On some Asymptotic Formulas and Ramanujan Identities: mathematical connections with ϕ, $\zeta(2)$ and various Fractal Hausdorff Dimensions values. I

Michele Nardelli[1], **Antonio Nardelli**[2]

Abstract

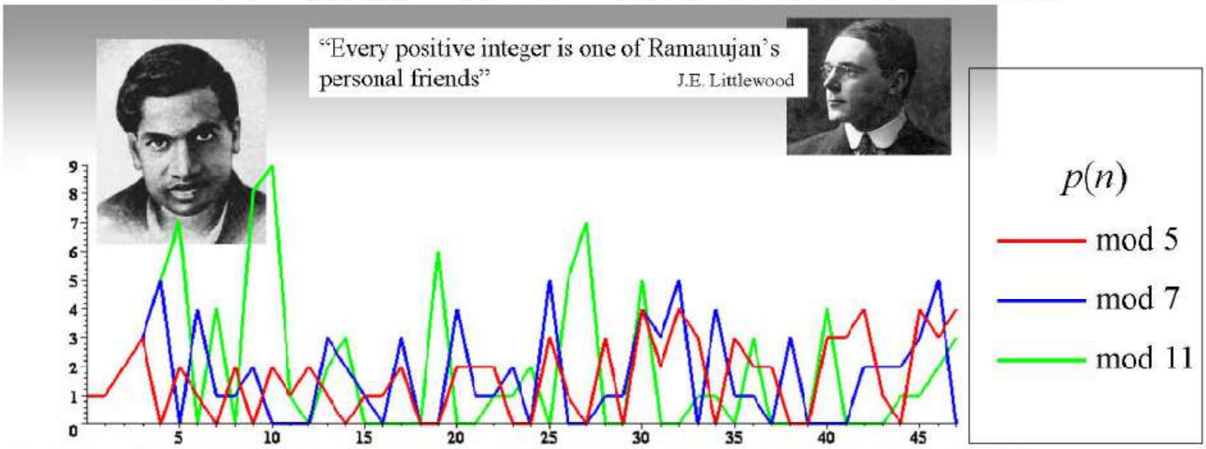
In this paper we have described some Asymptotic Formulas and Ramanujan Identities, and obtained several mathematical connections with ϕ*,* $\zeta(2)$ *and various Fractal Hausdorff Dimensions values*

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

[2] A. Nardelli studies at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

The Ramanujan Partition Congruences *Let n be a non-negative integer and let $p(n)$ denote the number of partitions of n (that is, the number of ways to write n as a sum of positive integers). Then $p(n)$ satisfies the congruence relations:*

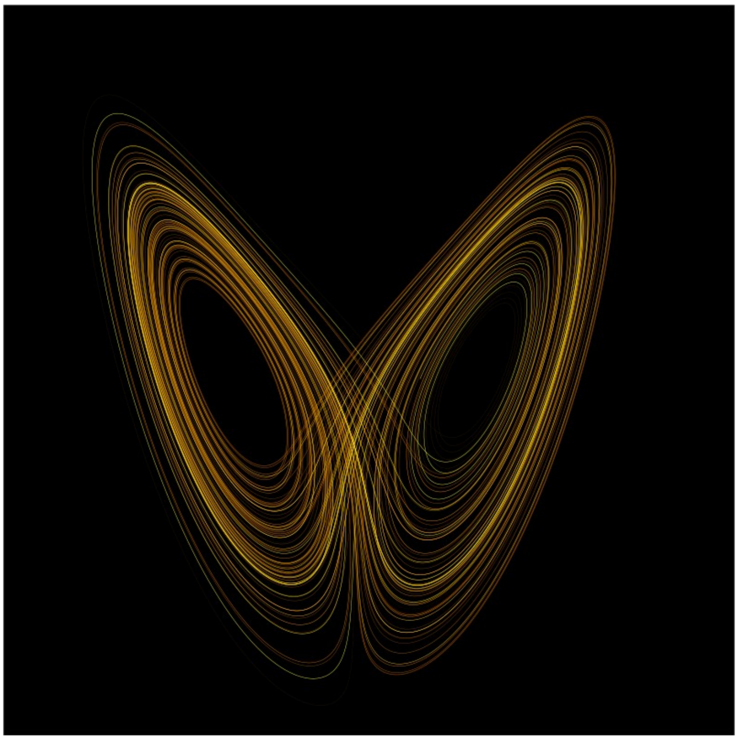
$$p(5t+4) \equiv 0 \pmod 5, \qquad p(7t+5) \equiv 0 \pmod 7, \quad \textit{and} \quad p(11t+6) \equiv 0 \pmod{11}.$$



Ramanujan's congruences tell us that, in the set of values of n for which $p(n) \bmod q = 0$, when q is 5, 7 or 11, there is an infinite arithmetic progression of common difference q. Thus we see that, in the above plot, the three graphs touch the horizontal axis at intervals which appear quite irregular but are certainly constrained by this arithmetic progression property. The property extends to all primes $q > 5$, a deep result published in 2000 by Ken Ono, but the common differences will not generally be q: the set of values of n for which $p(n) \bmod 31 = 0$, for instance, contains an infinite arithmetic progression whose common difference is not 31 but 31×107^4, and which starts at $n = 30064597$. For $q = 3$, the situation is very different—it is not even known if the values of n for which $p(n) \bmod 3 = 0$ form an infinite set!

Ramanujan published proofs of the congruences for 5 and 7 in 1919. His proof for mod 11 remained unpublished at the time of his death in 1920 and was written up by G.H. Hardy.

https://www.theoremoftheday.org/NumberTheory/Ramanujan/TotDRamanujan.pdf



https://en.wikipedia.org/wiki/Chaos_theory#/media/File:Lorenz_attractor_yb.svg

From:

ℓ-ADIC PROPERTIES OF THE PARTITION FUNCTION
AMANDA FOLSOM, ZACHARY A. KENT, AND KEN ONO
(Appendix by Nick Ramsey) - Celebrating the life of A. O. L. Atkin

Ramanujan's famous partition congruences modulo powers of 5; 7; and 11 imply that certain sequences of partition generating functions tend ℓ-adically to 0. little is known about the ℓ-adic behaviour of these sequences for primes ℓ ≥13. Using the classical theory of "modular forms mod p", as developed by Serre in the 1970s, we show that these sequences are governed by "fractal" behavior.

From:

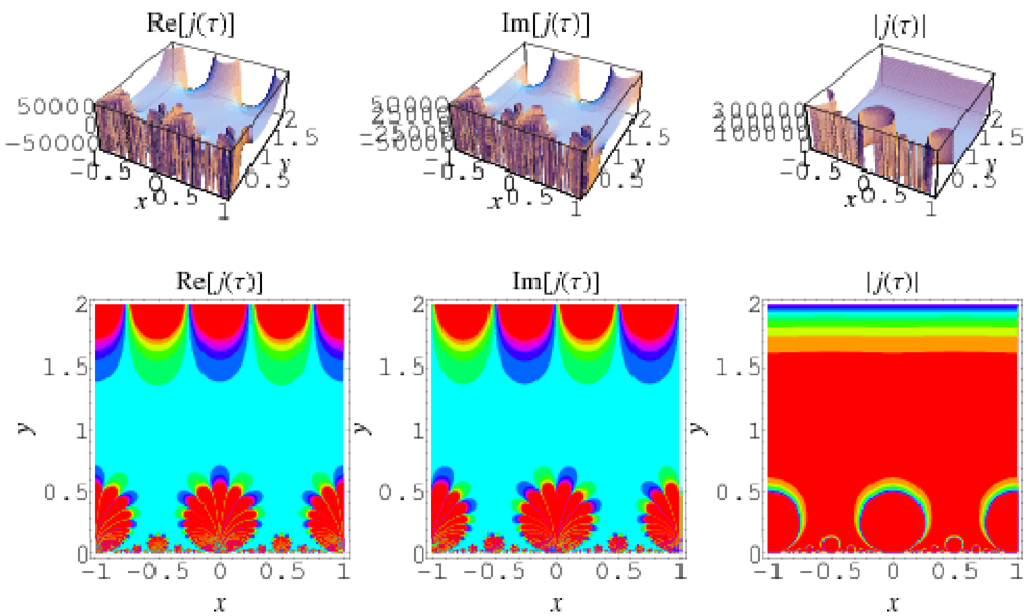
https://mathworld.wolfram.com/j-Function.html

In 1979, Conway and Norton discovered an unexpected intimate connection between the monster group M and the j-function. The Fourier expansion of $j(\tau)$ is given by

$$j(\tau) = \frac{1}{q} + 744 + 196884\, q + 21493760\, q^2 + 864299970\, q^3 + \ldots$$

j-Function

DOWNLOAD Wolfram Notebook



The j-function is the modular function defined by

$$j(\tau) \equiv 1728\, J(\tau), \tag{1}$$

where τ is the half-period ratio, $\mathrm{I}[\tau] > 0$,

$$J(\tau) \equiv \frac{4}{27} \frac{[1 - \lambda(\tau) + \lambda^2(\tau)]^3}{\lambda^2(\tau)[1 - \lambda(\tau)]^2} \tag{2}$$

is Klein's absolute invariant, $\lambda(\tau)$ is the elliptic lambda function

$$\lambda(\tau) \equiv \frac{\vartheta_2^4(e^{i\pi\tau})}{\vartheta_3^4(e^{i\pi\tau})}, \tag{3}$$

$\vartheta_i(0, q)$ are Jacobi theta functions,

$$q = e^{i\pi\tau} \tag{4}$$

is the nome, and $1728 = 12^3$.

Gauss was apparently aware of the j-function before 1800. Hermite used it in solving the quintic in about 1858. Dedekind gave a nice definition in about 1877, and Klein studied the function beginning in 1879 or 1880. The j-function is related to the factors of the group order of the monster group and to supersingular primes (Ogg 1980).

This function can also be specified in terms of the Weber functions f, f_1, f_2, γ_2, and γ_3 as

$$j(\tau) = \frac{[f^{24}(\tau) - 16]^3}{f^{24}(\tau)} \tag{5}$$

$$= \frac{[f_1^{24}(\tau) + 16]^3}{f_1^{24}(\tau)} \tag{6}$$

$$= \frac{[f_2^{24}(\tau) + 16]^3}{f_2^{24}(\tau)} \tag{7}$$

$$= \gamma_2^3(\tau) \tag{8}$$

$$= \gamma_3^2(\tau) + 1728 \tag{9}$$

(Weber 1979, p. 179; Atkin and Morain 1993).

The j-function is an analytic function on the upper half-plane which is invariant with respect to the special linear group $\mathrm{SL}(2, Z)$. It has a Fourier series

$$j(\bar{q}) = \sum_{n=-\infty}^{\infty} c(n)\, \bar{q}^n, \tag{10}$$

where

$$\bar{q} \equiv q^2 = e^{2\pi i \tau}. \tag{11}$$

$j(\bar{q})$ is therefore related $J(\tau)$ via

$$j(\bar{q}) = 1728\, J\left(-\frac{i \ln \bar{q}}{2\pi}\right). \tag{12}$$

The coefficients in the expansion of the j-function satisfy:

1. $c(n) = 0$ for $n < -1$ and $c(-1) = 1$,

2. all $c(n)$s are integers with fairly limited growth with respect to n, and

3. $j(\tau)$ is an algebraic number, sometimes a rational number, and sometimes even an integer at certain very special values of τ.

The latter result is the end result of the massive and beautiful theory of complex multiplication and the first step of Kronecker's so-called "Jugendtraum."

Therefore all of the coefficients in the Laurent series

$$\begin{aligned} j(\bar{q}) &= \frac{1}{\bar{q}} + 744 + 196\,884\,\bar{q} + 21\,493\,760\,\bar{q}^2 \\ &+ 864\,299\,970\,\bar{q}^3 + 20\,245\,856\,256\,\bar{q}^4 + 333\,202\,640\,600\,\bar{q}^5 + \ldots \end{aligned} \tag{13}$$

(OEIS A000521) are positive integers (Rankin 1977, Apostol 1997). Berwick (1916) calculated the first seven $c(n)$, Zuckerman (1939) found the first 24, and van Wijngaarden (193) gave the first 100.

Some remarkable sum formulas involving $j(\tau)$ for $\tau \in H$, where H is the upper half-plane, and $c(n)$ include

$$j(\bar{q}) = \frac{\left[1 + 240 \sum_{n=1}^{\infty} \sigma_3(n)\, \bar{q}^n\right]^3}{\bar{q} \prod_{n=1}^{\infty} (1 - \bar{q}^n)^{24}} \tag{14}$$

$$= \frac{E_4^3\left(\sqrt{\bar{q}}\right)}{q\,(\bar{q})_\infty^{24}} \tag{15}$$

$$= \frac{\left[\vartheta_2^8\left(\sqrt{\bar{q}}\right) + \vartheta_3^8\left(\sqrt{\bar{q}}\right) + \vartheta_2^4\left(\sqrt{\bar{q}}\right)\right]^3}{8\,\bar{q}\,(\bar{q})_\infty^{24}}, \tag{16}$$

where $E_4(q)$ is an Eisenstein series, $(q)_\infty$ is a q-Pochhammer symbol, and

$$\left[-1 + 504 \sum_{n=1}^{\infty} \sigma_5(n)\, \bar{q}^n\right]^2 = \left[j(\bar{q}) - 12^3\right] \sum_{n=1}^{\infty} \tau(n)\, \bar{q}^n, \tag{17}$$

where $\sigma_k(n)$ is the divisor function, and $\tau(n)$ is the tau function (not to be confused with the half-period ratio τ). In addition,

$$504^2\left[-\frac{2}{504}\sigma_5(n)+\sum_{k=1}^{n-1}\sigma_5(k)\,\sigma_5(n-k)\right]$$

$$=\tau(n+1)-984\,\tau(n)+\sum_{k=1}^{n-1}c(k)\,\tau(n-k) \tag{18}$$

$$\frac{65\,520}{691}\left[\sigma_{11}(n)-\tau(n)\right]$$

$$=\tau(n+1)+24\,\tau(n)+\sum_{k=1}^{n-1}c(k)\,\tau(n-k)$$

(Lehmer 1942; Apostol 1997, p. 92). These are closely related to Eisenstein series.

Equation (18) leads immediately to the remarkable congruence

$$\tau(n)\equiv\sigma_{11}(n)\ (\mathrm{mod}\ 691). \tag{19}$$

Lehmer (1942) showed that

$$(n+1)\,c(n)\equiv 0\ (\mathrm{mod}\ 24) \tag{20}$$

for all $n \geq 1$, and Lehner (1949ab) and Apostol (1997, pp. 22, 74, and 90-91) demonstrated that

$$c(2\,n)\equiv 0\left(\mathrm{mod}\ 2^{11}\right) \tag{21}$$

$$c(3\,n)\equiv 0\left(\mathrm{mod}\ 3^{5}\right) \tag{22}$$

$$c(5\,n)\equiv 0\left(\mathrm{mod}\ 5^{2}\right) \tag{23}$$

$$c(7\,n)\equiv 0\ (\mathrm{mod}\ 7) \tag{24}$$

$$c(11\,n)\equiv 0\ (\mathrm{mod}\ 11). \tag{25}$$

More generally,

$$c(2^{\alpha}\,n)\equiv 0\left(\mathrm{mod}\ 2^{3\,\alpha+8}\right) \tag{26}$$

$$c(3^{\alpha}\,n)\equiv 0\left(\mathrm{mod}\ 3^{2\,\alpha+3}\right) \tag{27}$$

$$c(5^{\alpha}\,n)\equiv 0\left(\mathrm{mod}\ 5^{\alpha+1}\right) \tag{28}$$

$$c(7^{\alpha}\,n)\equiv 0\ (\mathrm{mod}\ 7^{\alpha}) \tag{29}$$

(Lehner 1949ab; Apostol 1997, p. 91). Congruences of this type cannot exist for 13, but Newman (1958) showed

$$c(13\,n\,p)+c(13\,n)\,c(13\,p)+p^{-1}\,c\left(\frac{13\,n}{p}\right)\equiv 0\ (\mathrm{mod}\ 13), \tag{30}$$

where $p^{-1}\,p\equiv 1\ (\mathrm{mod}\ 13)$ and $c(x)=0$ if x is not an integer (Apostol 1997, p. 91). Congruences for $c(k\,n)$ have been generalized by Atkin and O'Brien (1967).

An asymptotic formula for $c(n)$ was discovered by Petersson (1932), and subsequently independently rediscovered by Rademacher (1938):

$$c(n) \sim \frac{e^{4\pi\sqrt{n}}}{\sqrt{2}\, n^{3/4}}. \tag{31}$$

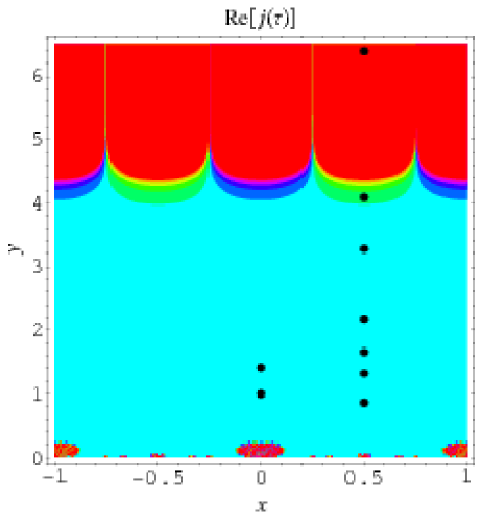
Let d be a squarefree positive integer, and define the half-period ratio by

$$\tau \equiv \begin{cases} i\sqrt{d} & \text{for } d \equiv 1 \text{ or } 2 \pmod 4 \\ \frac{1}{2}\left(1 + i\sqrt{d}\right) & \text{for } d \equiv 3 \pmod 4, \end{cases} \tag{32}$$

so

$$\bar{q} = \begin{cases} e^{-2\pi\sqrt{d}} & \text{for } d \equiv 1 \text{ or } 2 \pmod 4 \\ -e^{-\pi\sqrt{d}} & \text{for } d \equiv 3 \pmod 4. \end{cases} \tag{33}$$

It then turns out that $j(\tau)$ is an algebraic integer of degree $h(-d)$, where $h(-d)$ is the class number of the binary quadratic form discriminant $-d$ of the quadratic field $\mathbb{Q}\left(\sqrt{d}\right)$ (Silverman 1986; Berndt 1994, p. 90).



