

# **On a Ramanujan equation: mathematical connections with the golden ratio and various formulas concerning some arguments of Cosmology and Black Holes/Wormholes Physics X**

**Michele Nardelli<sup>1</sup>, Antonio Nardelli<sup>2</sup>**

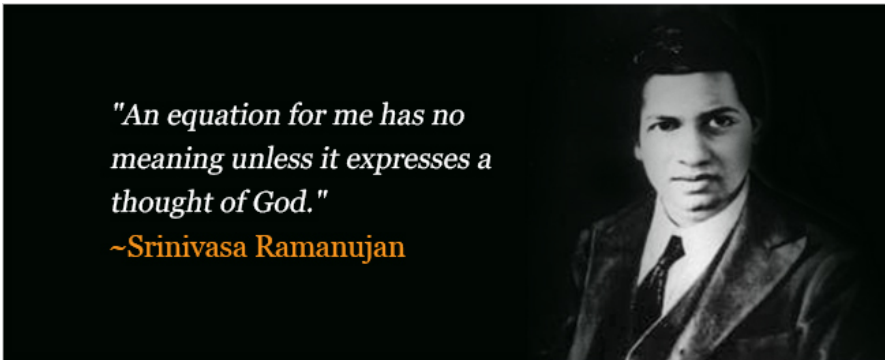
## **Abstract**

*In this paper we have described a Ramanujan formula and obtained some mathematical connections with the golden ratio and various equations concerning different sectors of Cosmology and Black Holes/Wormholes Physics.*

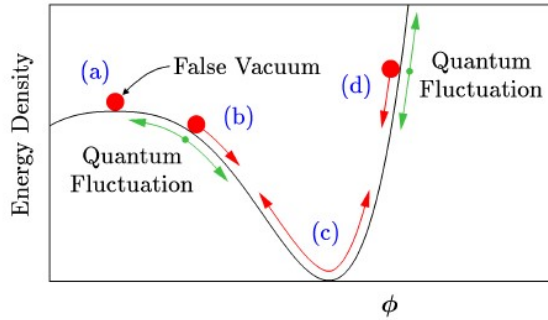
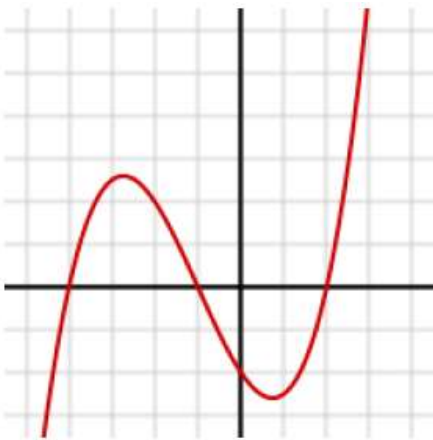
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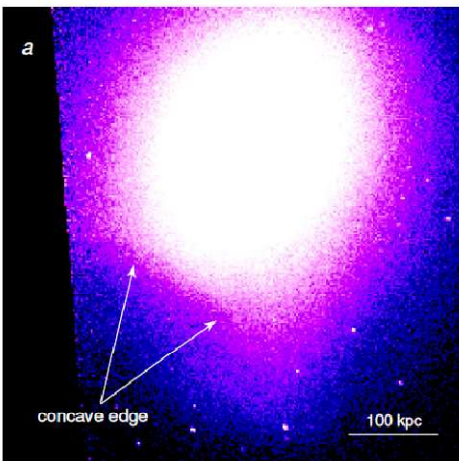
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<http://www.aicte-india.org/content/srinivasa-ramanujan>



(arXiv:2002.01291v1 [astro-ph.GA] 4 Feb 2020)



**Figure 9.** Chandra X-ray image of the Ophiuchus cluster in the 0.5-4 keV band, binned to 4'' pixels. (a) The concave edge, first reported by W16, is shown by arrows.

From:

## Static spherically symmetric wormholes with isotropic pressure

*Mauricio Cataldo, Luis Liempi and Pablo Rodriguez*

arXiv:1604.04578v1 [gr-qc] 15 Apr 2016

We have that:

$$ds^2 = \left(\frac{r}{r_0}\right)^{2\beta} dt^2 - \frac{(\beta^2 - 2\beta - 1) dr^2}{\left(\frac{r}{r_0}\right)^{\frac{2(1+2\beta-\beta^2)}{1+\beta}} - 1} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (22)$$

It becomes clear that this metric describes a Lorentzian spacetime for  $r \geq r_0$  if  $\beta^2 - 2\beta - 1 > 0$  and  $\frac{1+2\beta-\beta^2}{1+\beta} > 0$ , which implies that the condition  $\beta < -1$  must be required.

For the metric (22) the energy density and the isotropic pressure are given by

$$\kappa\rho = \frac{\left(\frac{r}{r_0}\right)^{-\frac{2(\beta^2-2\beta-1)}{1+\beta}} (2\beta+1)(\beta-3)}{(1+\beta)(\beta^2-2\beta-1)r^2} + \frac{\beta(\beta-2)}{(\beta^2-2\beta-1)r^2}, \quad (23)$$

$$\kappa p = \frac{\left(\frac{r}{r_0}\right)^{-\frac{2(\beta^2-2\beta-1)}{1+\beta}} (2\beta+1)}{(\beta^2-2\beta-1)r^2} - \frac{\beta^2}{(\beta^2-2\beta-1)r^2}. \quad (24)$$

$\beta \leq -1$  and  $r_0 = 1$ . The maximum value is reached at  $\beta = -4.745695219$  where  $r_{max} = 1.232835973$ . For  $\beta \rightarrow -\infty$  we have  $r_{max} \rightarrow 1$ .

From:

$$-(23 \cdot 0.915965594177219)/2 + 5 - (11\pi)/2 + \pi^2 + \pi \log(729/8)$$

where 0.915965594177219 is the Catalan Constant

**Input interpretation:**

$$-\frac{1}{2} (23 \times 0.915965594177219) + 5 - \frac{11 \pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)$$

log(x) is the natural logarithm

**Result:**

1.2328359737359...

1.2328359737359.... = r<sub>max</sub> = r

**Alternative representations:**

$$-\frac{1}{2} (23 \times 0.9159655941772190000) + 5 - \frac{11 \pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right) = -5.53360433303801850 - \frac{11 \pi}{2} + \pi \log_e\left(\frac{729}{8}\right) + \pi^2$$

$$-\frac{1}{2} (23 \times 0.9159655941772190000) + 5 - \frac{11 \pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right) = -5.53360433303801850 - \frac{11 \pi}{2} + \pi \log(a) \log_a\left(\frac{729}{8}\right) + \pi^2$$

$$-\frac{1}{2} (23 \times 0.9159655941772190000) + 5 - \frac{11 \pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right) = -5.53360433303801850 - \frac{11 \pi}{2} - \pi \operatorname{Li}_1\left(1 - \frac{729}{8}\right) + \pi^2$$

**Series representations:**

$$-\frac{1}{2} (23 \times 0.9159655941772190000) + 5 - \frac{11 \pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right) = -5.53360433303801850 - 5.500000000000000000 \pi + \pi^2 + \pi \log\left(\frac{721}{8}\right) - \pi \sum_{k=1}^{\infty} \frac{\left(-\frac{8}{721}\right)^k}{k}$$

$$-\frac{1}{2} (23 \times 0.9159655941772190000) + 5 - \frac{11 \pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right) = -5.53360433303801850 - 5.500000000000000000 \pi + \pi^2 + 2 i \pi^2 \left[ \frac{\arg\left(\frac{729}{8} - x\right)}{2 \pi} \right] + \pi \log(x) - \pi \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{729}{8} - x\right)^k x^{-k}}{k} \text{ for } x < 0$$

$$-\frac{1}{2} (23 \times 0.9159655941772190000) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right) =$$

$$-5.53360433303801850 - 5.500000000000000000 \pi + \pi^2 +$$

$$2i\pi^2 \left[ -\frac{-\pi + \arg\left(\frac{729}{8z_0}\right) + \arg(z_0)}{2\pi} \right] + \pi \log(z_0) - \pi \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{729}{8} - z_0\right)^k z_0^{-k}}{k}$$

**Integral representations:**

$$-\frac{1}{2} (23 \times 0.9159655941772190000) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right) =$$

$$-5.53360433303801850 - 5.500000000000000000 \pi + \pi^2 + \pi \int_1^{\frac{729}{8}} \frac{1}{t} dt$$

$$-\frac{1}{2} (23 \times 0.9159655941772190000) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right) =$$

$$-5.53360433303801850 - 5.500000000000000000 \pi +$$

$$\pi^2 + \frac{1}{2i} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{8}{721}\right)^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

And

$$1/12 (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi))$$

**Input:**

$$\frac{1}{12} (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi))$$

$\log(x)$  is the natural logarithm

$\tan^{-1}(x)$  is the inverse tangent function

**Exact Result:**

$$\frac{1}{12} (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi))$$

(result in radians)

**Decimal approximation:**

$$-4.74569521901099097005545362784961099786533667035332078487...$$

(result in radians)

$$-4.74569521901099... = \beta$$

**Alternate forms:**

$$\frac{1}{12} (-7 e^\pi + 8 \pi - 3 \log(2) + 65 \log(\pi) + 6 \tan^{-1}(\pi))$$

$$\frac{1}{12} (8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)) - \frac{7 e^\pi}{12}$$

$$-\frac{7 e^\pi}{12} + \frac{2 \pi}{3} - \frac{\log(8)}{12} + \frac{1}{4} i \log(1 - i \pi) - \frac{1}{4} i \log(1 + i \pi) + \frac{65 \log(\pi)}{12}$$

**Alternative representations:**

$$\frac{1}{12} (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)) =$$

$$\frac{1}{12} (8 \pi + 6 \tan^{-1}(1, \pi) - \log(8) + 65 \log(\pi) - 7 e^\pi)$$

$$\frac{1}{12} (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)) =$$

$$\frac{1}{12} (8 \pi + 6 \tan^{-1}(\pi) - \log(a) \log_a(8) + 65 \log(a) \log_a(\pi) - 7 e^\pi)$$

$$\frac{1}{12} (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)) =$$

$$\frac{1}{12} (8 \pi + 6 \tan^{-1}(\pi) - \log_e(8) + 65 \log_e(\pi) - 7 e^\pi)$$

**Series representations:**

$$\frac{1}{12} (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)) =$$

$$-\frac{7 e^\pi}{12} + \frac{2 \pi}{3} + \frac{1}{2} \tan^{-1}(\pi) - \frac{\log(7)}{12} + \frac{65}{12} \log(-1 + \pi) + \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{7}\right)^k - 65 \left(\frac{1}{1-\pi}\right)^k}{12 k}$$

$$\frac{1}{12} (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)) = \frac{1}{12}$$

$$\left( -7 e^\pi + 11 \pi - \log(7) + 65 \log(-1 + \pi) + 12 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{7}\right)^k - 65 \left(\frac{1}{1-\pi}\right)^k}{12 k} - 6 \sum_{k=0}^{\infty} \frac{(-1)^k \pi^{-1-2k}}{1 + 2k} \right)$$

$$\frac{1}{12} (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)) = -\frac{7 e^\pi}{12} + \frac{2 \pi}{3} +$$

$$\frac{1}{2} \tan^{-1}(z_0) - \frac{\log(8)}{12} + \frac{65 \log(\pi)}{12} + \frac{1}{4} i \sum_{k=1}^{\infty} \frac{\left(-(-i - z_0)^{-k} + (i - z_0)^{-k}\right) (\pi - z_0)^k}{k}$$

for  $(i z_0 \notin \mathbb{R} \text{ or } ((\text{not } 1 \leq i z_0 < \infty) \text{ and } (\text{not } -\infty < i z_0 \leq -1)))$

**Integral representations:**

$$\frac{1}{12} (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)) =$$

$$-\frac{7 e^\pi}{12} + \frac{2 \pi}{3} + \frac{\pi}{2} \int_0^1 \frac{1}{1 + \pi^2 t^2} dt - \frac{\log(8)}{12} + \frac{65 \log(\pi)}{12}$$

$$\frac{1}{12} (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)) =$$

$$-\frac{7 e^\pi}{12} + \frac{2 \pi}{3} + \int_0^1 \left( \frac{\pi}{2(1 + \pi^2 t^2)} + 7 \left( -\frac{1}{12(1 + 7t)} + \frac{65(-1 + \pi)}{12(7 - \pi - 7t + \pi(1 + 7t))} \right) \right) dt$$

$$\frac{1}{12} (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)) =$$

$$-\frac{7 e^\pi}{12} + \frac{2 \pi}{3} - \frac{i}{8 \sqrt{\pi}} \int_{-i\infty + \gamma}^{i\infty + \gamma} (1 + \pi^2)^{-s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1 - s) \Gamma(s)^2 ds -$$

$$\frac{\log(8)}{12} + \frac{65 \log(\pi)}{12} \text{ for } 0 < \gamma < \frac{1}{2}$$

**Continued fraction representations:**

$$\frac{1}{12} (-7 e^\pi + 8 \pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)) =$$

$$\frac{1}{12} \left( -7 e^\pi - \log(8) + 65 \log(\pi) + \pi \left( 8 + \frac{6}{1 + \mathbf{K}_{k=1}^{\infty} \frac{k^2 \pi^2}{1+2k}} \right) \right) =$$

$$\frac{1}{12} \left( -7 e^\pi - \log(8) + 65 \log(\pi) + \pi \left( 8 + \frac{6}{1 + \frac{\pi^2}{3 + \frac{4\pi^2}{5 + \frac{9\pi^2}{7 + \frac{16\pi^2}{9 + \dots}}}}} \right) \right)$$





**Input interpretation:**

$$1 + \frac{1}{\sqrt[3]{-\frac{1}{2} \left( 23 \times 0.915965594177219 \right) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)}} - (18 + 2) \times \frac{1}{10^3}$$

$\log(x)$  is the natural logarithm

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

1.61807023375046...

