximation:

1.618592304157537497421796307738672621538390233385770997612...

1.618592304…

Property:

$\sqrt[1250]{36 e^{46\pi} + 17 e^{94\pi} + 38 e^{142\pi} + 155 e^{190\pi}}$ is a transcendental number

Alternate form:

$$e^{(23\pi)/625} \sqrt[1250]{36 + 17 e^{48\pi} + 38 e^{96\pi} + 155 e^{144\pi}}$$

All 1250th roots of 36 e^(46 π) + 17 e^(94 π) + 38 e^(142 π) + 155 e^(190 π):

$\sqrt[1250]{36 e^{46\pi} + 17 e^{94\pi} + 38 e^{142\pi} + 155 e^{190\pi}}\; e^0 \approx 1.6186$ (real, principal root)

$\sqrt[1250]{36 e^{46\pi} + 17 e^{94\pi} + 38 e^{142\pi} + 155 e^{190\pi}}\; e^{(i\pi)/625} \approx 1.6186 + 0.008136\, i$

$$\sqrt[1250]{36 e^{46\pi} + 17 e^{94\pi} + 38 e^{142\pi} + 155 e^{190\pi}} \; e^{(2 i \pi)/625} \approx 1.6185 + 0.016272 i$$

$$\sqrt[1250]{36 e^{46\pi} + 17 e^{94\pi} + 38 e^{142\pi} + 155 e^{190\pi}} \; e^{(3 i \pi)/625} \approx 1.6184 + 0.024407 i$$

$$\sqrt[1250]{36 e^{46\pi} + 17 e^{94\pi} + 38 e^{142\pi} + 155 e^{190\pi}} \; e^{(4 i \pi)/625} \approx 1.6183 + 0.032542 i$$

Alternative representations:

$$\sqrt[1250]{36 (e^{2\pi})^{23} + 17 (e^{2\pi})^{47} + 38 (e^{2\pi})^{71} + 155 (e^{2\pi})^{95}} = \sqrt[1250]{36 \left(e^{360^\circ}\right)^{23} + 17 \left(e^{360^\circ}\right)^{47} + 38 \left(e^{360^\circ}\right)^{71} + 155 \left(e^{360^\circ}\right)^{95}}$$

$$\sqrt[1250]{36 (e^{2\pi})^{23} + 17 (e^{2\pi})^{47} + 38 (e^{2\pi})^{71} + 155 (e^{2\pi})^{95}} = \sqrt[1250]{36 \exp^{2\pi}(z)^{23} + 17 \exp^{2\pi}(z)^{47} + 38 \exp^{2\pi}(z)^{71} + 155 \exp^{2\pi}(z)^{95}} \quad \text{for } z = 1$$

$$\sqrt[1250]{36 (e^{2\pi})^{23} + 17 (e^{2\pi})^{47} + 38 (e^{2\pi})^{71} + 155 (e^{2\pi})^{95}} = \sqrt[1250]{36 \left(e^{-2 i \log(-1)}\right)^{23} + 17 \left(e^{-2 i \log(-1)}\right)^{47} + 38 \left(e^{-2 i \log(-1)}\right)^{71} + 155 \left(e^{-2 i \log(-1)}\right)^{95}}$$

Series representations:

$$\sqrt[1250]{36 (e^{2\pi})^{23} + 17 (e^{2\pi})^{47} + 38 (e^{2\pi})^{71} + 155 (e^{2\pi})^{95}} = \left(e^{184 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} \left(36 + 17 e^{192 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 38 e^{384 \sum_{k=0}^{\infty} (-1)^k/(1+2k)} + 155 e^{576 \sum_{k=0}^{\infty} (-1)^k/(1+2k)}\right)\right) \wedge (1/1250)$$

$$\sqrt[1250]{36 (e^{2\pi})^{23} + 17 (e^{2\pi})^{47} + 38 (e^{2\pi})^{71} + 155 (e^{2\pi})^{95}} = \sqrt[1250]{\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{46\pi} \left(36 + 17 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{48\pi} + 38 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{96\pi} + 155 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{144\pi}\right)}$$

$$\sqrt[1250]{36\left(e^{2\pi}\right)^{23}+17\left(e^{2\pi}\right)^{47}+38\left(e^{2\pi}\right)^{71}+155\left(e^{2\pi}\right)^{95}} =$$

$$\left(\left(36+17\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{48\pi}+38\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{96\pi}+155\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{144\pi}\right)\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{46\pi}\right)\hat{}\,(1/1250)$$