If $h(-d) = 1$, then $j(\tau)$ is an algebraic integer of degree 1, i.e., just a plain integer. Furthermore, the integer is a perfect cube. But these are precisely the Heegner numbers -1, -2, -3, -7, -11, -19, -43, -67, -163. The exact values of $j(\tau)$ corresponding to the Heegner numbers are

$$j(1 + i) = 12^3 \tag{34}$$

$$j\left(1 + i\sqrt{2}\right) = 20^3 \tag{35}$$

$$j\left(\tfrac{1}{2}\left(1 + i\sqrt{3}\right)\right) = 0^3 \tag{36}$$

$$j\left(\tfrac{1}{2}\left(1 + i\sqrt{7}\right)\right) = (-15)^3 \tag{37}$$

$$j\left(\tfrac{1}{2}\left(1 + i\sqrt{11}\right)\right) = (-32)^3 \tag{38}$$

$$j\left(\tfrac{1}{2}\left(1+i\sqrt{19}\right)\right)=(-96)^3 \tag{39}$$

$$j\left(\tfrac{1}{2}\left(1+i\sqrt{43}\right)\right)=(-960)^3 \tag{40}$$

$$j\left(\tfrac{1}{2}\left(1+i\sqrt{67}\right)\right)=(-5280)^3 \tag{41}$$

$$j\left(\tfrac{1}{2}\left(1+i\sqrt{163}\right)\right)=(-640\,320)^3. \tag{42}$$

The positions of these special values of τ are illustrated above. (Note the curious though not particularly significant fact that number 5280 is also the number of feet in a mile.)

The greater (in absolute value) the Heegner number d, the closer to an integer is the expression $e^{\pi\sqrt{-d}}$, since the initial term in $j(\tau)$ is the largest and subsequent terms are the smallest. The best approximations with $h(-d)=1$ are therefore

$$e^{\pi\sqrt{43}} \approx 960^3+744-2.2\times10^{-4} \tag{43}$$

$$e^{\pi\sqrt{67}} \approx 5280^3+744-1.3\times10^{-6} \tag{44}$$

$$e^{\pi\sqrt{163}} \approx 640\,320^3+744-7.5\times10^{-13} \tag{45}$$

(the latter of which appears in Trott 2004, p. 8). The almost integer generated by the last of these

, $e^{\pi\sqrt{163}}$ (corresponding to the field $\mathbb{Q}\left(\sqrt{-163}\right)$ and the imaginary quadratic field of maximal discriminant), is sometimes known as the Ramanujan constant. However, this attribution is historically fallacious since this amazing property of $e^{\pi\sqrt{163}}$ was first noted by Hermite (1859) and does not seem to appear in any of the works of Ramanujan.

There are 18 numbers having class number $h(-d)=2$, with the odd discriminants not divisible by three corresponding to the exact values

$$j\left(\tfrac{1}{2}\left(1+i\sqrt{35}\right)\right)=-16^3\left(15+7\sqrt{5}\right)^3 \tag{46}$$

$$j\left(\tfrac{1}{2}\left(1+i\sqrt{91}\right)\right)=-48^3\left(227+63\sqrt{13}\right)^3 \tag{47}$$

$$j\left(\tfrac{1}{2}\left(1+i\sqrt{115}\right)\right)=-48^3\left(785+351\sqrt{5}\right)^3 \tag{48}$$

$$j\left(\tfrac{1}{2}\left(1+i\sqrt{187}\right)\right)=-240^3\left(3451+837\sqrt{17}\right)^3 \tag{49}$$

$$j\left(\tfrac{1}{2}\left(1+i\sqrt{235}\right)\right)=-528^3\left(8875+3969\sqrt{5}\right)^3 \tag{50}$$

$$j\left(\tfrac{1}{2}\left(1+i\sqrt{403}\right)\right)=-240^3\left(2\,809\,615+779\,247\sqrt{13}\right)^3 \tag{51}$$

$$j\left(\tfrac{1}{2}\left(1+i\sqrt{427}\right)\right)=-5280^3\left(236\,674+30\,303\sqrt{61}\right)^3 \tag{52}$$

and even $d=4m$ for $m=5$, 10, 13, 22, 37, 58,

$$j\left(i\sqrt{5}\right)=2^3\left(25+13\sqrt{5}\right)^3 \tag{53}$$

$$j\left(i\sqrt{10}\right)=6^3\left(65+27\sqrt{5}\right)^3 \tag{54}$$

$$j\left(i\sqrt{13}\right) = 30^3\left(31 + 9\sqrt{13}\right)^3 \quad (55)$$

$$j\left(i\sqrt{22}\right) = 60^3\left(155 + 108\sqrt{2}\right)^3 \quad (56)$$

$$j\left(i\sqrt{37}\right) = 60^3\left(2837 + 468\sqrt{37}\right)^3 \quad (57)$$

$$j\left(i\sqrt{58}\right) = 30^3\left(140\,989 + 26\,163\sqrt{29}\right)^3 \quad (58)$$

and discriminants divisible by 3,

$$j\left(i\sqrt{6}\right) = 12^3\left(1 + \sqrt{2}\right)^2\left(5 + 2\sqrt{2}\right)^3 \quad (59)$$

$$j\left(\tfrac{1}{2}\left(1 + i\sqrt{15}\right)\right) = -3^3\left(\tfrac{1}{2}\left(1 + \sqrt{5}\right)\right)^2\left(5 + 4\sqrt{5}\right)^3 \quad (60)$$

$$j\left(\tfrac{1}{2}\left(1 + i\sqrt{51}\right)\right) = -48^3\left(4 + \sqrt{17}\right)^2\left(5 + \sqrt{17}\right)^3 \quad (61)$$

$$j\left(\tfrac{1}{2}\left(1 + i\sqrt{123}\right)\right) = -480^3\left(32 + 5\sqrt{41}\right)^2 \times \left(8 + \sqrt{41}\right)^3 \quad (62)$$

$$j\left(\tfrac{1}{2}\left(1 + i\sqrt{267}\right)\right) = -240^3\left(500 + 53\sqrt{89}\right)^2 \times \left(625 + 53\sqrt{89}\right)^3 \quad (63)$$

with the square factor being a fundamental unit.

The best approximations for $h(d) = 2$ are, for even discriminants,

$$e^{\pi\sqrt{232}} \approx 30^3\left(140\,989 + 26\,163\sqrt{29}\right)^3 - 744 - 3.2 \times 10^{-16}, \quad (64)$$

and for odd discriminants,

$$e^{\pi\sqrt{427}} \approx 5280^3\left(236\,674 + 30\,303\sqrt{61}\right)^3 + 744 - 1.3 \times 10^{-23}. \quad (65)$$

The numbers

$$e^{\pi\sqrt{22}} = \left(12\sqrt{11}\right)^4 - 104 - 1.7 \times 10^{-3} \quad (66)$$

$$e^{\pi\sqrt{37}} = \left(84\sqrt{2}\right)^4 + 104 - 2.2 \times 10^{-5} \quad (67)$$

$$e^{\pi\sqrt{58}} = 396^4 - 104 - 1.8 \times 10^{-7} \quad (68)$$

are also almost integers. These correspond to binary quadratic forms with discriminants -88, -148, and -232, which are the largest (in absolute value) discriminants with class number two that are divisible by 4. They were noted by Ramanujan (Berndt 1994, pp. 88-91).

SEE ALSO: Almost Integer, Heegner Number, Imaginary Quadratic Field, Klein's Absolute Invariant, Monster Group, Ramanujan Constant, Supersingular Prime, Weber Functions

Portions of this entry contributed by Tito Piezas III

From:

Congruence properties of partitions

Mathematische Zeitschrift, IX, 1921, 147 – 153

[Extracted from the manuscripts of the author by G. H. Hardy]*

1. Let

$$P = 1 - 24\left(\frac{x}{1-x} + \frac{2x^2}{1-x^2} + \frac{3x^3}{1-x^3} + \cdots\right), \tag{1.11}$$

$$Q = 1 + 240\left(\frac{x}{1-x} + \frac{2^3x^2}{1-x^2} + \frac{3^3x^3}{1-x^3} + \cdots\right), \tag{1.12}$$

$$R = 1 - 504\left(\frac{x}{1-x} + \frac{2^5x^2}{1-x^2} + \frac{3^5x^3}{1-x^3} + \cdots\right), \tag{1.13}$$

$$f(x) = (1-x)(1-x^2)(1-x^3)\cdots. \tag{1.2}$$

Then it is well known that

$$f(x) = 1 - x - x^2 + x^5 + x^7 - \cdots = 1 + \sum_{n=1}^{\infty}(-1)^n\left(x^{\frac{1}{2}n(3n-1)} + x^{\frac{1}{2}n(3n+1)}\right), \tag{1.3}$$

(1.4) $$Q^3 - R^2 = 1728x(f(x))^{24}.$$

Further, let

(1.51) $$\Phi_{r,s}(x) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} m^r n^s x^{mn} = \sum_{n=1}^{\infty} n^r \sigma_{s-r}(n) x^n,$$

where $\sigma_k(n)$ is the sum of the kth powers of the divisors of n; so that

(1.52) $$\Phi_{0,s}(x) = \frac{x}{1-x} + \frac{2^s x^2}{1-x^2} + \frac{3^s x^3}{1-x^3} + \cdots,$$

and in particular

(1.53) $$P = 1 - 24\Phi_{0,1}(x),\ Q = 1 + 240\Phi_{0,3}(x),\ R = 1 - 504\Phi_{0,5}(x).$$

Then [it may be deduced from the theory of the elliptic modular functions, and has been shewn by the author in a direct and elementary manner *, that, when $r+s$ is odd, and $r < s$, $\Phi_{r,s}(x)$ is expressible as a polynomial in $P, Q,$ and R, in the form

$$\Phi_{r,s}(x) = \sum k_{l,m,n} P^l Q^m R^n,$$

where

$$l - 1 \leq \text{ Min } (r,s),\ 2l + 4m + 6n = r + s + 1.$$

In particular †]

(1.61) $$Q^2 = 1 + 480\Phi_{0,7}(x) = 1 + 480\left(\frac{x}{1-x} + \frac{2^7 x^2}{1-x^2} + \cdots\right),$$

(1.62) $$QR = 1 - 264\Phi_{0,9}(x) = 1 - 264\left(\frac{x}{1-x} + \frac{2^9 x^2}{1-x^2} + \cdots\right),$$

(1.63) $$\begin{aligned}441Q^3 + 250R^2 &= 691 + 65520\Phi_{0,11}(x)\\ &= 691 + 65520\left(\frac{x}{1-x} + \frac{2^{11} x^2}{1-x^2} + \cdots\right),\end{aligned}$$

(1.71) $$Q - P^2 = 288\Phi_{1,2}(x),$$

(1.72) $$PQ - R = 720\Phi_{1,4}(x),$$

(1.73) $$Q^2 - PR = 1008\Phi_{1,6}(x),$$

(1.74) $$Q(PQ - R) = 720\Phi_{1,8}(x),$$

(1.81) $$3PQ - 2R - P^3 = 1728\Phi_{2,3}(x),$$

(1.82) $$P^2Q - 2PR + Q^2 = 1728\Phi_{2,5}(x),$$

(1.83) $$2PQ^2 - P^2R - QR = 1728\Phi_{2,7}(x),$$

(1.91) $$6P^2Q - 8PR + 3Q^2 - P^4 = 6912\Phi_{3,4}(x),$$

(1.92) $$P^3Q - 3P^2R + 3PQ^2 - QR = 3456\Phi_{3,6}(x),$$

(1.93) $$15PQ^2 - 20P^2R + 10P^3Q - 4QR - P^5 = 20736\Phi_{4,5}(x).$$

From:

Asymptotic formulæ in combinatory analysis – *Srinivasa Ramanujan*
Proceedings of the London Mathematical Society, 2, XVII, 1918, 75-115

From:

$$\chi(x) = \frac{x^{1/24}}{\sqrt{(2\pi)}}\sqrt{\left(\log\frac{1}{x}\right)}\left[\exp\left\{\frac{\pi^2}{6\log(1/x)}\right\} - 1\right] \qquad (1.54)$$

For x = 24, we obtain:

((24^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/24)) * [exp(Pi^2/(6 ln(1/24)))-1]

Input:

$$\frac{\sqrt[24]{24}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{24}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{24}\right)}\right)-1\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{i\sqrt[24]{3}\left(e^{-\pi^2/(6\log(24))}-1\right)\sqrt{\frac{\log(24)}{\pi}}}{2^{3/8}}$$

Decimal approximation:

$-0.328042564435867513975093189240330565296826802593634\,29748\ldots\, i$

Polar coordinates:

$r \approx 0.328043$ (radius), $\theta = -90°$ (angle)

0.328043

Alternate forms:

$$\frac{i\sqrt[24]{3}\left(e^{-\pi^2/(6(3\log(2)+\log(3)))}-1\right)\sqrt{\frac{3\log(2)+\log(3)}{\pi}}}{2^{3/8}}$$

$$-\frac{i\sqrt[24]{3}\,e^{-\pi^2/(6\log(24))}\left(e^{\pi^2/(6\log(24))}-1\right)\sqrt{\frac{\log(24)}{\pi}}}{2^{3/8}}$$

$$\frac{i\sqrt[24]{3}\,e^{-\pi^2/(6\log(24))}\sqrt{\frac{\log(24)}{\pi}}}{2^{3/8}}-\frac{i\sqrt[24]{3}\sqrt{\log(24)}}{2^{3/8}\sqrt{\pi}}$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{24}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{24}\right)}\right)-1\right)\right)\sqrt[24]{24}}{\sqrt{2\pi}}=\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{24}\right)}\right)\right)\sqrt[24]{24}\sqrt{\log_e\left(\frac{1}{24}\right)}}{\sqrt{2\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{24}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{24}\right)}\right)-1\right)\right)\sqrt[24]{24}}{\sqrt{2\pi}} = \frac{\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{24}\right)}\right)\right)\sqrt[24]{24}\sqrt{\log(a)\log_a\left(\frac{1}{24}\right)}}{\sqrt{2\pi}}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{24}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{24}\right)}\right)-1\right)\right)\sqrt[24]{24}}{\sqrt{2\pi}} = -\frac{i\sqrt[24]{3}\, e^{-\frac{\pi^2}{6\left(\log(23)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{23}\right)^k}{k}\right)}}\left(-1+e^{\frac{\pi^2}{6\left(\log(23)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{23}\right)^k}{k}\right)}}\right)\sqrt{\log(23)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{23}\right)^k}{k}}}{2^{3/8}\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{24}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{24}\right)}\right)-1\right)\right)\sqrt[24]{24}}{\sqrt{2\pi}} = \frac{i\sqrt[24]{3}\sqrt{23}\, e^{-\pi^2/(6\log(24))}\left(-1+e^{\pi^2/(6\log(24))}\right)\sum_{k=0}^{\infty}23^k\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}\frac{(-1)^j\binom{k}{j}p_{j,k}}{-1+2j}}{2^{3/8}\sqrt{\pi}}$$

$$\text{for}\left(c_k=\frac{(-1)^k}{(1+k)\pi}\text{ and }p_{j,0}=1\text{ and } p_{j,k}=\frac{\pi\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\text{ and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{24}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{24}\right)}\right)-1\right)\right)\sqrt[24]{24}}{\sqrt{2\pi}} =$$

$$-\frac{1}{2\times 2^{3/8}\sqrt{\pi}}\, i\sqrt[24]{3}\,\sqrt{23}\; e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(24)}}\left(-1+e^{\pi^2/(6\log(24))}\right)$$

$$\sum_{k=0}^{\infty} 23^k \binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k} -\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j} \text{ for } \left(c_k = \frac{(-1)^k}{(1+k)\pi} \text{ and } p_{j,0}=1\right.$$

$$\left.\text{and } p_{j,k} = \frac{\pi\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k} \text{ and } k\in\mathbb{Z} \text{ and } k>0\right)$$

Integral representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{24}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{24}\right)}\right)-1\right)\right)\sqrt[24]{24}}{\sqrt{2\pi}} =$$

$$-\frac{i\sqrt[24]{3}\; e^{-\pi^2/\left(6\int_1^{24}\frac{1}{t}dt\right)}\left(-1+e^{\pi^2/\left(6\int_1^{24}\frac{1}{t}dt\right)}\right)\sqrt{\int_1^{24}\frac{1}{t}\,dt}}{2^{3/8}\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{24}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{24}\right)}\right)-1\right)\right)\sqrt[24]{24}}{\sqrt{2\pi}} = -\frac{1}{2^{7/8}\pi}\, i\sqrt[24]{3}$$

$$\exp\left(-\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{23^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\left(-1+\exp\left(\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{23^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)$$

$$\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{23^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \text{ for } -1<\gamma<0$$

For x = 240

((240^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/240)) * [exp(Pi^2/(6 ln(1/240)))-1]

Input:

$$\frac{\sqrt[24]{240}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{240}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{240}\right)}\right)-1\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{i \sqrt[24]{15} \left(e^{-\pi^2/(6 \log(240))} - 1\right) \sqrt{\frac{\log(240)}{\pi}}}{\sqrt[3]{2}}$$

Decimal approximation:

$-0.30428020553255356032433772829069124740813150426972799145\ldots i$

Polar coordinates:

$r \approx 0.30428$ (radius), $\theta = -90°$ (angle)