(result in radians)

1.61807023375046.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

**Alternative representations:**

$$1 + \frac{1}{\sqrt[3]{-\frac{1}{2} \left( 23 \times 0.9159655941772190000 \right) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)}} - \frac{18 + 2}{10^3} =$$

$$1 - \frac{20}{10^3} + \frac{1}{\sqrt[3]{-\frac{8\pi + 6 \tan^{-1}(1,\pi) - \log(8) + 65 \log(\pi) - 7 e^\pi}{12 \left( -5.53360433303801850 - \frac{11\pi}{2} + \pi \log\left(\frac{729}{8}\right) + \pi^2 \right)}}$$

$$1 + \frac{1}{\sqrt[3]{-\frac{1}{2} \left( 23 \times 0.9159655941772190000 \right) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)}} - \frac{18 + 2}{10^3} =$$

$$1 - \frac{20}{10^3} + \frac{1}{\sqrt[3]{-\frac{8\pi + 6 \tan^{-1}(\pi) - \log_e(8) + 65 \log_e(\pi) - 7 e^\pi}{12 \left( -5.53360433303801850 - \frac{11\pi}{2} + \pi \log_e\left(\frac{729}{8}\right) + \pi^2 \right)}}$$

$$1 + \frac{1}{\sqrt[3]{-\frac{1}{2} \left( 23 \times 0.9159655941772190000 \right) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)}} - \frac{18 + 2}{10^3} =$$

$$1 - \frac{20}{10^3} + \frac{1}{\sqrt[3]{-\frac{8\pi + 6 \tan^{-1}(1,\pi) - \log_e(8) + 65 \log_e(\pi) - 7 e^\pi}{12 \left( -5.53360433303801850 - \frac{11\pi}{2} + \pi \log_e\left(\frac{729}{8}\right) + \pi^2 \right)}}$$

## Series representations:

$$1 + \frac{1}{\sqrt[3]{\frac{-7e^\pi + 8\pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)}{\left(-\frac{1}{2}(23 \times 0.9159655941772190000) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)\right)12}}} - \frac{18+2}{10^3} =$$

$$\frac{49}{50} + \frac{2^{2/3} \sqrt[3]{3}}{\sqrt[3]{\frac{7e^\pi - 8\pi + \log(8) - 65 \log(\pi) - 6 \left(\frac{\pi^2}{2\sqrt{\pi^2}} - \sum_{k=0}^{\infty} \frac{(-1)^k \pi^{-1-2k}}{1+2k}\right)}{-5.53360433303801850 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)}}$$

$$1 + \frac{1}{\sqrt[3]{\frac{-7e^\pi + 8\pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)}{\left(-\frac{1}{2}(23 \times 0.9159655941772190000) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)\right)12}}} - \frac{18+2}{10^3} =$$

$$\frac{49}{50} + \frac{2^{2/3} \sqrt[3]{3}}{\sqrt[3]{\frac{(-\frac{1}{5})^k 2^{1+2k} F_{1+2k} \left(\frac{\pi}{1+\sqrt{1+\frac{4\pi^2}{5}}}\right)^{1+2k}}{7e^\pi - 8\pi + \log(8) - 65 \log(\pi) - 6 \sum_{k=0}^{\infty} \frac{1+2k}{-5.53360433303801850 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)}}}}$$

$$1 + \frac{1}{\sqrt[3]{\frac{-7e^\pi + 8\pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)}{\left(-\frac{1}{2}(23 \times 0.9159655941772190000) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)\right)12}}} - \frac{18+2}{10^3} =$$

$$50 \times 2^{2/3} \sqrt[3]{3} + 49 \sqrt[3]{\frac{7e^\pi - 8\pi - 6 \tan^{-1}(\pi) + \log(7) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{7})^k}{k} - 65 \left(\log(-1+\pi) - \sum_{k=1}^{\infty} \frac{(-1)^k (-1+\pi)^{-k}}{k}\right)}{-5.53360433303801850 - \frac{11\pi}{2} + \pi^2 + \pi \left(\log\left(\frac{721}{8}\right) - \sum_{k=1}^{\infty} \frac{(-\frac{8}{721})^k}{k}\right)}}$$


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$$50 \sqrt[3]{\frac{7e^\pi - 8\pi - 6 \tan^{-1}(\pi) + \log(7) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{7})^k}{k} - 65 \left(\log(-1+\pi) - \sum_{k=1}^{\infty} \frac{(-1)^k (-1+\pi)^{-k}}{k}\right)}{-5.53360433303801850 - \frac{11\pi}{2} + \pi^2 + \pi \left(\log\left(\frac{721}{8}\right) - \sum_{k=1}^{\infty} \frac{(-\frac{8}{721})^k}{k}\right)}}$$

**Integral representations:**

$$1 + \frac{1}{\sqrt[3]{-\frac{-7e^\pi + 8\pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)}{\left(-\frac{1}{2}(23 \times 0.9159655941772190000) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)\right)12}}} - \frac{18+2}{10^3} =$$

$$\frac{49}{50} + \frac{2^{2/3} \sqrt[3]{3}}{\sqrt[3]{\frac{7e^\pi - 8\pi - 6\pi \int_0^1 \frac{1}{1+\pi^2 t^2} dt + \log(8) - 65 \log(\pi)}{-5.53360433303801850 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)}}$$

$$1 + \frac{1}{\sqrt[3]{-\frac{-7e^\pi + 8\pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)}{\left(-\frac{1}{2}(23 \times 0.9159655941772190000) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)\right)12}}} - \frac{18+2}{10^3} =$$

$$\frac{49}{50} + \frac{2^{2/3} \sqrt[3]{3}}{\sqrt[3]{\frac{7e^\pi - 8\pi + \frac{3i}{2\sqrt{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} (1+\pi^2)^{-s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)^2 ds + \log(8) - 65 \log(\pi)}{-5.53360433303801850 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)}} \text{ for } 0 < \gamma < \frac{1}{2}$$

$$1 + \frac{1}{\sqrt[3]{-\frac{-7e^\pi + 8\pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)}{\left(-\frac{1}{2}(23 \times 0.9159655941772190000) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)\right)12}}} - \frac{18+2}{10^3} =$$

$$\frac{49}{50} + \frac{2^{2/3} \sqrt[3]{3}}{\sqrt[3]{\frac{7e^\pi - 8\pi - \frac{3}{2i} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(\pi^2)^{-s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds + \log(8) - 65 \log(\pi)}{-5.53360433303801850 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)}} \text{ for } 0 < \gamma < \frac{1}{2}$$

**Continued fraction representations:**

$$1 + \frac{1}{\sqrt[3]{-\frac{-7e^\pi + 8\pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)}{\left(-\frac{1}{2}(23 \times 0.9159655941772190000) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)\right)12}}} - \frac{18+2}{10^3} =$$

$$\frac{49}{50} + \frac{2^{2/3} \sqrt[3]{3}}{\sqrt[3]{\frac{7e^\pi - 8\pi + \log(8) - 65 \log(\pi) - \frac{6\pi}{1 + \sum_{k=1}^{\infty} \frac{k^2 \pi^2}{1+2k}}}{-5.53360433303801850 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)}}} =$$

$$\frac{49}{50} + \frac{2.454871976036789600}{\sqrt[3]{7e^\pi - 8\pi + \log(8) - 65 \log(\pi) - \frac{6\pi}{1 + \frac{\pi^2}{3 + \frac{4\pi^2}{5 + \frac{9\pi^2}{7 + \frac{16\pi^2}{9 + \dots}}}}}}}$$

$$\begin{aligned}
& 1 + \frac{1}{\sqrt[3]{-\frac{-7e^\pi + 8\pi - \log(8) + 65\log(\pi) + 6\tan^{-1}(\pi)}{\left(-\frac{1}{2}(23 \times 0.9159655941772190000) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)\right)12}}} - \frac{18+2}{10^3} = \\
& \frac{49}{50} + \frac{2^{2/3} \sqrt[3]{3}}{\sqrt[3]{\frac{7e^\pi - 8\pi + \log(8) - 65\log(\pi) - 6\left(\pi - \frac{\pi^3}{3 + \sum_{k=1}^{\infty} \frac{(1+(-1)^{1+k+k})^2 \pi^2}{3+2k}}\right)}{-5.53360433303801850 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)}}}} = \\
& \frac{49}{50} + \frac{2.454871976036789600}{\sqrt[3]{\frac{7e^\pi - 8\pi + \log(8) - 65\log(\pi) - 6\left(\pi - \frac{\pi^3}{3 + \frac{9\pi^2}{5 + \frac{4\pi^2}{7 + \frac{25\pi^2}{9 + \frac{16\pi^2}{11 + \dots}}}}}\right)}}{\sqrt[3]{\dots}}}
\end{aligned}$$

$$\begin{aligned}
& 1 + \frac{1}{\sqrt[3]{\frac{-7e^\pi + 8\pi - \log(8) + 65 \log(\pi) + 6 \tan^{-1}(\pi)}{\left(-\frac{1}{2}(23 \times 0.9159655941772190000) + 5 - \frac{11\pi}{2} + \pi^2 + \pi \log\left(\frac{729}{8}\right)\right)12}}} - \frac{18+2}{10^3} = \\
& \frac{49}{50} + \frac{2^{2/3} \sqrt[3]{3}}{\sqrt[3]{\frac{7e^\pi - 8\pi - \frac{6\pi}{1 + \prod_{k=1}^{\infty} \frac{k^2 \pi^2}{1+2k}} + \frac{7}{1 + \prod_{k=1}^{\infty} \frac{7 \left|\frac{1+k}{2}\right|^2}{1+k}} - \frac{65(-1+\pi)}{1 + \prod_{k=1}^{\infty} \frac{(-1+\pi) \left|\frac{1+k}{2}\right|^2}{1+k}}}{-5.53360433303801850 - \frac{11\pi}{2} + \pi^2 + \frac{721\pi}{8 \left(1 + \prod_{k=1}^{\infty} \frac{721 \left|\frac{1+k}{2}\right|^2}{8 \left(\frac{1+k}{2}\right)}\right)}}}} = \\
& \frac{49}{50} + \frac{2^{2/3} \sqrt[3]{3}}{\sqrt[3]{\frac{7e^\pi - 8\pi - \frac{6\pi}{1 + \frac{\pi^2}{3 + \frac{4\pi^2}{5 + \frac{9\pi^2}{7 + \frac{16\pi^2}{9 + \dots}}}}} + \frac{7}{1 + \frac{7}{2 + \frac{28}{3 + \frac{28}{4 + \frac{28}{5 + \dots}}}}} - \frac{65(-1+\pi)}{1 + \frac{-1+\pi}{2 + \frac{-1+\pi}{3 + \frac{4(-1+\pi)}{4 + \frac{4(-1+\pi)}{5 + \dots}}}}}{-12.94275952669252269 + \frac{721\pi}{8 \left(1 + \frac{721}{8 \left(2 + \frac{721}{8 \left(3 + \frac{721}{2 \left(4 + \frac{721}{2(5 + \dots)}\right)}\right)}\right)}\right)}}}}}}
\end{aligned}$$

$\prod_{k=k_1}^{k_2} a_k/b_k$  is a continued fraction

$$1.2328359737359\dots = r_{\max} = r$$

$$-4.74569521901099\dots = \beta ; r_0 = 1$$

Thence, we from

$$\begin{aligned}
\kappa\rho = & \frac{\left(\frac{r}{r_0}\right)^{-\frac{2(\beta^2 - 2\beta - 1)}{1+\beta}} (2\beta + 1)(\beta - 3)}{(1 + \beta)(\beta^2 - 2\beta - 1)r^2} + \\
& \frac{\beta(\beta - 2)}{(\beta^2 - 2\beta - 1)r^2}, \tag{23}
\end{aligned}$$

Where there is:

$$-\frac{2(\beta^2 - 2\beta - 1)}{1 + \beta}$$

We obtain:

$$-((2(-4.74569521901099^2 - 2*(-4.74569521901099) - 1))/(1 - 4.74569521901099))$$

**Input interpretation:**

$$\frac{2(-4.74569521901099^2 - 2 \times (-4.74569521901099) - 1)}{1 - 4.74569521901099}$$

**Result:**

$$-7.49139043802198$$

$$-7.49139043802198$$

$$(1.23283597)^{-7.491390438} 2((-4.74569) + 1)((-4.74569) - 3) * 1 / (((1 - 4.74569)((-4.74569)^2 - 2(-4.74569) - 1)1.232835^2))) + (-4.74569)((-4.74569) - 2) * 1 / (((1 - 4.74569)((-4.74569)^2 - 2(-4.74569) - 1)1.232835^2)))$$

**Input interpretation:**

$$\frac{2(-4.74569 + 1) \left( (-4.74569 - 3) \times \frac{1}{(1 - 4.74569)((-4.74569)^2 - 2 \times (-4.74569) - 1) \times 1.232835^2} \right)}{1.23283597^{7.491390438} - 4.74569(-4.74569 - 2) \times \frac{1}{(1 - 4.74569)((-4.74569)^2 - 2 \times (-4.74569) - 1) \times 1.232835^2}}$$

**Result:**

$$0.552222...$$

$$0.5522202162668 = \kappa p$$

And:

$$\kappa p = \frac{\left(\frac{r}{r_0}\right)^{-\frac{2(\beta^2 - 2\beta - 1)}{1 + \beta}} (2\beta + 1)}{(\beta^2 - 2\beta - 1)r^2 - \frac{\beta^2}{(\beta^2 - 2\beta - 1)r^2}} \quad (24)$$

$$1.2328359^{(-7.49139043)} \frac{2((-4.74569521)+1) \cdot 1 / (((-4.74569521)^2 - 2(-4.74569521) - 1) \cdot 1.2328359^2)}{((-4.74569521)^2 \cdot 1 / (((-4.74569521)^2 - 2(-4.74569521) - 1) \cdot 1.2328359^2))} -$$

**Input interpretation:**

$$\frac{2(-4.74569521 + 1) \times \frac{1}{(-4.74569521^2 - 2 \times (-4.74569521) - 1) \times 1.2328359^2}}{1.2328359^{7.49139043}} - \frac{(-4.74569521)^2 \times \frac{1}{(-4.74569521^2 - 2 \times (-4.74569521) - 1) \times 1.2328359^2}}{1}$$

**Result:**

1.1293753...

$$1.1293750829053726 = \kappa p$$

We have also that:

$$1.12937 \left[ \frac{(1.23283)^{(-7.49139)} \frac{2((-4.74569)+1)((-4.74569)-3)}{(((1-4.74569)((-4.74569)^2 - 2(-4.74569) - 1) \cdot 1.23283^2))} + (-4.74569)((-4.74569)-2)}{((1-4.74569)((-4.74569)^2 - 2(-4.74569) - 1) \cdot 1.23283^2)} \right] - \frac{5}{10^3} + 1$$

**Input interpretation:**

$$1.12937 \left( \frac{2(-4.74569 + 1) \times \frac{-4.74569 - 3}{(1 - 4.74569)((-4.74569)^2 - 2 \times (-4.74569) - 1) \times 1.23283^2}}{1.23283^{7.49139}} - 4.74569 \times \frac{-4.74569 - 2}{(1 - 4.74569)((-4.74569)^2 - 2 \times (-4.74569) - 1) \times 1.23283^2} \right) - \frac{5}{10^3} + 1$$

**Result:**

1.618674618852830645137857088596483583952016125654378746359...