Integral representations:

$$\sqrt[1250]{36\left(e^{2\pi}\right)^{23}+17\left(e^{2\pi}\right)^{47}+38\left(e^{2\pi}\right)^{71}+155\left(e^{2\pi}\right)^{95}} =$$

$$\left(e^{92\int_0^\infty 1/(1+t^2)\,dt}\left(36+17\,e^{96\int_0^\infty 1/(1+t^2)\,dt}+38\,e^{192\int_0^\infty 1/(1+t^2)\,dt}+155\,e^{288\int_0^\infty 1/(1+t^2)\,dt}\right)\right)\hat{}\,(1/1250)$$

$$\sqrt[1250]{36\left(e^{2\pi}\right)^{23}+17\left(e^{2\pi}\right)^{47}+38\left(e^{2\pi}\right)^{71}+155\left(e^{2\pi}\right)^{95}} =$$

$$\sqrt[1250]{e^{92\int_0^\infty \sin(t)/t\,dt}\left(36+17\,e^{96\int_0^\infty \sin(t)/t\,dt}+38\,e^{192\int_0^\infty \sin(t)/t\,dt}+155\,e^{288\int_0^\infty \sin(t)/t\,dt}\right)}$$

$$\sqrt[1250]{36\left(e^{2\pi}\right)^{23}+17\left(e^{2\pi}\right)^{47}+38\left(e^{2\pi}\right)^{71}+155\left(e^{2\pi}\right)^{95}} =$$

$$\left(e^{184\int_0^1 \sqrt{1-t^2}\,dt}\left(36+17\,e^{192\int_0^1 \sqrt{1-t^2}\,dt}+38\,e^{384\int_0^1 \sqrt{1-t^2}\,dt}+155\,e^{576\int_0^1 \sqrt{1-t^2}\,dt}\right)\right)\hat{}\,(1/1250)$$

From:

Congruence properties of partitions – *S. Ramanujan*
Mathematische Zeitschrift, IX, 1921, 147 – 153
[Extracted from the manuscripts of the author by G. H. Hardy]

We have that:

$$Q = 1 + 240\left(\frac{x}{1-x}+\frac{2^3x^2}{1-x^2}+\frac{3^3x^3}{1-x^3}+\cdots\right),$$

$$P = 1 - 24\Phi_{0,1}(x),\ Q = 1 + 240\Phi_{0,3}(x),\ R = 1 - 504\Phi_{0,5}(x).$$

In particular [†]]

$$Q^2 = 1 + 480\Phi_{0,7}(x) = 1 + 480\left(\frac{x}{1-x} + \frac{2^7x^2}{1-x^2} + \cdots\right), \tag{1.61}$$

$$QR = 1 - 264\Phi_{0,9}(x) = 1 - 264\left(\frac{x}{1-x} + \frac{2^9x^2}{1-x^2} + \cdots\right), \tag{1.62}$$

$$\begin{aligned} 441Q^3 + 250R^2 &= 691 + 65520\Phi_{0,11}(x) \\ &= 691 + 65520\left(\frac{x}{1-x} + \frac{2^{11}x^2}{1-x^2} + \cdots\right), \end{aligned} \tag{1.63}$$

[†]Ramanujan, pp. 163 – 165 [pp. 180 – 181] (Tables I to III). Ramanujan carried the calculation of formulæ of this kind to considerable lengths, the formula of Table I being

$$\begin{aligned} 7709321041217 + 32640\Phi_{0,31}(x) &= 764412173217Q^8 \\ &+ 5323905468000Q^5R^2 + 1621003400000Q^2R^4. \end{aligned}$$

It is worth while to quote one such formula; for it is impossible to understand Ramanujan without realising his love of numbers for their own sake.

$$(1.71) \qquad Q - P^2 = 288\Phi_{1,2}(x),$$

$$(1.72) \qquad PQ - R = 720\Phi_{1,4}(x),$$

$$(1.73) \qquad Q^2 - PR = 1008\Phi_{1,6}(x),$$

$$(1.74) \qquad Q(PQ - R) = 720\Phi_{1,8}(x),$$

$$(1.81) \qquad 3PQ - 2R - P^3 = 1728\Phi_{2,3}(x),$$

$$(1.82) \qquad P^2Q - 2PR + Q^2 = 1728\Phi_{2,5}(x),$$

$$(1.83) \qquad 2PQ^2 - P^2R - QR = 1728\Phi_{2,7}(x),$$

$$(1.91) \qquad 6P^2Q - 8PR + 3Q^2 - P^4 = 6912\Phi_{3,4}(x),$$

$$(1.92) \qquad P^3Q - 3P^2R + 3PQ^2 - QR = 3456\Phi_{3,6}(x),$$

$$(1.93) \qquad 15PQ^2 - 20P^2R + 10P^3Q - 4QR - P^5 = 20736\Phi_{4,5}(x).$$

Now, we have that:

(20736 + 3456 + 6912 + 1728 + 1728 + 1728 + 720 + 1008 + 720 + 288) =

= 39024; (39024 * 12) / 271 = 1728; (271 * 1728) / 12 = 39024

(we note that 1008 + 720 = 1728)

