

# CONJECTURES ABOUT MIN/MAX PRIME DIVISOR OF THE NATURAL NUMBERS AND PRIME NUMBERS

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*Applying maxrad,minrad or its difference to the sequence of natural numbers and taking the differences of this sequence you can split this in subsequences by the sum equals length rule so that you can associate each subsequence to each gap between consecutive prime numbers*

## Premise

All this research has been done using Mathematica©, a symbolic computation environment that uses a programming language called Wolfram Language.

## Some useful function

Define rad(n) as the product of all prime numbers that are divisors of n. Now define minrad(n) as the lower of such prime numbers, maxrad(n) the greater of such prime numbers, minmaxdiff(n) the difference of the twos if positive, minrad otherwise, minmaxamean1 the floor of the arithmetic mean of the twos and minmaxamean2 the ceiling of the arithmetic mean, minmaxgmean1 the floor of the geometric mean of the twos and minmaxgmean2 the ceiling of the geometric mean:

```
In[190]:= rad[n_Integer] := Times@@(First/@FactorInteger[n])
minrad[n_Integer] := Min[First/@FactorInteger[n]]
maxrad[n_Integer] := Max[First/@FactorInteger[n]]
minmaxdiff[n_Integer] := If[minrad[n] != maxrad[n],
    maxrad[n] - minrad[n],
    minrad[n]
]
minmaxamean1[n_Integer] := Floor[(minrad[n] + maxrad[n]) / 2]
minmaxamean2[n_Integer] := Ceiling[(minrad[n] + maxrad[n]) / 2]
minmaxgmean1[n_Integer] := Floor[Sqrt[maxrad[n] * minrad[n]]]
minmaxgmean2[n_Integer] := Ceiling[Sqrt[maxrad[n] * minrad[n]]]
```

Above function FactorInteger(n) returns a list of couples. Each couple is done by a prime number p and an exponent number e where  $p^e$  is a divisor of n but not  $p^{(e+1)}$ . As an example:

```
In[146]:= FactorInteger[60]
```

```
Out[146]= {{2, 2}, {3, 1}, {5, 1}}
```

Now apply the functions to the natural number starting with 2:

```
In[147]:= r1=minrad/@Range[2,300]
r2=maxrad/@Range[2,300]
r3=minmaxdiff/@Range[2,300]
r4=minmaxamean1/@Range[2,300]
r5=minmaxamean2/@Range[2,300]
r6=minmaxgmean1/@Range[2,300]
r7=minmaxgmean2/@Range[2,300]
```

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Out[147]= {2, 3, 2, 5, 2, 7, 2, 3, 2, 11, 2, 13, 2, 3, 2, 17, 2, 19, 2, 3, 2, 23, 2, 5, 2, 3, 2, 29, 2,
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3, 2, 89, 2, 7, 2, 3, 2, 5, 2, 97, 2, 3, 2, 101, 2, 103, 2, 3, 2, 107, 2, 109, 2, 3, 2, 113,
2, 5, 2, 3, 2, 7, 2, 11, 2, 3, 2, 5, 2, 127, 2, 3, 2, 131, 2, 7, 2, 3, 2, 137, 2, 139, 2,
3, 2, 11, 2, 5, 2, 3, 2, 149, 2, 151, 2, 3, 2, 5, 2, 157, 2, 3, 2, 7, 2, 163, 2, 3, 2, 167,
2, 13, 2, 3, 2, 173, 2, 5, 2, 3, 2, 179, 2, 181, 2, 3, 2, 5, 2, 11, 2, 3, 2, 191, 2, 193,
2, 3, 2, 197, 2, 199, 2, 3, 2, 7, 2, 5, 2, 3, 2, 11, 2, 211, 2, 3, 2, 5, 2, 7, 2, 3, 2, 13,
2, 223, 2, 3, 2, 227, 2, 229, 2, 3, 2, 233, 2, 5, 2, 3, 2, 239, 2, 241, 2, 3, 2, 5, 2, 13,
2, 3, 2, 251, 2, 11, 2, 3, 2, 257, 2, 7, 2, 3, 2, 263, 2, 5, 2, 3, 2, 269, 2, 271, 2, 3,
2, 5, 2, 277, 2, 3, 2, 281, 2, 283, 2, 3, 2, 7, 2, 17, 2, 3, 2, 293, 2, 5, 2, 3, 2, 13, 2}
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11, 7, 19, 29, 59, 5, 61, 31, 7, 2, 13, 11, 67, 17, 23, 7, 71, 3, 73, 37, 5, 19, 11, 13,
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103, 13, 7, 53, 107, 3, 109, 11, 37, 7, 113, 19, 23, 29, 13, 59, 17, 5, 11, 61, 41, 31,
5, 7, 127, 2, 43, 13, 131, 11, 19, 67, 5, 17, 137, 23, 139, 7, 47, 71, 13, 3, 29, 73, 7,
37, 149, 5, 151, 19, 17, 11, 31, 13, 157, 79, 53, 5, 23, 3, 163, 41, 11, 83, 167, 7, 13,
17, 19, 43, 173, 29, 7, 11, 59, 89, 179, 5, 181, 13, 61, 23, 37, 31, 17, 47, 7, 19, 191,
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47, 59, 79, 17, 239, 5, 241, 11, 3, 61, 7, 41, 19, 31, 83, 5, 251, 7, 23, 127, 17, 2, 257,
43, 37, 13, 29, 131, 263, 11, 53, 19, 89, 67, 269, 5, 271, 17, 13, 137, 11, 23, 277, 139,
31, 7, 281, 47, 283, 71, 19, 13, 41, 3, 17, 29, 97, 73, 293, 7, 59, 37, 11, 149, 23, 5}
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Out[149]= {2, 3, 2, 5, 1, 7, 2, 3, 3, 11, 1, 13, 5, 2, 2, 17, 1, 19, 3, 4, 9, 23, 1, 5, 11, 3, 5, 29, 3, 31, 2,
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3, 61, 29, 4, 2, 8, 9, 67, 15, 20, 5, 71, 1, 73, 35, 2, 17, 4, 11, 79, 3, 3, 39, 83, 5, 12, 41,
26, 9, 89, 3, 6, 21, 28, 45, 14, 1, 97, 5, 8, 3, 101, 15, 103, 11, 4, 51, 107, 1, 109, 9, 34,
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21, 32, 29, 6, 45, 4, 17, 191, 1, 193, 95, 10, 5, 197, 9, 199, 3, 64, 99, 22, 15, 36, 101, 20,
11, 8, 5, 211, 51, 68, 105, 38, 1, 24, 107, 70, 9, 4, 35, 223, 5, 2, 111, 227, 17, 229, 21,
8, 27, 233, 11, 42, 57, 76, 15, 239, 3, 241, 9, 3, 59, 2, 39, 6, 29, 80, 3, 251, 5, 12, 125,
14, 2, 257, 41, 30, 11, 26, 129, 263, 9, 48, 17, 86, 65, 269, 3, 271, 15, 10, 135, 6, 21, 277,
137, 28, 5, 281, 45, 283, 69, 16, 11, 34, 1, 17, 27, 94, 71, 293, 5, 54, 35, 8, 147, 10, 3}
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Out[150]= {2, 3, 2, 5, 2, 7, 2, 3, 3, 11, 2, 13, 4, 4, 2, 17, 2, 19, 3, 5, 6, 23, 2, 5, 7, 3, 4, 29, 3, 31, 2,
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16, 6, 89, 3, 10, 12, 17, 24, 12, 2, 97, 4, 7, 3, 101, 9, 103, 7, 5, 27, 107, 2, 109, 6, 20,
4, 113, 10, 14, 15, 8, 30, 12, 3, 11, 31, 22, 16, 5, 4, 127, 2, 23, 7, 131, 6, 13, 34, 4, 9,
137, 12, 139, 4, 25, 36, 12, 2, 17, 37, 5, 19, 149, 3, 151, 10, 10, 6, 18, 7, 157, 40, 28,
3, 15, 2, 163, 21, 7, 42, 167, 4, 13, 9, 11, 22, 173, 15, 6, 6, 31, 45, 179, 3, 181, 7, 32,
12, 21, 16, 14, 24, 5, 10, 191, 2, 193, 49, 8, 4, 197, 6, 199, 3, 35, 51, 18, 9, 23, 52, 13,
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13, 21, 17, 14, 25, 5, 11, 191, 3, 193, 50, 8, 5, 197, 7, 199, 4, 35, 52, 18, 10, 23, 53,
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13, 7, 16, 233, 8, 26, 31, 41, 10, 239, 4, 241, 7, 3, 32, 6, 22, 16, 17, 43, 4, 251, 5, 17,
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Out[153]= {2, 3, 2, 5, 3, 7, 2, 3, 4, 11, 3, 13, 4, 4, 2, 17, 3, 19, 4, 5, 5, 23, 3, 5, 6, 3, 4, 29, 4,
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8, 2, 257, 10, 17, 6, 10, 17, 263, 5, 17, 7, 17, 12, 269, 4, 271, 6, 7, 17, 8, 7, 277,
17, 10, 4, 281, 10, 283, 12, 8, 6, 17, 3, 17, 8, 18, 13, 293, 4, 18, 9, 6, 18, 18, 4}