1.61867461885283..... result that is a very good approximation to the value of the golden ratio 1.618033988749...

That is:

$$(1.1293750829053726 \times 0.5522202162668) - \frac{5}{10^3} + 1$$

**Input interpretation:**

$$1.1293750829053726 \times 0.5522202162668 - \frac{5}{10^3} + 1$$

**Result:**

1.61866375252834003685163500968

1.61866375252834..... result that is a very good approximation to the value of the golden ratio 1.618033988749...

We have also:

$$(1.1293750829053726 \times 0.5522202162668) + 1 + \frac{123 - 11 - 4}{10^3}$$

**Input interpretation:**

$$1.1293750829053726 \times 0.5522202162668 + 1 + \frac{123 - 11 - 4}{10^3}$$

**Result:**

1.73166375252834003685163500968

1.73166375.....  $\approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

1.7316

**Possible closed forms:**

$$\sqrt{3} \approx 1.7320508$$

$$\frac{71}{41} \approx 1.73170731$$



$$\frac{70\pi}{127} \approx 1.73158650$$

$$e^{1/e+e+1/\pi} \pi^{-e} \csc(e\pi) \approx 1.73157286$$

$$\sqrt{\frac{21\pi}{22}} \approx 1.73170233$$

$$2\sqrt{\frac{6}{79}}\pi \approx 1.73157755$$

$$\frac{49}{9\pi} \approx 1.7330204$$

$$e \log^2(2) \log^3(3) \approx 1.73172434$$

$$\frac{1}{2} e \log(2) \log(2\pi) \approx 1.73143585$$

$$\frac{4\pi^4}{225} \approx 1.73171717$$

$$\frac{400}{231} \approx 1.731601731$$

$$-4 - \frac{3}{\pi} + 2\sqrt{\pi} + \pi \approx 1.73157069$$

$$\sqrt{\frac{2}{65}}\pi^2 \approx 1.73124313$$

$$\frac{2(\pi-2)\pi}{1+\pi} \approx 1.73190334$$

$$\frac{5}{34} (3 + \sqrt{77}) \approx 1.731612409$$

Now, for

$$1.2328359737359\dots = r_{\max} = r$$

$$-4.74569521901099\dots = \beta$$

and

$$\frac{2(1+2\beta-\beta^2)}{\beta+1}$$

$$\frac{(((2(1+2*(-4.74569521901099)+(4.74569521901099^2)))))/(-4.74569521901099+1)}$$

**Input interpretation:**

$$\frac{2(1 + 2 \times (-4.74569521901099) + 4.74569521901099^2)}{-4.74569521901099 + 1}$$

**Result:**

-7.49139043802198

**-7.49139043802198**

From

$$\frac{dz(r)}{dr} = \sqrt{\frac{\beta(\beta - 2) - \left(\frac{r}{r_0}\right)^{\frac{2(1+2\beta-\beta^2)}{\beta+1}}}{\left(\frac{r}{r_0}\right)^{\frac{2(1+2\beta-\beta^2)}{\beta+1}} - 1}} \quad (27)$$

for:

$$1.2328359737359\dots = r_{\max} = r$$

$$-4.74569521901099\dots = \beta$$

we obtain:

$$\text{sqrt}[((( -4.74569521901099(-4.74569521901099-2)-(1.2328359737359)^{-7.49139043802198}))))/((((((1.2328359737359)^{-7.49139043802198})-1)))]$$

**Input interpretation:**

$$\sqrt{\frac{-4.74569521901099(-4.74569521901099-2) - \frac{1}{1.2328359737359^{7.49139043802198}}}{\frac{1}{1.2328359737359^{7.49139043802198}} - 1}}$$

**Result:**

6.3387611881459... *i*

**Polar coordinates:**

*r* = 6.3387611881459 (radius), *θ* = 90° (angle)

**6.3387611881459**

We note that for  $C = 2\pi r$ , we have

$$(1.00884517617248i \cdot 2\pi)$$

**Input interpretation:**

$$1.00884517617248 i \times 2 \pi$$

$i$  is the imaginary unit

**Result:**

$$6.33876118814593... i$$

Thence,  $1.00884517617248 i$  is an imaginary radius of the above circle

**Polar coordinates:**

$$r = 6.33876118814593 \text{ (radius), } \theta = 90^\circ \text{ (angle)}$$

$$6.33876118814593...$$

We obtain:

$$1 + 1 / (1.00884517617248 \cdot 2\pi)^{1/4} - (11 + 1) / 10^3$$

**Input interpretation:**

$$1 + \frac{1}{\sqrt[4]{1.00884517617248 \times 2 \pi}} - (11 + 1) \times \frac{1}{10^3}$$

**Result:**

$$1.618229753306224...$$

$1.618229753306224...$  result that is a very good approximation to the value of the golden ratio  $1.618033988749...$

**Alternative representations:**

$$1 + \frac{1}{\sqrt[4]{1.008845176172480000 \times 2 \pi}} - \frac{11 + 1}{10^3} =$$

$$1 - \frac{12}{10^3} + \frac{1}{\sqrt[4]{363.1842634220928000^\circ}}$$

$$1 + \frac{1}{\sqrt[4]{1.008845176172480000 \times 2 \pi}} - \frac{11+1}{10^3} =$$

$$1 - \frac{12}{10^3} + \frac{1}{\sqrt[4]{-2.017690352344960000 i \log(-1)}}$$

$$1 + \frac{1}{\sqrt[4]{1.008845176172480000 \times 2 \pi}} - \frac{11+1}{10^3} =$$

$$1 - \frac{12}{10^3} + \frac{1}{\sqrt[4]{2.017690352344960000 \cos^{-1}(-1)}}$$

### Series representations:

$$1 + \frac{1}{\sqrt[4]{1.008845176172480000 \times 2 \pi}} - \frac{11+1}{10^3} = \frac{247}{250} + \frac{0.5932959350834001353}{\sqrt[4]{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}$$

$$1 + \frac{1}{\sqrt[4]{1.008845176172480000 \times 2 \pi}} - \frac{11+1}{10^3} = \frac{247}{250} + \frac{0.7055517473033719571}{\sqrt[4]{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}}$$

$$1 + \frac{1}{\sqrt[4]{1.008845176172480000 \times 2 \pi}} - \frac{11+1}{10^3} = \frac{247}{250} + \frac{0.8390471578957718482}{\sqrt[4]{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}}}$$

### Integral representations:

$$1 + \frac{1}{\sqrt[4]{1.008845176172480000 \times 2 \pi}} - \frac{11+1}{10^3} = \frac{247}{250} + \frac{0.7055517473033719571}{\sqrt[4]{\int_0^{\infty} \frac{1}{1+t^2} dt}}$$

$$1 + \frac{1}{\sqrt[4]{1.008845176172480000 \times 2 \pi}} - \frac{11+1}{10^3} = \frac{247}{250} + \frac{0.5932959350834001353}{\sqrt[4]{\int_0^1 \sqrt{1-t^2} dt}}$$

$$1 + \frac{1}{\sqrt[4]{1.008845176172480000 \times 2 \pi}} - \frac{11+1}{10^3} = \frac{247}{250} + \frac{0.7055517473033719571}{\sqrt[4]{\int_0^{\infty} \frac{\sin(t)}{t} dt}}$$

And:

$$(1.00884517617248)^{55} - 5 \times \frac{1}{10^3}$$

**Input interpretation:**

$$1.00884517617248^{55} - 5 \times \frac{1}{10^3}$$

**Result:**

1.618112774901604768989718131296982777057750814695412630524...

1.6181127749.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

Now, from

$$\frac{d^2 r/dz^2 = -r(\beta^2 - 2\beta - 1)^2 \left(\frac{r}{r_0}\right)^{2\beta}}{r_0^2(1 + \beta) \left( \left(\frac{r}{r_0}\right)^{\frac{2\beta^2}{1+\beta}} (\beta^2 - 2\beta) - \left(\frac{r}{r_0}\right)^{\frac{2(2\beta+1)}{1+\beta}} \right)^2}, \quad (29)$$

For:

1.2328359737359.... =  $r_{\max} = r$ ; -4.74569521901099.... =  $\beta$ ;  $r_0 = 1$  we obtain:

$$((2(-4.74569521901099^2)))/(((1-4.74569521901099)))$$

**Input interpretation:**

$$\frac{2(-4.74569521901099^2)}{1 - 4.74569521901099}$$

**Result:**

12.02533671049208187949452423943848784847130698130228740756...

12.02533671.... result very near to the black hole entropy 12.1904 that is equal to  $\ln(196884)$

$$((2(2(-4.74569521901099)+1))) / ((1-4.74569521901099))$$

**Input interpretation:**

$$\frac{2(2 \times (-4.74569521901099) + 1)}{1 - 4.74569521901099}$$

**Result:**

4.533946272470101879494524239438487848471306981302287407563...  
**4.53394627....**

From

$$\frac{d^2 r / dz^2 =}{r_0^2(1 + \beta) \left( \left( \frac{r}{r_0} \right)^{\frac{2\beta^2}{1+\beta}} (\beta^2 - 2\beta) - \left( \frac{r}{r_0} \right)^{\frac{2(2\beta+1)}{1+\beta}} \right)^2}, \quad (29)$$

we obtain:

$$\frac{-1.23283597(-4.745695219^2 - 2(-4.745695219) - 1)^2 * 1.23283597^{(2(-4.745695219))}}{(((1-4.745695219)*(((((((1.23283597^{(12.02533671)} * ((-4.745695219^2 - 2(-4.745695219)) - 1.23283597^{(4.53394627)}))))))))))^2}$$

**Input interpretation:**

$$\frac{-1.23283597(-4.745695219^2 - 2 \times (-4.745695219) - 1)^2 \times 1.23283597^{2 \times (-4.745695219)}}{((1 - 4.745695219) (1.23283597^{12.02533671} ((-4.745695219^2 - 2 \times (-4.745695219)) - 1.23283597^{4.53394627}))))^2}$$

**Result:**

-0.00006336092896100333263626117514844283222895205254187829...  
**-0.000063360928961.....**

And

$$(-1/-0.00006336093)$$

**Input interpretation:**

$$\frac{-1}{-0.00006336093}$$

**Result:**

15782.59662539675475091669266849460700782011880191783801153...

15782.596625396...

We note that:

$$\frac{1}{6} \ln(-1/-0.00006336093) + 7/10^3$$

**Input interpretation:**

$$\frac{1}{6} \log\left(\frac{-1}{-0.00006336093}\right) + \frac{7}{10^3}$$

$\log(x)$  is the natural logarithm

**Result:**

1.6181105...

1.6181105.... result that is a very good approximation to the value of the golden ratio  
1.618033988749...

**Alternative representations:**

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{7}{10^3} = \frac{1}{6} \log_e\left(\frac{-1}{-0.0000633609}\right) + \frac{7}{10^3}$$

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{7}{10^3} = \frac{1}{6} \log_{(a)} \log_a\left(\frac{-1}{-0.0000633609}\right) + \frac{7}{10^3}$$

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{7}{10^3} = -\frac{1}{6} \text{Li}_1\left(1 + -\frac{1}{0.0000633609}\right) + \frac{7}{10^3}$$

**Series representations:**

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{7}{10^3} = \frac{7}{1000} + \frac{\log(15781.6)}{6} - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-9.6666k}}{k}$$

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{7}{10^3} = \frac{7}{1000} + \frac{1}{3} i \pi \left[ \frac{\arg(15782.6 - x)}{2\pi} \right] + \frac{\log(x)}{6} - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k (15782.6 - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{7}{10^3} = \frac{7}{1000} + \frac{1}{6} \left[ \frac{\arg(15782.6 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \frac{\log(z_0)}{6} + \frac{1}{6} \left[ \frac{\arg(15782.6 - z_0)}{2\pi} \right] \log(z_0) - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k (15782.6 - z_0)^k z_0^{-k}}{k}$$

**Integral representations:**

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{7}{10^3} = \frac{7}{1000} + \frac{1}{6} \int_1^{15782.6} \frac{1}{t} dt$$

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{7}{10^3} = \frac{7}{1000} + \frac{1}{12 i \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-9.6666s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

Or:

$$(-1/-0.00006336093)^{1/20} - 3/10^3$$

**Input interpretation:**

$$\sqrt[20]{\frac{-1}{-0.00006336093} - \frac{3}{10^3}}$$

**Result:**

1.6184700...

1.61847.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

And:

$$1/6 \ln(-1/-0.00006336093) + (89+34-2)1/10^3$$

**Input interpretation:**

$$\frac{1}{6} \log\left(\frac{-1}{-0.00006336093}\right) + (89 + 34 - 2) \times \frac{1}{10^3}$$



**Result:**

1.732110522089253648530726095461566979031815441162327844913...

