For all $s \in \mathbb{C}$,

$$\xi(s) = \xi(0) \prod_{\rho} \left(1 - \frac{s}{\rho}\right)$$

where if we combine the factors $\left(1 - \frac{s}{\rho}\right)$ and $\left(1 - \frac{s}{1-\rho}\right)$ the product converges absolutely and uniformly on compact subsets of $\mathbb{C}$.

Also, $\xi(0) = \frac{1}{2}$

$$\xi(s) = \xi(0) \prod_{\rho} \left(1 - \frac{s}{\rho}\right)$$

$$\xi(s) = \xi(0) \prod_{\rho} \left(1 - \frac{s}{\rho}\right)\left(1 - \frac{s}{1-\rho}\right)$$

Let, $\xi(s) = 0$

$$\Rightarrow \quad \xi(0) \prod \left(1 - \frac{s}{\rho}\right)\left(1 - \frac{s}{1-\rho}\right) = 0$$

$$\Rightarrow \quad \frac{1}{2} \prod \left(1 - \frac{s}{\rho}\right)\left(1 - \frac{s}{1-\rho}\right) = 0$$

$$\Rightarrow \quad \prod \left[1 - \frac{s}{1-\rho} - \frac{s}{\rho} + \frac{s^2}{(1-\rho)\rho}\right] = 0$$

$$\Rightarrow \quad \prod 1 + \frac{[-s\rho - s(1-\rho) + s^2]}{\rho(1-\rho)} = 0$$

$$\Rightarrow \quad \prod 1 + \frac{(-s\rho - s + s\rho + s^2)}{\rho(1-\rho)} = 0$$

$$\Rightarrow \prod_{\rho} 1 + \frac{s^2 - s}{\rho(1-\rho)} = 0$$

$$\Rightarrow \prod_{\rho} 1 - \frac{s(1-s)}{\rho(1-\rho)} = 0$$

$$\because \left| \prod 1 - \frac{s(1-s)}{\rho(1-\rho)} \right| < \infty$$

$$\therefore \prod_{\rho} 1 - \frac{s(1-s)}{\rho(1-\rho)} = 0 \Rightarrow 1 - \frac{s(1-s)}{\rho_0(1-\rho_0)} = 0 \text{ for some } \rho_0 \in \mathbb{C}$$

let $s = \sigma + it$.

$\rho_0 = a_0 + i b_0$

$$1 - \frac{s(1-s)}{\rho_0(1-\rho_0)} = 0$$

$$\Rightarrow \rho_0(1-\rho_0) - s(1-s) = 0$$

$$\Rightarrow (a_0 + i b_0)(1 - a_0 - i b_0) - (\sigma + it)(1 - \sigma - it) = 0$$

$$\Rightarrow \left[a_0(1-a_0) + b_0^2\right] + i\left[b_0(1-a_0) - a_0 b_0\right] - \left[\sigma(1-\sigma) + t^2 + i\{t(1-\sigma) - t\sigma\}\right] = 0$$

$$\Rightarrow \left[a_0(1-a_0) - \sigma(1-\sigma) + b_0^2 - t^2\right] + i\left[b_0(1-2a_0) - t(1-2\sigma)\right] = 0$$

Equating Real & Imaginary parts to zero,

$$a_0(1-a_0) - \sigma(1-\sigma) + b_0^2 t^2 = 0 \quad \text{--- (1)}$$

$$b_0(1-2a_0) - t(1-2\sigma) = 0 \quad \text{--- (2)}$$

We discuss 2 cases $\rightarrow a_0 = \frac{1}{2}$ & $a_0 \neq \frac{1}{2}$

Case 1:- $a_0 = \frac{1}{2}$

(2) $\Rightarrow \quad t(2\sigma-1) = 0$

$\Rightarrow \quad t = 0 \quad$ or $\quad 2\sigma - 1 = 0$

Putting $t=0$ in (1), $\quad$ or $\quad \boxed{\sigma = \frac{1}{2}}$

$a_0(1-a_0) - \sigma(1-\sigma) + b_0^2 = 0$

$\because a_0 = \frac{1}{2} \quad$ or $\quad \sigma = \frac{1}{2}$

$\frac{1}{4} - \sigma(1-\sigma) + b_0^2 = 0 \quad$ or $\quad \sigma = \frac{1}{2}$

$\Rightarrow \sigma^2 - \sigma + \frac{1}{4} + b_0^2 = 0 \quad$ or $\quad \sigma = \frac{1}{2}$

$\Rightarrow \left(\sigma - \frac{1}{2}\right)^2 + b_0^2 = 0 \quad$ or $\quad \sigma = \frac{1}{2}$

$\Rightarrow \left(\sigma - \frac{1}{2}\right)^2 = 0$ & $b_0^2 = 0 \quad$ or $\quad \sigma = \frac{1}{2}$

$\Rightarrow \boxed{\sigma = \frac{1}{2}}$ & $b_0 = 0 \quad$ or $\quad \boxed{\sigma = \frac{1}{2}}$

Case 2 $\boxed{a_0 \neq \frac{1}{2}}$

$$② \Rightarrow b_0(1-2a_0) - t(1-2\sigma) = 0$$

$$\Rightarrow b_0 = \frac{t(1-2\sigma)}{1-2a_0}, \quad a_0 \neq \frac{1}{2}$$

Putting $b_0 = \frac{t(1-2\sigma)}{1-2a_0}$ in ①

$$\Rightarrow a_0(1-a_0) - \sigma(1-\sigma) + \frac{t^2(1-2\sigma)^2}{(1-2a_0)^2} - t^2 = 0$$

$$\Rightarrow (1-2a_0)^2\left[a_0 - a_0^2 - \sigma + \sigma^2\right] + t^2\left[(1-2\sigma)^2 - (1-2a_0)^2\right] = 0$$

$$\Rightarrow (1-2a_0)^2\left[(a_0-\sigma) - (a_0^2-\sigma^2)\right] + t^2\left[(2-2\sigma-2a_0)(2a_0-2\sigma)\right] = 0$$

$$\Rightarrow (1-2a_0)^2\left[(a_0-\sigma)(1-a_0-\sigma)\right] + 4t^2(1-\sigma-a_0)(a_0-\sigma) = 0$$

$$\Rightarrow (a_0-\sigma)(1-a_0-\sigma)\left[(1-2a_0)^2 + 4t^2\right] = 0$$

$$\Rightarrow (a_0-\sigma) = 0 \text{ or } 1-a_0-\sigma = 0 \text{ or } (1-2a_0)^2 + 4t^2 = 0$$

$$\Rightarrow a_0 = \sigma \text{ or } a_0 = 1-\sigma \text{ or } 1-2a_0 = 0,\ a_0 = \frac{1}{2}$$

$\sigma = a_0$ or $\sigma = 1 - a_0$ or (Contradiction $\because a_0 \neq \frac{1}{2}$)

$\boxed{\sigma = a_0 \neq \frac{1}{2}}$ or $\sigma = 1 - a_0$

$\because a_0 \neq \frac{1}{2}$

$-a_0 \neq -\frac{1}{2}$

$\sigma = 1 - a_0 \neq 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow \boxed{\sigma \neq \frac{1}{2}}$

Thus if $a_0 = \frac{1}{2}$ Riemann Hypothesis is true i.e $\sigma = \frac{1}{2}$

& if $a_0 \neq \frac{1}{2}$, Riemann Hypothesis is not true i.e $\sigma \neq \frac{1}{2}$