0.30428

Alternate forms:

$$-\frac{i \sqrt[24]{15}\, e^{-\pi^2/(6 \log(240))} \left(e^{\pi^2/(6 \log(240))} - 1\right) \sqrt{\frac{\log(240)}{\pi}}}{\sqrt[3]{2}}$$

$$\frac{i \sqrt[24]{15} \left(e^{-\pi^2/(6 (4 \log(2)+\log(3)+\log(5)))} - 1\right) \sqrt{\frac{4 \log(2)+\log(3)+\log(5)}{\pi}}}{\sqrt[3]{2}}$$

$$\frac{i \sqrt[24]{15}\, e^{-\pi^2/(6 \log(240))} \sqrt{\frac{\log(240)}{\pi}}}{\sqrt[3]{2}} - \frac{i \sqrt[24]{15} \sqrt{\log(240)}}{\sqrt[3]{2} \sqrt{\pi}}$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{240}\right)} \left(\exp\left(\frac{\pi^2}{6 \log\left(\frac{1}{240}\right)}\right) - 1\right)\right) \sqrt[24]{240}}{\sqrt{2 \pi}} = \frac{\left(-1 + \exp\left(\frac{\pi^2}{6 \log_e\left(\frac{1}{240}\right)}\right)\right) \sqrt[24]{240} \sqrt{\log_e\left(\frac{1}{240}\right)}}{\sqrt{2 \pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{240}\right)} \left(\exp\left(\frac{\pi^2}{6 \log\left(\frac{1}{240}\right)}\right) - 1\right)\right) \sqrt[24]{240}}{\sqrt{2 \pi}} = \frac{\left(-1 + \exp\left(\frac{\pi^2}{6 \log(a) \log_a\left(\frac{1}{240}\right)}\right)\right) \sqrt[24]{240} \sqrt{\log(a) \log_a\left(\frac{1}{240}\right)}}{\sqrt{2 \pi}}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{240}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{240}\right)}\right)-1\right)\right)\sqrt[24]{240}}{\sqrt{2\pi}} = -\frac{1}{\sqrt[3]{2}\sqrt{\pi}} i \sqrt[24]{15} e^{-\frac{\pi^2}{6\left(\log(239)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{239}\right)^k}{k}\right)}}\left(-1+e^{\frac{\pi^2}{6\left(\log(239)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{239}\right)^k}{k}\right)}}\right)\sqrt{\log(239)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{239}\right)^k}{k}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{240}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{240}\right)}\right)-1\right)\right)\sqrt[24]{240}}{\sqrt{2\pi}} = -\frac{1}{2\sqrt[3]{2}\sqrt{\pi}} i \sqrt[24]{15}\sqrt{239}\, e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(240)}}\left(-1+e^{\pi^2/(6\log(240))}\right) \sum_{k=0}^{\infty} 239^k \binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k} -\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j} \text{ for } \left(c_k = \frac{(-1)^k}{(1+k)\pi} \text{ and } p_{j,0}=1 \text{ and } p_{j,k} = \frac{\pi\sum_{m=1}^{k}(-k+m+jm)c_m\, p_{j,k-m}}{k} \text{ and } k\in\mathbb{Z} \text{ and } k>0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{240}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{240}\right)}\right)-1\right)\right)\sqrt[24]{240}}{\sqrt{2\pi}} = -\frac{1}{\sqrt[3]{2}\sqrt{\pi}} i\sqrt[24]{15}\exp\left(-\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(240-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(240-x)^k x^{-k}}{k}\right)}\right)\left(-1+\exp\left(\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(240-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(240-x)^k x^{-k}}{k}\right)}\right)\right)\sqrt{2i\pi\left\lfloor\frac{\arg(240-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(240-x)^k x^{-k}}{k}} \text{ for } x<0$$

Integral representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{240}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{240}\right)}\right)-1\right)\right)\sqrt[24]{240}}{\sqrt{2\pi}} = -\frac{i\sqrt[24]{15}\,e^{-\pi^2/\left(6\int_1^{240}\frac{1}{t}dt\right)}\left(-1+e^{\pi^2/\left(6\int_1^{240}\frac{1}{t}dt\right)}\right)\sqrt{\int_1^{240}\frac{1}{t}\,dt}}{\sqrt[3]{2}\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{240}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{240}\right)}\right)-1\right)\right)\sqrt[24]{240}}{\sqrt{2\pi}} = -\frac{1}{2^{5/6}\pi}\,i\sqrt[24]{15}$$
$$\exp\left(-\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{239^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\left(-1+\exp\left(\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{239^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)$$
$$\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{239^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\quad \text{for } -1<\gamma<0$$

For x = 504

((504^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/504)) * [exp(Pi^2/(6 ln(1/504)))-1]

Input:

$$\frac{\sqrt[24]{504}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{504}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{504}\right)}\right)-1\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{i\sqrt[12]{3}\sqrt[24]{7}\left(e^{-\pi^2/(6\log(504))}-1\right)\sqrt{\frac{\log(504)}{\pi}}}{2^{3/8}}$$

Decimal approximation:

$-0.299595841005938327963843879628007498049929997067653656 09\ldots\, i$

Polar coordinates:

$r\approx 0.299596$ (radius), $\theta = -90°$ (angle)

0.299596

Alternate forms:

$$-\frac{i\sqrt[12]{3}\,\sqrt[24]{7}\,e^{-\pi^2/(6\log(504))}\left(e^{\pi^2/(6\log(504))}-1\right)\sqrt{\frac{\log(504)}{\pi}}}{2^{3/8}}$$

$$\frac{i\sqrt[12]{3}\,\sqrt[24]{7}\left(e^{-\frac{\pi^2}{6\,(3\log(2)+2\log(3)+\log(7))}}-1\right)\sqrt{\frac{3\log(2)+2\log(3)+\log(7)}{\pi}}}{2^{3/8}}$$

$$i\left(\frac{\sqrt[12]{3}\,\sqrt[24]{7}\,e^{-\pi^2/(6\log(504))}\sqrt{\frac{\log(504)}{\pi}}}{2^{3/8}}-\frac{\sqrt[12]{3}\,\sqrt[24]{7}\,\sqrt{\log(504)}}{2^{3/8}\sqrt{\pi}}\right)$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{504}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{504}\right)}\right)-1\right)\right)\sqrt[24]{504}}{\sqrt{2\pi}}=\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{504}\right)}\right)\right)\sqrt[24]{504}\,\sqrt{\log_e\left(\frac{1}{504}\right)}}{\sqrt{2\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{504}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{504}\right)}\right)-1\right)\right)\sqrt[24]{504}}{\sqrt{2\pi}}=\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{504}\right)}\right)\right)\sqrt[24]{504}\,\sqrt{\log(a)\log_a\left(\frac{1}{504}\right)}}{\sqrt{2\pi}}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{504}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{504}\right)}\right)-1\right)\right)\sqrt[24]{504}}{\sqrt{2\pi}}=-\frac{1}{2^{3/8}\sqrt{\pi}}\,i\sqrt[12]{3}\,\sqrt[24]{7}\,e^{-\frac{\pi^2}{6\left(\log(503)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{503}\right)^k}{k}\right)}}\left(-1+e^{\frac{\pi^2}{6\left(\log(503)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{503}\right)^k}{k}\right)}}\right)\sqrt{\log(503)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{503}\right)^k}{k}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{504}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{504}\right)}\right)-1\right)\right)\sqrt[24]{504}}{\sqrt{2\pi}} =$$

$$-\frac{1}{2^{3/8}}\, i \sqrt[12]{3}\, \sqrt[24]{7}\, e^{-\pi^2/(6\log(504))}\left(1-e^{\pi^2/(6\log(504))}\right)\sqrt{\frac{503}{\pi}}$$

$$\sum_{k=0}^{\infty} 503^k \binom{-\frac{1}{2}+k}{k} \sum_{j=0}^{k} \frac{(-1)^j \binom{k}{j} p_{j,k}}{-1+2j} \quad \text{for } \left(c_k = \frac{(-1)^k}{(1+k)\pi} \text{ and } p_{j,0} = 1\right.$$

$$\left.\text{and } p_{j,k} = \frac{\pi \sum_{m=1}^{k} (-k+m+j\,m)\, c_m\, p_{j,k-m}}{k} \text{ and } k \in \mathbb{Z} \text{ and } k > 0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{504}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{504}\right)}\right)-1\right)\right)\sqrt[24]{504}}{\sqrt{2\pi}} =$$

$$-\frac{1}{2\times 2^{3/8}}\, i \sqrt[12]{3}\, \sqrt[24]{7}\, e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(504)}}\left(-1+e^{\pi^2/(6\log(504))}\right)\sqrt{\frac{503}{\pi}}$$

$$\sum_{k=0}^{\infty} 503^k \binom{-\frac{1}{2}+k}{k} \sum_{j=0}^{k} -\frac{2(-1)^j \binom{k}{j} p_{j,k}}{-1+2j} \quad \text{for } \left(c_k = \frac{(-1)^k}{(1+k)\pi} \text{ and } p_{j,0} = 1\right.$$

$$\left.\text{and } p_{j,k} = \frac{\pi \sum_{m=1}^{k} (-k+m+j\,m)\, c_m\, p_{j,k-m}}{k} \text{ and } k \in \mathbb{Z} \text{ and } k > 0\right)$$

Integral representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{504}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{504}\right)}\right)-1\right)\right)\sqrt[24]{504}}{\sqrt{2\pi}} =$$

$$-\frac{i \sqrt[12]{3}\, \sqrt[24]{7}\, e^{-\pi^2/\left(6\int_1^{504}\frac{1}{t}\,dt\right)}\left(-1+e^{\pi^2/\left(6\int_1^{504}\frac{1}{t}\,dt\right)}\right)\sqrt{\int_1^{504}\frac{1}{t}\,dt}}{2^{3/8}\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{504}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{504}\right)}\right)-1\right)\right)\sqrt[24]{504}}{\sqrt{2\pi}} = -\frac{1}{2^{7/8}\pi}\, i \sqrt[12]{3}\, \sqrt[24]{7}$$

$$\exp\left(-\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{503^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\left(-1+\exp\left(\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{503^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)$$

$$\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{503^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

For x = 1728

((1728^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/1728)) * [exp(Pi^2/(6 ln(1/1728)))-1]

Input:

$$\frac{\sqrt[24]{1728}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{1728}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{1728}\right)}\right)-1\right)$$

log(x) is the natural logarithm

Exact result:

$$\frac{i\sqrt[8]{3}\left(e^{-\pi^2/(6\log(1728))}-1\right)\sqrt{\frac{\log(1728)}{\pi}}}{\sqrt[4]{2}}$$

Decimal approximation:

$-0.294243080317364078968366093831848360880901490919337934\ldots\, i$

Polar coordinates:

$r \approx 0.294243$ (radius), $\theta = -90°$ (angle)

0.294243 result very near to the following Ramanujan continued fraction:

https://sites.google.com/site/marelv83/fisica-moderna/serie-infinite-per-p **(in Italian)**

Come esempio di uno dei suoi risultati, Ramanujan fornì questa frazione continua,

$$\sqrt{\phi+2}-\phi=\cfrac{e^{-2\pi/5}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-4\pi}}{1+\cfrac{e^{-6\pi}}{1+\cdots}}}}=0{,}2840\ldots$$

fra le altre, dove $\phi=(1+\sqrt{5})/2$ è la sezione aurea.

Alternate forms:

$$\frac{i\sqrt[8]{3}\left(e^{-\pi^2/(18\log(12))}-1\right)\sqrt{\frac{\log(1728)}{\pi}}}{\sqrt[4]{2}}$$

$$-\frac{i\sqrt[8]{3}\ e^{-\pi^2/(6\log(1728))}\left(e^{\pi^2/(6\log(1728))}-1\right)\sqrt{\frac{\log(1728)}{\pi}}}{\sqrt[4]{2}}$$

$$\frac{i\sqrt[8]{3}\left(e^{-\pi^2/(18\,(2\log(2)+\log(3)))}-1\right)\sqrt{\frac{6\log(2)+3\log(3)}{\pi}}}{\sqrt[4]{2}}$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{1728}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{1728}\right)}\right)-1\right)\right)\sqrt[24]{1728}}{\sqrt{2\pi}}=$$
$$\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{1728}\right)}\right)\right)\sqrt[24]{1728}\ \sqrt{\log_e\left(\frac{1}{1728}\right)}}{\sqrt{2\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{1728}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{1728}\right)}\right)-1\right)\right)\sqrt[24]{1728}}{\sqrt{2\pi}}=$$
$$\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{1728}\right)}\right)\right)\sqrt[24]{1728}\ \sqrt{\log(a)\log_a\left(\frac{1}{1728}\right)}}{\sqrt{2\pi}}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{1728}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{1728}\right)}\right)-1\right)\right)\sqrt[24]{1728}}{\sqrt{2\pi}}=$$
$$-\frac{1}{\sqrt[4]{2}\ \sqrt{\pi}}i\sqrt[8]{3}\ e^{-\frac{\pi^2}{6\left(\log(1727)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{1727}\right)^k}{k}\right)}}\left(-1+e^{\frac{\pi^2}{6\left(\log(1727)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{1727}\right)^k}{k}\right)}}\right)\sqrt{\log(1727)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{1727}\right)^k}{k}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{1728}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{1728}\right)}\right)-1\right)\right)\sqrt[24]{1728}}{\sqrt{2\pi}} =$$

$$-\frac{1}{2\sqrt[4]{2}\sqrt{\pi}}\, i\sqrt[8]{3}\sqrt{1727}\, e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(1728)}}\left(-1+e^{\pi^2/(6\log(1728))}\right)$$

$$\sum_{k=0}^{\infty} 1727^k \binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k} -\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j} \text{ for } \left(c_k = \frac{(-1)^k}{(1+k)\pi} \text{ and } p_{j,0}=1\right.$$

$$\left.\text{and } p_{j,k} = \frac{\pi\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k} \text{ and } k\in\mathbb{Z} \text{ and } k>0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{1728}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{1728}\right)}\right)-1\right)\right)\sqrt[24]{1728}}{\sqrt{2\pi}} =$$

$$-\frac{1}{\sqrt[4]{2}\sqrt{\pi}}\, i\sqrt[8]{3}\exp\left(-\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(1728-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(1728-x)^k x^{-k}}{k}\right)}\right)$$

$$\left(-1+\exp\left(\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(1728-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(1728-x)^k x^{-k}}{k}\right)}\right)\right)$$

$$\sqrt{2i\pi\left\lfloor\frac{\arg(1728-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(1728-x)^k x^{-k}}{k}} \quad \text{for } x<0$$

Integral representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{1728}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{1728}\right)}\right)-1\right)\right)\sqrt[24]{1728}}{\sqrt{2\pi}} =$$

$$-\frac{i\sqrt[8]{3}\, e^{-\pi^2/\left(6\int_1^{1728}\frac{1}{t}\,dt\right)}\left(-1+e^{\pi^2/\left(6\int_1^{1728}\frac{1}{t}\,dt\right)}\right)\sqrt{\int_1^{1728}\frac{1}{t}\,dt}}{\sqrt[4]{2}\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{1728}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{1728}\right)}\right)-1\right)\right)\sqrt[24]{1728}}{\sqrt{2\pi}} = -\frac{1}{2^{3/4}\,\pi}\, i\,\sqrt[8]{3}$$

$$\exp\left(-\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{1727^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\left(-1+\exp\left(\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{1727^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)$$

$$\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{1727^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

From

$$j(\bar{q}) = \frac{1}{\bar{q}} + 744 + 196\,884\,\bar{q} + 21\,493\,760\,\bar{q}^2$$
$$+864\,299\,970\,\bar{q}^3 + 20\,245\,856\,256\,\bar{q}^4 + 333\,202\,640\,600\,\bar{q}^5 + \ldots$$

we obtain, for x = 744:

((744^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/744)) * [exp(Pi^2/(6 ln(1/744)))-1]

Input:

$$\frac{\sqrt[24]{744}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{744}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{744}\right)}\right)-1\right)$$

log(x) is the natural logarithm

Exact result:

$$\frac{i\sqrt[24]{93}\left(e^{-\pi^2/(6\log(744))}-1\right)\sqrt{\frac{\log(744)}{\pi}}}{2^{3/8}}$$

Decimal approximation:

$-0.29760310519689345433382267421016365607927897267120475075\ldots\, i$

Polar coordinates:

$r \approx 0.297603$ (radius), $\theta = -90°$ (angle)

0.297603

Alternate forms:

$$-\frac{i\sqrt[24]{93}\,e^{-\pi^2/(6\log(744))}\left(e^{\pi^2/(6\log(744))}-1\right)\sqrt{\frac{\log(744)}{\pi}}}{2^{3/8}}$$

$$\frac{i\sqrt[24]{93}\left(e^{-\pi^2/(6(3\log(2)+\log(3)+\log(31)))}-1\right)\sqrt{\frac{3\log(2)+\log(3)+\log(31)}{\pi}}}{2^{3/8}}$$

$$\frac{i\sqrt[24]{93}\,e^{-\pi^2/(6\log(744))}\sqrt{\frac{\log(744)}{\pi}}}{2^{3/8}}-\frac{i\sqrt[24]{93}\sqrt{\log(744)}}{2^{3/8}\sqrt{\pi}}$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{744}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{744}\right)}\right)-1\right)\right)\sqrt[24]{744}}{\sqrt{2\pi}}=\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{744}\right)}\right)\right)\sqrt[24]{744}\sqrt{\log_e\left(\frac{1}{744}\right)}}{\sqrt{2\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{744}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{744}\right)}\right)-1\right)\right)\sqrt[24]{744}}{\sqrt{2\pi}}=$$
$$\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{744}\right)}\right)\right)\sqrt[24]{744}\sqrt{\log(a)\log_a\left(\frac{1}{744}\right)}}{\sqrt{2\pi}}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{744}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{744}\right)}\right)-1\right)\right)\sqrt[24]{744}}{\sqrt{2\pi}}=-\frac{1}{2^{3/8}\sqrt{\pi}}i\sqrt[24]{93}$$
$$e^{-\frac{\pi^2}{6\left(\log(743)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{743}\right)^k}{k}\right)}}\left(-1+e^{\frac{\pi^2}{6\left(\log(743)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{743}\right)^k}{k}\right)}}\right)\sqrt{\log(743)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{743}\right)^k}{k}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{744}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{744}\right)}\right)-1\right)\right)\sqrt[24]{744}}{\sqrt{2\pi}} =$$

$$-\frac{1}{2\times 2^{3/8}\sqrt{\pi}}\, i\sqrt[24]{93}\,\sqrt{743}\, e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(744)}}\left(-1+e^{\pi^2/(6\log(744))}\right)$$

$$\sum_{k=0}^{\infty}743^k\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j}\quad\text{for}\left(c_k=\frac{(-1)^k}{(1+k)\pi}\text{ and }p_{j,0}=1\right.$$

$$\left.\text{and }p_{j,k}=\frac{\pi\sum_{m=1}^{k}(-k+m+jm)\,c_m\,p_{j,k-m}}{k}\text{ and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{744}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{744}\right)}\right)-1\right)\right)\sqrt[24]{744}}{\sqrt{2\pi}} =$$

$$-\frac{1}{2^{3/8}\sqrt{\pi}}\, i\sqrt[24]{93}\,\exp\left(-\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(744-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(744-x)^k x^{-k}}{k}\right)}\right)$$

$$\left(-1+\exp\left(\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(744-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(744-x)^k x^{-k}}{k}\right)}\right)\right)$$

$$\sqrt{2i\pi\left\lfloor\frac{\arg(744-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(744-x)^k x^{-k}}{k}}\quad\text{for } x<0$$