$1.732110522089\dots \approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

**Alternative representations:**

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{89 + 34 - 2}{10^3} = \frac{1}{6} \log_e\left(\frac{-1}{-0.0000633609}\right) + \frac{121}{10^3}$$

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{89 + 34 - 2}{10^3} = \frac{1}{6} \log(a) \log_a\left(\frac{-1}{-0.0000633609}\right) + \frac{121}{10^3}$$

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{89 + 34 - 2}{10^3} = -\frac{1}{6} \text{Li}_1\left(1 + -\frac{1}{0.0000633609}\right) + \frac{121}{10^3}$$

**Series representations:**

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{89 + 34 - 2}{10^3} = \frac{121}{1000} + \frac{\log(15781.6)}{6} - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-9.6666k}}{k}$$

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{89 + 34 - 2}{10^3} = \frac{121}{1000} + \frac{1}{3} i\pi \left\lfloor \frac{\arg(15782.6 - x)}{2\pi} \right\rfloor + \frac{\log(x)}{6} - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k (15782.6 - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{89 + 34 - 2}{10^3} = \frac{121}{1000} + \frac{1}{6} \left\lfloor \frac{\arg(15782.6 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \frac{\log(z_0)}{6} + \frac{1}{6} \left\lfloor \frac{\arg(15782.6 - z_0)}{2\pi} \right\rfloor \log(z_0) - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k (15782.6 - z_0)^k z_0^{-k}}{k}$$

**Integral representations:**

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{89 + 34 - 2}{10^3} = \frac{121}{1000} + \frac{1}{6} \int_1^{15782.6} \frac{1}{t} dt$$

$$\frac{1}{6} \log\left(\frac{-1}{-0.0000633609}\right) + \frac{89 + 34 - 2}{10^3} = \frac{121}{1000} + \frac{1}{12 i \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-9.6666s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

Or:

$$(-1/-0.00006336093)^{1/18} + 21/10^3$$

**Input interpretation:**

$$\sqrt[18]{\frac{-1}{-0.00006336093}} + \frac{21}{10^3}$$

**Result:**

1.7319296...

1.7319296...  $\approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

Now, we have that:

$$\rho + p = \frac{2 \left(\frac{r}{r_0}\right)^{-\frac{2(\beta^2 - 2\beta - 1)}{1 + \beta}} (2\beta + 1)(\beta - 1)}{(1 + \beta)(\beta^2 - 2\beta - 1)r^2} - \frac{2\beta}{(\beta^2 - 2\beta - 1)r^2}, \quad (31)$$

$$(-4.745695219^2 - 2(-4.745695219) - 1)$$

**Input interpretation:**

$$-4.745695219^2 - 2 \times (-4.745695219) - 1$$

**Result:**

$$-14.030232673639457961$$

$$\text{-14.030232673639457961}$$

$$(-4.745695219^2 - 2(-4.745695219) - 1) \times (1.23283597)^2$$

**Input interpretation:**

$$(-4.745695219^2 - 2 \times (-4.745695219) - 1) \times 1.23283597^2$$

**Result:**

$$-21.3243335778944488510020894421244049$$

$$\text{-21.3243335778944488510020894421244049}$$

From

$$\rho + p = \frac{2 \left( \frac{r}{r_0} \right)^{-\frac{2(\beta^2 - 2\beta - 1)}{1 + \beta}} (2\beta + 1)(\beta - 1)}{(1 + \beta)(\beta^2 - 2\beta - 1)r^2} - \frac{2\beta}{(\beta^2 - 2\beta - 1)r^2}, \quad (31)$$

we obtain:

$$-2 \times (-14.03023) \times 1 / (1 - 4.745695219)$$

**Input interpretation:**

$$-2 \times (-14.03023) \times \frac{1}{1 - 4.745695219}$$

**Result:**

$$-7.49138901042017748855182549757794375421125260538716564488...$$

$$\text{-7.49138901042.....}$$

$$\frac{((2*(1.23283597)^{7.491389}*(2(-4.745695219)+1)(-4.745695219-1))))}{(((1-4.745695219)(-21.32433357))))} - \frac{((2(-4.745695219))))}{(((1-21.32433357))))}$$

**Input interpretation:**

$$\frac{2 \times 1.23283597^{7.491389} (2 \times (-4.745695219) + 1) (-4.745695219 - 1)}{(1 - 4.745695219) \times (-21.32433357)} - \frac{2 \times (-4.745695219)}{21.32433357}$$

**Result:**

5.415615974706405720283835837207099148869118134524180425140...

5.4156159747064.....

From the ratio with the previous result, we obtain:

$$\frac{-\frac{(((((2*(1.23283597)^{7.491389}*(2(-4.745695219)+1)(-4.745695219-1))))}{(((1-4.745695219)(-21.32433357))))} - \frac{((2(-4.745695219))))}{(((1-21.32433357))))))}{*1/(-0.000063360928961)}$$

**Input interpretation:**

$$-\left( \frac{2 \times 1.23283597^{7.491389} (2 \times (-4.745695219) + 1) (-4.745695219 - 1)}{(1 - 4.745695219) \times (-21.32433357)} - \frac{2 \times (-4.745695219)}{21.32433357} \right) \left( -\frac{1}{0.000063360928961} \right)$$

**Result:**

85472.48...

85472.48...

From which:

$$\frac{1}{6} \left[ -\frac{(((((2*(1.23283597)^{7.491389}*(2(-4.745695219)+1)(-4.745695219-1))))}{(((1-4.745695219)(-21.32433357))))} - \frac{((2(-4.745695219))))}{(((1-21.32433357))))))}{*1/(-0.000063360928961)} \right]^{1/5}$$

**Input interpretation:**

$$\frac{1}{6} \left( -\left( \frac{2 \times 1.23283597^{7.491389} (2 \times (-4.745695219) + 1) (-4.745695219 - 1)}{(1 - 4.745695219) \times (-21.32433357)} - \frac{2 \times (-4.745695219)}{21.32433357} \right) \left( -\frac{1}{0.000063360928961} \right) \right)^{(1/5)}$$

**Result:**

1.615154287289698838554963228509607279156351474527693144424...

1.6151542872896.... result that is an approximation to the value of the golden ratio  
1.618033988749...

From:

**Dilatonic Black Holes in Higher Curvature String Gravity**

*P. Kanti, N.E. Mavromatos, J. Rizos, K. Tamvakis and E. Winstanley*

arXiv:hep th/9511071v1 10 Nov 1995

We now observe that in order to keep  $\phi_h''$  finite we have to impose the boundary condition  $6e^\phi + e^\phi \phi'^2 r^2 + 2\phi' r^3 = 0$  which relates  $\phi_h'$  with  $\phi_h$

$$\phi_h' = r_h e^{-\phi_h} \left( -1 + \sigma \sqrt{1 - 6 \frac{e^{2\phi_h}}{r_h^4}} \right), \sigma = \pm 1$$

$$\phi_h < \log\left(\frac{r_h^2}{\sqrt{6}}\right) \tag{60}$$

$$\ln \left( \frac{(1.949322 \times 10^{13})^2}{\sqrt{6}} \right)$$

**Input interpretation:**

$$\log\left(\frac{(1.949322 \times 10^{13})^2}{\sqrt{6}}\right)$$

log(x) is the natural logarithm

**Result:**

60.306296...

60.306296...

$$\phi_h = 2\pi\sqrt{58} = 47.85131368 \dots < 60.306296\dots$$

$$(((1.949322e+13)*e^{(-2\pi*\sqrt{58}))})*(-1+\sqrt{((1-6*e^{(2*2\pi*\sqrt{58})})/(1.949322e+13)^4))))$$

**Input interpretation:**

$$\left(1.949322 \times 10^{13} e^{-2\pi\sqrt{58}}\right) \left(-1 + \sqrt{1 - 6 \times \frac{e^{2 \times 2(\pi\sqrt{58})}}{(1.949322 \times 10^{13})^4}}\right)$$

**Result:**

$$-2.44924... \times 10^{-19}$$

$$-2.44924... * 10^{-19}$$

From:

**Black Holes in Einstein-Gauss-Bonnet-Dilaton Theory**  
*Jutta Kunz* - 12th-16th September 2016, Ljubljana (Slovenia)

We have:

$$\phi'_h = \frac{r_h}{\alpha'} e^{-\phi_h} \left( -1 \pm \sqrt{1 - 6 \frac{\alpha'^2}{r_h^4} e^{2\phi_h}} \right)$$

With  $r_h = 1.949322e+13$  and  $\alpha' = 0.988-1.18 = 0.9991104684$ . We note that 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}}$$

$$\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\varphi^5 4 \sqrt{5^3} - 1}}} - \varphi + 1 \approx 0.9991104684$$

and  $e^{2\pi\sqrt{58}}$  is obtained from the following Ramanujan expression

**(Modular equations and approximations to  $\pi$  – Srinivasa Ramanujan** - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \dots = 64 \left\{ \left( \frac{5 + \sqrt{29}}{2} \right)^{12} + \left( \frac{5 - \sqrt{29}}{2} \right)^{12} \right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982 \dots$$

$$\left( \frac{((1.949322e+13) * e^{(-2\pi * \sqrt{58})})}{(0.9991104684) * [-1 + ((((((1 - ((((((6 * 0.9991104684^2 * e^{(2 * 2\pi * \sqrt{58})})))))))/(((1.949322e+13)^4))))))^{1/2})] \right)$$

**Input interpretation:**

$$\frac{1.949322 \times 10^{13} e^{-2\pi\sqrt{58}}}{0.9991104684} \left( -1 + \sqrt{1 - \frac{6 \times 0.9991104684^2 e^{2 \times 2(\pi\sqrt{58})}}{(1.949322 \times 10^{13})^4}} \right)$$

**Result:**

$$-2.44706... \times 10^{-19}$$

$$-2.44706... * 10^{-19}$$

From which:

$$3 \left( \frac{(-((((((1.949322e+13) * e^{(-2\pi * \sqrt{58})})}{(0.9991104684) * [-1 + ((((((1 - ((((((6 * 0.9991104684^2 * e^{(2 * 2\pi * \sqrt{58})})))))))/(((1.949322e+13)^4))))))^{1/2})] \right))^{11-8}}$$

**Input interpretation:**

$$3 \sqrt[11]{ \frac{1}{\frac{1.949322 \times 10^{13} e^{-2\pi\sqrt{58}}}{0.9991104684} \left( -1 + \sqrt{1 - \frac{6 \times 0.9991104684^2 e^{2 \times 2(\pi\sqrt{58})}}{(1.949322 \times 10^{13})^4}} \right)} - 8}$$

**Result:**

$$139.5919...$$

139.5919.... result practically equal to the rest mass of Pion meson 139.57 MeV

$$3\left(\frac{(((((1.949322e+13) \cdot e^{-2\pi\sqrt{58}}) / (0.9991104684) \cdot [-1 + (((1 - (((6 \cdot 0.9991104684^2 \cdot e^{2 \cdot 2(\pi\sqrt{58})} / ((1.949322e+13)^4))))^{1/2}))))^{1/11-18-4}}\right)^{1/11-18-4}}$$

**Input interpretation:**

$$3 \sqrt[11]{\frac{1}{\frac{1.949322 \times 10^{13} e^{-2\pi\sqrt{58}}}{0.9991104684} \left( -1 + \sqrt{1 - \frac{6 \cdot 0.9991104684^2 e^{2 \cdot 2(\pi\sqrt{58})}}{(1.949322 \times 10^{13})^4}} \right)} - 18 - 4}}$$

**Result:**

125.5919...

125.5919.... result very near to the Higgs boson mass 125.18 GeV

$$27 \cdot \frac{1}{2} \cdot \left( \left( \frac{(((((1.949322e+13) \cdot e^{-2\pi\sqrt{58}}) / (0.9991104684) \cdot [-1 + (((1 - (((6 \cdot 0.9991104684^2 \cdot e^{2 \cdot 2(\pi\sqrt{58})} / ((1.949322e+13)^4))))^{1/2}))))^{1/11-18-3/2}}\right)^{1/11-18-3/2}} \right)$$

**Input interpretation:**

$$27 \times \frac{1}{2} \left( 3 \sqrt[11]{\frac{1}{\frac{1.949322 \times 10^{13} e^{-2\pi\sqrt{58}}}{0.9991104684} \left( -1 + \sqrt{1 - \frac{6 \cdot 0.9991104684^2 e^{2 \cdot 2(\pi\sqrt{58})}}{(1.949322 \times 10^{13})^4}} \right)} - 18 - \frac{3}{2}}}} \right)$$

**Result:**

1729.241...

1729.241...

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)



For  $r_h = 2.26$ , we obtain.

$$\left( \frac{((2.26) \cdot e^{(-2\pi \sqrt{58}))})}{(0.9991104684)} \cdot [-1 + \left( \frac{1 - \left( \frac{6 \cdot 0.9991104684^2 \cdot e^{(2 \cdot 2\pi \sqrt{58})}}{2.26^4} \right)}{\left( (2.26)^4 \right)} \right)^{1/2}} \right]$$

**Input interpretation:**

$$\frac{2.26 e^{-2\pi \sqrt{58}}}{0.9991104684} \left( -1 + \sqrt{1 - \frac{6 \times 0.9991104684^2 e^{2 \cdot 2(\pi \sqrt{58})}}{2.26^4}} \right)$$

**Result:**

$$-3.74053... \times 10^{-21} + 1.08385... i$$

**Polar coordinates:**

$$r = 1.08385 \text{ (radius), } \theta = 90^\circ \text{ (angle)}$$

1.08385

**Series representations:**

$$\frac{\left( -1 + \sqrt{1 - \frac{6 \cdot 0.99911^2 e^{2 \cdot 2(\pi \sqrt{58})}}{2.26^4}} \right) (2.26 e^{-2\pi \sqrt{58}})}{0.99911} = 2.26201 e^{-2\pi \sqrt{57} \sum_{k=0}^{\infty} 57^{-k} \binom{1/2}{k}} \left( -1 + \sqrt{1 - 0.229586 e^{4\pi \sqrt{57} \sum_{k=0}^{\infty} 57^{-k} \binom{1/2}{k}}} \right)$$

$$\frac{\left( -1 + \sqrt{1 - \frac{6 \cdot 0.99911^2 e^{2 \cdot 2(\pi \sqrt{58})}}{2.26^4}} \right) (2.26 e^{-2\pi \sqrt{58}})}{0.99911} = 2.26201 \exp \left( -2\pi \sqrt{57} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{57}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \left( -1 + \sqrt{1 - 0.229586 \exp \left( 4\pi \sqrt{57} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{57}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)} \right)$$

$$\frac{\left(-1 + \sqrt{1 - \frac{6 \cdot 0.99911^2 e^{2 \times 2(\pi \sqrt{58})}}{2.26^4}}\right) \left(2.26 e^{-2\pi \sqrt{58}}\right)}{0.99911} =$$

$$2.26201 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 57^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}\right)$$

$$\left(-1 + \sqrt{1 - 0.229586 \exp\left(\frac{2\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 57^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}\right)}\right)$$