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These sequences appear quite unpredictable but more can be conjectured on the difference sequence. The operator difference simply substitute an item in the sequence with the difference between the following item and the item itself for every item; it's like a derivative of the sequence.

In[154]=

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s1=Differences [r1]
s2=Differences [r2]
s3=Differences [r3]
s4=Differences [r4]
s5=Differences [r5]
s6=Differences [r6]
s7=Differences [r7]

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Out[154]= {1, -1, 3, -3, 5, -5, 1, -1, 9, -9, 11, -11, 1, -1, 15, -15, 17, -17, 1, -1, 21, -21, 3, -3,
1, -1, 27, -27, 29, -29, 1, -1, 3, -3, 35, -35, 1, -1, 39, -39, 41, -41, 1, -1, 45, -45,
5, -5, 1, -1, 51, -51, 3, -3, 1, -1, 57, -57, 59, -59, 1, -1, 3, -3, 65, -65, 1, -1, 69,
-69, 71, -71, 1, -1, 5, -5, 77, -77, 1, -1, 81, -81, 3, -3, 1, -1, 87, -87, 5, -5, 1,
-1, 3, -3, 95, -95, 1, -1, 99, -99, 101, -101, 1, -1, 105, -105, 107, -107, 1, -1, 111,
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-1, 135, -135, 137, -137, 1, -1, 9, -9, 3, -3, 1, -1, 147, -147, 149, -149, 1, -1, 3, -3,
155, -155, 1, -1, 5, -5, 161, -161, 1, -1, 165, -165, 11, -11, 1, -1, 171, -171, 3, -3,
1, -1, 177, -177, 179, -179, 1, -1, 3, -3, 9, -9, 1, -1, 189, -189, 191, -191, 1, -1,
195, -195, 197, -197, 1, -1, 5, -5, 3, -3, 1, -1, 9, -9, 209, -209, 1, -1, 3, -3, 5, -5,
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237, -237, 239, -239, 1, -1, 3, -3, 11, -11, 1, -1, 249, -249, 9, -9, 1, -1, 255, -255,
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Out[155]= {1, -1, 3, -2, 4, -5, 1, 2, 6, -8, 10, -6, -2, -3, 15, -14, 16, -14, 2, 4, 12, -20, 2, 8, -10,
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-2, 12, -4, 40, -50, 8, -4, 12, 10, 30, -54, 56, -30, -24, -5, 11, -2, 56, -50, 6, -16, 64,
-68, 70, -36, -32, 14, -8, 2, 66, -74, -2, 38, 42, -76, 10, 26, -14, -18, 78, -84, 8, 10,
8, 16, -28, -16, 94, -90, 4, -6, 96, -84, 86, -90, -6, 46, 54, -104, 106, -98, 26, -30,
106, -94, 4, 6, -16, 46, -42, -12, 6, 50, -20, -10, -26, 2, 120, -125, 41, -30, 118, -120,
8, 48, -62, 12, 120, -114, 116, -132, 40, 24, -58, -10, 26, 44, -66, 30, 112, -144, 146,
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-12, 204, -158, 18, 36, -64, -40, 28, 78, -36, -62, 6, 20, 186, -216, -2, 108, 114, -208,
210, -206, -12, 18, 204, -220, 34, 12, 20, -62, 222, -234, 236, -230, -8, 58, -54, 34, -22,
12, 52, -78, 246, -244, 16, 104, -110, -15, 255, -214, -6, -24, 16, 102, 132, -252, 42,
-34, 70, -22, 202, -264, 266, -254, -4, 124, -126, 12, 254, -138, -108, -24, 274, -234,
236, -212, -52, -6, 28, -38, 14, 12, 68, -24, 220, -286, 52, -22, -26, 138, -126, -18}