Integral representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{744}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{744}\right)}\right)-1\right)\right)\sqrt[24]{744}}{\sqrt{2\pi}} =$$

$$-\frac{i\sqrt[24]{93}\, e^{-\pi^2/\left(6\int_1^{744}\frac{1}{t}dt\right)}\left(-1+e^{\pi^2/\left(6\int_1^{744}\frac{1}{t}dt\right)}\right)\sqrt{\int_1^{744}\frac{1}{t}\,dt}}{2^{3/8}\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{744}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{744}\right)}\right)-1\right)\right)\sqrt[24]{744}}{\sqrt{2\pi}} = -\frac{1}{2^{7/8}\,\pi}\, i\,\sqrt[24]{93}$$

$$\exp\left(-\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{743^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\left(-1+\exp\left(\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{743^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)$$

$$\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{743^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

for x = 196884

((196884^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/196884)) * [exp(Pi^2/(6 ln(1/196884)))-1]

Input:

$$\frac{\sqrt[24]{196\,884}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{196\,884}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{196\,884}\right)}\right)-1\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{i\sqrt[8]{3}\,\sqrt[24]{1823}\left(e^{-\pi^2/(6\log(196884))}-1\right)\sqrt{\frac{\log(196884)}{\pi}}}{2^{5/12}}$$

Decimal approximation:

$-0.29219341129005178775413131956451594212613434652282079099\ldots\, i$

Polar coordinates:

$r \approx 0.292193$ (radius), $\theta = -90°$ (angle)

0.292193 result very near to the following Ramanujan continued fraction:

https://sites.google.com/site/marelv83/fisica-moderna/serie-infinite-per-p **(in Italian)**

Come esempio di uno dei suoi risultati, Ramanujan fornì questa frazione continua,

$$\sqrt{\phi+2}-\phi = \cfrac{e^{-2\pi/5}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-4\pi}}{1+\cfrac{e^{-6\pi}}{1+\cdots}}}} = 0{,}2840\ldots$$

fra le altre, dove $\phi = (1+\sqrt{5})/2$ è la sezione aurea.

Alternate forms:

$$-\frac{i\sqrt[8]{3}\,\sqrt[24]{1823}\,e^{-\pi^2/(6\log(196884))}\left(e^{\pi^2/(6\log(196884))}-1\right)\sqrt{\frac{\log(196884)}{\pi}}}{2^{5/12}}$$

$$\frac{i\sqrt[8]{3}\,\sqrt[24]{1823}\left(e^{-\frac{\pi^2}{6(2\log(2)+3\log(3)+\log(1823))}}-1\right)\sqrt{\frac{2\log(2)+3\log(3)+\log(1823)}{\pi}}}{2^{5/12}}$$

$$i\left(\frac{\sqrt[8]{3}\,\sqrt[24]{1823}\,e^{-\pi^2/(6\log(196884))}\sqrt{\frac{\log(196884)}{\pi}}}{2^{5/12}}-\frac{\sqrt[8]{3}\,\sqrt[24]{1823}\,\sqrt{\log(196884)}}{2^{5/12}\sqrt{\pi}}\right)$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{196884}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{196884}\right)}\right)-1\right)\right)\sqrt[24]{196884}}{\sqrt{2\pi}}=\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{196884}\right)}\right)\right)\sqrt[24]{196884}\sqrt{\log_e\left(\frac{1}{196884}\right)}}{\sqrt{2\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{196884}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{196884}\right)}\right)-1\right)\right)\sqrt[24]{196884}}{\sqrt{2\pi}}=\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{196884}\right)}\right)\right)\sqrt[24]{196884}\sqrt{\log(a)\log_a\left(\frac{1}{196884}\right)}}{\sqrt{2\pi}}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{196884}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{196\,884}\right)}\right)-1\right)\right)\sqrt[24]{196\,884}}{\sqrt{2\pi}} = -\frac{1}{2^{5/12}\sqrt{\pi}} i\sqrt[8]{3}\sqrt[24]{1823}\, e^{-\frac{\pi^2}{6\left(\log(196\,883)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{196\,883}\right)^k}{k}\right)}} \left(-1+e^{\frac{\pi^2}{6\left(\log(196\,883)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{196\,883}\right)^k}{k}\right)}}\right)\sqrt{\log(196\,883)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{196\,883}\right)^k}{k}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{196884}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{196\,884}\right)}\right)-1\right)\right)\sqrt[24]{196\,884}}{\sqrt{2\pi}} = -\frac{1}{2\times 2^{5/12}} i\sqrt[8]{3}\sqrt[24]{1823}\, e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(196\,884)}}\left(-1+e^{\pi^2/(6\log(196884))}\right) \sqrt{\frac{196\,883}{\pi}}\sum_{k=0}^{\infty}196\,883^k\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j}$$

$$\text{for}\left(c_k=\frac{(-1)^k}{(1+k)\pi}\text{ and }p_{j,0}=1\text{ and }p_{j,k}=\frac{\pi\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\text{ and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{196884}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{196\,884}\right)}\right)-1\right)\right)\sqrt[24]{196\,884}}{\sqrt{2\pi}} = -\frac{1}{2^{5/12}\sqrt{\pi}} i\sqrt[8]{3}\sqrt[24]{1823}\exp\left(-\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(196884-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(196884-x)^k x^{-k}}{k}\right)}\right) \left(-1+\exp\left(\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(196884-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(196884-x)^k x^{-k}}{k}\right)}\right)\right) \sqrt{2i\pi\left\lfloor\frac{\arg(196\,884-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(196\,884-x)^k x^{-k}}{k}}\quad\text{for } x<0$$

Integral representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{196884}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{196\,884}\right)}\right)-1\right)\right)\sqrt[24]{196\,884}}{\sqrt{2\pi}} = -\frac{i\sqrt[8]{3}\sqrt[24]{1823}\,e^{-\pi^2/\left(6\int_1^{196\,884}\frac{1}{t}\,dt\right)}\left(-1+e^{\pi^2/\left(6\int_1^{196\,884}\frac{1}{t}\,dt\right)}\right)\sqrt{\int_1^{196\,884}\frac{1}{t}\,dt}}{2^{5/12}\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{196884}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{196\,884}\right)}\right)-1\right)\right)\sqrt[24]{196\,884}}{\sqrt{2\pi}} = -\frac{1}{2^{11/12}\,\pi}\,i\sqrt[8]{3}\sqrt[24]{1823}$$
$$\exp\left(-\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{196\,883^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\left(-1+\exp\left(\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{196\,883^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)$$
$$\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{196\,883^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\quad \text{for } -1<\gamma<0$$

for x = 21493760

((21493760^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/21493760)) * [exp(Pi^2/(6 ln(1/21493760)))-1]

Input:

$$\frac{\sqrt[24]{21\,493\,760}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{21\,493\,760}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{21\,493\,760}\right)}\right)-1\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$i\sqrt[24]{\frac{10\,495}{2}}\left(e^{-\pi^2/(6\log(21\,493\,760))}-1\right)\sqrt{\frac{\log(21\,493\,760)}{\pi}}$$

Decimal approximation:

$-0.307509402302859809018945637545509023441464170053627144 78\ldots\, i$

Polar coordinates:

$r \approx 0.307509$ (radius), $\theta = -90°$ (angle)

0.307509

Alternate forms:

$$-i \sqrt[24]{\frac{10\,495}{2}}\; e^{-\pi^2/(6\log(21\,493\,760))} \left(e^{\pi^2/(6\log(21\,493\,760))} - 1\right) \sqrt{\frac{\log(21\,493\,760)}{\pi}}$$

$$i \sqrt[24]{\frac{10\,495}{2}}\; e^{-\pi^2/(6\log(21\,493\,760))} \sqrt{\frac{\log(21\,493\,760)}{\pi}} - i \sqrt[24]{\frac{10\,495}{2}} \sqrt{\frac{\log(21\,493\,760)}{\pi}}$$

$$i \left(\sqrt[24]{\frac{10\,495}{2}}\; e^{-\pi^2/(6\log(21\,493\,760))} \sqrt{\frac{\log(21\,493\,760)}{\pi}} - \sqrt[24]{\frac{10\,495}{2}} \sqrt{\frac{\log(21\,493\,760)}{\pi}}\right)$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{21\,493\,760}\right)} \left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{21\,493\,760}\right)}\right) - 1\right)\right) \sqrt[24]{21\,493\,760}}{\sqrt{2\pi}} = \frac{\left(-1 + \exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{21\,493\,760}\right)}\right)\right) \sqrt[24]{21\,493\,760}\, \sqrt{\log_e\left(\frac{1}{21\,493\,760}\right)}}{\sqrt{2\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{21\,493\,760}\right)} \left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{21\,493\,760}\right)}\right) - 1\right)\right) \sqrt[24]{21\,493\,760}}{\sqrt{2\pi}} = \frac{\left(-1 + \exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{21\,493\,760}\right)}\right)\right) \sqrt[24]{21\,493\,760}\, \sqrt{\log(a)\log_a\left(\frac{1}{21\,493\,760}\right)}}{\sqrt{2\pi}}$$

for x = 864299970

((864299970^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/864299970)) * [exp(Pi^2/(6 ln(1/864299970)))-1]

Input:

$$\frac{\sqrt[24]{864\,299\,970}}{\sqrt{2\,\pi}}\sqrt{\log\left(\frac{1}{864\,299\,970}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{864\,299\,970}\right)}\right)-1\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{i\,3^{5/24}\sqrt[24]{1\,778\,395}\left(e^{-\pi^2/(6\log(864\,299\,970))}-1\right)\sqrt{\frac{\log(864\,299\,970)}{\pi}}}{2^{11/24}}$$

Decimal approximation:

$-0.32770356028644679463560592588733047188001558717673423257\ldots\,i$

Polar coordinates:

$r\approx 0.327704$ (radius), $\theta=-90°$ (angle)

0.327704

Alternate forms:

$$-\frac{i\,3^{5/24}\sqrt[24]{1\,778\,395}\,e^{-\pi^2/(6\log(864\,299\,970))}\left(e^{\pi^2/(6\log(864\,299\,970))}-1\right)\sqrt{\frac{\log(864\,299\,970)}{\pi}}}{2^{11/24}}$$

$$\frac{1}{2^{11/24}}\,i\,3^{5/24}\sqrt[24]{1\,778\,395}\left(e^{-\frac{\pi^2}{6(\log(2)+5\log(3)+\log(5)+\log(355\,679))}}-1\right)\sqrt{\frac{\log(2)+5\log(3)+\log(5)+\log(355\,679)}{\pi}}$$

$$\frac{i\,3^{5/24}\sqrt[24]{1\,778\,395}\,e^{-\pi^2/(6\log(864\,299\,970))}\sqrt{\frac{\log(864\,299\,970)}{\pi}}}{2^{11/24}}-\frac{i\,3^{5/24}\sqrt[24]{1\,778\,395}\sqrt{\log(864\,299\,970)}}{2^{11/24}\sqrt{\pi}}$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{864299970}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{864299970}\right)}\right)-1\right)\right)\sqrt[24]{864299970}}{\sqrt{2\pi}} = \frac{\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{864299970}\right)}\right)\right)\sqrt[24]{864299970}\sqrt{\log_e\left(\frac{1}{864299970}\right)}}{\sqrt{2\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{864299970}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{864299970}\right)}\right)-1\right)\right)\sqrt[24]{864299970}}{\sqrt{2\pi}} = \frac{\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{864299970}\right)}\right)\right)\sqrt[24]{864299970}\sqrt{\log(a)\log_a\left(\frac{1}{864299970}\right)}}{\sqrt{2\pi}}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{864299970}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{864299970}\right)}\right)-1\right)\right)\sqrt[24]{864299970}}{\sqrt{2\pi}} = -\frac{1}{2^{11/24}\sqrt{\pi}} i\, 3^{5/24}\sqrt[24]{1778395}\, e^{-\frac{\pi^2}{6\left(\log(864299969)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{864299969}\right)^k}{k}\right)}} \left(-1+e^{\frac{\pi^2}{6\left(\log(864299969)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{864299969}\right)^k}{k}\right)}}\right)\sqrt{\log(864299969)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{864299969}\right)^k}{k}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{864299970}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{864299970}\right)}\right)-1\right)\right)\sqrt[24]{864299970}}{\sqrt{2\pi}} = -\frac{1}{2\times 2^{11/24}}\, 13\, i\, 3^{5/24}$$

$$\sqrt[24]{1778395}\; e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(864299970)}}\left(-1+e^{\pi^2/(6\log(864299970))}\right)$$

$$\sqrt{\frac{5114201}{\pi}}\sum_{k=0}^{\infty}864299969^k\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j}$$

$$\text{for}\left(c_k=\frac{(-1)^k}{(1+k)\pi}\text{ and }p_{j,0}=1\text{ and }p_{j,k}=\frac{\pi\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\right.$$

$$\left.\text{and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{864299970}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{864299970}\right)}\right)-1\right)\right)\sqrt[24]{864299970}}{\sqrt{2\pi}} =$$

$$-\frac{1}{2^{11/24}\sqrt{\pi}}\, i\, 3^{5/24}\sqrt[24]{1778395}$$

$$\exp\left(-\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(864299970-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(864299970-x)^k x^{-k}}{k}\right)}\right)$$

$$\left(-1+\exp\left(\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(864299970-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(864299970-x)^k x^{-k}}{k}\right)}\right)\right)$$

$$\sqrt{2i\pi\left\lfloor\frac{\arg(864299970-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(864299970-x)^k x^{-k}}{k}}\quad\text{for }x<0$$

For x = 20245856256

((20245856256^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/20245856256)) * [exp(Pi^2/(6 ln(1/20245856256)))-1]

Input:

$$\frac{\sqrt[24]{20245856256}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{20245856256}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{20245856256}\right)}\right)-1\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$i \sqrt[12]{2} \sqrt[8]{3} \sqrt[24]{45767} \left(e^{-\pi^2/(6 \log(20245856256))} - 1\right) \sqrt{\frac{\log(20245856256)}{\pi}}$$

Decimal approximation:

$$-0.349836058380351221705660421997131894039726585390599680 49\ldots i$$

Polar coordinates:

$r \approx 0.349836$ (radius), $\theta = -90°$ (angle)