From: "SQUARE SERIES GENERATING FUNCTION TRANSFORMATIONS"  
 MAXIE D. SCHMIDT - <https://arxiv.org/abs/1609.02803v2>

**Corollary 4.7** (Special Values of Ramanujan's  $\varphi$ -Function). *For any  $k \in \mathbb{R}^+$ , the variant of the Ramanujan  $\varphi$ -function,  $\varphi(e^{-k\pi}) \equiv \vartheta_3(e^{-k\pi})$ , has the integral representation*

$$\varphi(e^{-k\pi}) = 1 + \int_0^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[ \frac{4e^{k\pi} (e^{2k\pi} - \cos(\sqrt{2\pi kt}))}{e^{4k\pi} - 2e^{2k\pi} \cos(\sqrt{2\pi kt}) + 1} \right] dt. \quad (33)$$

Moreover, the special values of this function corresponding to the particular cases of  $k \in \{1, 2, 3, 5\}$  in (33) have the respective integral representations

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = 1 + \int_0^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[ \frac{4e^{\pi} (e^{2\pi} - \cos(\sqrt{2\pi t}))}{e^{4\pi} - 2e^{2\pi} \cos(\sqrt{2\pi t}) + 1} \right] dt \quad (34)$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{\sqrt{2}+2}}{2} = 1 + \int_0^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[ \frac{4e^{2\pi} (e^{4\pi} - \cos(2\sqrt{\pi t}))}{e^{8\pi} - 2e^{4\pi} \cos(2\sqrt{\pi t}) + 1} \right] dt$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{\sqrt{3}+1}}{2^{1/4} 3^{3/8}} = 1 + \int_0^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[ \frac{4e^{3\pi} (e^{6\pi} - \cos(\sqrt{6\pi t}))}{e^{12\pi} - 2e^{6\pi} \cos(\sqrt{6\pi t}) + 1} \right] dt$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{5+2\sqrt{5}}}{5^{3/4}} = 1 + \int_0^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[ \frac{4e^{5\pi} (e^{10\pi} - \cos(\sqrt{10\pi t}))}{e^{20\pi} - 2e^{10\pi} \cos(\sqrt{10\pi t}) + 1} \right] dt.$$

From the first of (34):

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = 1 + \int_0^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[ \frac{4e^{\pi} (e^{2\pi} - \cos(\sqrt{2\pi t}))}{e^{4\pi} - 2e^{2\pi} \cos(\sqrt{2\pi t}) + 1} \right] dt$$

we have:

$$\Gamma\left(\frac{3}{4}\right) = \frac{\pi\sqrt{2}}{\Gamma\left(\frac{1}{4}\right)} = \frac{4,44288293815}{3,625609908} = 1,2254167025$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = \frac{1,3313353638}{1,2254167025} = 1,08643481 \dots$$

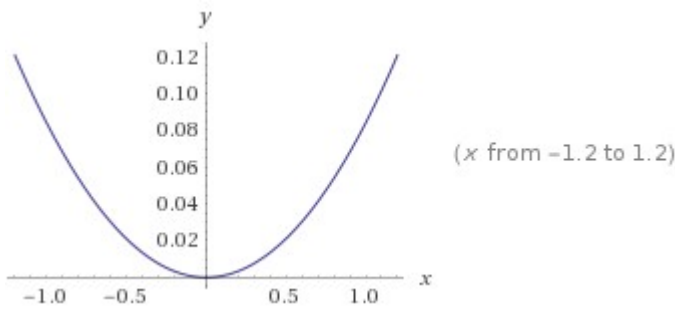
For the integral, we have calculate as follows:

integrate [(2.71828^0.89)/(sqrt6.283185307)][4e^3.14159265 \* (e^6.283185307 - cos((sqrt6.283185307)1.33416))]/[e^12.56637 - 2e^6.283185307 (cos(sqrt6.283185307)1.33416))+1]x

**Indefinite integral:**

$$\int \frac{2.71828^{0.89} \left( 4 e^{3.14159265} \left( e^{6.283185307} - \cos\left(\sqrt{6.283185307} \cdot 1.33416\right)\right) \right) x}{\sqrt{6.283185307} \left( e^{12.56637} - \left( 2 e^{6.283185307} \right) \left( \cos\left(\sqrt{6.283185307}\right) \cdot 1.33416 \right) + 1 \right)} dx = 0.0837798 x^2 + \text{constant}$$

**Plot of the integral:**



**Alternate form assuming x is real:**

$$0.0837798 x^2 + 0 + \text{constant}$$

Thence:  $1 + 0.0837798 = 1.0837798$  value very near to the previous solution

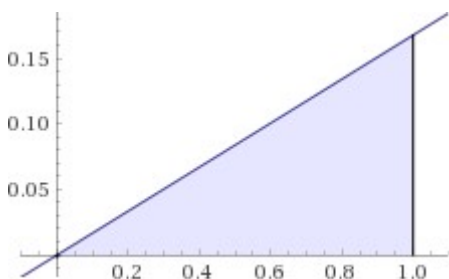
and:

integrate  $[(2.71828^{0.89})/(\text{sqrt}6.283185307)][4e^{3.14159265 * (e^{6.283185307} - \cos((\text{sqrt}6.283185307)1.33416))}]/[e^{12.56637} - 2e^{6.283185307} (\cos(\text{sqrt}6.283185307)1.33416))+1] x, [0, 1]$

**Definite integral:**

$$\int_0^1 \frac{2.71828^{0.89} \left( 4 e^{3.14159265} \left( e^{6.283185307} - \cos\left(\sqrt{6.283185307} 1.33416\right)\right)\right) x}{\sqrt{6.283185307} \left( e^{12.56637} - (2 e^{6.283185307}) \left( \cos\left(\sqrt{6.283185307}\right) 1.33416\right) + 1\right)} dx = 0.0837798$$

**Visual representation of the integral:**



**Riemann sums:**

left sum	$0.0837798 - \frac{0.0837798}{n} = 0.0837798 - \frac{0.0837798}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$
----------	--

(assuming subintervals of equal length)

**Indefinite integral:**

$$\int \frac{2.71828^{0.89} \left( 4 e^{3.14159265} \left( e^{6.283185307} - \cos\left(\sqrt{6.283185307} 1.33416\right)\right)\right) x}{\sqrt{6.283185307} \left( e^{12.56637} - (2 e^{6.283185307}) \left( \cos\left(\sqrt{6.283185307}\right) 1.33416\right) + 1\right)} dx = 0.0837798 x^2 + \text{constant}$$

Thence:  $1 + 0.0837798 = 1.0837798$

With regard the integral, from 0 to 0,58438 for  $t = 2$ , where  $(2.71828^2)/(\text{sqrt}6.283185307) = 2,94780$  for  $t=2$ , we have:

integrate (2.94780)[4e^3.14159265 \* (e^6.283185307 - cos((sqrt(6.283185307\*2))))]/[e^12.56637 - 2e^6.283185307 (cos(sqrt(6.283185307\*2))+1)] x, [0,0.58438]

$$\int_0^{0.58438} \frac{2.94780 \left( 4 e^{3.14159265} \left( e^{6.283185307} - \cos\left(\sqrt{6.283185307 \cdot 2}\right) \right) \right) x}{e^{12.56637} - (2 e^{6.283185307}) \left( \cos\left(\sqrt{6.283185307 \cdot 2}\right) + 1 \right)} dx = 0.0864364$$

Thence,  $1 + 0,0864364 = 1,0864364$ ;  $1,08643481 \cong 1,0864364$ .

In conclusion, the value of this, defined by us, "New Ramanujan's Constant" is 1.08643.

We have also:

$$-\left[ \frac{(2.26) \cdot e^{-2\pi \sqrt{58}}}{(0.9991104684)} \cdot \left[ -1 + \left( \frac{1 - \left( \frac{6 \cdot 0.9991104684^2 \cdot e^{2 \cdot 2\pi \sqrt{58}}}{(2.26)^4} \right)^{1/2}}{(2.26)^4} \right) \right] \right]^6$$

**Input interpretation:**

$$-\left[ \frac{2.26 e^{-2\pi \sqrt{58}}}{0.9991104684} \left( -1 + \sqrt{1 - \frac{6 \times 0.9991104684^2 e^{2 \times 2(\pi \sqrt{58})}}{2.26^4}} \right) \right]^6$$

**Result:**

$$1.62108... + 3.35676... \times 10^{-20} i$$

**Alternate form:**

$$1.62108$$

$$1.62108$$

**Series representations:**

$$\begin{aligned}
 & \left[ \frac{\left( -1 + \sqrt{1 - \frac{6 \times 0.99911^2 e^{2 \times 2 (\pi \sqrt{58})}}{2.26^4}} \right) (2.26 e^{-2\pi \sqrt{58}})}{0.99911} \right]^6 \\
 &= -133.958 e^{-12\pi \sqrt{57} \sum_{k=0}^{\infty} 57^{-k} \binom{1/2}{k}} \left( -1 + \sqrt{1 - 0.229586 e^{4\pi \sqrt{57} \sum_{k=0}^{\infty} 57^{-k} \binom{1/2}{k}}} \right)^6
 \end{aligned}$$

$$\begin{aligned}
 & \left[ \frac{\left( -1 + \sqrt{1 - \frac{6 \times 0.99911^2 e^{2 \times 2 (\pi \sqrt{58})}}{2.26^4}} \right) (2.26 e^{-2\pi \sqrt{58}})}{0.99911} \right]^6 \\
 &= -133.958 \exp \left( -12\pi \sqrt{57} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{57}\right)^k \binom{-1/2}{k}}{k!} \right) \\
 & \left( -1 + \sqrt{1 - 0.229586 \exp \left( 4\pi \sqrt{57} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{57}\right)^k \binom{-1/2}{k}}{k!} \right)} \right)^6
 \end{aligned}$$

$$\begin{aligned}
 & \left[ \frac{\left( -1 + \sqrt{1 - \frac{6 \times 0.99911^2 e^{2 \times 2 (\pi \sqrt{58})}}{2.26^4}} \right) (2.26 e^{-2\pi \sqrt{58}})}{0.99911} \right]^6 \\
 &= -133.958 \exp \left( -\frac{6\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 57^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) \\
 & \left( -1 + \sqrt{1 - 0.229586 \exp \left( \frac{2\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 57^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right)} \right)^6
 \end{aligned}$$

and:

$$-\left[\frac{(2.26)^{2\pi\sqrt{58}}}{0.9991104684} \left[-1 + \sqrt{1 - \frac{6 \times 0.9991104684^2 e^{2 \times 2(\pi\sqrt{58})}}{(2.26)^4}}\right]\right]^6 - \frac{3}{10^3}$$

**Input interpretation:**

$$-\left[\frac{2.26 e^{-2\pi\sqrt{58}}}{0.9991104684} \left(-1 + \sqrt{1 - \frac{6 \times 0.9991104684^2 e^{2 \times 2(\pi\sqrt{58})}}{2.26^4}}\right)\right]^6 - \frac{3}{10^3}$$

**Result:**

$$1.61808... + 3.35676... \times 10^{-20} i$$

**Alternate form:**

$$1.61808$$

1.61808 result that is a very good approximation to the value of the golden ratio 1.618033988749...

**Series representations:**

$$-\left[\frac{\left(-1 + \sqrt{1 - \frac{6 \times 0.99911^2 e^{2 \times 2(\pi\sqrt{58})}}{2.26^4}}\right) (2.26 e^{-2\pi\sqrt{58}})}{0.99911}\right]^6 - \frac{3}{10^3} = -\frac{3}{1000}$$

$$133.958 e^{-12\pi\sqrt{57} \sum_{k=0}^{\infty} 57^{-k} \binom{1/2}{k}} \left(-1 + \sqrt{1 - 0.229586 e^{4\pi\sqrt{57} \sum_{k=0}^{\infty} 57^{-k} \binom{1/2}{k}}}\right)^6$$

$$\begin{aligned}
& - \left[ \frac{\left( -1 + \sqrt{1 - \frac{6 \times 0.99911^2 e^{2 \times 2 (\pi \sqrt{58})}}{2.26^4}} \right) (2.26 e^{-2\pi \sqrt{58}})}{0.99911} \right]^6 - \frac{3}{10^3} = \\
& - \frac{3}{1000} - 133.958 \exp \left( -12 \pi \sqrt{57} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{57}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \\
& \left( -1 + \sqrt{1 - 0.229586 \exp \left( 4 \pi \sqrt{57} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{57}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)} \right)^6 \\
& - \left[ \frac{\left( -1 + \sqrt{1 - \frac{6 \times 0.99911^2 e^{2 \times 2 (\pi \sqrt{58})}}{2.26^4}} \right) (2.26 e^{-2\pi \sqrt{58}})}{0.99911} \right]^6 - \frac{3}{10^3} = \\
& - \frac{3}{1000} - 133.958 \exp \left( - \frac{6 \pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 57^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) \\
& \left( -1 + \sqrt{1 - 0.229586 \exp \left( \frac{2 \pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 57^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right)} \right)^6
\end{aligned}$$

From:

**Euclidean Wormholes, Baby Universes, and Their Impact on Particle Physics and Cosmology** - *Arthur Hebecker\**, *Thomas Mikhail* and *Pablo Soler*

Institute for Theoretical Physics, University of Heidelberg, Heidelberg, Germany

published: 08 October 2018 - doi: 10.3389/fspas.2018.00035

Now, we have that:

The phase  $\delta$  is redefined accordingly and is generically non-zero



In the regime of small wormhole corrections, one can expand the potential (72) and obtain the axion mass and effective  $\theta_{\text{eff}}$  parameter as functions of  $f$ ,  $\Lambda_{\text{QCD}}$  and  $M_P$ :

$$m^2 \approx \frac{\Lambda_{\text{QCD}}^4}{f^2} + 24\pi^4 M_P^2 \cos(\delta) \exp\left(-\frac{\pi\sqrt{6} M_P}{8 f}\right) \quad (73)$$

$$\theta_{\text{eff}} \approx 24\pi^4 \sin(\delta) \frac{f^2 M_P^2}{\Lambda_{\text{QCD}}^4} \exp\left(-\frac{\pi\sqrt{6} M_P}{8 f}\right). \quad (74)$$