With regard:

$$7709321041217 + 32640\Phi_{0,31}(x) = 764412173217Q^8 + 5323905468000Q^5R^2 + 1621003400000Q^2R^4.$$

We note that:

764412173217+5323905468000+1621003400000 = 7709321041217

Thence, considering P = Q = R = 1, we obtain:

$7709321041217 + 32640\Phi_{0,31}(x) = 7709321041217$

$7709321041217 - 7709321041217 + 32640\Phi_{0,31}(x) =$

$= 32640\Phi_{0,31}(x)$

Now:

20736 + 3456 + 6912 + 1008 + 288 + 240 = 32640

Where

20736 = 1728*12; 3456 = 1728*2; 6912 = 1728*4; 1008 = 504*2 = 63*1728 / 108; 288 = 1728 / 6 ; 720 = 1728 / 36 * 15 ; 240 = 1728 / 72 * 10

We note also that:

39024 – 32640 – 3456 – 1728 – 1008 = 192;

1728 / 9 = 192; 89 + 55 + 34 + 13 + 1 = 192

32640 / 960 = 3264 / 96 = 34 = (3264/24) / (96/24) = 136 / 4 = 34; 21 + 8 + 5 = 34

We have also:

$$Q - P^2 = 288\Phi_{1,2}(x),$$

$$PQ - R = 720\Phi_{1,4}(x),$$

$$Q^2 - PR = 1008\Phi_{1,6}(x),$$

$$Q(PQ - R) = 720\Phi_{1,8}(x),$$

$$3PQ - 2R - P^3 = 1728\Phi_{2,3}(x),$$

$$P^2Q - 2PR + Q^2 = 1728\Phi_{2,5}(x),$$

$$2PQ^2 - P^2R - QR = 1728\Phi_{2,7}(x),$$

$$6P^2Q - 8PR + 3Q^2 - P^4 = 6912\Phi_{3,4}(x),$$

$$P^3Q - 3P^2R + 3PQ^2 - QR = 3456\Phi_{3,6}(x),$$

$$15PQ^2 - 20P^2R + 10P^3Q - 4QR - P^5 = 20736\Phi_{4,5}(x).$$

504-240+24 = 288; 288+720 = 1008; 1008+720 = 1728; 1728+1728 = 3456; 3456+1728+1728 = 6912; 6912 + 3456 + 3456 + 1728 + 1728 + 1728 + 1728 = 20736

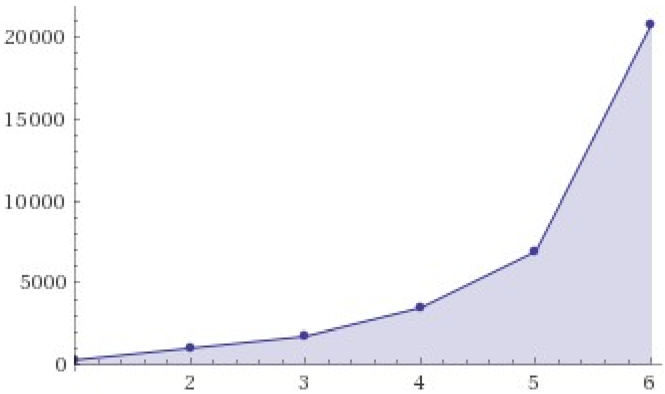
We have that: $20736 = 144^2$; 144 = 89 + 34 + 13 + 5 + 3

plot (504-240+24), (288+720), (1008+720), (1728+1728), (3456+1728+1728), (6912 + 3456 + 3456 + 1728 + 1728 + 1728 + 1728)

Input interpretation:

plot	{504 – 240 + 24, 288 + 720, 1008 + 720, 1728 + 1728, 3456 + 1728 + 1728, 6912 + 3456 + 3456 + 1728 + 1728 + 1728 + 1728}

Plot:



((504-240+24+720+720)+1728+1728+1728+3456+3456+1728+1728+1728+1728)

Input:

(504 – 240 + 24 + 720 + 720) + 1728 + 1728 + 1728 + 3456 + 3456 + 1728 + 1728 + 1728 + 1728

Result:

20 736

20736

(504-240+24+720+720)

Input:

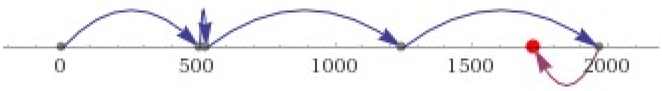
504 – 240 + 24 + 720 + 720

Result:

1728

1728

Number line:



(1728+1728+1728+1728+2*1728+2*1728+1728+1728+1728+1728)

Input:

$1728 + 1728 + 1728 + 1728 + 2 \times 1728 + 2 \times 1728 + 1728 + 1728 + 1728 + 1728$