0.349836

Alternate forms:

$$-i \sqrt[12]{2} \sqrt[8]{3} \sqrt[24]{45767} e^{-\pi^2/(6 \log(20245856256))} \left(e^{\pi^2/(6 \log(20245856256))} - 1\right) \sqrt{\frac{\log(20245856256)}{\pi}}$$

$$i \sqrt[12]{2} \sqrt[8]{3} \sqrt[24]{45767} \left(e^{-\frac{\pi^2}{6(14 \log(2)+3 \log(3)+\log(45767))}} - 1\right) \sqrt{\frac{14 \log(2) + 3 \log(3) + \log(45767)}{\pi}}$$

$$i \sqrt[12]{2} \sqrt[8]{3} \sqrt[24]{45767} e^{-\pi^2/(6 \log(20245856256))} \sqrt{\frac{\log(20245856256)}{\pi}} - \frac{i \sqrt[12]{2} \sqrt[8]{3} \sqrt[24]{45767} \sqrt{\log(20245856256)}}{\sqrt{\pi}}$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{20245856256}\right)} \left(\exp\left(\frac{\pi^2}{6 \log\left(\frac{1}{20245856256}\right)}\right) - 1\right)\right) \sqrt[24]{20245856256}}{\sqrt{2 \pi}} = \frac{\left(-1 + \exp\left(\frac{\pi^2}{6 \log_e\left(\frac{1}{20245856256}\right)}\right)\right) \sqrt[24]{20245856256} \sqrt{\log_e\left(\frac{1}{20245856256}\right)}}{\sqrt{2 \pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{20245856256}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{20245856256}\right)}\right)-1\right)\right)\sqrt[24]{20245856256}}{\sqrt{2\pi}} = \frac{\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{20245856256}\right)}\right)\right)\sqrt[24]{20245856256}\sqrt{\log(a)\log_a\left(\frac{1}{20245856256}\right)}}{\sqrt{2\pi}}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{20245856256}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{20245856256}\right)}\right)-1\right)\right)\sqrt[24]{20245856256}}{\sqrt{2\pi}} = -\frac{1}{\sqrt{\pi}} i \sqrt[12]{2}\sqrt[8]{3}\sqrt[24]{45767}\, e^{-\frac{\pi^2}{6\left(\log(20245856255)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{20245856255}\right)^k}{k}\right)}} \left(-1+e^{\frac{\pi^2}{6\left(\log(20245856255)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{20245856255}\right)^k}{k}\right)}}\right)\sqrt{\log(20245856255)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{20245856255}\right)^k}{k}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{20245856256}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{20245856256}\right)}\right)-1\right)\right)\sqrt[24]{20245856256}}{\sqrt{2\pi}} = -\frac{1}{2^{11/12}} i \sqrt[8]{3}\sqrt[24]{45767}\, e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(20245856256)}} \left(-1+e^{\pi^2/(6\log(20245856256))}\right)\sqrt{\frac{20245856255}{\pi}} \sum_{k=0}^{\infty} 20245856255^k \binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j} \text{ for } \left(c_k=\frac{(-1)^k}{(1+k)\pi} \text{ and } p_{j,0}=1 \text{ and } p_{j,k}=\frac{\pi\sum_{m=1}^{k}(-k+m+jm)c_m\,p_{j,k-m}}{k} \text{ and } k\in\mathbb{Z} \text{ and } k>0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{20245856256}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{20245856256}\right)}\right)-1\right)\right)\sqrt[24]{20245856256}}{\sqrt{2\pi}} =$$

$$-\frac{1}{\sqrt{\pi}}\, i \sqrt[12]{2}\ \sqrt[8]{3}\ \sqrt[24]{45767}$$
$$\exp\left(-\frac{\pi^2}{6\left(2 i \pi \left\lfloor \frac{\arg(20245856256-x)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{(-1)^k (20245856256-x)^k x^{-k}}{k}\right)}\right)$$
$$\left(-1+\exp\left(\frac{\pi^2}{6\left(2 i \pi \left\lfloor \frac{\arg(20245856256-x)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{(-1)^k (20245856256-x)^k x^{-k}}{k}\right)}\right)\right)$$
$$\sqrt{2 i \pi \left\lfloor \frac{\arg(20245856256-x)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{(-1)^k (20245856256-x)^k x^{-k}}{k}}$$

for $x < 0$

Integral representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{20245856256}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{20245856256}\right)}\right)-1\right)\right)\sqrt[24]{20245856256}}{\sqrt{2\pi}} =$$

$$-\frac{1}{\sqrt{\pi}} i \sqrt[12]{2}\ \sqrt[8]{3}\ \sqrt[24]{45767}\ e^{-\pi^2/\left(6\int_1^{20245856256}\frac{1}{t}dt\right)}$$
$$\left(-1+e^{\pi^2/\left(6\int_1^{20245856256}\frac{1}{t}dt\right)}\right)\sqrt{\int_1^{20245856256}\frac{1}{t}\,dt}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{20245856256}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{20245856256}\right)}\right)-1\right)\right)\sqrt[24]{20245856256}}{\sqrt{2\pi}} =$$

$$-\frac{1}{2^{5/12}\pi}\, i \sqrt[8]{3}\ \sqrt[24]{45767}\ \exp\left(-\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{20245856255^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)$$
$$\left(-1+\exp\left(\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{20245856255^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)$$
$$\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{20245856255^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

For x = 333202640600

((333202640600^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/333202640600)) * [exp(Pi^2/(6 ln(1/333202640600)))-1]

Input:

$$\frac{\sqrt[24]{333\,202\,640\,600}}{\sqrt{2\,\pi}}\sqrt{\log\left(\frac{1}{333\,202\,640\,600}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{333\,202\,640\,600}\right)}\right)-1\right)$$

log(x) is the natural logarithm

Exact result:

$$\frac{i\sqrt[12]{5}\sqrt[24]{1\,666\,013\,203}\left(e^{-\pi^2/(6\log(333\,202\,640\,600))}-1\right)\sqrt{\frac{\log(333\,202\,640\,600)}{\pi}}}{2^{3/8}}$$

Decimal approximation:

$-0.37315831440200484353970945067699330892968302939426322345\ldots\,i$

Polar coordinates:

$r \approx 0.373158$ (radius), $\theta = -90°$ (angle)

0.373158

Alternate forms:

$$-\frac{1}{2^{3/8}}i\sqrt[12]{5}\sqrt[24]{1\,666\,013\,203}\,e^{-\pi^2/(6\log(333\,202\,640\,600))}\left(e^{\pi^2/(6\log(333\,202\,640\,600))}-1\right)\sqrt{\frac{\log(333\,202\,640\,600)}{\pi}}$$

$$\frac{i\sqrt[12]{5}\sqrt[24]{1\,666\,013\,203}\,e^{-\pi^2/(6\log(333\,202\,640\,600))}\sqrt{\frac{\log(333\,202\,640\,600)}{\pi}}}{2^{3/8}}-\frac{i\sqrt[12]{5}\sqrt[24]{1\,666\,013\,203}\sqrt{\log(333\,202\,640\,600)}}{2^{3/8}\sqrt{\pi}}$$

$$i\left(\frac{\sqrt[12]{5}\ \sqrt[24]{1666013203}\ e^{-\pi^2/(6\log(333202640600))}\sqrt{\frac{\log(333202640600)}{\pi}}}{2^{3/8}} - \frac{\sqrt[12]{5}\ \sqrt[24]{1666013203}\ \sqrt{\log(333202640600)}}{2^{3/8}\sqrt{\pi}}\right)$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{333202640600}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{333202640600}\right)}\right)-1\right)\right)\sqrt[24]{333202640600}}{\sqrt{2\pi}} = \frac{\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{333202640600}\right)}\right)\right)\sqrt[24]{333202640600}\ \sqrt{\log_e\left(\frac{1}{333202640600}\right)}}{\sqrt{2\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{333202640600}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{333202640600}\right)}\right)-1\right)\right)\sqrt[24]{333202640600}}{\sqrt{2\pi}} = \frac{1}{\sqrt{2\pi}}\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{333202640600}\right)}\right)\right)\sqrt[24]{333202640600}\ \sqrt{\log(a)\log_a\left(\frac{1}{333202640600}\right)}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{333\,202\,640\,600}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{333\,202\,640\,600}\right)}\right)-1\right)\right)\sqrt[24]{333\,202\,640\,600}}{\sqrt{2\pi}} = -\frac{1}{2^{3/8}\sqrt{\pi}} i \sqrt[12]{5}\ \sqrt[24]{1\,666\,013\,203}\ e^{-\frac{\pi^2}{6\left(\log(333\,202\,640\,599)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{333\,202\,640\,599}\right)^k}{k}\right)}} \left(-1+e^{\frac{\pi^2}{6\left(\log(333\,202\,640\,599)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{333\,202\,640\,599}\right)^k}{k}\right)}}\right) \sqrt{\log(333\,202\,640\,599)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{333\,202\,640\,599}\right)^k}{k}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{333\,202\,640\,600}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{333\,202\,640\,600}\right)}\right)-1\right)\right)\sqrt[24]{333\,202\,640\,600}}{\sqrt{2\pi}} = -\frac{1}{2\times 2^{3/8}} i \sqrt[12]{5}\ \sqrt[24]{1\,666\,013\,203}\ e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(333\,202\,640\,600)}} \left(-1+e^{\pi^2/(6\log(333\,202\,640\,600))}\right)\sqrt{\frac{333\,202\,640\,599}{\pi}} \sum_{k=0}^{\infty} 333\,202\,640\,599^k \binom{-\frac{1}{2}+k}{k} \sum_{j=0}^{k} -\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j}$$

$$\text{for}\left(c_k=\frac{(-1)^k}{(1+k)\pi}\text{ and }p_{j,0}=1\text{ and }p_{j,k}=\frac{\pi\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\text{ and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{333\,202\,640\,600}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{333\,202\,640\,600}\right)}\right)-1\right)\right)\sqrt[24]{333\,202\,640\,600}}{\sqrt{2\,\pi}} =$$

$$-\frac{1}{2^{3/8}\sqrt{\pi}}\,i\,\sqrt[12]{5}\,\sqrt[24]{1\,666\,013\,203}$$
$$\exp\left(-\frac{\pi^2}{6\left(2\,i\,\pi\left\lfloor\frac{\arg(333\,202\,640\,600-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(333\,202\,640\,600-x)^k\,x^{-k}}{k}\right)}\right)$$
$$\left(-1+\exp\left(\frac{\pi^2}{6\left(2\,i\,\pi\left\lfloor\frac{\arg(333\,202\,640\,600-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(333\,202\,640\,600-x)^k\,x^{-k}}{k}\right)}\right)\right)$$
$$\sqrt{\left(2\,i\,\pi\left\lfloor\frac{\arg(333\,202\,640\,600-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(333\,202\,640\,600-x)^k\,x^{-k}}{k}\right)} \quad \text{for } x<0$$

Integral representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{333\,202\,640\,600}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{333\,202\,640\,600}\right)}\right)-1\right)\right)\sqrt[24]{333\,202\,640\,600}}{\sqrt{2\,\pi}} =$$

$$-\frac{1}{2^{3/8}\sqrt{\pi}}\,i\,\sqrt[12]{5}\,\sqrt[24]{1\,666\,013\,203}\;e^{-\pi^2/\left(6\int_1^{333\,202\,640\,600}\frac{1}{t}\,dt\right)}$$
$$\left(-1+e^{\pi^2/\left(6\int_1^{333\,202\,640\,600}\frac{1}{t}\,dt\right)}\right)\sqrt{\int_1^{333\,202\,640\,600}\frac{1}{t}\,dt}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{333\,202\,640\,600}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{333\,202\,640\,600}\right)}\right)-1\right)\right)\sqrt[24]{333\,202\,640\,600}}{\sqrt{2\,\pi}} =$$

$$-\frac{1}{2^{7/8}\,\pi}\,i\,\sqrt[12]{5}\,\sqrt[24]{1\,666\,013\,203}\;\exp\left(-\frac{i\,\pi^3}{3\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{333\,202\,640\,599^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)$$
$$\left(-1+\exp\left(\frac{i\,\pi^3}{3\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{333\,202\,640\,599^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)$$
$$\sqrt{-i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{333\,202\,640\,599^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

Now, we have that:

x = 24   0.328043

x = 240   0.30428

x = 504   0.299596

x = 1728 0.294243   Arithmetic mean = 0.3065405

x = 744   0.297603

x = 196884   0.292193

x = 21493760   0.307509

x = 864299970   0.327704   Aritmetic mean = 0.30625225

x = 20245856256   0.349836

x = 333202640600   0.373158

Total arithmetic mean = 0.3174165

Minimal value = 0.292193 ;   Maximal value = 0.373158 ;   Mean = 0.3326755

We note that obtain also:

2*1/(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)

Input interpretation:

$2 \times 1/(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)$

Result:

0.630086967753724207783779356145632000856918276145064922585...

0.630086967… result very near to the Hausdorff dimension of Cantor set that is equal to 0.6309

And:

(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)^1/(2Pi)

Input interpretation:

$$(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158) \text{^} \left(\frac{1}{2\pi}\right)$$

Result:

1.20181...

1.20181….result very near to the Hausdorff dimension of Fibonacci word fractal 60° that is equal to 1.2083

Alternative representations:

$$(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158) \text{^} \left(\frac{1}{2\pi}\right) = \sqrt[360^\circ]{3.17417}$$

$$(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158) \text{^} \left(\frac{1}{2\pi}\right) = 3.17417^{-1/(2 i \log(-1))}$$

$$(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158) \text{^} \left(\frac{1}{2\pi}\right) = \sqrt[2\cos^{-1}(-1)]{3.17417}$$

Series representations:

$$(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)^\wedge\left(\frac{1}{2\pi}\right) = \sqrt[8\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}]{3.17417}$$

$$(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)^\wedge\left(\frac{1}{2\pi}\right) = \sqrt[4\left(-1+\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}\right)]{3.17417}$$

$$(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)^\wedge\left(\frac{1}{2\pi}\right) = \sqrt[2x+4\sum_{k=1}^{\infty}\frac{\sin(kx)}{k}]{3.17417} \quad \text{for } (x \in \mathbb{R} \text{ and } x > 0)$$

Integral representations:

$$(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)^\wedge\left(\frac{1}{2\pi}\right) = e^{0.288761/\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)}$$

$$(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)^\wedge\left(\frac{1}{2\pi}\right) = e^{0.144381/\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)}$$

$$(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)^\wedge\left(\frac{1}{2\pi}\right) = e^{0.288761/\left(\int_0^{\infty}\frac{\sin(t)}{t}\,dt\right)}$$

Now, we perform the following calculations:

(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)^1/6

Input interpretation:

$(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)^{\wedge}(1/6)$

Result:

1.212286...

1.212286…. result very near to the Hausdorff dimension of Boundary of the tame twindragon, that is equal to 1.2108

And:

2^(1/sqrt2)(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)^1/13

where √2^√2 = 2^(1/√2) is the square root of Gelfond - Schneider constant

Input interpretation:

$\sqrt[\sqrt{2}]{2}\,(0.328043 + 0.30428 + 0.299596 + 0.294243 + 0.297603 + 0.292193 + 0.307509 + 0.327704 + 0.349836 + 0.373158)^{\wedge}(1/13)$

Result:

1.784215...

1.784215..... result very near to the Hausdorff dimension of Von Koch curve 85° that is equal to 1.7848

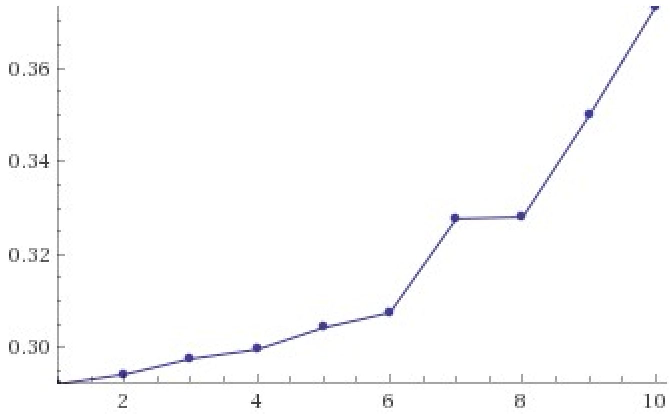
Now, we show the plots regarding the results that we have obtained.

plot(0.292193, 0.294243, 0.297603, 0.299596, 0.30428, 0.307509, 0.327704, 0.328043, 0.349836, 0.373158)

Input interpretation:

plot	{0.292193, 0.294243, 0.297603, 0.299596, 0.30428, 0.307509, 0.327704, 0.328043, 0.349836, 0.373158}

Plot:

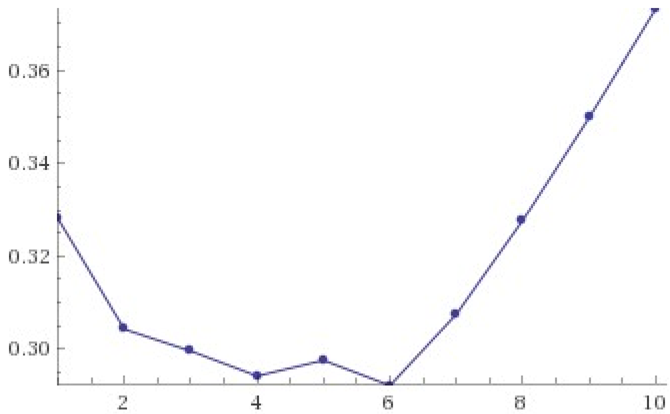


plot(0.328043, 0.30428, 0.299596, 0.294243, 0.297603, 0.292193, 0.307509, 0.327704, 0.349836, 0.373158)

Input interpretation:

plot	{0.328043, 0.30428, 0.299596, 0.294243, 0.297603, 0.292193, 0.307509, 0.327704, 0.349836, 0.373158}

Plot:



Now, we have that:

(24 + 240 + 504 + 1728 + 744 + 196884 + 21493760 + 864299970 + 20245856256 + 333202640600)/10

Input:

$$\frac{1}{10}(24 + 240 + 504 + 1728 + 744 + 196\,884 + 21\,493\,760 + 864\,299\,970 + 20\,245\,856\,256 + 333\,202\,640\,600)$$

Result:

35 433 449 071

35433449071

From which, for the principal formula, we obtain:

((35433449071^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/35433449071)) * [exp(Pi^2/(6 ln(1/35433449071)))-1]

Input:

$$\frac{\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)$$

log(x) is the natural logarithm

Exact result:

$$i\sqrt[24]{35\,433\,449\,071}\left(e^{-\pi^2/(6\log(35\,433\,449\,071))}-1\right)\sqrt{\frac{\log(35\,433\,449\,071)}{2\pi}}$$

Decimal approximation:

$-0.354220459523040301753909021714647143401665285793821023 08\ldots\, i$

Polar coordinates:

$r \approx 0.35422$ (radius), $\theta = -90°$ (angle)