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Out[156]= {1, -1, 3, -4, 6, -5, 1, 0, 8, -10, 12, -8, -3, 0, 15, -16, 18, -16, 1, 5, 14, -22, 4, 6, -8, 2,
24, -26, 28, -29, 6, 7, -13, -1, 36, -20, -7, -7, 38, -36, 38, -34, -7, 19, 26, -46, 6,
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108, -96, 1, 9, -17, 47, -47, -7, 8, 48, -21, -9, -24, 0, 122, -125, 38, -29, 120, -122,
3, 53, -63, 13, 122, -116, 118, -134, 39, 25, -67, -1, 23, 47, -67, 31, 114, -146, 148,
-134, -3, -5, 17, -15, 146, -80, -27, -47, 13, -15, 162, -124, -31, 73, 86, -162, 8, 2,
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174, -190, 192, -98, -85, -5, 192, -188, 190, -196, 61, 35, -77, -7, 21, 65, -81, -9, -3,
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212, -208, -13, 19, 206, -222, 31, 15, 19, -61, 224, -236, 238, -232, -6, 56, -57, 37, -33,
23, 51, -77, 248, -246, 7, 113, -111, -12, 255, -216, -11, -19, 15, 103, 134, -254, 39,
-31, 69, -21, 204, -266, 268, -256, -5, 125, -129, 15, 256, -140, -109, -23, 276, -236,
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-4, 2, 11, 151, -158, -9, 0, 25, 14, 134, -176, 178, -174, 25, -20, 9, -5, -2, 10, -19,
5, 181, -189, 191, -144, -41, -4, 193, -191, 193, -196, 32, 16, -33, -9, 14, 29, -39,
-6, 8, -11, 207, -184, 10, 17, -30, -22, 17, 36, -17, -32, 9, 4, 204, -219, 0, 53, 170,
-217, 219, -217, -5, 8, 218, -226, 19, 4, 11, -32, 230, -236, 238, -235, -3, 28, -25,
15, -5, 0, 27, -40, 248, -247, 13, 47, -54, -8, 255, -235, 0, -15, 9, 50, 197, -257,
23, -19, 36, -12, 235, -266, 268, -262, -1, 61, -61, 4, 265, -207, -53, -13, 277, -257,
259, -247, -25, -4, 17, -22, 15, -2, 35, -13, 256, -289, 28, -13, -12, 68, -57, -15}
```

```
Out[158]= {1, -1, 3, -2, 4, -5, 1, 1, 7, -8, 10, -8, -1, -2, 15, -14, 16, -15, 1, 2, 16, -20, 2, 3, -5,
2, 24, -25, 27, -29, 5, 3, -4, -3, 34, -26, -3, -4, 37, -36, 38, -36, -3, 9, 34, -44, 4,
-3, 6, -2, 45, -50, 5, -3, 6, 5, 43, -55, 57, -44, -12, -3, 7, -2, 60, -57, 3, -8, 66,
-68, 70, -53, -16, 7, -2, -1, 71, -75, -1, 19, 61, -78, 6, 12, -7, -9, 82, -85, 6, 3,
4, 8, -13, -9, 94, -92, 2, -3, 97, -91, 93, -95, -3, 23, 79, -104, 106, -102, 13, -15,
108, -102, 3, 2, -8, 23, -19, -8, 7, 21, -10, -5, -12, 0, 122, -125, 21, -15, 123, -124,
6, 22, -31, 6, 127, -124, 126, -134, 20, 12, -25, -9, 14, 21, -33, 15, 129, -145, 147,
-140, -1, -3, 11, -10, 149, -116, -13, -24, 11, -12, 160, -141, -15, 36, 124, -162, 8,
-3, 1, 12, 150, -157, -10, 1, 24, 15, 133, -175, 177, -173, 24, -19, 8, -4, -3, 11, -20,
6, 180, -188, 190, -143, -42, -3, 192, -190, 192, -195, 31, 17, -34, -8, 13, 30, -40,
-5, 7, -10, 206, -183, 9, 18, -31, -21, 16, 37, -18, -31, 8, 5, 203, -218, -1, 54, 169,
-216, 218, -216, -6, 9, 217, -225, 18, 5, 10, -31, 229, -235, 237, -234, -4, 29, -26,
16, -6, 1, 26, -39, 247, -246, 12, 48, -55, -8, 255, -234, -1, -14, 8, 51, 196, -256,
22, -18, 35, -11, 234, -265, 267, -261, -2, 62, -62, 5, 264, -206, -54, -12, 276, -256,
258, -246, -26, -3, 16, -21, 14, -1, 34, -12, 255, -288, 27, -12, -13, 69, -58, -14}
```

```
Out[159]= {1, -1, 3, -3, 5, -5, 1, 0, 8, -9, 11, -10, 0, -1, 15, -15, 17, -16, 1, 0, 19, -21, 3, 0, -2,
0, 26, -26, 28, -29, 3, 0, 0, -3, 35, -31, 0, -3, 38, -38, 40, -39, -1, 3, 41, -45, 5, -4,
4, -2, 48, -51, 5, -4, 4, 0, 52, -56, 58, -54, -3, -2, 6, -4, 63, -62, 3, -5, 68, -69, 71,
-65, -5, 3, 2, -3, 74, -76, 0, 6, 74, -80, 6, 0, 0, -5, 85, -86, 6, -3, 3, 0, 0, -7, 95,
-94, 2, -2, 98, -96, 98, -98, -1, 6, 97, -105, 107, -105, 6, -7, 110, -107, 4, -3, -1,
4, 0, -7, 8, 0, 0, -4, -2, -2, 124, -125, 9, -6, 126, -127, 7, 0, -8, 2, 132, -131, 133,
-136, 8, 0, 0, -9, 10, 0, -8, 4, 141, -146, 148, -145, 1, -3, 8, -7, 152, -145, 0, -9, 9,
-10, 161, -154, -4, 7, 155, -164, 10, -8, 2, 2, 164, -166, -2, -1, 9, 0, 166, -176, 178,
-176, 8, -7, 7, -6, 6, -4, -5, 2, 185, -189, 191, -180, -7, -3, 194, -193, 195, -196,
11, 0, 0, -9, 9, 0, -6, -3, 9, -11, 208, -201, 4, 0, 0, -12, 12, 0, 0, -10, 10, -6, 215,
-220, 0, 12, 212, -221, 223, -223, -1, 2, 226, -228, 10, -5, 5, -10, 234, -236, 238,
-237, -1, 8, -6, 4, 6, -8, 8, -12, 248, -248, 12, 0, -8, -5, 255, -248, 7, -11, 4, 7,
247, -259, 12, -10, 10, -5, 258, -266, 268, -266, 1, 10, -9, -1, 271, -261, -7, -6, 278,
-272, 274, -272, -4, -2, 11, -14, 15, -10, 10, -5, 281, -290, 14, -9, -3, 12, 0, -14}
```