For  $M_P = 2.435 \times 10^{18} \text{ GeV}/c^2$  that is the reduced Planck mass,  $\delta = \sqrt{2}$  and:  $f \lesssim 1.2 \times 10^{16} \text{ GeV}$

Here  $\lambda_1 = \Lambda$  and  $\lambda_2 = -M_P^2/2$  characterize the cosmological constant and the Planck scale. As discussed before, including

$$\Lambda = 4.33 \times 10^{-66} \text{ eV}^2 \text{ in natural units.}$$

We obtain:

$$\left( \frac{(4.33 \times 10^{-66})^4}{(1.2 \times 10^{16})^2} + 24\pi^4 (2.435 \times 10^{18})^2 \cos(\sqrt{2}) \exp\left(\frac{-\pi\sqrt{6} \times 2.435 \times 10^{18}}{8 \times 1.2 \times 10^{16}}\right) \right)^{1/21 - 2\pi}$$

**Input interpretation:**

$$\frac{(4.33 \times 10^{-66})^4}{(1.2 \times 10^{16})^2} + (24 \pi^4 (2.435 \times 10^{18})^2) \cos(\sqrt{2}) \exp\left(-\frac{\pi \sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right)$$

**Result:**

$$3.67859897616516... \times 10^{-46}$$

$$3.67859897616516... * 10^{-46}$$

From which:

$$\left[ \frac{(4.33 \times 10^{-66})^4}{(1.2 \times 10^{16})^2} + 24\pi^4 (2.435 \times 10^{18})^2 \cos(\sqrt{2}) \exp\left(\frac{-\pi\sqrt{6} \times 2.435 \times 10^{18}}{8 \times 1.2 \times 10^{16}}\right) \right]^{1/21 - 2\pi}$$

**Input interpretation:**

$$\frac{1}{\left( \frac{(4.33 \times 10^{-66})^4}{(1.2 \times 10^{16})^2} + (24 \pi^4 (2.435 \times 10^{18})^2) \cos(\sqrt{2}) \exp\left(-\frac{\pi \sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right) \right)^{21 - 2\pi}}$$

**Result:**

139.4434478522446...

139.4434478522446.... result practically equal to the rest mass of Pion meson  
139.57 MeV

and:

$$1/\left[\left(\frac{(4.33 \times 10^{-66})^4}{(1.2 \times 10^{16})^2}\right) + 24\pi^4 (2.435 \times 10^{18})^2 \cos(\sqrt{2}) \exp\left(\frac{-\pi \sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right)\right]^{1/21-2\pi-(11+3)}$$

**Input interpretation:**

$$\frac{1}{\sqrt[21]{\frac{(4.33 \times 10^{-66})^4}{(1.2 \times 10^{16})^2} + (24 \pi^4 (2.435 \times 10^{18})^2) \cos(\sqrt{2}) \exp\left(-\frac{\pi \sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right)}} - 2\pi - (11 + 3)}$$

**Result:**

125.4434478522446...

125.4434478522446... result very near to the Higgs boson mass 125.18 GeV

Now:

**Input interpretation:**

$4.33 \times 10^{-66} \text{ eV}^2$  (electronvolts squared)

**Unit conversion:**

$1.111 \times 10^{-103} \text{ J}^2$  (joules squared)

**Input interpretation:**

convert  $1.111 \times 10^{-103} \text{ J}^2$  (joules squared) to gigaelectronvolts

**Result:**

$4.328 \times 10^{-84} \text{ GeV}^2$  (gigaelectronvolts squared)

$4.328 \times 10^{-84} \text{ GeV}^2$

Thence:

$$m^2 \approx \frac{\Lambda_{\text{QCD}}^4}{f^2} + 24\pi^4 M_p^2 \cos(\delta) \exp\left(-\frac{\pi \sqrt{6} M_p}{8 f}\right)$$

$$\left( \frac{(4.328 \times 10^{-84})^4}{(1.2 \times 10^{16})^2} + 24\pi^4 (2.435 \times 10^{18})^2 \cos(\sqrt{2}) \exp\left(\frac{-\pi\sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right) \right)$$

**Input interpretation:**

$$\frac{(4.328 \times 10^{-84})^4}{(1.2 \times 10^{16})^2} + (24\pi^4 (2.435 \times 10^{18})^2) \cos(\sqrt{2}) \exp\left(-\frac{\pi\sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right)$$

**Result:**

$$3.67859897616516... \times 10^{-46}$$

$$3.67859897616516... * 10^{-46}$$

From

$$\theta_{\text{eff}} \approx 24\pi^4 \sin(\delta) \frac{f^2 M_P^2}{\Lambda_{\text{QCD}}^4} \exp\left(-\frac{\pi\sqrt{6}}{8} \frac{M_P}{f}\right).$$

we obtain:

$$24\pi^4 \sin(\sqrt{2}) \left( \frac{(1.2 \times 10^{16})^2 (2.435 \times 10^{18})^2}{(4.328 \times 10^{-84})^4} \right) \exp\left(\frac{-\pi\sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right)$$

**Input interpretation:**

$$24\pi^4 \sin(\sqrt{2}) \times \frac{(1.2 \times 10^{16})^2 (2.435 \times 10^{18})^2}{(4.328 \times 10^{-84})^4} \exp\left(-\frac{\pi\sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right)$$

**Result:**

$$9.5627358404274069101035159950062110468136456906882172... \times 10^{320}$$

$$9.5627358404274... * 10^{320}$$

From which:

$$\left[ 24\pi^4 \sin(\sqrt{2}) \left( \frac{(1.2 \times 10^{16})^2 (2.435 \times 10^{18})^2}{(4.328 \times 10^{-84})^4} \right) \exp\left(\frac{-\pi\sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right) \right]^{\left(\frac{\sqrt{257/10}}{70\pi^4}\right)}$$

**Input interpretation:**

$$\left( 24\pi^4 \sin(\sqrt{2}) \left( (1.2 \times 10^{16})^2 \times \frac{(2.435 \times 10^{18})^2}{(4.328 \times 10^{-84})^4} \right) \exp\left(-\frac{\pi\sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right) \right)^{\sqrt{\frac{257}{10}} / (70\pi^4)}$$

**Result:**

1.7323774...

1.7323774....  $\approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

$$[24\pi^4 \sin(\sqrt{2}) * (((1.2e+16)^2 * (2.435e+18)^2 / (4.328e-84)^4)) * \exp(((((-\pi * \sqrt{6})/8) * ((2.435e+18)/(1.2e+16))))))]^{1/147-13}$$

**Input interpretation:**

$$\sqrt[147]{24 \pi^4 \sin(\sqrt{2}) \left( (1.2 \times 10^{16})^2 \times \frac{(2.435 \times 10^{18})^2}{(4.328 \times 10^{-84})^4} \right) \exp\left( -\frac{\pi \sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}} \right) - 13}$$

**Result:**

139.59538...

139.59538... result practically equal to the rest mass of Pion meson 139.57 MeV

$$[24\pi^4 \sin(\sqrt{2}) * (((1.2e+16)^2 * (2.435e+18)^2 / (4.328e-84)^4)) * \exp(((((-\pi * \sqrt{6})/8) * ((2.435e+18)/(1.2e+16))))))]^{1/147-(29-2)}$$

**Input interpretation:**

$$\sqrt[147]{24 \pi^4 \sin(\sqrt{2}) \left( (1.2 \times 10^{16})^2 \times \frac{(2.435 \times 10^{18})^2}{(4.328 \times 10^{-84})^4} \right) \exp\left( -\frac{\pi \sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}} \right) - (29 - 2)}$$

**Result:**

125.5954...

125.5954... result very near to the Higgs boson mass 125.18 GeV

From the ratio of the two results, we obtain:

$$(9.5627358404274e+320)/((((((4.328e-84)^4 / (1.2e+16)^2)) + 24\pi^4*(2.435e+18)^2 \cos(\sqrt{2}) \exp(((((-\pi*\sqrt{6})/8)*((2.435e+18)/(1.2e+16))))))))))$$

**Input interpretation:**

$$\frac{9.5627358404274 \times 10^{320}}{\frac{(4.328 \times 10^{-84})^4}{(1.2 \times 10^{16})^2} + (24 \pi^4 (2.435 \times 10^{18})^2) \cos(\sqrt{2}) \exp\left(-\frac{\pi \sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right)}$$

**Result:**

$$2.5995592078363... \times 10^{366}$$

$$2.5995592078363... * 10^{366}$$

From which:

$$\sqrt[4]{(9.5627358404274e+320)/((((((4.328e-84)^4 / (1.2e+16)^2)) + 24\pi^4*(2.435e+18)^2 \cos(\sqrt{2}) \exp(((((-\pi*\sqrt{6})/8)*((2.435e+18)/(1.2e+16))))))))))}$$

**Input interpretation:**

$$\sqrt[4]{\frac{9.5627358404274 \times 10^{320}}{\frac{(4.328 \times 10^{-84})^4}{(1.2 \times 10^{16})^2} + (24 \pi^4 (2.435 \times 10^{18})^2) \cos(\sqrt{2}) \exp\left(-\frac{\pi \sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right)}}$$

**Result:**

$$1.6123148600185... \times 10^{183}$$

$$1.6123148600185... * 10^{183}$$

And:

$$(4+2)*1/10^3+1/10^183 \sqrt[4]{(9.5627358404274e+320)/((((((4.328e-84)^4 / (1.2e+16)^2)) + 24\pi^4*(2.435e+18)^2 \cos(\sqrt{2}) \exp(((((-\pi*\sqrt{6})/8)*((2.435e+18)/(1.2e+16))))))))))}$$

**Input interpretation:**

$$(4+2) \times \frac{1}{10^3} + \frac{1}{10^{183}} \sqrt[4]{\frac{9.5627358404274 \times 10^{320}}{\frac{(4.328 \times 10^{-84})^4}{(1.2 \times 10^{16})^2} + (24 \pi^4 (2.435 \times 10^{18})^2) \cos(\sqrt{2}) \exp\left(-\frac{\pi \sqrt{6}}{8} \times \frac{2.435 \times 10^{18}}{1.2 \times 10^{16}}\right)}}$$

**Result:**

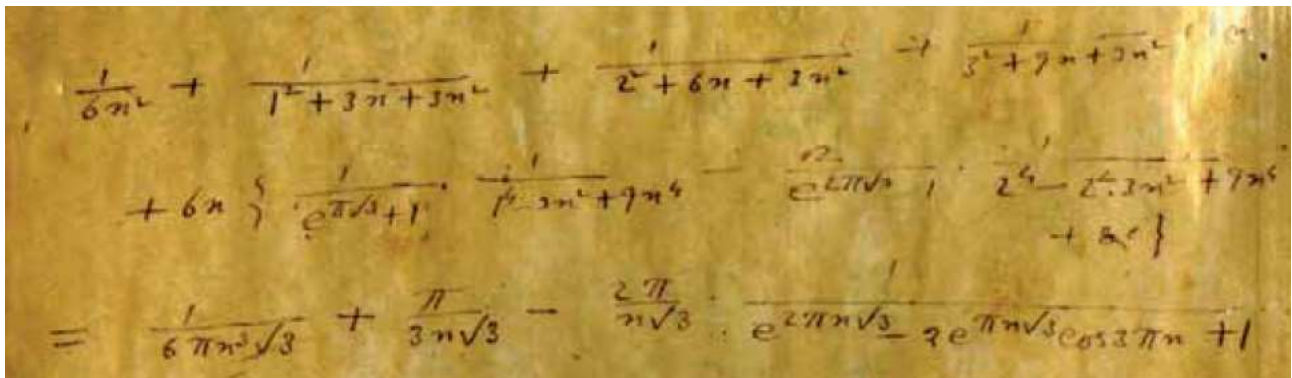
1.6183148600185...

1.6183148600185.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

From: **Manuscript Book 1 of Srinivasa Ramanujan**

We have that:

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For n = 2, we obtain:

$$\left[ \frac{1}{(6\pi \cdot 8 \cdot \sqrt{3})} + \frac{\pi}{(6\sqrt{3})} - \frac{2\pi}{(2\sqrt{3})} \cdot \frac{1}{((e^{(4\pi(\sqrt{3}))} - 2e^{(2\pi(\sqrt{3}))} \cos(6\pi) + 1))} \right]$$

**Input:**

$$\frac{1}{6\pi \times 8 \sqrt{3}} + \frac{\pi}{6 \sqrt{3}} - 2 \times \frac{\pi}{2 \sqrt{3}} \times \frac{1}{e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}}) \cos(6\pi) + 1}$$

**Exact result:**

$$\frac{1}{48 \sqrt{3} \pi} + \frac{\pi}{6 \sqrt{3}} - \frac{\pi}{\sqrt{3} (1 - 2e^{2\sqrt{3}\pi} + e^{4\sqrt{3}\pi})}$$

**Decimal approximation:**

0.306128566284279762069260090232527986234402500052541797685...

0.3061285662842...