0.35422

Alternate forms:

$$-i\sqrt[24]{35\,433\,449\,071}\;e^{-\pi^2/(6\log(35\,433\,449\,071))}\left(e^{\pi^2/(6\log(35\,433\,449\,071))}-1\right)\sqrt{\frac{\log(35\,433\,449\,071)}{2\pi}}$$

$$i\sqrt[24]{35\,433\,449\,071}\left(e^{-\pi^2/(6(\log(269)+\log(4157)+\log(31\,687)))}-1\right)\sqrt{\frac{\frac{\log(269)}{2}+\frac{\log(4157)}{2}+\frac{\log(31\,687)}{2}}{\pi}}$$

$$\frac{i\sqrt[24]{35\,433\,449\,071}\;e^{-\pi^2/(6\log(35\,433\,449\,071))}\sqrt{\frac{\log(35\,433\,449\,071)}{\pi}}}{\sqrt{2}}-i\sqrt[24]{35\,433\,449\,071}\sqrt{\frac{\log(35\,433\,449\,071)}{2\pi}}$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\pi}}=\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{35\,433\,449\,071}\right)}\right)\right)\sqrt[24]{35\,433\,449\,071}\sqrt{\log_e\left(\frac{1}{35\,433\,449\,071}\right)}}{\sqrt{2\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\pi}}=\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{35\,433\,449\,071}\right)}\right)\right)\sqrt[24]{35\,433\,449\,071}\sqrt{\log(a)\log_a\left(\frac{1}{35\,433\,449\,071}\right)}}{\sqrt{2\pi}}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{35433449071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35433449071}\right)}\right)-1\right)\right)\sqrt[24]{35433449071}}{\sqrt{2\pi}} = -\frac{1}{\sqrt{2\pi}} i \sqrt[24]{35433449071}\, e^{-\frac{\pi^2}{6\left(\log(35433449070)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{35433449070}\right)^k}{k}\right)}} \left(-1+e^{\frac{\pi^2}{6\left(\log(35433449070)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{35433449070}\right)^k}{k}\right)}}\right) \sqrt{\log(35433449070)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{35433449070}\right)^k}{k}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{35433449071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35433449071}\right)}\right)-1\right)\right)\sqrt[24]{35433449071}}{\sqrt{2\pi}} = -\frac{1}{2\sqrt{\pi}} i \sqrt{17716724535}\, \sqrt[24]{35433449071}\, e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(35433449071)}} \left(-1+e^{\pi^2/(6\log(35433449071))}\right)\sum_{k=0}^{\infty} 35433449070^k \binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j}$$

$$\text{for }\left(k\in\mathbb{Z}\text{ and }\frac{(-1)^k}{1+k}=2\pi c_k\text{ and }p_{j,0}=1\text{ and }p_{j,k}=\frac{2\pi\sum_{m=1}^{k}(-k+m+jm)\,c_m\,p_{j,k-m}}{k}\text{ and }k>0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\pi}} =$$

$$-\frac{1}{2\sqrt{\pi}}\, i\sqrt{17\,716\,724\,535}\,\sqrt[24]{35\,433\,449\,071}\, e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(35\,433\,449\,071)}}$$
$$\left(-1+e^{\pi^2/(6\log(35\,433\,449\,071))}\right)\sum_{k=0}^{\infty}35\,433\,449\,070^k\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j}$$
$$\text{for }\left(c_k=\frac{(-1)^k}{2(1+k)\pi}\text{ and }p_{j,0}=1\text{ and } p_{j,k}=\frac{2\pi\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\text{ and }k\in\mathbb{Z}\text{ and }k>0\right)$$

Integral representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\pi}} =$$
$$-\frac{1}{\sqrt{2\pi}}\, i\sqrt[24]{35\,433\,449\,071}\, e^{-\pi^2/\left(6\int_1^{35\,433\,449\,071}\frac{1}{t}\,dt\right)}$$
$$\left(-1+e^{\pi^2/\left(6\int_1^{35\,433\,449\,071}\frac{1}{t}\,dt\right)}\right)\sqrt{\int_1^{35\,433\,449\,071}\frac{1}{t}\,dt}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\pi}} =$$
$$-\frac{1}{2\pi}\, i\sqrt[24]{35\,433\,449\,071}\,\exp\left(-\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{35\,433\,449\,070^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)$$
$$\left(-1+\exp\left(\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{35\,433\,449\,070^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)$$
$$\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{35\,433\,449\,070^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\quad\text{for } -1<\gamma<0$$

From which:

5*((35433449071^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/35433449071)) * [exp(Pi^2/(6 ln(1/35433449071)))-1]

Input:

$$5 \times \frac{\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\,\pi}} \sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)} \left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right) - 1\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$5\,i\,\sqrt[24]{35\,433\,449\,071}\,\left(e^{-\pi^2/(6\log(35\,433\,449\,071))} - 1\right)\sqrt{\frac{\log(35\,433\,449\,071)}{2\,\pi}}$$

Decimal approximation:

$-1.77110229761520150876954510857323571700832642896910511542\ldots\,i$

Polar coordinates:

$r \approx 1.7711$ (radius), $\theta = -90°$ (angle)

1.7711

Alternate forms:

$$-5\,i\,\sqrt[24]{35\,433\,449\,071}\;e^{-\pi^2/(6\log(35\,433\,449\,071))}\left(e^{\pi^2/(6\log(35\,433\,449\,071))} - 1\right)\sqrt{\frac{\log(35\,433\,449\,071)}{2\,\pi}}$$

$$5\,i\,\sqrt[24]{35\,433\,449\,071}\,\left(e^{-\pi^2/(6(\log(269)+\log(4157)+\log(31\,687)))} - 1\right)\sqrt{\frac{\frac{\log(269)}{2}+\frac{\log(4157)}{2}+\frac{\log(31\,687)}{2}}{\pi}}$$

$$\frac{5\,i\,\sqrt[24]{35\,433\,449\,071}\;e^{-\pi^2/(6\log(35\,433\,449\,071))}\sqrt{\frac{\log(35\,433\,449\,071)}{\pi}}}{\sqrt{2}} - 5\,i\,\sqrt[24]{35\,433\,449\,071}\,\sqrt{\frac{\log(35\,433\,449\,071)}{2\,\pi}}$$

Alternative representations:

$$\frac{5\sqrt[24]{35433449071}\sqrt{\log\left(\frac{1}{35433449071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35433449071}\right)}\right)-1\right)}{\sqrt{2\pi}} = \frac{5\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{35433449071}\right)}\right)\right)\sqrt[24]{35433449071}\sqrt{\log_e\left(\frac{1}{35433449071}\right)}}{\sqrt{2\pi}}$$

$$\frac{5\sqrt[24]{35433449071}\sqrt{\log\left(\frac{1}{35433449071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35433449071}\right)}\right)-1\right)}{\sqrt{2\pi}} = \frac{5\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{35433449071}\right)}\right)\right)\sqrt[24]{35433449071}\sqrt{\log(a)\log_a\left(\frac{1}{35433449071}\right)}}{\sqrt{2\pi}}$$

Ln(7) / ln(3)

Input:

$$\frac{\log(7)}{\log(3)}$$

$\log(x)$ is the natural logarithm

Decimal approximation:

1.7712437491614222600679283070824577180664713345942434793 68...

1.7712437491…

The above result is equal to **hexaflake**, that is a fractal constructed by iteratively exchanging hexagons by a flake of seven hexagons. The Hausdorff dimension of the hexaflake is equal to ln(7)/ln(3), approximately 1.7712. It may also be constructed by projecting the Cantor cube onto the plane orthogonal to its main diagonal.

Alternative representations:

$$\frac{\log(7)}{\log(3)} = \frac{\log_e(7)}{\log_e(3)}$$

$$\frac{\log(7)}{\log(3)} = \frac{\log(a)\log_a(7)}{\log(a)\log_a(3)}$$

$$\frac{\log(7)}{\log(3)} = \frac{-\text{Li}_1(-6)}{-\text{Li}_1(-2)}$$

Series representations:

$$\frac{\log(7)}{\log(3)} = \frac{\log(6) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{6}\right)^k}{k}}{\log(2) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k}{k}}$$

$$\frac{\log(7)}{\log(3)} = \frac{2\pi \left\lfloor \frac{\arg(7-x)}{2\pi} \right\rfloor - i\log(x) + i\sum_{k=1}^{\infty} \frac{(-1)^k (7-x)^k x^{-k}}{k}}{2\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor - i\log(x) + i\sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{\log(7)}{\log(3)} = \frac{2\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor - i\log(z_0) + i\sum_{k=1}^{\infty} \frac{(-1)^k (7-z_0)^k z_0^{-k}}{k}}{2\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor - i\log(z_0) + i\sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{\log(7)}{\log(3)} = \frac{\int_1^7 \frac{1}{t}\,dt}{\int_1^3 \frac{1}{t}\,dt}$$

$$\frac{\log(7)}{\log(3)} = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{6^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1 < \gamma < 0$$

We have also:

-2.3219*2*((35433449071^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/35433449071)) * [exp(Pi^2/(6 ln(1/35433449071)))-1]

where 2.3219 is the Hausdorff dimension of fractal Pyramid, that is equal to ln (5) / ln (2)

Input interpretation:

$$-2.3219\times 2\times \frac{\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\,\pi}}\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)$$

log(x) is the natural logarithm

Result:

$1.64493\ldots\, i$

Polar coordinates:

$r = 1.64493$ (radius), $\theta = 90°$ (angle)

1.64493

Alternative representations:

$$\frac{\left(2\,(-1)\,2.3219\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\,\pi}} = -\frac{4.6438\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{35\,433\,449\,071}\right)}\right)\right)\sqrt[24]{35\,433\,449\,071}\sqrt{\log_e\left(\frac{1}{35\,433\,449\,071}\right)}}{\sqrt{2\,\pi}}$$

$$\frac{\left(2\,(-1)\,2.3219\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\,\pi}} = -\frac{1}{\sqrt{2\,\pi}}\,4.6438\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{35\,433\,449\,071}\right)}\right)\right)\sqrt[24]{35\,433\,449\,071}\sqrt{\log(a)\log_a\left(\frac{1}{35\,433\,449\,071}\right)}$$

Series representations:

$$\frac{\left(2(-1)\,2.3219\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\pi}} =$$

$$-\left(\left(12.7771\left(-\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+\log\left(\frac{1}{35\,433\,449\,071}\right)\right)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\log\left(\frac{1}{35\,433\,449\,071}\right)\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+\log\left(\frac{1}{35\,433\,449\,071}\right)\right)}{2\pi}\right\rfloor\right)\exp\left(-\frac{\pi^2}{6\sum_{k=1}^{\infty}\frac{(-1)^k\left(-\frac{35\,433\,449\,070}{35\,433\,449\,071}\right)^k}{k}}\right)\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\log\left(\frac{1}{35\,433\,449\,071}\right)\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\right) \Big/ \left(\exp\left(i\pi\left\lfloor\frac{\arg(2\pi-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(2\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\left(2\,(-1)\,2.3219\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\pi}} =$$

$$-\left(\left(12.7771\left(\frac{1}{z_0}\right)^{-1/2\left\lfloor\arg(2\pi-z_0)/(2\pi)\right\rfloor+1/2\left\lfloor\arg\left(\log\left(\frac{1}{35\,433\,449\,071}\right)-z_0\right)/(2\pi)\right\rfloor}\right.\right.$$

$$z_0^{-1/2\left\lfloor\arg(2\pi-z_0)/(2\pi)\right\rfloor}\left(-z_0^{1/2\left\lfloor\arg\left(\log\left(\frac{1}{35\,433\,449\,071}\right)-z_0\right)/(2\pi)\right\rfloor}\right.$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\log\left(\frac{1}{35\,433\,449\,071}\right)-z_0\right)^k z_0^{-k}}{k!}+$$

$$\exp\left(-\frac{\pi^2}{6\sum_{k=1}^{\infty}\frac{(-1)^k\left(-\frac{35\,433\,449\,070}{35\,433\,449\,071}\right)^k}{k}}\right)z_0^{1/2\left\lfloor\arg\left(\log\left(\frac{1}{35\,433\,449\,071}\right)-z_0\right)/(2\pi)\right\rfloor}$$

$$\left.\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\log\left(\frac{1}{35\,433\,449\,071}\right)-z_0\right)^k z_0^{-k}}{k!}\right)\right)\right)\Big/$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\pi-z_0)^k z_0^{-k}}{k!}\right)$$

Integral representation:

$$\frac{\left(2\,(-1)\,2.3219\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\pi}} =$$

$$-\frac{12.7771\left(-\sqrt{\int_1^{\frac{1}{35\,433\,449\,071}}\frac{1}{t}\,dt}+\exp\left(\frac{\pi^2}{6\int_1^{\frac{1}{35\,433\,449\,071}}\frac{1}{t}\,dt}\right)\sqrt{\int_1^{\frac{1}{35\,433\,449\,071}}\frac{1}{t}\,dt}\right)}{\sqrt{2\pi}}$$

Or:

-ln(5)/ln(2)*2*((35433449071^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/35433449071)) * [exp(Pi^2/(6 ln(1/35433449071)))-1]

Input:

$$-\frac{\log(5)}{\log(2)}\times 2\times\frac{\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)$$

log(x) is the natural logarithm

Exact result:

$$-\frac{i\sqrt[24]{35\,433\,449\,071}\left(e^{-\pi^2/(6\log(35\,433\,449\,071))}-1\right)\log(5)\sqrt{\frac{2\log(35\,433\,449\,071)}{\pi}}}{\log(2)}$$

Decimal approximation:

1.644948873500918031125414645649588480483416976932232116261... i

Polar coordinates:

$r \approx 1.64495$ (radius), $\theta = 90°$ (angle)

1.64495

Alternate forms:

$$\frac{1}{\log(2)} i\sqrt[24]{35\,433\,449\,071}\; e^{-\pi^2/(6\log(35\,433\,449\,071))}\left(e^{\pi^2/(6\log(35\,433\,449\,071))}-1\right)\log(5)\sqrt{\frac{2\log(35\,433\,449\,071)}{\pi}}$$

$$-\frac{1}{\log(2)} i\sqrt[24]{35\,433\,449\,071}\left(e^{-\pi^2/(6(\log(269)+\log(4157)+\log(31\,687)))}-1\right)\log(5)\sqrt{\frac{2\log(269)+2\log(4157)+2\log(31\,687)}{\pi}}$$

$$\frac{i\sqrt[24]{35\,433\,449\,071}\log(5)\sqrt{\frac{2\log(35\,433\,449\,071)}{\pi}}}{\log(2)}-\frac{i\sqrt[24]{35\,433\,449\,071}\; e^{-\pi^2/(6\log(35\,433\,449\,071))}\log(5)\sqrt{\frac{2\log(35\,433\,449\,071)}{\pi}}}{\log(2)}$$

Alternative representations:

$$\frac{\left(2\sqrt[24]{35\,433\,449\,071}\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)(-1)\log(5)}{\log(2)\sqrt{2\pi}} =$$

$$-\left(\left(2\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{35\,433\,449\,071}\right)}\right)\right)\right)\log(a)\log_a(5)\sqrt[24]{35\,433\,449\,071}\sqrt{\log(a)\log_a\left(\frac{1}{35\,433\,449\,071}\right)}\right)\Big/\left((\log(a)\log_a(2))\sqrt{2\pi}\right)$$

$$\frac{\left(2\sqrt[24]{35\,433\,449\,071}\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)(-1)\log(5)}{\log(2)\sqrt{2\pi}} =$$

$$-\frac{2\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{35\,433\,449\,071}\right)}\right)\right)\log_e(5)\sqrt[24]{35\,433\,449\,071}\sqrt{\log_e\left(\frac{1}{35\,433\,449\,071}\right)}}{\log_e(2)\sqrt{2\pi}}$$

Series representations:

$$\frac{\left(2\sqrt[24]{35\,433\,449\,071}\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)(-1)\log(5)}{\log(2)\sqrt{2\pi}} =$$

$$\frac{1}{\sqrt{\pi}\log(2)}\,i\sqrt{17\,716\,724\,535}\sqrt[24]{35\,433\,449\,071}\; e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(35\,433\,449\,071)}}\left(-1+e^{\pi^2/(6\log(35\,433\,449\,071))}\right)\log(5)\sum_{k=0}^{\infty}35\,433\,449\,070^k\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j}$$

$$\text{for}\left(\pi c_k=\frac{2(-1)^k}{1+k}\text{ and }p_{j,0}=1\text{ and } 2p_{j,k}=\frac{\pi\sum_{m=1}^{k}(-k+m+jm)c_m\,p_{j,k-m}}{k}\text{ and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$\frac{\left(2\sqrt[24]{35433449071}\left(\sqrt{\log\left(\frac{1}{35433449071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35433449071}\right)}\right)-1\right)\right)\right)(-1)\log(5)}{\log(2)\sqrt{2\pi}} =$$

$$\Bigg(\sqrt{2}\sqrt[24]{35433449071}\exp\left(-\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(35433449071-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(35433449071-x)^k x^{-k}}{k}\right)}\right)$$

$$\left(-1+\exp\left(\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(35433449071-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(35433449071-x)^k x^{-k}}{k}\right)}\right)\right)$$

$$\left(2\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor-i\log(x)+i\sum_{k=1}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}}{k}\right)$$

$$\sqrt{\left(2i\pi\left\lfloor\frac{\arg(35433449071-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(35433449071-x)^k x^{-k}}{k}\right)}\Bigg)\Big/$$

$$\left(\sqrt{\pi}\left(-2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor-\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)\right) \quad \text{for } x<0$$

$$\frac{\left(2\sqrt[24]{35433449071}\left(\sqrt{\log\left(\frac{1}{35433449071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35433449071}\right)}\right)-1\right)\right)\right)(-1)\log(5)}{\log(2)\sqrt{2\pi}} =$$

$$\left(\sqrt{2}\sqrt[24]{35433449071}\right.$$

$$\exp\left(-\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(35433449071-z_0)^k z_0^{-k}}{k}\right)}\right)$$

$$\left(-1+\exp\left(\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(35433449071-z_0)^k z_0^{-k}}{k}\right)}\right)\right)$$

$$\left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-i\log(z_0)+i\sum_{k=1}^{\infty}\frac{(-1)^k(5-z_0)^k z_0^{-k}}{k}\right)$$

$$\left.\sqrt{2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(35433449071-z_0)^k z_0^{-k}}{k}}\right)$$

$$\Bigg/\left(\sqrt{\pi}\left(-2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)\right)$$

$$\frac{\left(2\sqrt[24]{35\,433\,449\,071}\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)(-1)\log(5)}{\log(2)\sqrt{2\pi}} =$$

$$\left(i\sqrt{2}\sqrt[24]{35\,433\,449\,071}\exp\left(-\left(\pi^2\Big/\left(6\left(\log(z_0)+\left\lfloor\frac{\arg(35\,433\,449\,071-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(35\,433\,449\,071-z_0)^k z_0^{-k}}{k}\right)\right)\right)\right)\right.$$
$$\left(-1+\exp\left(\pi^2\Big/\left(6\left(\log(z_0)+\left\lfloor\frac{\arg(35\,433\,449\,071-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(35\,433\,449\,071-z_0)^k z_0^{-k}}{k}\right)\right)\right)\right)$$
$$\left(\left\lfloor\frac{\arg(5-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(5-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(5-z_0)^k z_0^{-k}}{k}\right)$$
$$\left.\sqrt{\left(\left\lfloor\frac{\arg(35\,433\,449\,071-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(35\,433\,449\,071-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(35\,433\,449\,071-z_0)^k z_0^{-k}}{k}\right)}\right)\Big/$$
$$\left(\sqrt{\pi}\left(\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)\right)$$

Integral representations:

$$\frac{\left(2\sqrt[24]{35\,433\,449\,071}\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)(-1)\log(5)}{\log(2)\sqrt{2\pi}} =$$

$$\frac{1}{\sqrt{\pi}\int_1^2\frac{1}{t}\,dt}\,i\sqrt{2}\sqrt[24]{35\,433\,449\,071}\;e^{-\pi^2/\left(6\int_1^{35\,433\,449\,071}\frac{1}{t}\,dt\right)}\left(-1+e^{\pi^2/\left(6\int_1^{35\,433\,449\,071}\frac{1}{t}\,dt\right)}\right)\left(\int_1^5\frac{1}{t}\,dt\right)\sqrt{\int_1^{35\,433\,449\,071}\frac{1}{t}\,dt}$$

$$\frac{\left(2\sqrt[24]{35\,433\,449\,071}\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)(-1)\log(5)}{\log(2)\sqrt{2\pi}} =$$

$$\left(i\sqrt[24]{35\,433\,449\,071}\exp\left(-\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{35\,433\,449\,070^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right.$$
$$\left(-1+\exp\left(\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{35\,433\,449\,070^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)$$
$$\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{4^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)$$
$$\left.\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{35\,433\,449\,070^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\Big/$$
$$\left(\pi\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\quad\text{for } -1<\gamma<0$$

And:

-ln(5)/ln(2)*2*((35433449071^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/35433449071)) * [exp(Pi^2/(6 ln(1/35433449071)))-1] - (29-2)i1/10^3

Input:

$$-\frac{\log(5)}{\log(2)}\times 2\times\frac{\sqrt[24]{35\,433\,449\,071}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)-(29-2)\,i\times\frac{1}{10^3}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Exact result:

$$-\frac{27\,i}{1000}-\frac{i\sqrt[24]{35\,433\,449\,071}\left(e^{-\pi^2/(6\log(35\,433\,449\,071))}-1\right)\log(5)\sqrt{\frac{2\log(35\,433\,449\,071)}{\pi}}}{\log(2)}$$

Decimal approximation:

1.617948873500918031125414645649588480483416976932232116261... i

1.6179488735009….