```
Out[160]= {1, -1, 3, -2, 4, -5, 1, 1, 7, -8, 10, -9, 0, -2, 15, -14, 16, -15, 1, 0, 18, -20, 2, 1, -3, 1,
25, -25, 27, -29, 4, 0, 0, -3, 34, -30, 0, -3, 37, -37, 39, -38, -1, 3, 40, -44, 4, -3,
4, -2, 47, -50, 5, -4, 4, 0, 51, -55, 57, -53, -3, -3, 7, -4, 62, -61, 3, -5, 67, -68,
70, -64, -5, 3, 2, -3, 73, -75, -1, 7, 73, -79, 6, 0, 0, -5, 84, -85, 6, -3, 3, 0, 0, -7,
94, -93, 2, -2, 97, -95, 97, -97, -1, 6, 96, -104, 106, -104, 6, -7, 109, -106, 4, -3,
-1, 4, 0, -7, 7, 1, 0, -4, -3, -1, 123, -125, 10, -6, 125, -126, 7, 0, -8, 2, 131, -130,
132, -135, 8, 0, 0, -9, 10, 0, -8, 4, 140, -145, 147, -144, 1, -3, 8, -7, 151, -144, 0,
-9, 9, -10, 160, -153, -4, 7, 154, -163, 9, -7, 2, 2, 163, -165, -2, -1, 9, 0, 165, -175,
177, -175, 8, -7, 7, -6, 6, -4, -5, 2, 184, -188, 190, -179, -7, -3, 193, -192, 194,
-195, 11, 0, 0, -9, 9, 0, -6, -3, 9, -11, 207, -200, 4, 0, 0, -12, 12, 0, 0, -10, 10, -6,
214, -219, 0, 12, 211, -220, 222, -222, -1, 2, 225, -227, 10, -5, 5, -10, 233, -235,
237, -236, -2, 9, -6, 4, 6, -8, 8, -12, 247, -247, 12, 0, -8, -6, 255, -247, 7, -11, 4,
7, 246, -258, 12, -10, 10, -5, 257, -265, 267, -265, 1, 10, -9, -1, 270, -260, -7, -6,
277, -271, 273, -271, -4, -2, 11, -14, 14, -9, 10, -5, 280, -289, 14, -9, -3, 12, 0, -14}
```

At the first sight it seems that this step added more variability with the introduction of negative items. Now apply a simple rule to each sequence. **Collect items in a subsequence until the sum of items equals its length so write it and restart with a new one.** This algorithm can be described with this function:

```
In[161]:= splitseq[l_List] := Module[{i, parz = {}, res = {}},
  For[i = 1, i <= Length[l], i++,
    AppendTo[parz, l[[i]]];
    If[Total[parz] == Length[parz],
      AppendTo[res, parz];
      parz = {}
    ]
  ];
  Return[res]
]
```

So you can apply the above function to the sequences and you will get:

```
In[162]:= t1=splitseq[s1]
t2=splitseq[s2]
t3=splitseq[s3]
t4=splitseq[s4]
t5=splitseq[s5]
t6=splitseq[s6]
t7=splitseq[s7]
```

```
Out[162]= {{1}, {-1, 3}, {-3, 5}, {-5, 1, -1, 9}, {-9, 11}, {-11, 1, -1, 15}, {-15, 17}, {-17, 1, -1, 21},
{-21, 3, -3, 1, -1, 27}, {-27, 29}, {-29, 1, -1, 3, -3, 35}, {-35, 1, -1, 39},
{-39, 41}, {-41, 1, -1, 45}, {-45, 5, -5, 1, -1, 51}, {-51, 3, -3, 1, -1, 57},
{-57, 59}, {-59, 1, -1, 3, -3, 65}, {-65, 1, -1, 69}, {-69, 71}, {-71, 1, -1, 5, -5, 77},
{-77, 1, -1, 81}, {-81, 3, -3, 1, -1, 87}, {-87, 5, -5, 1, -1, 3, -3, 95},
{-95, 1, -1, 99}, {-99, 101}, {-101, 1, -1, 105}, {-105, 107}, {-107, 1, -1, 111},
{-111, 3, -3, 1, -1, 5, -5, 9, -9, 1, -1, 3, -3, 125}, {-125, 1, -1, 129},
{-129, 5, -5, 1, -1, 135}, {-135, 137}, {-137, 1, -1, 9, -9, 3, -3, 1, -1, 147},
{-147, 149}, {-149, 1, -1, 3, -3, 155}, {-155, 1, -1, 5, -5, 161}, {-161, 1, -1, 165},
{-165, 11, -11, 1, -1, 171}, {-171, 3, -3, 1, -1, 177}, {-177, 179},
{-179, 1, -1, 3, -3, 9, -9, 1, -1, 189}, {-189, 191}, {-191, 1, -1, 195}, {-195, 197},
{-197, 1, -1, 5, -5, 3, -3, 1, -1, 9, -9, 209}, {-209, 1, -1, 3, -3, 5, -5, 1, -1, 11, -11, 221},
{-221, 1, -1, 225}, {-225, 227}, {-227, 1, -1, 231}, {-231, 3, -3, 1, -1, 237},
{-237, 239}, {-239, 1, -1, 3, -3, 11, -11, 1, -1, 249}, {-249, 9, -9, 1, -1, 255},
{-255, 5, -5, 1, -1, 261}, {-261, 3, -3, 1, -1, 267}, {-267, 269}, {-269, 1, -1, 3, -3, 275},
{-275, 1, -1, 279}, {-279, 281}, {-281, 1, -1, 5, -5, 15, -15, 1, -1, 291}}
```

