The Goldbach Conjecture

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1 Introduction

The Goldbach Conjecture states that every even number can be represented as the sum of two primes. The following is a proof by induction of this conjecture. Mathematical induction can be used to prove that a statement, f(n), holds for all natural numbers n. If I were to show the base case(s) and the inductive step to be true then I have proven the conjecture as shown below.

2 Proof by Induction

PROPOSITION: Every even number 2n can be represented as the sum of two primes; p, q. n $\in \mathbb{N}$; p, q $\in \mathbb{P}$. (n is natural; p, q are primes). Let $f(n)$ denote the nth Goldbach number/even number then equations can be formed as follows:

$$f(n) = 2n = p + q.$$

$$f(n+1) = 2n + 2 = p + q + 2$$

BASE CASES:

The following equations are the sums of two primes; $f(n)$, $f(n+1)$:

$$f(3) = 6 = 3 + 3$$

$$f(4) = 8 = (3 + 3) + 2 = 3 + 3$$

INDUCTIVE STEP:

If we show that $f(n+2)$ is the sum of two primes based on $f(n)$, $f(n+1)$ or any true mathematical axiom then the proposition is true for any $(f(n+2) = 2n + 4) \geq 10$. We have:

$$f(n) = 2n = p + q$$

$$f(n+1) = 2n + 2 = p + q + 2$$

$$f(n+2) = 2n + 4 = p + q + 4$$

$$\Rightarrow f(n+2) = (f(n+1) - p) + (p + 2)$$

$$\Rightarrow f(n+2) = (f(n+1) - p) + (f(n+1) - q)$$

We know that $f(n+1)-\ p=q+2=3+2=5\ (n=3,\ q=3,\ p=3)$ is prime and $f(n+1)-q=p+2=3+2=5\ (n=3,\ p=3,\ q=3)$ is prime due to the base cases listed above. This shows $f(n+2)$ can be represented as the sum of two primes. It is also true that every even number $2n\geq 4$ can be represented in form $f(n+2)=2n+4$ therefore, by induction every even number of form $f(n+2)=2n+4$ which is greater than the base cases $f(n),\ f(n+1)$ can be represented as the sum of two primes. The remaining trivial case $f(2)=4=2+2$ is the sum of two primes.

This paper wholly proves all numbers greater than or equal to four can be represented as the sum of two primes.

QED