Polar coordinates:

$r \approx 1.61795$ (radius), $\theta = 90°$ (angle)

1.61795

Alternate forms:

$$\frac{1}{1000\log(2)} i\, e^{-\pi^2/(6\log(35433449071))} \left(-27\, e^{\pi^2/(6\log(35433449071))}\log(2) - 1000\sqrt[24]{35433449071}\log(5)\sqrt{\frac{2\log(35433449071)}{\pi}} + 1000\sqrt[24]{35433449071}\, e^{\pi^2/(6\log(35433449071))}\log(5)\sqrt{\frac{2\log(35433449071)}{\pi}}\right)$$

$$-\frac{1}{1000\log(2)} i \left(27\log(2) - 1000\sqrt[24]{35433449071}\log(5)\sqrt{\frac{2\log(269)+2\log(4157)+2\log(31687)}{\pi}} + 1000\sqrt[24]{35433449071}\, e^{-\pi^2/(6(\log(269)+\log(4157)+\log(31687)))}\log(5)\sqrt{\frac{2\log(269)+2\log(4157)+2\log(31687)}{\pi}}\right)$$

$$-\frac{27i}{1000} - \frac{1}{\log(2)} i\sqrt[24]{35433449071}\left(e^{-\pi^2/(6(\log(269)+\log(4157)+\log(31687)))} - 1\right)\log(5)\sqrt{\frac{2(\log(269)+\log(4157)+\log(31687))}{\pi}}$$

Expanded form:

$$-\frac{27i}{1000} + \frac{i\sqrt[24]{35433449071}\log(5)\sqrt{\frac{2\log(35433449071)}{\pi}}}{\log(2)} - \frac{i\sqrt[24]{35433449071}\, e^{-\pi^2/(6\log(35433449071))}\log(5)\sqrt{\frac{2\log(35433449071)}{\pi}}}{\log(2)}$$

Alternative representations:

$$\frac{\left(2\sqrt[24]{35\,433\,449\,071}\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)(-1)\log(5)}{\log(2)\sqrt{2\pi}}-\frac{i(29-2)}{10^3}=$$

$$-\frac{27\,i}{10^3}-\left(2\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{35\,433\,449\,071}\right)}\right)\right)\log(a)\log_a(5)\sqrt[24]{35\,433\,449\,071}\sqrt{\log(a)\log_a\left(\frac{1}{35\,433\,449\,071}\right)}\right)\Big/\left((\log(a)\log_a(2))\sqrt{2\pi}\right)$$

$$\frac{\left(2\sqrt[24]{35\,433\,449\,071}\left(\sqrt{\log\left(\frac{1}{35\,433\,449\,071}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{35\,433\,449\,071}\right)}\right)-1\right)\right)\right)(-1)\log(5)}{\log(2)\sqrt{2\pi}}-\frac{i(29-2)}{10^3}=-\frac{27\,i}{10^3}-$$

$$\frac{2\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{35\,433\,449\,071}\right)}\right)\right)\log_e(5)\sqrt[24]{35\,433\,449\,071}\sqrt{\log_e\left(\frac{1}{35\,433\,449\,071}\right)}}{\log_e(2)\sqrt{2\pi}}$$

Now, we have for:

x = 24

x = 240

x = 504

x = 1728

x = 744

x = 196884

x = 21493760

x = 864299970

x = 20245856256

x = 333202640600

From:

$$g(x) \sim \frac{(1-x)^{3/2}}{\sqrt{2\pi}} \exp\left\{\frac{\pi^2}{6(1-x)}\right\},$$

For x = 24, we obtain

(((1-24)^1.5))/(2Pi) exp(((Pi^2)/(6(1-24))))

Input:

$$\frac{(1-24)^{1.5}}{\sqrt{2\pi}} \exp\left(\frac{\pi^2}{6(1-24)}\right)$$

Result:

$-40.9677\ldots i$

Polar coordinates:

$r = 40.9677$ (radius), $\theta = -90.°$ (angle)

40.9677

Series representations:

$$\frac{\exp\left(\frac{\pi^2}{6(1-24)}\right)(1-24)^{1.5}}{\sqrt{2\pi}} = -\frac{(2.02619\times 10^{-14} + 110.304\, i)\exp\left(-\frac{\pi^2}{138}\right)}{\sqrt{-1+2\pi}\sum_{k=0}^{\infty}(-1+2\pi)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(\frac{\pi^2}{6(1-24)}\right)(1-24)^{1.5}}{\sqrt{2\pi}} = -\frac{(2.02619\times 10^{-14} + 110.304\, i)\exp\left(-\frac{\pi^2}{138}\right)}{\sqrt{-1+2\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+2\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(\frac{\pi^2}{6(1-24)}\right)(1-24)^{1.5}}{\sqrt{2\pi}} = -\frac{(2.02619\times 10^{-14} + 110.304\, i)\exp\left(-\frac{\pi^2}{138}\right)}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2\pi - z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

(((1-240)^1.5))/sqrt(2Pi) exp(((Pi^2)/(6(1-240))))

Input:

$$\frac{(1-240)^{1.5}}{\sqrt{2\pi}}\exp\left(\frac{\pi^2}{6(1-240)}\right)$$

Result:

$-1463.92\ldots\, i$

Polar coordinates:

$r = 1463.92$ (radius), $\theta = -90.°$ (angle)

1463.92

Series representations:

$$\frac{\exp\left(\frac{\pi^2}{6(1-240)}\right)(1-240)^{1.5}}{\sqrt{2\pi}} = -\frac{(6.78711\times 10^{-13} + 3694.85\, i)\exp\left(-\frac{\pi^2}{1434}\right)}{\sqrt{-1+2\pi}\sum_{k=0}^{\infty}(-1+2\pi)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(\frac{\pi^2}{6(1-240)}\right)(1-240)^{1.5}}{\sqrt{2\pi}} = -\frac{(6.78711\times 10^{-13} + 3694.85\, i)\exp\left(-\frac{\pi^2}{1434}\right)}{\sqrt{-1+2\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+2\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(\frac{\pi^2}{6(1-240)}\right)(1-240)^{1.5}}{\sqrt{2\pi}} = -\frac{(6.78711\times 10^{-13} + 3694.85\, i)\exp\left(-\frac{\pi^2}{1434}\right)}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2\pi - z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$\binom{n}{m}$ is the binomial coefficient

(((1-504)^1.5))/sqrt(2Pi) exp(((Pi^2)/(6(1-504))))

Input:

$$\frac{(1-504)^{1.5}}{\sqrt{2\pi}} \exp\left(\frac{\pi^2}{6(1-504)}\right)$$

Result:

$-4485.82\ldots i$

Polar coordinates:

$r = 4485.82$ (radius), $\theta = -90.°$ (angle)

4485.42

Series representations:

$$\frac{\exp\left(\frac{\pi^2}{6(1-504)}\right)(1-504)^{1.5}}{\sqrt{2\pi}} = -\frac{(2.07224\times 10^{-12} + 11\,281.1\,i)\exp\left(-\frac{\pi^2}{3018}\right)}{\sqrt{-1+2\pi}\,\sum_{k=0}^{\infty}(-1+2\pi)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(\frac{\pi^2}{6(1-504)}\right)(1-504)^{1.5}}{\sqrt{2\pi}} = -\frac{(2.07224\times 10^{-12} + 11\,281.1\,i)\exp\left(-\frac{\pi^2}{3018}\right)}{\sqrt{-1+2\pi}\,\sum_{k=0}^{\infty}\frac{(-1)^k(-1+2\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(\frac{\pi^2}{6(1-504)}\right)(1-504)^{1.5}}{\sqrt{2\pi}} = -\frac{(2.07224\times 10^{-12} + 11\,281.1\,i)\exp\left(-\frac{\pi^2}{3018}\right)}{\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\pi-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

(((1-1728)^1.5))/sqrt(2Pi) exp(((Pi^2)/(6(1-1728))))

Input:

$$\frac{(1-1728)^{1.5}}{\sqrt{2\pi}}\exp\left(\frac{\pi^2}{6(1-1728)}\right)$$

Result:

$-28604.5\ldots i$

Polar coordinates:

$r = 28\,604.5$ (radius), $\theta = -90.°$ (angle)

28604.5

Series representations:

$$\frac{\exp\left(\frac{\pi^2}{6(1-1728)}\right)(1-1728)^{1.5}}{\sqrt{2\pi}} = -\frac{\left(1.31834\times 10^{-11}+71\,769.3\,i\right)\exp\left(-\frac{\pi^2}{10\,362}\right)}{\sqrt{-1+2\pi}\sum_{k=0}^{\infty}(-1+2\pi)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(\frac{\pi^2}{6(1-1728)}\right)(1-1728)^{1.5}}{\sqrt{2\pi}} = -\frac{\left(1.31834\times 10^{-11}+71\,769.3\,i\right)\exp\left(-\frac{\pi^2}{10\,362}\right)}{\sqrt{-1+2\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+2\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(\frac{\pi^2}{6(1-1728)}\right)(1-1728)^{1.5}}{\sqrt{2\pi}} = -\frac{\left(1.31834\times 10^{-11}+71\,769.3\,i\right)\exp\left(-\frac{\pi^2}{10\,362}\right)}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\pi-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

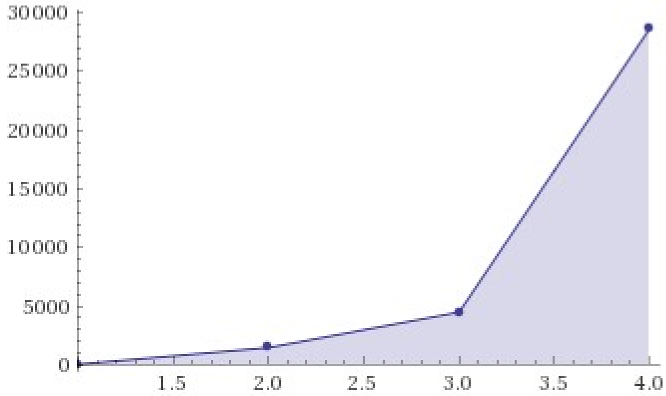
With regard the plot, we have:

plot (40.9677 , 1463.92, 4485.82, 28604.5)

Input interpretation:

plot	{40.9677, 1463.92, 4485.82, 28 604.5}

Plot:



Download Page

From the results, we obtain:

(40.9677+1463.92 +4485.82+ 28604.5)^1/21

Input interpretation:

$$\sqrt[21]{40.9677 + 1463.92 + 4485.82 + 28\,604.5}$$

Result:

1.644916...

1.644916…

and:

(40.9677+1463.92 +4485.82+ 28604.5)^1/21-(29-2)1/10^3

Input interpretation:

$$\sqrt[21]{40.9677 + 1463.92 + 4485.82 + 28\,604.5} - (29 - 2) \times \frac{1}{10^3}$$

Result:

1.6179155883726674487724367795083294820974702766826561803850...

1.617915588372….

We have also:

(24 + 240 + 504 + 1728)/4

Input:

$$\frac{1}{4}(24+240+504+1728)$$

Result:

624

624

From the above expression, we obtain:

(((1-624)^1.5))/sqrt(2Pi) exp(((Pi^2)/(6(1-624))))

Input:

$$\frac{(1-624)^{1.5}}{\sqrt{2\pi}}\exp\left(\frac{\pi^2}{6(1-624)}\right)$$

Result:

$-6187.22\ldots i$

Polar coordinates:

$r = 6187.22$ (radius), $\theta = -90.°$ (angle)

6187.22

Series representations:

$$\frac{\exp\left(\frac{\pi^2}{6(1-624)}\right)(1-624)^{1.5}}{\sqrt{2\pi}} = -\frac{(2.85641\times10^{-12}+15\,550.1\,i)\exp\left(-\frac{\pi^2}{3738}\right)}{\sqrt{-1+2\pi}\sum_{k=0}^{\infty}(-1+2\pi)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(\frac{\pi^2}{6(1-624)}\right)(1-624)^{1.5}}{\sqrt{2\pi}} = -\frac{(2.85641\times10^{-12}+15\,550.1\,i)\exp\left(-\frac{\pi^2}{3738}\right)}{\sqrt{-1+2\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+2\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(\frac{\pi^2}{6(1-624)}\right)(1-624)^{1.5}}{\sqrt{2\pi}} = -\frac{(2.85641\times10^{-12}+15\,550.1\,i)\exp\left(-\frac{\pi^2}{3738}\right)}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\pi-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

From which:

(((1-624)^1.5))/sqrt(2Pi) exp(((Pi^2)/(6(1-624))))-89i

Input:

$$\frac{(1-624)^{1.5}}{\sqrt{2\pi}}\exp\left(\frac{\pi^2}{6(1-624)}\right)-89\,i$$

i is the imaginary unit

Result:

$-6276.22\ldots\,i$

Polar coordinates:

$r = 6276.22$ (radius), $\theta = -90.°$ (angle)

6276.22 result practically equal to the rest mass of Charmed B meson 6276

Series representations:

$$\frac{\exp\left(\frac{\pi^2}{6(1-624)}\right)(1-624)^{1.5}}{\sqrt{2\pi}} - i\,89 = -89\,i - \frac{\left(2.85641\times 10^{-12} + 15\,550.1\,i\right)\exp\left(-\frac{\pi^2}{3738}\right)}{\sqrt{-1+2\pi}\,\sum_{k=0}^{\infty}(-1+2\pi)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(\frac{\pi^2}{6(1-624)}\right)(1-624)^{1.5}}{\sqrt{2\pi}} - i\,89 = -89\,i - \frac{\left(2.85641\times 10^{-12} + 15\,550.1\,i\right)\exp\left(-\frac{\pi^2}{3738}\right)}{\sqrt{-1+2\pi}\,\sum_{k=0}^{\infty}\frac{(-1)^k(-1+2\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(\frac{\pi^2}{6(1-624)}\right)(1-624)^{1.5}}{\sqrt{2\pi}} - i\,89 = -89\,i - \frac{\left(2.85641\times 10^{-12} + 15\,550.1\,i\right)\exp\left(-\frac{\pi^2}{3738}\right)}{\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\pi-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

[(((1-624)^1.5))/sqrt(2Pi) exp(((Pi^2)/(6(1-624))))]^1/18 - 7/10^3

Input:

$$\sqrt[18]{\frac{(1-624)^{1.5}}{\sqrt{2\pi}}\exp\left(\frac{\pi^2}{6(1-624)}\right)} - \frac{7}{10^3}$$

Result:

1.61102... −
0.141558... i

Polar coordinates:

$r = 1.61722$ (radius), $\theta = -5.02161°$ (angle)