```
Out[163]= {{1}, {-1, 3}, {-2, 4}, {-5, 1, 2, 6}, {-8, 10}, {-6, -2, -3, 15}, {-14, 16}, {-14, 2, 4, 12},
{-20, 2, 8, -10, 4, 22}, {-24, 26}, {-29, 9, 6, -10, -4, 34}, {-18, -6, -8, 36}, {-34, 36},
{-32, -6, 18, 24}, {-44, 4, -2, 12, -4, 40}, {-50, 8, -4, 12, 10, 30}, {-54, 56},
{-30, -24, -5, 11, -2, 56}, {-50, 6, -16, 64}, {-68, 70}, {-36, -32, 14, -8, 2, 66},
{-74, -2, 38, 42}, {-76, 10, 26, -14, -18, 78}, {-84, 8, 10, 8, 16, -28, -16, 94},
{-90, 4, -6, 96}, {-84, 86}, {-90, -6, 46, 54}, {-104, 106}, {-98, 26, -30, 106},
{-94, 4, 6, -16, 46, -42, -12, 6, 50, -20, -10, -26, 2, 120}, {-125, 41, -30, 118},
{-120, 8, 48, -62, 12, 120}, {-114, 116}, {-132, 40, 24, -58, -10, 26, 44, -66, 30, 112},
{-144, 146}, {-132, -2, -6, 20, -18, 144}, {-78, -26, -48, 18, -20, 160},
{-122, -30, 72, 84}, {-160, 6, 4, 2, 24, 130}, {-144, -22, 4, 48, 30, 90}, {-174, 176},
{-168, 48, -38, 14, -6, -14, 30, -40, 12, 172}, {-188, 190}, {-96, -84, -6, 190},
{-186, 188}, {-194, 62, 34, -72, -12, 24, 62, -80, -10, 6, -12, 204},
{-158, 18, 36, -64, -40, 28, 78, -36, -62, 6, 20, 186}, {-216, -2, 108, 114},
{-208, 210}, {-206, -12, 18, 204}, {-220, 34, 12, 20, -62, 222}, {-234, 236},
{-230, -8, 58, -54, 34, -22, 12, 52, -78, 246}, {-244, 16, 104, -110, -15, 255},
{-214, -6, -24, 16, 102, 132}, {-252, 42, -34, 70, -22, 202},
{-264, 266}, {-254, -4, 124, -126, 12, 254}, {-138, -108, -24, 274},
{-234, 236}, {-212, -52, -6, 28, -38, 14, 12, 68, -24, 220}}
```

```
Out[164]= {{1}, {-1, 3}, {-4, 6}, {-5, 1, 0, 8}, {-10, 12}, {-8, -3, 0, 15}, {-16, 18}, {-16, 1, 5, 14},
{-22, 4, 6, -8, 2, 24}, {-26, 28}, {-29, 6, 7, -13, -1, 36}, {-20, -7, -7, 38}, {-36, 38},
{-34, -7, 19, 26}, {-46, 6, -4, 11, -3, 42}, {-52, 5, -1, 11, 11, 32}, {-56, 58},
{-32, -25, -2, 6, 1, 58}, {-52, 5, -15, 66}, {-70, 72}, {-38, -33, 15, -13, 7, 68},
{-76, 0, 36, 44}, {-78, 7, 29, -15, -17, 80}, {-86, 3, 15, 7, 17, -31, -13, 96},
{-92, 3, -5, 98}, {-86, 88}, {-92, -7, 47, 56}, {-106, 108}, {-100, 25, -29, 108},
{-96, 1, 9, -17, 47, -47, -7, 8, 48, -21, -9, -24, 0, 122}, {-125, 38, -29, 120},
{-122, 3, 53, -63, 13, 122}, {-116, 118}, {-134, 39, 25, -67, -1, 23, 47, -67, 31, 114},
{-146, 148}, {-134, -3, -5, 17, -15, 146}, {-80, -27, -47, 13, -15, 162},
{-124, -31, 73, 86}, {-162, 8, 2, 1, 25, 132}, {-146, -25, 7, 47, 31, 92},
{-176, 178}, {-170, 47, -37, 11, -3, -23, 39, -41, 13, 174}, {-190, 192},
{-98, -85, -5, 192}, {-188, 190}, {-196, 61, 35, -77, -7, 21, 65, -81, -9, -3, -3, 206},
{-160, 17, 37, -67, -37, 23, 83, -37, -61, -5, 31, 188}, {-218, -3, 109, 116},
{-210, 212}, {-208, -13, 19, 206}, {-222, 31, 15, 19, -61, 224}, {-236, 238},
{-232, -6, 56, -57, 37, -33, 23, 51, -77, 248}, {-246, 7, 113, -111, -12, 255},
{-216, -11, -19, 15, 103, 134}, {-254, 39, -31, 69, -21, 204},
{-266, 268}, {-256, -5, 125, -129, 15, 256}, {-140, -109, -23, 276},
{-236, 238}, {-214, -53, -5, 23, -33, 16, 10, 67, -23, 222}}
```

```
Out[165]= {{1}, {-1, 3}, {-3, 5}, {-5, 1, 0, 8}, {-9, 11}, {-9, 0, -2, 15}, {-15, 17}, {-16, 2, 1, 17},
{-21, 3, 2, -4, 1, 25}, {-26, 28}, {-29, 5, 2, -3, -4, 35}, {-27, -2, -5, 38},
{-37, 39}, {-37, -2, 8, 35}, {-45, 5, -4, 7, -3, 46}, {-51, 6, -4, 7, 4, 44}, {-56, 58},
{-45, -11, -3, 7, -3, 61}, {-58, 4, -9, 67}, {-69, 71}, {-54, -15, 6, -1, -2, 72},
{-76, 0, 18, 62}, {-79, 7, 11, -6, -10, 83}, {-86, 7, 2, 5, 7, -12, -10, 95},
{-93, 3, -4, 98}, {-92, 94}, {-96, -2, 22, 80}, {-105, 107}, {-103, 14, -16, 109},
{-103, 4, 1, -7, 22, -18, -9, 8, 20, -9, -6, -11, -1, 123}, {-125, 21, -16, 124},
{-125, 7, 21, -30, 5, 128}, {-125, 127}, {-135, 21, 11, -24, -10, 15, 20, -32, 14, 130},
{-146, 148}, {-141, 0, -4, 12, -11, 150}, {-117, -12, -25, 12, -13, 161},
{-142, -14, 35, 125}, {-163, 9, -4, 2, 11, 151}, {-158, -9, 0, 25, 14, 134},
{-176, 178}, {-174, 25, -20, 9, -5, -2, 10, -19, 5, 181}, {-189, 191},
{-144, -41, -4, 193}, {-191, 193}, {-196, 32, 16, -33, -9, 14, 29, -39, -6, 8, -11, 207},
{-184, 10, 17, -30, -22, 17, 36, -17, -32, 9, 4, 204}, {-219, 0, 53, 170},
{-217, 219}, {-217, -5, 8, 218}, {-226, 19, 4, 11, -32, 230}, {-236, 238},
{-235, -3, 28, -25, 15, -5, 0, 27, -40, 248}, {-247, 13, 47, -54, -8, 255},
{-235, 0, -15, 9, 50, 197}, {-257, 23, -19, 36, -12, 235},
{-266, 268}, {-262, -1, 61, -61, 4, 265}, {-207, -53, -13, 277},
{-257, 259}, {-247, -25, -4, 17, -22, 15, -2, 35, -13, 256}}
```