Result:

20 736

20736

(1728+1728+1728+1728+1728+1728+1728+1728+1728+1728+1728+1728)

Input:

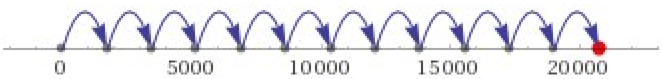
$1728 + 1728 + 1728 + 1728 + 1728 + 1728 + 1728 + 1728 + 1728 + 1728 + 1728 + 1728$

Result:

20 736

20736

Number line:



12(1728)

Input:

12×1728

Result:

20 736

20736

We note that:

sqrt(12(1728))

Input:

$\sqrt{12\times 1728}$

Result:

144

144

(((504-240+24)-55)) / ((sqrt(12(1728))))

Input:

$$\frac{(504-240+24)-55}{\sqrt{12\times 1728}}$$

Exact result:

$$\frac{233}{144}$$

Decimal approximation:

1.6180555555555555555555555555555555555555555555555555555...

1.618055555…

Note that:

1728/20736

Input:

$$\frac{1728}{20\,736}$$

Exact result:

$$\frac{1}{12}$$

Decimal approximation:

0.08333333333333333333333333333333333333333333333333...

0.083333…

Prime factorization:

$2^{-2} \times 3^{-1}$

Occurrence in convergents:

$$\frac{8}{\pi^4} \approx 0, \frac{1}{12}, \frac{5}{61}, \frac{6}{73}, \ldots$$

(simple continued fraction convergent sequence)

In Superstring Theory we have 1/12 in various expressions. For example:

From:

CERN-TH/97-218 hep-th/9709062

INTRODUCTION TO SUPERSTRING THEORY

Elias Kiritsis

There is further evidence for this duality. The effective action of type-IIA theory on K3 has a string solution, singular at the core. The zero mode structure of the string is similar to the perturbative type-IIA string. There is also a string solution that is regular at the core. This is a solitonic string and analysis of its zero modes indicates that it has the same (chiral) word-sheet structure as the heterotic string.[27] The string-string duality map (14.9.3)-(14.9.4) exchanges the roles of the two strings. The type-IIA string now becomes regular (solitonic), while the heterotic string solution becomes singular.

We will now further compactify both theories on a two-torus down to four dimensions and examine the consequences of the duality. In both cases we use the standard Kaluza-Klein ansatz described in Appendix C. The four-dimensional dilaton becomes, as usual,

$$\phi = \Phi - \frac{1}{2}\log[\det G_{\alpha\beta}]\,, \tag{14.9.5}$$

where $G_{\alpha\beta}$ is the metric of T^2 and $B_{\alpha\beta} = \epsilon_{\alpha\beta}B$ is the antisymmetric tensor. We obtain

$$S^{\text{het}}_{D=4} = \int d^4x\sqrt{-g}e^{-\phi}\left[R + L_B + L_{\text{gauge}} + L_{\text{scalar}}\right]\,, \tag{14.9.6}$$

where

$$\mathcal{L}_{g+\phi} = R + \partial^\mu\phi\partial_\mu\phi\,, \tag{14.9.7}$$

$$\mathcal{L}_B = -\frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho}\,, \tag{14.9.8}$$

with

$$\begin{aligned} H_{\mu\nu\rho} &= \partial_\mu B_{\nu\rho} - \frac{1}{2}\left[B_{\mu\alpha}F^{A,\alpha}_{\nu\rho} + A^\alpha_\mu F^B_{a,\nu\rho} + \hat{L}_{ij}A^i_\mu F^j_{\nu\rho}\right] + \text{cyclic} \\ &\equiv \partial_\mu B_{\nu\rho} - \frac{1}{2}L_{IJ}A^I_\mu F^J_{\nu\rho} + \text{cyclic}\,. \end{aligned} \tag{14.9.9}$$

We see that in eq.(14.9.8) there is -1/12

$$\mathcal{L}_B = -\frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho}$$

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64 g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276 e^{-\pi\sqrt{22}} - \cdots, \\ 64 g_{22}^{-24} &= 4096 e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64 g_{22}^{-24} \quad = \quad 4096 e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] *The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] *That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies*[3] *- golden spirals are one special case of these logarithmic spirals*

From Wikipedia

*The **Rössler attractor** is the attractor for the **Rössler system**, a system of three non-linear ordinary differential equations originally studied by Otto Rössler. These differential equations define a continuous-time dynamical system that exhibits chaotic dynamics associated with the fractal properties of the attractor.*

The Lorenz attractor was the first example of a low-dimensional differential equations system capable of generating chaotic behavior.(fractal)

In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749..., that is also a Hausdorff dimension and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

References

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[Extracted from the manuscripts of the author by G. H. Hardy]