1.6172

Series representations:

$$\sqrt[18]{\frac{\exp\left(\frac{\pi^2}{6(1-624)}\right)(1-624)^{1.5}}{\sqrt{2\pi}}}-\frac{7}{10^3}=$$
$$-\frac{7}{1000}+\sqrt[18]{-\frac{(2.85641\times10^{-12}+15\,550.1\,i)\exp\left(-\frac{\pi^2}{3738}\right)}{\sqrt{-1+2\pi}\sum_{k=0}^{\infty}(-1+2\pi)^{-k}\binom{\frac{1}{2}}{k}}}$$

$$\sqrt[18]{\frac{\exp\left(\frac{\pi^2}{6(1-624)}\right)(1-624)^{1.5}}{\sqrt{2\pi}}}-\frac{7}{10^3}=$$
$$-\frac{7}{1000}+\sqrt[18]{-\frac{(2.85641\times10^{-12}+15\,550.1\,i)\exp\left(-\frac{\pi^2}{3738}\right)}{\sqrt{-1+2\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+2\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}}$$

$$\sqrt[18]{\frac{\exp\left(\frac{\pi^2}{6(1-624)}\right)(1-624)^{1.5}}{\sqrt{2\pi}}}-\frac{7}{10^3}=$$
$$-\frac{7}{1000}+\sqrt[18]{-\frac{(5.71281\times10^{-12}+31\,100.1\,i)\exp\left(-\frac{\pi^2}{3738}\right)\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}(-1+2\pi)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the previous analyzed expression:

$$\chi(x)=\frac{x^{1/24}}{\sqrt{(2\pi)}}\sqrt{\left(\log\frac{1}{x}\right)}\left[\exp\left\{\frac{\pi^2}{6\log(1/x)}\right\}-1\right]$$

for x = 480, 264 and 66211 , we obtain:

((480^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/480)) * [exp(Pi^2/(6 ln(1/480)))-1]

Input:

$$\frac{\sqrt[24]{480}}{\sqrt{2\pi}} \sqrt{\log\left(\frac{1}{480}\right)} \left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{480}\right)}\right) - 1\right)$$

log(x) is the natural logarithm

Exact result:

$$\frac{i \sqrt[24]{15} \left(e^{-\pi^2/(6\log(480))} - 1\right) \sqrt{\frac{\log(480)}{\pi}}}{2^{7/24}}$$

Decimal approximation:

$-0.29986696045013438564250319054733878543145924929337437153\ldots i$

Polar coordinates:

$r \approx 0.299867$ (radius), $\theta = -90°$ (angle)

0.299867

Alternate forms:

$$-\frac{i \sqrt[24]{15}\, e^{-\pi^2/(6\log(480))} \left(e^{\pi^2/(6\log(480))} - 1\right) \sqrt{\frac{\log(480)}{\pi}}}{2^{7/24}}$$

$$\frac{i \sqrt[24]{15} \left(e^{-\pi^2/(6(5\log(2)+\log(3)+\log(5)))} - 1\right) \sqrt{\frac{5\log(2)+\log(3)+\log(5)}{\pi}}}{2^{7/24}}$$

$$\frac{i \sqrt[24]{15}\, e^{-\pi^2/(6\log(480))} \sqrt{\frac{\log(480)}{\pi}}}{2^{7/24}} - \frac{i \sqrt[24]{15} \sqrt{\log(480)}}{2^{7/24} \sqrt{\pi}}$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{480}\right)} \left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{480}\right)}\right) - 1\right)\right) \sqrt[24]{480}}{\sqrt{2\pi}} = \frac{\left(-1 + \exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{480}\right)}\right)\right) \sqrt[24]{480} \sqrt{\log_e\left(\frac{1}{480}\right)}}{\sqrt{2\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{480}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{480}\right)}\right)-1\right)\right)\sqrt[24]{480}}{\sqrt{2\pi}} = \frac{\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{480}\right)}\right)\right)\sqrt[24]{480}\sqrt{\log(a)\log_a\left(\frac{1}{480}\right)}}{\sqrt{2\pi}}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{480}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{480}\right)}\right)-1\right)\right)\sqrt[24]{480}}{\sqrt{2\pi}} = -\frac{1}{2^{7/24}\sqrt{\pi}} i \sqrt[24]{15}\, e^{-\frac{\pi^2}{6\left(\log(479)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{479}\right)^k}{k}\right)}}\left(-1+e^{\frac{\pi^2}{6\left(\log(479)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{479}\right)^k}{k}\right)}}\right)\sqrt{\log(479)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{479}\right)^k}{k}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{480}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{480}\right)}\right)-1\right)\right)\sqrt[24]{480}}{\sqrt{2\pi}} = -\frac{1}{2\times 2^{7/24}\sqrt{\pi}} i \sqrt[24]{15}\sqrt{479}\, e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(480)}}\left(-1+e^{\pi^2/(6\log(480))}\right)\sum_{k=0}^{\infty}479^k\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j} \text{ for } \left(c_k=\frac{(-1)^k}{(1+k)\pi} \text{ and } p_{j,0}=1 \text{ and } p_{j,k}=\frac{\pi\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k} \text{ and } k\in\mathbb{Z} \text{ and } k>0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{480}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{480}\right)}\right)-1\right)\right)\sqrt[24]{480}}{\sqrt{2\pi}} =$$

$$-\frac{1}{2^{7/24}\sqrt{\pi}}\, i\,\sqrt[24]{15}\,\exp\left(-\frac{\pi^2}{6\left(2 i\pi\left\lfloor\frac{\arg(480-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(480-x)^k x^{-k}}{k}\right)}\right)$$
$$\left(-1+\exp\left(\frac{\pi^2}{6\left(2 i\pi\left\lfloor\frac{\arg(480-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(480-x)^k x^{-k}}{k}\right)}\right)\right)$$
$$\sqrt{2 i\pi\left\lfloor\frac{\arg(480-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(480-x)^k x^{-k}}{k}} \quad \text{for } x<0$$

Integral representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{480}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{480}\right)}\right)-1\right)\right)\sqrt[24]{480}}{\sqrt{2\pi}} =$$
$$-\frac{i\sqrt[24]{15}\, e^{-\pi^2/\left(6\int_1^{480}\frac{1}{t}dt\right)}\left(-1+e^{\pi^2/\left(6\int_1^{480}\frac{1}{t}dt\right)}\right)\sqrt{\int_1^{480}\frac{1}{t}dt}}{2^{7/24}\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{480}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{480}\right)}\right)-1\right)\right)\sqrt[24]{480}}{\sqrt{2\pi}} = -\frac{1}{2^{19/24}\pi}\, i\,\sqrt[24]{15}$$
$$\exp\left(-\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{479^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}ds}\right)\left(-1+\exp\left(\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{479^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}ds}\right)\right)$$
$$\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{479^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}ds} \quad \text{for } -1<\gamma<0$$

((264^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/264)) * [exp(Pi^2/(6 ln(1/264)))-1]

Input:

$$\frac{\sqrt[24]{264}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{264}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{264}\right)}\right)-1\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{i\sqrt[24]{33}\left(e^{-\pi^2/(6\log(264))}-1\right)\sqrt{\frac{\log(264)}{\pi}}}{2^{3/8}}$$

Decimal approximation:

$-0.30360816042340200468385112254219528157829022579126663420\ldots\, i$

Polar coordinates:

$r \approx 0.303608$ (radius), $\theta = -90°$ (angle)

0.303608

Alternate forms:

$$-\frac{i\sqrt[24]{33}\,e^{-\pi^2/(6\log(264))}\left(e^{\pi^2/(6\log(264))}-1\right)\sqrt{\frac{\log(264)}{\pi}}}{2^{3/8}}$$

$$\frac{i\sqrt[24]{33}\left(e^{-\pi^2/(6(3\log(2)+\log(3)+\log(11)))}-1\right)\sqrt{\frac{3\log(2)+\log(3)+\log(11)}{\pi}}}{2^{3/8}}$$

$$\frac{i\sqrt[24]{33}\,e^{-\pi^2/(6\log(264))}\sqrt{\frac{\log(264)}{\pi}}}{2^{3/8}}-\frac{i\sqrt[24]{33}\sqrt{\log(264)}}{2^{3/8}\sqrt{\pi}}$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{264}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{264}\right)}\right)-1\right)\right)\sqrt[24]{264}}{\sqrt{2\pi}}=\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{264}\right)}\right)\right)\sqrt[24]{264}\sqrt{\log_e\left(\frac{1}{264}\right)}}{\sqrt{2\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{264}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{264}\right)}\right)-1\right)\right)\sqrt[24]{264}}{\sqrt{2\pi}}=\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{264}\right)}\right)\right)\sqrt[24]{264}\sqrt{\log(a)\log_a\left(\frac{1}{264}\right)}}{\sqrt{2\pi}}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{264}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{264}\right)}\right)-1\right)\right)\sqrt[24]{264}}{\sqrt{2\pi}} = -\frac{1}{2^{3/8}\sqrt{\pi}} i \sqrt[24]{33}\, e^{-\frac{\pi^2}{6\left(\log(263)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{263}\right)^k}{k}\right)}}\left(-1+e^{\frac{\pi^2}{6\left(\log(263)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{263}\right)^k}{k}\right)}}\right)\sqrt{\log(263)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{263}\right)^k}{k}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{264}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{264}\right)}\right)-1\right)\right)\sqrt[24]{264}}{\sqrt{2\pi}} = -\frac{1}{2\times 2^{3/8}\sqrt{\pi}} i \sqrt[24]{33}\sqrt{263}\, e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(264)}}\left(-1+e^{\pi^2/(6\log(264))}\right) \sum_{k=0}^{\infty} 263^k\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j} \quad \text{for } \left(c_k=\frac{(-1)^k}{(1+k)\pi} \text{ and } p_{j,0}=1 \text{ and } p_{j,k}=\frac{\pi\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k} \text{ and } k\in\mathbb{Z} \text{ and } k>0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{264}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{264}\right)}\right)-1\right)\right)\sqrt[24]{264}}{\sqrt{2\pi}} = -\frac{1}{2^{3/8}\sqrt{\pi}} i \sqrt[24]{33}\exp\left(-\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(264-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(264-x)^k x^{-k}}{k}\right)}\right)\left(-1+\exp\left(\frac{\pi^2}{6\left(2i\pi\left\lfloor\frac{\arg(264-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(264-x)^k x^{-k}}{k}\right)}\right)\right)\sqrt{2i\pi\left\lfloor\frac{\arg(264-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(264-x)^k x^{-k}}{k}} \quad \text{for } x<0$$

Integral representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{264}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{264}\right)}\right)-1\right)\right)\sqrt[24]{264}}{\sqrt{2\pi}} = -\frac{i\sqrt[24]{33}\, e^{-\pi^2/\left(6\int_1^{264}\frac{1}{t}\,dt\right)}\left(-1+e^{\pi^2/\left(6\int_1^{264}\frac{1}{t}\,dt\right)}\right)\sqrt{\int_1^{264}\frac{1}{t}\,dt}}{2^{3/8}\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{264}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{264}\right)}\right)-1\right)\right)\sqrt[24]{264}}{\sqrt{2\pi}} = -\frac{1}{2^{7/8}\,\pi}\, i\sqrt[24]{33}$$
$$\exp\left(-\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{263^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\left(-1+\exp\left(\frac{i\pi^3}{3\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{263^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)$$
$$\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{263^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

((66211^1/24 / (sqrt(2Pi)))) * sqrt(ln(1/66211)) * [exp(Pi^2/(6 ln(1/66211)))-1]

Input:

$$\frac{\sqrt[24]{66\,211}}{\sqrt{2\pi}}\sqrt{\log\left(\frac{1}{66\,211}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{66\,211}\right)}\right)-1\right)$$

log(x) is the natural logarithm

Exact result:

$$i\sqrt[24]{66\,211}\left(e^{-\pi^2/(6\log(66\,211))}-1\right)\sqrt{\frac{\log(66\,211)}{2\pi}}$$

Decimal approximation:

$-0.29072131348352288268104320118380251768319422488326753194\ldots\, i$

Polar coordinates:

$r \approx 0.290721$ (radius), $\theta = -90°$ (angle)

0.290721 result very near to the following Ramanujan continued fraction:

https://sites.google.com/site/marelv83/fisica-moderna/serie-infinite-per-p **(in Italian)**

Come esempio di uno dei suoi risultati, Ramanujan fornì questa frazione continua,

$$\sqrt{\phi+2}-\phi=\cfrac{e^{-2\pi/5}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-4\pi}}{1+\cfrac{e^{-5\pi}}{1+\cdots}}}}=0{,}2840\ldots$$

fra le altre, dove $\phi=(1+\sqrt{5})/2$ è la sezione aurea.

Alternate forms:

$$-i\sqrt[24]{66211}\,e^{-\pi^2/(6\log(66211))}\left(e^{\pi^2/(6\log(66211))}-1\right)\sqrt{\frac{\log(66211)}{2\pi}}$$

$$i\sqrt[24]{66211}\left(e^{-\pi^2/(6(\log(73)+\log(907)))}-1\right)\sqrt{\frac{\frac{\log(73)}{2}+\frac{\log(907)}{2}}{\pi}}$$

$$\frac{i\sqrt[24]{66211}\,e^{-\pi^2/(6\log(66211))}\sqrt{\frac{\log(66211)}{\pi}}}{\sqrt{2}}-i\sqrt[24]{66211}\sqrt{\frac{\log(66211)}{2\pi}}$$

Alternative representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{66211}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{66211}\right)}\right)-1\right)\right)\sqrt[24]{66211}}{\sqrt{2\pi}}=\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log_e\left(\frac{1}{66211}\right)}\right)\right)\sqrt[24]{66211}\sqrt{\log_e\left(\frac{1}{66211}\right)}}{\sqrt{2\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{66211}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{66211}\right)}\right)-1\right)\right)\sqrt[24]{66211}}{\sqrt{2\pi}}=\frac{\left(-1+\exp\left(\frac{\pi^2}{6\log(a)\log_a\left(\frac{1}{66211}\right)}\right)\right)\sqrt[24]{66211}\sqrt{\log(a)\log_a\left(\frac{1}{66211}\right)}}{\sqrt{2\pi}}$$

Series representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{66211}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{66211}\right)}\right)-1\right)\right)\sqrt[24]{66211}}{\sqrt{2\pi}}=$$
$$-\frac{1}{\sqrt{2\pi}}i\sqrt[24]{66211}\,e^{-\frac{\pi^2}{6\left(\log(66210)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{66210}\right)^k}{k}\right)}}$$
$$\left(-1+e^{\frac{\pi^2}{6\left(\log(66210)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{66210}\right)^k}{k}\right)}}\right)\sqrt{\log(66210)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{66210}\right)^k}{k}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{66211}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{66211}\right)}\right)-1\right)\right)\sqrt[24]{66211}}{\sqrt{2\pi}}=-\frac{1}{2\sqrt{\pi}}i\sqrt{33105}\,\sqrt[24]{66211}$$
$$e^{-\pi^2/(6\log(66211))}\left(-1+e^{\pi^2/(6\log(66211))}\right)\sum_{k=0}^{\infty}66210^k\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j}$$
$$\text{for }\left(k\in\mathbb{Z}\text{ and }\frac{(-1)^k}{1+k}=2\pi c_k\text{ and }p_{j,0}=1\text{ and }p_{j,k}=\frac{2\pi\sum_{m=1}^{k}(-k+m+jm)c_m\,p_{j,k-m}}{k}\text{ and }k>0\right)$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{66211}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{66211}\right)}\right)-1\right)\right)\sqrt[24]{66211}}{\sqrt{2\pi}}=$$
$$-\frac{1}{2\sqrt{\pi}}i\sqrt{33105}\,\sqrt[24]{66211}\,e^{i\pi\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{1}{e}\right)}{2\pi}\right\rfloor-\frac{\pi^2}{6\log(66211)}}$$
$$\left(-1+e^{\pi^2/(6\log(66211))}\right)\sum_{k=0}^{\infty}66210^k\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j}$$
$$\text{for }\left(k\in\mathbb{Z}\text{ and }\frac{(-1)^k}{1+k}=2\pi c_k\text{ and }p_{j,0}=1\text{ and }p_{j,k}=\frac{2\pi\sum_{m=1}^{k}(-k+m+jm)c_m\,p_{j,k-m}}{k}\text{ and }k>0\right)$$

Integral representations:

$$\frac{\left(\sqrt{\log\left(\frac{1}{66211}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{66\,211}\right)}\right)-1\right)\right)\sqrt[24]{66\,211}}{\sqrt{2\,\pi}} = -\frac{i\sqrt[24]{66\,211}\; e^{-\pi^2/\left(6\int_1^{66\,211}\frac{1}{t}\,dt\right)}\left(-1+e^{\pi^2/\left(6\int_1^{66\,211}\frac{1}{t}\,dt\right)}\right)\sqrt{\int_1^{66\,211}\frac{1}{t}\,dt}}{\sqrt{2\,\pi}}$$

$$\frac{\left(\sqrt{\log\left(\frac{1}{66211}\right)}\left(\exp\left(\frac{\pi^2}{6\log\left(\frac{1}{66\,211}\right)}\right)-1\right)\right)\sqrt[24]{66\,211}}{\sqrt{2\,\pi}} = -\frac{1}{2\,\pi}\,i\sqrt[24]{66\,211}$$
$$\exp\left(-\frac{i\,\pi^3}{3\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{66\,210^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\left(-1+\exp\left(\frac{i\,\pi^3}{3\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{66\,210^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)$$
$$\sqrt{-i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{66\,210^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

From the three results, we obtain:

(1.2683+0.538) (0.290721 + 0.299867 + 0.303608)

where 1.2683 and 0.538 are Hausdorff dimensions of Julia set z^2-1 and Feigenbaum attractor respectively

Input interpretation:

(1.2683 + 0.538) (0.290721 + 0.299867 + 0.303608)

Result:

1.6151862348

1.6151862348

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} \quad = \qquad 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms *themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

From Wikipedia

*The **Rössler attractor** is the attractor for the **Rössler system**, a system of three non-linear ordinary differential equations originally studied by Otto Rössler. These differential equations define a continuous-time dynamical system that exhibits chaotic dynamics associated with the fractal properties of the attractor.*

The Lorenz attractor was the first example of a low-dimensional differential equations system capable of generating chaotic behavior.(fractal)

In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749..., that is also a Hausdorff dimension and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

Refences

Asymptotic formulæ in combinatory analysis – *Srinivasa Ramanujan*
Proceedings of the London Mathematical Society, 2, XVII, 1918, 75-115

Congruence properties of partitions – *Srinivasa Ramanujan*
Mathematische Zeitschrift, IX, 1921, 147 – 153
[Extracted from the manuscripts of the author by G. H. Hardy]