```
Out[166]= {{1}, {-1, 3}, {-2, 4}, {-5, 1, 1, 7}, {-8, 10}, {-8, -1, -2, 15}, {-14, 16}, {-15, 1, 2, 16},
{-20, 2, 3, -5, 2, 24}, {-25, 27}, {-29, 5, 3, -4, -3, 34}, {-26, -3, -4, 37},
{-36, 38}, {-36, -3, 9, 34}, {-44, 4, -3, 6, -2, 45}, {-50, 5, -3, 6, 5, 43}, {-55, 57},
{-44, -12, -3, 7, -2, 60}, {-57, 3, -8, 66}, {-68, 70}, {-53, -16, 7, -2, -1, 71},
{-75, -1, 19, 61}, {-78, 6, 12, -7, -9, 82}, {-85, 6, 3, 4, 8, -13, -9, 94},
{-92, 2, -3, 97}, {-91, 93}, {-95, -3, 23, 79}, {-104, 106}, {-102, 13, -15, 108},
{-102, 3, 2, -8, 23, -19, -8, 7, 21, -10, -5, -12, 0, 122}, {-125, 21, -15, 123},
{-124, 6, 22, -31, 6, 127}, {-124, 126}, {-134, 20, 12, -25, -9, 14, 21, -33, 15, 129},
{-145, 147}, {-140, -1, -3, 11, -10, 149}, {-116, -13, -24, 11, -12, 160},
{-141, -15, 36, 124}, {-162, 8, -3, 1, 12, 150}, {-157, -10, 1, 24, 15, 133},
{-175, 177}, {-173, 24, -19, 8, -4, -3, 11, -20, 6, 180}, {-188, 190},
{-143, -42, -3, 192}, {-190, 192}, {-195, 31, 17, -34, -8, 13, 30, -40, -5, 7, -10, 206},
{-183, 9, 18, -31, -21, 16, 37, -18, -31, 8, 5, 203}, {-218, -1, 54, 169},
{-216, 218}, {-216, -6, 9, 217}, {-225, 18, 5, 10, -31, 229}, {-235, 237},
{-234, -4, 29, -26, 16, -6, 1, 26, -39, 247}, {-246, 12, 48, -55, -8, 255},
{-234, -1, -14, 8, 51, 196}, {-256, 22, -18, 35, -11, 234},
{-265, 267}, {-261, -2, 62, -62, 5, 264}, {-206, -54, -12, 276},
{-256, 258}, {-246, -26, -3, 16, -21, 14, -1, 34, -12, 255}}
```

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Out[167]= {{1}, {-1, 3}, {-3, 5}, {-5, 1, 0, 8}, {-9, 11}, {-10, 0, -1, 15}, {-15, 17}, {-16, 1, 0, 19},
{-21, 3, 0, -2, 0, 26}, {-26, 28}, {-29, 3, 0, 0, -3, 35}, {-31, 0, -3, 38}, {-38, 40},
{-39, -1, 3, 41}, {-45, 5, -4, 4, -2, 48}, {-51, 5, -4, 4, 0, 52}, {-56, 58},
{-54, -3, -2, 6, -4, 63}, {-62, 3, -5, 68}, {-69, 71}, {-65, -5, 3, 2, -3, 74},
{-76, 0, 6, 74}, {-80, 6, 0, 0, -5, 85}, {-86, 6, -3, 3, 0, 0, -7, 95},
{-94, 2, -2, 98}, {-96, 98}, {-98, -1, 6, 97}, {-105, 107}, {-105, 6, -7, 110},
{-107, 4, -3, -1, 4, 0, -7, 8, 0, 0, -4, -2, -2, 124}, {-125, 9, -6, 126},
{-127, 7, 0, -8, 2, 132}, {-131, 133}, {-136, 8, 0, 0, -9, 10, 0, -8, 4, 141},
{-146, 148}, {-145, 1, -3, 8, -7, 152}, {-145, 0, -9, 9, -10, 161},
{-154, -4, 7, 155}, {-164, 10, -8, 2, 2, 164}, {-166, -2, -1, 9, 0, 166}, {-176, 178},
{-176, 8, -7, 7, -6, 6, -4, -5, 2, 185}, {-189, 191}, {-180, -7, -3, 194}, {-193, 195},
{-196, 11, 0, 0, -9, 9, 0, -6, -3, 9, -11, 208}, {-201, 4, 0, 0, -12, 12, 0, 0, -10, 10, -6, 215},
{-220, 0, 12, 212}, {-221, 223}, {-223, -1, 2, 226}, {-228, 10, -5, 5, -10, 234},
{-236, 238}, {-237, -1, 8, -6, 4, 6, -8, 8, -12, 248}, {-248, 12, 0, -8, -5, 255},
{-248, 7, -11, 4, 7, 247}, {-259, 12, -10, 10, -5, 258}, {-266, 268}, {-266, 1, 10, -9, -1, 271},
{-261, -7, -6, 278}, {-272, 274}, {-272, -4, -2, 11, -14, 15, -10, 10, -5, 281}}
```



```
Out[168]= {{1}, {-1, 3}, {-2, 4}, {-5, 1, 1, 7}, {-8, 10}, {-9, 0, -2, 15}, {-14, 16}, {-15, 1, 0, 18},
{-20, 2, 1, -3, 1, 25}, {-25, 27}, {-29, 4, 0, 0, -3, 34}, {-30, 0, -3, 37}, {-37, 39},
{-38, -1, 3, 40}, {-44, 4, -3, 4, -2, 47}, {-50, 5, -4, 4, 0, 51}, {-55, 57},
{-53, -3, -3, 7, -4, 62}, {-61, 3, -5, 67}, {-68, 70}, {-64, -5, 3, 2, -3, 73},
{-75, -1, 7, 73}, {-79, 6, 0, 0, -5, 84}, {-85, 6, -3, 3, 0, 0, -7, 94},
{-93, 2, -2, 97}, {-95, 97}, {-97, -1, 6, 96}, {-104, 106}, {-104, 6, -7, 109},
{-106, 4, -3, -1, 4, 0, -7, 7, 1, 0, -4, -3, -1, 123}, {-125, 10, -6, 125},
{-126, 7, 0, -8, 2, 131}, {-130, 132}, {-135, 8, 0, 0, -9, 10, 0, -8, 4, 140},
{-145, 147}, {-144, 1, -3, 8, -7, 151}, {-144, 0, -9, 9, -10, 160},
{-153, -4, 7, 154}, {-163, 9, -7, 2, 2, 163}, {-165, -2, -1, 9, 0, 165}, {-175, 177},
{-175, 8, -7, 7, -6, 6, -4, -5, 2, 184}, {-188, 190}, {-179, -7, -3, 193}, {-192, 194},
{-195, 11, 0, 0, -9, 9, 0, -6, -3, 9, -11, 207}, {-200, 4, 0, 0, -12, 12, 0, 0, -10, 10, -6, 214},
{-219, 0, 12, 211}, {-220, 222}, {-222, -1, 2, 225}, {-227, 10, -5, 5, -10, 233},
{-235, 237}, {-236, -2, 9, -6, 4, 6, -8, 8, -12, 247}, {-247, 12, 0, -8, -6, 255},
{-247, 7, -11, 4, 7, 246}, {-258, 12, -10, 10, -5, 257}, {-265, 267}, {-265, 1, 10, -9, -1, 270},
{-260, -7, -6, 277}, {-271, 273}, {-271, -4, -2, 11, -14, 14, -9, 10, -5, 280}}
```