**Alternate forms:**

$$\frac{1 + \left( 8 - \frac{48}{(e^{2\sqrt{3}\pi} - 1)^2} \right) \pi^2}{48\sqrt{3}\pi}$$

$$\frac{(1 - 2e^{2\sqrt{3}\pi} + e^{4\sqrt{3}\pi} - 40\pi^2 - 16e^{2\sqrt{3}\pi}\pi^2 + 8e^{4\sqrt{3}\pi}\pi^2)\sqrt{3}}{144(e^{2\sqrt{3}\pi} - 1)^2\pi}$$

$$\frac{1 - 2e^{2\sqrt{3}\pi} + e^{4\sqrt{3}\pi} - 40\pi^2 - 16e^{2\sqrt{3}\pi}\pi^2 + 8e^{4\sqrt{3}\pi}\pi^2}{48\sqrt{3}(e^{\sqrt{3}\pi} - 1)^2(1 + e^{\sqrt{3}\pi})^2\pi}$$

**Alternative representations:**

$$\frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi^2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} =$$

$$- \frac{2\pi}{(1 - 2\cosh(-6i\pi)e^{2\pi\sqrt{3}} + e^{4\pi\sqrt{3}})(2\sqrt{3})} + \frac{\pi}{6\sqrt{3}} + \frac{1}{48\pi\sqrt{3}}$$

$$\frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi^2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} =$$

$$- \frac{2\pi}{(1 - 2\cosh(6i\pi)e^{2\pi\sqrt{3}} + e^{4\pi\sqrt{3}})(2\sqrt{3})} + \frac{\pi}{6\sqrt{3}} + \frac{1}{48\pi\sqrt{3}}$$

$$\frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi^2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} =$$

$$- \frac{2\pi}{\left(1 + e^{4\pi\sqrt{3}} - \frac{2e^{2\pi\sqrt{3}}}{\sec(6\pi)}\right)(2\sqrt{3})} + \frac{\pi}{6\sqrt{3}} + \frac{1}{48\pi\sqrt{3}}$$

### Series representations:

$$\frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} =$$

$$\left( 1 + e^{4\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} - 40\pi^2 + 8e^{4\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \pi^2 - \right.$$

$$2e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} J_0(6\pi) - 16e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \pi^2 J_0(6\pi) -$$

$$4e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \sum_{k=1}^{\infty} (-1)^k J_{2k}(6\pi) -$$

$$\left. 32e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \pi^2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(6\pi) \right) /$$

$$\left( 48\pi\sqrt{2} \left( 1 + e^{4\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} - 2e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} J_0(6\pi) - \right. \right.$$

$$\left. \left. 4e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \sum_{k=1}^{\infty} (-1)^k J_{2k}(6\pi) \right) \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k} \right)$$

$$\frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} =$$

$$\left( 1 + \exp\left( 4\pi \exp\left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \binom{-1/2}{k}}{k!} \right) - 40\pi^2 + \right.$$

$$8 \exp\left( 4\pi \exp\left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \binom{-1/2}{k}}{k!} \right) \pi^2 - 2 \exp\left( \right.$$

$$2\pi \exp\left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \binom{-1/2}{k}}{k!} \left. \right) \sum_{k=0}^{\infty} \frac{(-36)^k \pi^{2k}}{(2k)!} -$$

$$16 \exp\left( 2\pi \exp\left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \binom{-1/2}{k}}{k!} \right)$$

$$\pi^2 \sum_{k=0}^{\infty} \frac{(-36)^k \pi^{2k}}{(2k)!} \left. \right) / \left( 48\pi \exp\left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \sqrt{x} \right.$$

$$\left( 1 + \exp\left( 4\pi \exp\left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \binom{-1/2}{k}}{k!} \right) - \right.$$

$$2 \exp\left( 2\pi \exp\left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \binom{-1/2}{k}}{k!} \right)$$

$$\left. \sum_{k=0}^{\infty} \frac{(-36)^k \pi^{2k}}{(2k)!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \binom{-1/2}{k}}{k!} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$



$$\begin{aligned}
& \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} = \\
& \left( 1 + \exp\left(4\pi \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi}\right]\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) - 40\pi^2 + \\
& 8 \exp\left(4\pi \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi}\right]\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \pi^2 - \\
& 2 \exp\left(2\pi \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi}\right]\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \\
& \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) (6\pi - z_0)^k}{k!} - \\
& 16 \exp\left(2\pi \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi}\right]\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \\
& \pi^2 \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) (6\pi - z_0)^k}{k!} \Bigg/ \\
& \left( 48\pi \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi}\right]\right) \sqrt{x} \left( \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right. \\
& \left. \left( 1 + \exp\left(4\pi \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi}\right]\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) - \right. \\
& \left. 2 \exp\left(2\pi \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi}\right]\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right. \\
& \left. \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) (6\pi - z_0)^k}{k!} \right) \Bigg) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} = \\
& \frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{\left(1 + e^{4\pi\sqrt{3}} + 2e^{2\pi\sqrt{3}} \int_{\frac{\pi}{2}}^{6\pi} \sin(t) dt\right)\sqrt{3}}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} = \\
& \frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{\left(1 + e^{4\pi\sqrt{3}} - 2e^{2\pi\sqrt{3}} (1 - 6\pi \int_0^1 \sin(6\pi t) dt)\right)\sqrt{3}}
\end{aligned}$$

$$\frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} =$$

$$\frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{\sqrt{3} \left( 1 + e^{4\pi\sqrt{3}} - \frac{e^{2\pi\sqrt{3}}\sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\mathcal{A}^{-(9\pi^2)/s+s}}{\sqrt{s}} ds \right)} \quad \text{for } \gamma > 0$$

$$\frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} = \frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} -$$

$$\frac{\pi}{\sqrt{3} \left( 1 + e^{4\pi\sqrt{3}} - \frac{e^{2\pi\sqrt{3}}\sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{3^{-2s}\pi^{-2s}\Gamma(s)}{\Gamma(\frac{1}{2}-s)} ds \right)} \quad \text{for } 0 < \gamma < \frac{1}{2}$$

### Multiple-argument formulas:

$$\frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} =$$

$$\frac{1 + \pi^2 \left( 8 - \frac{48}{(1 + e^{2\pi\sqrt{3}})^2 - 4e^{2\pi\sqrt{3}}\cos^2(3\pi)} \right)}{48\pi\sqrt{3}}$$

$$\frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} =$$

$$\frac{1 + \pi^2 \left( 8 - \frac{48}{(-1 + e^{2\pi\sqrt{3}})^2 + 4e^{2\pi\sqrt{3}}\sin^2(3\pi)} \right)}{48\pi\sqrt{3}}$$

$$\frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} =$$

$$\frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{\left( 1 + e^{4\pi\sqrt{3}} - 2e^{2\pi\sqrt{3}}(-1 + 2\cos^2(3\pi)) \right) \sqrt{3}}$$

From which:

$$2e\left[\frac{1}{(6\pi \times 8\sqrt{3})} + \frac{\pi}{(6\sqrt{3})} - \frac{2\pi}{(2\sqrt{3})} \times \frac{1}{\left(\left(\frac{e^{(4\pi\sqrt{3})}}{e^{(2\pi\sqrt{3})}}\right) - 2e^{(2\pi\sqrt{3})}\right) \cos(6\pi) + 1}\right] - 47 \times \frac{1}{10^3}$$

**Input:**

$$2e\left(\frac{1}{6\pi \times 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - 2 \times \frac{\pi}{2\sqrt{3}} \times \frac{1}{e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1}\right) - 47 \times \frac{1}{10^3}$$

**Exact result:**

$$2e\left(\frac{1}{48\sqrt{3}\pi} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{\sqrt{3}(1 - 2e^{2\sqrt{3}\pi} + e^{4\sqrt{3}\pi})}\right) - \frac{47}{1000}$$

**Decimal approximation:**

1.617287437805556038123015895849855582464794006043936435218...

1.617287437805.... result that is a good approximation to the value of the golden ratio 1.618033988749...

**Alternate forms:**

$$-\frac{47}{1000} + \frac{e}{24\sqrt{3}\pi} + \frac{e\pi}{3\sqrt{3}} - \frac{2e\pi}{\sqrt{3}(1 - 2e^{2\sqrt{3}\pi} + e^{4\sqrt{3}\pi})}$$

$$\frac{e(1 - 2e^{2\sqrt{3}\pi} + e^{4\sqrt{3}\pi} - 40\pi^2 - 16e^{2\sqrt{3}\pi}\pi^2 + 8e^{4\sqrt{3}\pi}\pi^2)\sqrt{3}}{72(e^{2\sqrt{3}\pi} - 1)^2\pi} - \frac{47}{1000}$$

$$-\frac{e\pi}{2\sqrt{3}(e^{\sqrt{3}\pi} - 1)^2} + \frac{e\pi}{2\sqrt{3}(e^{\sqrt{3}\pi} - 1)} - \frac{e\pi}{2\sqrt{3}(1 + e^{\sqrt{3}\pi})^2} -$$

$$\frac{e\pi}{2\sqrt{3}(1 + e^{\sqrt{3}\pi})} + \frac{125\sqrt{3}e - 423\pi + 1000\sqrt{3}e\pi^2}{9000\pi}$$

**Alternative representations:**

$$2e\left(\frac{1}{6\pi \times 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi^2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)}\right) - \frac{47}{10^3} =$$

$$-\frac{47}{10^3} + 2e\left(-\frac{2\pi}{(1 - 2\cosh(-6i\pi)e^{2\pi\sqrt{3}} + e^{4\pi\sqrt{3}})(2\sqrt{3})} + \frac{\pi}{6\sqrt{3}} + \frac{1}{48\pi\sqrt{3}}\right)$$

$$2e \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1))} \right) - \frac{47}{10^3} =$$

$$-\frac{47}{10^3} + 2e \left( -\frac{2\pi}{(1 - 2\cosh(6i\pi)e^{2\pi\sqrt{3}} + e^{4\pi\sqrt{3}})(2\sqrt{3})} + \frac{\pi}{6\sqrt{3}} + \frac{1}{48\pi\sqrt{3}} \right)$$

$$2e \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1))} \right) - \frac{47}{10^3} =$$

$$-\frac{47}{10^3} + 2e \left( -\frac{2\pi}{\left(1 + e^{4\pi\sqrt{3}} - \frac{2e^{2\pi\sqrt{3}}}{\sec(6\pi)}\right)(2\sqrt{3})} + \frac{\pi}{6\sqrt{3}} + \frac{1}{48\pi\sqrt{3}} \right)$$

**Series representations:**

$$2e \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1))} \right) - \frac{47}{10^3} =$$

$$-\frac{47}{1000} + \frac{e}{24\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}} +$$

$$\frac{e\pi}{3\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}} - (2e\pi) / \left( \sqrt{2} \left( 1 + e^{4\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} - \right. \right.$$

$$\left. \left. 2e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \left( J_0(6\pi) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(6\pi) \right) \right) \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)$$

$$\begin{aligned}
& 2e \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) - \frac{47}{10^3} = \\
& -\frac{47}{1000} + \frac{e}{24\pi \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} + \\
& \frac{e\pi}{3 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} - (2e\pi) / \left( \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \right) \\
& \sqrt{x} \left( 1 + \exp\left(4\pi \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) - \\
& 2 \exp\left(2\pi \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \\
& \left( J_0(6\pi) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(6\pi) \right) \\
& \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& 2e \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) - \frac{47}{10^3} = \\
& -\frac{47}{1000} + \frac{e \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} z_0^{1/2 (-1 - \lfloor \arg(3-z_0)/(2\pi) \rfloor)}}{24\pi \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}} + \\
& \frac{e\pi \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} z_0^{1/2 (-1 - \lfloor \arg(3-z_0)/(2\pi) \rfloor)}}{3 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}} - \\
& \left( 2e\pi \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} z_0^{1/2 (-1 - \lfloor \arg(3-z_0)/(2\pi) \rfloor)} \right) / \\
& \left( \left( 1 + \exp\left(4\pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} z_0^{1/2 (1 + \lfloor \arg(3-z_0)/(2\pi) \rfloor)} \right) \right. \right. \\
& \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right) - 2 \exp\left(2\pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} z_0^{1/2 (1 + \lfloor \arg(3-z_0)/(2\pi) \rfloor)} \right) \\
& \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right) \\
& \left( J_0(6\pi) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(6\pi) \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}
\end{aligned}$$

**Integral representations:**

$$2e \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) - \frac{47}{10^3} =$$

$$-\frac{47}{1000} + \frac{e}{24\pi\sqrt{3}} + \frac{e\pi}{3\sqrt{3}} - \frac{2e\pi}{\left(1 + e^{4\pi\sqrt{3}} + 2e^{2\pi\sqrt{3}} \int_{\frac{\pi}{2}}^{6\pi} \sin(t) dt\right)\sqrt{3}}$$

$$2e \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) - \frac{47}{10^3} =$$

$$-\frac{47}{1000} + \frac{e}{24\pi\sqrt{3}} + \frac{e\pi}{3\sqrt{3}} - \frac{2e\pi}{\left(1 + e^{4\pi\sqrt{3}} - 2e^{2\pi\sqrt{3}} (1 - 6\pi \int_0^1 \sin(6\pi t) dt)\right)\sqrt{3}}$$

$$2e \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) - \frac{47}{10^3} = -\frac{47}{1000} +$$

$$\frac{e}{24\pi\sqrt{3}} + \frac{e\pi}{3\sqrt{3}} - \frac{2e\pi}{\sqrt{3} \left(1 + e^{4\pi\sqrt{3}} - \frac{e^{2\pi\sqrt{3}}\sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\mathcal{A}^{-(9\pi^2)/s+s}}{\sqrt{s}} ds\right)} \text{ for } \gamma > 0$$

$$2e \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) - \frac{47}{10^3} =$$

$$-\frac{47}{1000} + \frac{e}{24\pi\sqrt{3}} + \frac{e\pi}{3\sqrt{3}} - \frac{2e\pi}{\sqrt{3} \left(1 + e^{4\pi\sqrt{3}} - \frac{e^{2\pi\sqrt{3}}\sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\varrho^{-s}\pi^{-2s}\Gamma(s)}{\Gamma(\frac{1}{2}-s)} ds\right)} \text{ for } 0 < \gamma < \frac{1}{2}$$

**Multiple-argument formulas:**

$$2e \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) - \frac{47}{10^3} =$$

$$-\frac{47}{1000} + 2e \left( \frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{\left(1 + e^{4\pi\sqrt{3}} - 2e^{2\pi\sqrt{3}} (-1 + 2\cos^2(3\pi))\right)\sqrt{3}} \right)$$







**Decimal approximation:**

9.562315245801503163823373958479389421319547513870580...  $\times 10^{-320}$

9.5623152458...  $\times 10^{-320}$

From which:

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left[ \frac{1}{6\pi \times 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - 2 \times \frac{\pi}{2\sqrt{3}} \times \frac{1}{e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1} \right]$$

**Input:**

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{6\pi \times 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - 2 \times \frac{\pi}{2\sqrt{3}} \times \frac{1}{e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1} \right)$$

**Exact result:**

$$\left( \frac{1}{48\sqrt{3}\pi} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{\sqrt{3}(1 - 2e^{2\sqrt{3}\pi} + e^{4\sqrt{3}\pi})} \right) \left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right)$$

**Decimal approximation:**

1.731835245901638811125418084101596121743662044569116398305...