As you can see comparing above sequences, **corresponding subsequences have same length**. If you couple each natural number starting with 2 with the corresponding item in the sequences **each prime corresponds with each first item of each subsequence**.

```
In[169]=
```

```
Length/@t1
Length/@t2
Length/@t3
Length/@t4
Length/@t5
Length/@t6
Length/@t7
Differences [Prime/@Range [62] ]
```

```
Out[169]= {1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 6, 2, 6, 4, 2, 6, 4, 6, 8, 4, 2, 4, 2, 4, 14, 4, 6,
2, 10, 2, 6, 6, 4, 6, 6, 2, 10, 2, 4, 2, 12, 12, 4, 2, 4, 6, 2, 10, 6, 6, 6, 2, 6, 4, 2, 10}
Out[170]= {1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 6, 2, 6, 4, 2, 6, 4, 6, 8, 4, 2, 4, 2, 4, 14, 4, 6,
2, 10, 2, 6, 6, 4, 6, 6, 2, 10, 2, 4, 2, 12, 12, 4, 2, 4, 6, 2, 10, 6, 6, 6, 2, 6, 4, 2, 10}
Out[171]= {1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 6, 2, 6, 4, 2, 6, 4, 6, 8, 4, 2, 4, 2, 4, 14, 4, 6,
2, 10, 2, 6, 6, 4, 6, 6, 2, 10, 2, 4, 2, 12, 12, 4, 2, 4, 6, 2, 10, 6, 6, 6, 2, 6, 4, 2, 10}
Out[172]= {1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 6, 2, 6, 4, 2, 6, 4, 6, 8, 4, 2, 4, 2, 4, 14, 4, 6,
2, 10, 2, 6, 6, 4, 6, 6, 2, 10, 2, 4, 2, 12, 12, 4, 2, 4, 6, 2, 10, 6, 6, 6, 2, 6, 4, 2, 10}
Out[173]= {1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 6, 2, 6, 4, 2, 6, 4, 6, 8, 4, 2, 4, 2, 4, 14, 4, 6,
2, 10, 2, 6, 6, 4, 6, 6, 2, 10, 2, 4, 2, 12, 12, 4, 2, 4, 6, 2, 10, 6, 6, 6, 2, 6, 4, 2, 10}
Out[174]= {1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 6, 2, 6, 4, 2, 6, 4, 6, 8, 4, 2, 4, 2, 4, 14, 4, 6,
2, 10, 2, 6, 6, 4, 6, 6, 2, 10, 2, 4, 2, 12, 12, 4, 2, 4, 6, 2, 10, 6, 6, 6, 2, 6, 4, 2, 10}
Out[175]= {1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 6, 2, 6, 4, 2, 6, 4, 6, 8, 4, 2, 4, 2, 4, 14, 4, 6,
2, 10, 2, 6, 6, 4, 6, 6, 2, 10, 2, 4, 2, 12, 12, 4, 2, 4, 6, 2, 10, 6, 6, 6, 2, 6, 4, 2, 10}
Out[176]= {1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 6, 2, 6, 4, 2, 6, 4, 6, 8, 4, 2, 4, 2, 4, 14, 4, 6,
2, 10, 2, 6, 6, 4, 6, 6, 2, 10, 2, 4, 2, 12, 12, 4, 2, 4, 6, 2, 10, 6, 6, 6, 2, 6, 4, 2, 10}
```

Clearly applying each function above f to each prime return the same prime and hence  $f(\text{Prime}(i+1)) - f(\text{Prime}(i)) = \text{Prime}(i+1) - \text{Prime}(i)$  but f return Prime(i) even when applied to  $\text{Prime}(i)^e$  but **the sum of items never ends before reaching next prime**. Moreover only if you use the subset of natural number starting from a prime number the subset splitting algorithm works.

Now let's analyze deeply sequence t1. Every subsequence has the pattern  $\{-p+2, c1, -c1, c2, -c2, \dots, cn, -cn, p+subslen-2\}$  where  $p=Prime(i)$  and  $subslen=Prime(i+1)-Prime(i)$ .

We couple each number with the corresponding natural number:

```
In[177]:= z1=Transpose [ {s1, Range [2, 299] } ]

Out[177]= {{1, 2}, {-1, 3}, {3, 4}, {-3, 5}, {5, 6}, {-5, 7}, {1, 8}, {-1, 9}, {9, 10}, {-9, 11}, {11, 12},
{-11, 13}, {1, 14}, {-1, 15}, {15, 16}, {-15, 17}, {17, 18}, {-17, 19}, {1, 20}, {-1, 21},
{21, 22}, {-21, 23}, {3, 24}, {-3, 25}, {1, 26}, {-1, 27}, {27, 28}, {-27, 29}, {29, 30},
{-29, 31}, {1, 32}, {-1, 33}, {3, 34}, {-3, 35}, {35, 36}, {-35, 37}, {1, 38}, {-1, 39},
{39, 40}, {-39, 41}, {41, 42}, {-41, 43}, {1, 44}, {-1, 45}, {45, 46}, {-45, 47}, {5, 48},
{-5, 49}, {1, 50}, {-1, 51}, {51, 52}, {-51, 53}, {3, 54}, {-3, 55}, {1, 56}, {-1, 57},
{57, 58}, {-57, 59}, {59, 60}, {-59, 61}, {1, 62}, {-1, 63}, {3, 64}, {-3, 65}, {65, 66},
{-65, 67}, {1, 68}, {-1, 69}, {69, 70}, {-69, 71}, {71, 72}, {-71, 73}, {1, 74}, {-1, 75},
{5, 76}, {-5, 77}, {77, 78}, {-77, 79}, {1, 80}, {-1, 81}, {81, 82}, {-81, 83}, {3, 84},
{-3, 85}, {1, 86}, {-1, 87}, {87, 88}, {-87, 89}, {5, 90}, {-5, 91}, {1, 92}, {-1, 93},
{3, 94}, {-3, 95}, {95, 96}, {-95, 97}, {1, 98}, {-1, 99}, {99, 100}, {-99, 101}, {101, 102},
{-101, 103}, {1, 104}, {-1, 105}, {105, 106}, {-105, 107}, {107, 108}, {-107, 109}, {1, 110},
{-1, 111}, {111, 112}, {-111, 113}, {3, 114}, {-3, 115}, {1, 116}, {-1, 117}, {5, 118},
{-5, 119}, {9, 120}, {-9, 121}, {1, 122}, {-1, 123}, {3, 124}, {-3, 125}, {125, 126},
{-125, 127}, {1, 128}, {-1, 129}, {129, 130}, {-129, 131}, {5, 132}, {-5, 133}, {1, 134},
{-1, 135}, {135, 136}, {-135, 137}, {137, 138}, {-137, 139}, {1, 140}, {-1, 141}, {9, 142},
{-9, 143}, {3, 144}, {-3, 145}, {1, 146}, {-1, 147}, {147, 148}, {-147, 149}, {149, 150},
{-149, 151}, {1, 152}, {-1, 153}, {3, 154}, {-3, 155}, {155, 156}, {-155, 157}, {1, 158},
{-1, 159}, {5, 160}, {-5, 161}, {161, 162}, {-161, 163}, {1, 164}, {-1, 165}, {165, 166},
{-165, 167}, {11, 168}, {-11, 169}, {1, 170}, {-1, 171}, {171, 172}, {-171, 173}, {3, 174},
{-3, 175}, {1, 176}, {-1, 177}, {177, 178}, {-177, 179}, {179, 180}, {-179, 181}, {1, 182},
{-1, 183}, {3, 184}, {-3, 185}, {9, 186}, {-9, 187}, {1, 188}, {-1, 189}, {189, 190},
{-189, 191}, {191, 192}, {-191, 193}, {1, 194}, {-1, 195}, {195, 196}, {-195, 197},
{197, 198}, {-197, 199}, {1, 200}, {-1, 201}, {5, 202}, {-5, 203}, {3, 204}, {-3, 205},
{1, 206}, {-1, 207}, {9, 208}, {-9, 209}, {209, 210}, {-209, 211}, {1, 212}, {-1, 213},
{3, 214}, {-3, 215}, {5, 216}, {-5, 217}, {1, 218}, {-1, 219}, {11, 220}, {-11, 221},
{221, 222}, {-221, 223}, {1, 224}, {-1, 225}, {225, 226}, {-225, 227}, {227, 228},
{-227, 229}, {1, 230}, {-1, 231}, {231, 232}, {-231, 233}, {3, 234}, {-3, 235}, {1, 236},
{-1, 237}, {237, 238}, {-237, 239}, {239, 240}, {-239, 241}, {1, 242}, {-1, 243}, {3, 244},
{-3, 245}, {11, 246}, {-11, 247}, {1, 248}, {-1, 249}, {249, 250}, {-249, 251}, {9, 252},
{-9, 253}, {1, 254}, {-1, 255}, {255, 256}, {-255, 257}, {5, 258}, {-5, 259}, {1, 260},
{-1, 261}, {261, 262}, {-261, 263}, {3, 264}, {-3, 265}, {1, 266}, {-1, 267}, {267, 268},
{-267, 269}, {269, 270}, {-269, 271}, {1, 272}, {-1, 273}, {3, 274}, {-3, 275}, {275, 276},
{-275, 277}, {1, 278}, {-1, 279}, {279, 280}, {-279, 281}, {281, 282}, {-281, 283},
{1, 284}, {-1, 285}, {5, 286}, {-5, 287}, {15, 288}, {-15, 289}, {1, 290}, {-1, 291},
{291, 292}, {-291, 293}, {3, 294}, {-3, 295}, {1, 296}, {-1, 297}, {11, 298}, {-11, 299}}
```

```
In[178]:= Map [Last, Select [z1, # [ [2] ] == 2 - # [ [1] ] & ]  
Prime / @ Range [2, 62]
```

```
Out[178]= {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101,
103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193,
197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293}
```

```
Out[179]= {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101,
103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193,
197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293}
```

We can see that all and only the primes are present and for the odd primes all the couples

are  $(-p+2,p)$

```
In[180]:= k=Map [Last,Select [z1,# [ [1] ] ==1&] ]  
Differences [k]
```

```
Out[180]= {2, 8, 14, 20, 26, 32, 38, 44, 50, 56, 62, 68, 74, 80, 86, 92, 98, 104,  
110, 116, 122, 128, 134, 140, 146, 152, 158, 164, 170, 176, 182, 188, 194, 200,  
206, 212, 218, 224, 230, 236, 242, 248, 254, 260, 266, 272, 278, 284, 290, 296}
```

```
Out[181]= {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6,  
6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6}
```

```
In[182]:= k=Map [Last,Select [z1,# [ [1] ] ==3&] ]  
Differences [k]
```

```
Out[182]= {4, 24, 34, 54, 64, 84, 94, 114, 124, 144, 154, 174, 184, 204, 214, 234, 244, 264, 274, 294}
```

```
Out[183]= {20, 10, 20, 10, 20, 10, 20, 10, 20, 10, 20, 10, 20, 10, 20, 10, 20, 10, 20}
```

```
In[184]:= k=Map [Last,Select [z1,# [ [1] ] ==5&] ]  
Differences [k]
```

```
Out[184]= {6, 48, 76, 90, 118, 132, 160, 202, 216, 258, 286}
```

```
Out[185]= {42, 28, 14, 28, 14, 28, 42, 14, 42, 28}
```

```
In[186]:= k=Map [Last,Select [z1,# [ [1] ] ==9&] ]  
Differences [k]
```

```
Out[186]= {10, 120, 142, 186, 208, 252}
```

```
Out[187]= {110, 22, 44, 22, 44}
```

```
In[188]:= k=Map [Last,Select [z1,# [ [1] ] ==11&] ]  
Differences [k]
```

```
Out[188]= {12, 168, 220, 246, 298}
```

```
Out[189]= {156, 52, 26, 52}
```

So the gap between equal values follows a rule: if  $n$  is the number (always odd) the gap is a multiple of  $2(n+2)$ .

### Main Results

\*Each subsequence length corresponds to a gap between primes in each sequence.

\*With minrad each subsequence has other properties:

+every first item of a subsequence is equal to last item of the preceding subsequence but with changed sign

+excluding first and last item of a subsequence, other items are in couples  $(n,-n)$