1.7318352459...  $\approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3})M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

**Alternate forms:**

$$\frac{\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left(1 + \left(8 - \frac{48}{(e^{2\sqrt{3}\pi} - 1)^2}\right)\pi^2\right)}{48\sqrt{3}\pi}$$

$$\frac{(1 - 3\pi + 2\pi^2) \left(1 - 2e^{2\sqrt{3}\pi} + e^{4\sqrt{3}\pi} - 40\pi^2 - 16e^{2\sqrt{3}\pi}\pi^2 + 8e^{4\sqrt{3}\pi}\pi^2\right)\sqrt{3}}{288(e^{2\sqrt{3}\pi} - 1)^2\pi}$$

$$\frac{(\pi - 1)(2\pi - 1) \left( 1 - 2e^{2\sqrt{3}\pi} + e^{4\sqrt{3}\pi} - 40\pi^2 - 16e^{2\sqrt{3}\pi}\pi^2 + 8e^{4\sqrt{3}\pi}\pi^2 \right)}{96\sqrt{3} \left( e^{\sqrt{3}\pi} - 1 \right)^2 \left( 1 + e^{\sqrt{3}\pi} \right)^2 \pi}$$

**Alternative representations:**

$$\left( \frac{1}{2} - \frac{3\pi}{2} + \pi^2 \right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3}) \left( e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}}) \cos(6\pi) + 1 \right)} \right) =$$

$$\left( \frac{1}{2} - \frac{3\pi}{2} + \pi^2 \right) \left( -\frac{2\pi}{\left( 1 - 2 \cosh(-6i\pi) e^{2\pi\sqrt{3}} + e^{4\pi\sqrt{3}} \right) (2\sqrt{3})} + \frac{\pi}{6\sqrt{3}} + \frac{1}{48\pi\sqrt{3}} \right)$$

$$\left( \frac{1}{2} - \frac{3\pi}{2} + \pi^2 \right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3}) \left( e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}}) \cos(6\pi) + 1 \right)} \right) =$$

$$\left( \frac{1}{2} - \frac{3\pi}{2} + \pi^2 \right) \left( -\frac{2\pi}{\left( 1 - 2 \cosh(6i\pi) e^{2\pi\sqrt{3}} + e^{4\pi\sqrt{3}} \right) (2\sqrt{3})} + \frac{\pi}{6\sqrt{3}} + \frac{1}{48\pi\sqrt{3}} \right)$$

$$\left( \frac{1}{2} - \frac{3\pi}{2} + \pi^2 \right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3}) \left( e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}}) \cos(6\pi) + 1 \right)} \right) =$$

$$\left( \frac{1}{2} - \frac{3\pi}{2} + \pi^2 \right) \left( -\frac{2\pi}{\left( 1 + e^{4\pi\sqrt{3}} - \frac{2e^{2\pi\sqrt{3}}}{\sec(6\pi)} \right) (2\sqrt{3})} + \frac{\pi}{6\sqrt{3}} + \frac{1}{48\pi\sqrt{3}} \right)$$

**Series representations:**

$$\begin{aligned}
 & \left( \frac{1}{2} - \frac{3\pi}{2} + \pi^2 \right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi^2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) = \\
 & \left( (-1 + \pi)(-1 + 2\pi) \left( 1 + e^{4\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} - 40\pi^2 + \right. \right. \\
 & \quad 8e^{4\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \pi^2 - 2e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} J_0(6\pi) - \\
 & \quad 16e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \pi^2 J_0(6\pi) - 4e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \sum_{k=1}^{\infty} (-1)^k J_{2k}(6\pi) - \\
 & \quad \left. \left. 32e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \pi^2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(6\pi) \right) \right) / \\
 & \left( 96\pi\sqrt{2} \left( 1 + e^{4\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} - 2e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} J_0(6\pi) - \right. \right. \\
 & \quad \left. \left. 4e^{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \sum_{k=1}^{\infty} (-1)^k J_{2k}(6\pi) \right) \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k} \right)
 \end{aligned}$$

$$\begin{aligned}
& \left( \frac{1}{2} - \frac{3\pi}{2} + \pi^2 \right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) = \\
& \left( (-1 + \pi)(-1 + 2\pi) \right. \\
& \quad \left( 1 + \exp \left[ 4\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) - 40\pi^2 + \\
& \quad 8 \exp \left[ 4\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) \pi^2 - \\
& \quad 2 \exp \left[ 2\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) \\
& \quad \sum_{k=0}^{\infty} \frac{(-36)^k \pi^{2k}}{(2k)!} - \\
& \quad 16 \exp \left[ 2\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) \\
& \quad \left. \pi^2 \sum_{k=0}^{\infty} \frac{(-36)^k \pi^{2k}}{(2k)!} \right) \Bigg/ \left( 96\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \sqrt{x} \right. \\
& \quad \left( 1 + \exp \left[ 4\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) - \\
& \quad 2 \exp \left[ 2\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-36)^k \pi^{2k}}{(2k)!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \Bigg) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{1}{2} - \frac{3\pi}{2} + \pi^2 \right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3}) \left( e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}}) \cos(6\pi) + 1 \right)} \right) = \\
& \left( (-1 + \pi)(-1 + 2\pi) \right. \\
& \quad \left( 1 + \exp \left[ 4\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) - 40\pi^2 + \\
& \quad 8 \exp \left[ 4\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) \pi^2 - \\
& \quad 2 \exp \left[ 2\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) \\
& \quad \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) (6\pi - z_0)^k}{k!} - 16 \exp \left[ 2\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right] \sqrt{x} \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) \pi^2 \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) (6\pi - z_0)^k}{k!} \right) \Big/ \\
& \left( 96\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right) \sqrt{x} \left( \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) \\
& \quad \left( 1 + \exp \left[ 4\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) - \\
& \quad 2 \exp \left[ 2\pi \exp \left( i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) \\
& \quad \left. \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) (6\pi - z_0)^k}{k!} \right) \Big) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& \left( \frac{1}{2} - \frac{3\pi}{2} + \pi^2 \right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3}) \left( e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}}) \cos(6\pi) + 1 \right)} \right) = \\
& \left( \frac{1}{2} - \frac{3\pi}{2} + \pi^2 \right) \left( \frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{\left( 1 + e^{4\pi\sqrt{3}} + 2e^{2\pi\sqrt{3}} \int_{\frac{\pi}{2}}^{6\pi} \sin(t) dt \right) \sqrt{3}} \right)
\end{aligned}$$

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) =$$

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{(1 + e^{4\pi\sqrt{3}} - 2e^{2\pi\sqrt{3}}(1 - 6\pi \int_0^1 \sin(6\pi t) dt))\sqrt{3}} \right)$$

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) =$$

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{\sqrt{3} \left( 1 + e^{4\pi\sqrt{3}} - \frac{e^{2\pi\sqrt{3}}\sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\pi^{-(9\pi^2)/s+s}}{\sqrt{s}} ds \right)} \right) \text{ for } \gamma > 0$$

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) =$$

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{\sqrt{3} \left( 1 + e^{4\pi\sqrt{3}} - \frac{e^{2\pi\sqrt{3}}\sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{3^{-2s}\pi^{-2s}\Gamma(s)}{\Gamma(\frac{1}{2}-s)} ds \right)} \right) \text{ for } 0 < \gamma < \frac{1}{2}$$

**Multiple-argument formulas:**

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi 2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) =$$

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{(1 + e^{4\pi\sqrt{3}} - 2e^{2\pi\sqrt{3}}(-1 + 2\cos^2(3\pi)))\sqrt{3}} \right)$$

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi^2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) =$$

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{(1 + e^{4\pi\sqrt{3}} - 2e^{2\pi\sqrt{3}}(1 - 2\sin^2(3\pi)))\sqrt{3}} \right)$$

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{6\pi 8\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi^2}{(2\sqrt{3})(e^{4\pi\sqrt{3}} - (2e^{2\pi\sqrt{3}})\cos(6\pi) + 1)} \right) =$$

$$\left(\frac{1}{2} - \frac{3\pi}{2} + \pi^2\right) \left( \frac{1}{48\pi\sqrt{3}} + \frac{\pi}{6\sqrt{3}} - \frac{\pi}{(1 + e^{4\pi\sqrt{3}} - 2e^{2\pi\sqrt{3}} T_6(\cos(\pi)))\sqrt{3}} \right)$$

## Observations

Figs.

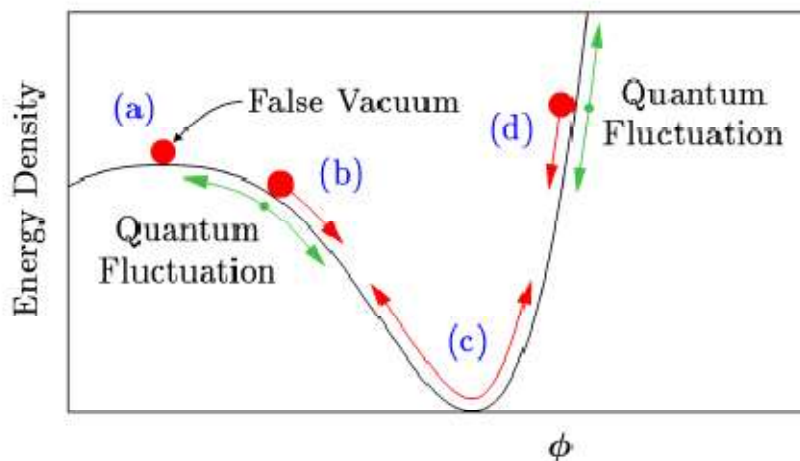
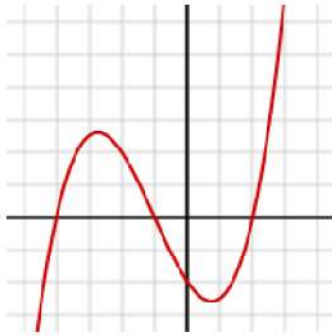


FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field,  $\phi$ . Red arrows show the classical motion of  $\phi$ . When  $\phi$  is near region (a), the energy density will remain nearly constant,  $\rho \cong \rho_f$ , even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of  $\phi$ : Even near regions (b) and (d),  $\phi$  behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of “slow roll,”  $\rho$  remains nearly constant. Only after  $\phi$  has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.



Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at  $y = 0$ ). The case shown has two critical points. Here the function is  $f(x) = (x^3 + 3x^2 - 6x - 8)/4$ .

The ratio between  $M_0$  and  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

i.e. the gravitating mass  $M_0$  and the Wheelerian mass  $q$  of the wormhole, is equal to:

$$\frac{\sqrt{3(2.17049 \times 10^{37})^2 - 0.001^2}}{\frac{1}{2}((3\sqrt{3})(4.2 \times 10^6 \times 1.9891 \times 10^{30}))}$$

1.732050787905194420703947625671018160083566548802082460520...

1.7320507879

$1.7320507879 \approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$  of the wormhole



We note that:

$$\left(-\frac{1}{2} + \frac{i}{2} \sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2} \sqrt{3}\right)$$

$$i\sqrt{3}$$

$i$  is the imaginary unit

$$1.732050807568877293527446341505872366942805253810380628055... i$$

$$r \approx 1.73205 \text{ (radius), } \theta = 90^\circ \text{ (angle)}$$

$$1.73205$$

This result is very near to the ratio between  $M_0$  and  $q$ , that is equal to  $1.7320507879 \approx \sqrt{3}$

With regard  $\sqrt{3}$ , we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result

$$\left(-\frac{1}{2} + \frac{i}{2} \sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2} \sqrt{3}\right) = i\sqrt{3} =$$

$$= 1.732050807568877293527446341505872366942805253810380628055... i$$

$$r \approx 1.73205 \text{ (radius), } \theta = 90^\circ \text{ (angle)}$$

can be related with:

$$u^2(-u)\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right) + v^2(-v)\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right) = q$$

Considering:

$$(-1)\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) - (-1)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) = q$$

$$= i\sqrt{3} = 1.732050807568877293527446341505872366942805253810380628055... i$$

$r \approx 1.73205$  (radius),  $\theta = 90^\circ$  (angle)

Thence:

$$\left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right) \Rightarrow$$

$$\Rightarrow (-1)\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) - (-1)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) = q = 1.73205 \approx \sqrt{3}$$

**We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.**

*From:*

[https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn\\_RpOSvJ1QxWsVLBcJ6KVgd\\_Af\\_hrmDYBNyU8mpSjRs1BDeremA](https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA)

*Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that  $p(9) = 30$ ,  $p(9 + 5) = 135$ ,  $p(9 + 10) = 490$ ,  $p(9 + 15) = 1,575$  and so on are all divisible by 5. Note that here the  $n$ 's come at intervals of five units.*

*Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of  $p(n)$  that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.*

*Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of  $n$ 's separated by  $5^3 = 125$  units, saying that the corresponding  $p(n)$ 's should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field  $\phi$  and a Dirac field  $\psi$ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982 \dots$$

Thence:

$$64g_{22}^{-24} - 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

*That are connected with 64, 128, 256, 512, 1024 and 4096 = 64<sup>2</sup>*

*(Modular equations and approximations to  $\pi$  - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)*

*All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

*In mathematics, the Fibonacci numbers, commonly denoted  $F_n$ , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the  $n$ th Fibonacci number in terms of  $n$  and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as  $n$  increases.*

*Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences*

*The beginning of the sequence is thus:*

*0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...*

*The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

*The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.<sup>[1]</sup> The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

*The sequence of Lucas numbers is:*

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

*All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

*A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:*

*2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).*

*In geometry, a golden spiral is a logarithmic spiral whose growth factor is  $\phi$ , the golden ratio.<sup>[1]</sup> That is, a golden spiral gets wider (or further from its origin) by a factor of  $\phi$  for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies<sup>[3]</sup> - golden spirals are one special case of these logarithmic spirals*

**We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.**

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published: 08 October 2018 - doi: 10.3389/fspas.2018.00035

### **Manuscript Book 1 of Srinivasa Ramanujan**